Liquidity Risk in Credit Default Swap Markets*

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Abstract

We show that liquidity risk is priced in the cross section of returns on credit default swaps (CDSs). Liquidity risk is defined as covariation between CDS returns and a liquidity factor that captures innovations to CDS market liquidity. Market-wide CDS illiquidity is measured by aggregating deviations of credit index levels from their no-arbitrage values implied by the index constituents’ CDS spreads, and the liquidity factor is the return on a diversified portfolio of index arbitrage strategies. Liquidity risk increases CDS spreads and the expected excess returns earned by sellers of credit protection. Our benchmark model implies that liquidity risk accounts for approximately 20% of CDS spreads, on average.

JEL Classification: G12, G13

Keywords: CDS, Credit Index, Index Arbitrage, Liquidity Risk

This version: June 26, 2014

*We thank Yakov Amihud, Jack Bao, John Cochrane, Pierre Collin-Dufresne, Peter DeMarzo, Frank de Jong, Hitesh Doshi, Gregory Duffee, Darrell Duffie, Andrea Eisfeld, Peter Feldhütter, Christopher Hennessy, Jingzhi Huang, Nikunj Kapadia, Andrew Karolyi, Leonid Kogan, David Lando, Semyon Malamud, Moran Ofir, Lasse Pedersen, Norman Schürhoff, Denitsa Stefanova, René Stulz, Christopher Trevisan, Fabio Trojani, and seminar participants at Copenhagen Business School, London Business School, the 6th Finance UC International Conference, the 6th Financial Risks International Forum, the 3rd International Conference of the Financial Engineering and Banking Society, the 2nd ITAM Finance Conference, the 2013 Princeton-Lausanne workshop on Quantitative Finance, the 2013 SFI Research Day, the 4th World Finance Conference, the 2013 Asian Finance Association conference, the 2013 European Finance Association conference, the 2013 Northern Finance Association conference, and the 2014 Western Finance Association conference for comments and suggestions. We are grateful to Moody’s Analytics, Inc. for providing Moody’s Custom EDF Data. Both authors gratefully acknowledge research support from the Swiss Finance Institute.

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1 Introduction

A recent literature starting with Pástor and Stambaugh (2003) and Acharya and Pedersen (2005) has shown that liquidity risk is priced within a variety of asset classes including stocks, Treasuries, corporate bonds, hedge funds, and private equity.\(^1\) In this paper, we study whether liquidity risk is priced in credit default swaps (CDSs). These contracts are particularly interesting as they trade in a relatively opaque, dealer-dominated, decentralized market and, therefore, are exposed to many potential sources of illiquidity.\(^2\) CDS contracts also differ by being in zero net supply, implying that the sign on any liquidity risk premium is not given a priori. Moreover, from a practical perspective, liquidity risk is important for the trading, pricing, hedging, and risk-management of CDS contracts—recently illustrated by J.P. Morgan’s six billion dollar trading loss on relatively illiquid CDS market strategies\(^3\)—and, from a regulatory perspective, liquidity risk is important given the potential systemic nature of the CDS market.

We show that liquidity risk is indeed priced in the cross section of returns on CDS contracts. Specifically, we first construct a model-independent measure of CDS market illiquidity by aggregating deviations of credit index levels from their no-arbitrage values implied by the index constituents’ CDS spreads. Second, we construct a tradable liquidity factor, based on index arbitrage strategies, that is highly negatively correlated with innovations to the CDS market illiquidity measure. Third, we define liquidity risk as covariation between CDS returns and the liquidity factor and show that liquidity risk is both statistically and

\(^1\)As discussed by Acharya and Pedersen (2005), liquidity risk can be defined in several ways. The notion of liquidity risk used in this paper (covariation between returns and a market-wide liquidity factor) has been shown to be priced in stocks (see, e.g., Sadka (2006) and Korajczyk and Sadka (2008) in addition to Pástor and Stambaugh (2003) and Acharya and Pedersen (2005)), Treasuries (see, e.g., Li, Wang, Wu, and He (2009)), corporate bonds (see, e.g., Lin, Wang, and Wu (2011) and Bongaerts, de Jong, and Driessen (2012)), hedge funds (see, e.g., Sadka (2010) and Hu, Pan, and Wang (2013)), and private equity (see, e.g., Franzoni, Nowak, and Phalippou (2012)).

\(^2\)The credit derivatives market is currently undergoing regulatory changes which will require certain credit derivatives to be traded on so-called swap execution facilities. This will likely bring more transparency to parts of the credit derivatives market.

economically important for the pricing of CDSs. In particular, liquidity risk increases CDS spreads and the expected excess returns earned by sellers of credit protection.

Why is the liquidity risk premium earned by sellers of credit protection? We find that the liquidity risk exposures of CDS contracts are such that CDS spreads tend to widen when market-wide liquidity deteriorates. Because the majority of CDS contracts are collateralized and marked to market on a daily basis, widening CDS spreads cause losses for sellers of credit protection.\footnote{From the annual “Margin Surveys” by the International Swaps and Derivatives Association (ISDA), we infer that the fraction of credit derivatives trades covered by collateral agreements averaged more than 80\% over the sample period.} If credit protection sellers are capital and funding constrained, such losses may eventually result in contract liquidations, which are particularly costly when market liquidity is low. This suggests that sellers of credit protection require a premium for bearing the risk associated with covariation between CDS returns and innovations to market-wide liquidity.\footnote{Kondor and Vayanos (2014) provide a dynamic equilibrium model in which assets are in zero net supply, liquidity varies due to capital-constrained liquidity providers, and exposure to market-wide liquidity is priced in the cross section of asset returns.}

Because CDSs trade in over-the-counter (OTC) markets with no readily available transaction data, it is difficult to apply standard measures of liquidity. Instead, we follow Brunnermeier and Pedersen (2009) and capture illiquidity by the extent to which market prices deviate from fundamental values. In particular, we consider the law-of-one-price relation that exists between a credit index and a basket of single-name CDSs that replicates the cash flow of the index. We denote the difference between the level of the index and its CDS-implied level as the \textit{index-to-CDS basis}, and we construct a market-wide CDS illiquidity measure as a weighted average of absolute values of index-to-CDS bases. The average is taken over ten credit indices that reference the most liquid North American and European names of both the investment grade and high yield universes, and cover a substantial part of the overall CDS market.

The rationale behind the construction of our illiquidity measure is index arbitrage. Hedge funds and trading desks at investment banks or other financial institutions active in the credit
derivatives market usually engage in index arbitrage strategies that keep the index levels in line with their CDS-implied levels. Therefore, nonzero index-to-CDS bases indicate that market participants are temporarily unable or unwilling to execute index arbitrage trades. As such, our measure captures illiquidity not only in terms of the transaction costs and margin requirements of these trades but also, in a broader sense, in terms of capital and funding constraints, and other limits to arbitrage. In other words, we view our illiquidity measure as a summary statistic of the impact of all the different dimensions of illiquidity that are present in the CDS market.

We find time-varying index-to-CDS bases across all ten credit indices during our sample period from September 20, 2006, to February 1, 2012. In particular, bases widened considerably in the period following the collapse of Lehman Brothers and AIG. For instance, bases of credit indices referencing North American investment grade and high yield names dropped to -61 basis points (bps) and -452 bps, respectively (corresponding to -25% and -38%, respectively, of the index levels). The market-wide measure suggests that CDS market liquidity was relatively high and stable in the early part of the sample period, deteriorated somewhat around the time of the collapse of two Bear Stearns structured credit hedge funds in late June 2007, deteriorated significantly in the aftermath of the Lehman Brothers default and the AIG bailout in September 2008, and then recovered substantially since early 2009—although not reaching the level of liquidity that prevailed prior to the crisis. Consistent with its interpretation as an illiquidity measure in a broad sense, we show that it correlates not only with bid-ask spreads and measures of price impact of trades in the CDS market but also with measures of illiquidity in the corporate bond and Treasury markets as well as measures of the cost and supply of capital.

To investigate whether liquidity risk is priced in the cross section of CDS returns, we set up a factor pricing model which accounts for default and liquidity risk. In principle, counterparty risk could also be a determinant of CDS returns. However, Arora, Gandhi, and Longstaff (2012) find that the effect of counterparty risk on CDS spreads is negligible, which is consistent

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uidity (default) risk is captured by the beta with respect to a market-wide liquidity (default) factor. The market-wide liquidity factor is the return on a diversified portfolio of credit index arbitrage trades. Each of these trades is based on a simple strategy that holds both the credit index and its replicating basket to profit from a narrowing of the index-to-CDS basis. Consequently, the liquidity factor is tradable and highly negatively correlated with innovations to the CDS market illiquidity measure.

The model is estimated on a large panel data set of single-name CDS contracts referencing 666 North American and European entities. The CDS contracts are sorted into portfolios that exhibit variation in credit quality and the level of illiquidity. Sample means of realized excess returns on CDSs are very imprecise estimates of expected excess returns because of the short sample period and the peso problem that arises when computing returns on securities that are subject to default risk (as defaults are rare events that have a dramatic impact on returns). Instead, we follow Bongaerts, de Jong, and Driessen (2011, henceforth BDD) in obtaining forward-looking estimates of conditional expected excess returns by using Moody’s KMV Expected Default Frequencies (EDFs) to calculate expected default losses. Across all portfolios, unconditional expected excess returns are positive from a credit protection seller’s perspective, ranging from 0.35% per year for a portfolio of the most liquid high-credit-quality CDSs to 5.80% per year for a portfolio of the most illiquid low-credit-quality CDSs.

A standard two-step estimation approach is applied. In the first step, we estimate factor betas from time-series regressions. The CDS portfolios have significant betas with respect to the default and liquidity factors, which together explain between 18% and 76% of the variation in realized excess returns. The betas are such that sellers of credit protection tend to realize negative excess returns when default risk or CDS market illiquidity increases. In the second step, we estimate factor prices of risk from a cross-sectional regression of

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with the widespread use of collateralization and netting agreements. Hence, we do not take counterparty risk into account in our factor pricing model.
expected excess returns on first-step betas.\textsuperscript{7} We find that liquidity risk is both a statistically significant and economically important determinant of expected excess returns. For instance, considering the difference in the expected excess return between the portfolio of the most illiquid low-credit-quality CDSs and the portfolio of the most liquid high-credit-quality CDSs, 2.66\% per year is due to liquidity risk, while 3.33\% is due to default risk. Alternatively, considering the average expected excess return across test portfolios, 0.52\% per year is due to liquidity risk, while 1.11\% is due to default risk. We also decompose the CDS spreads instead of the expected excess returns. Averaging the relative contributions across test portfolios, liquidity risk accounts for 21\% of CDS spreads, while default risk and expected default losses account for 54\% and 32\%, respectively, with the residual contribution being due to pricing errors.

The finding that liquidity risk is priced in the CDS market is robust to the inclusion of the level of illiquidity as a characteristic, to an alternative construction of the CDS market liquidity factor, and to the inclusion of additional risk factors in the asset pricing model such as liquidity factors from the Treasury, corporate bond, and stock markets, a factor that mimics balance sheet constraints of financial intermediaries, and stock market and volatility factors. Across all robustness checks, the contribution of liquidity risk to the difference in the expected excess return between the two above-mentioned test portfolios ranges from 1.74\% to 3.18\% per year.

BDD also investigate liquidity effects in CDS markets, but use a very different definition of liquidity risk. They define liquidity risk as covariation between innovations to transaction costs on individual CDS contracts and systematic default risk, but find that the liquidity risk premium is economically negligible. When including betas in our asset pricing model that capture BDD’s notion of liquidity risk, we confirm that the associated premium is minuscule,

\textsuperscript{7}Standard errors are adjusted for errors-in-variables, heteroscedastic and autocorrelated errors, and potential model misspecification as in Kan, Robotti, and Shanken (2013). They show, along with other recent papers, that taking into account potential model misspecification is crucial for reliable statistical inference.
while the pricing of our liquidity factor is virtually unaffected. Furthermore, BDD find that the level of illiquidity of CDS contracts is significantly related to expected excess returns, while we find a much weaker relation in the presence of our liquidity factor.

Two other papers on liquidity effects in CDS markets are Tang and Yan (2007) and Bühler and Trapp (2009). Tang and Yan (2007) regress CDS spreads on the level of illiquidity and liquidity betas inspired by Acharya and Pedersen’s (2005) liquidity-adjusted CAPM. They find suggestive evidence that liquidity risk affects CDS spreads. In contrast, we estimate a formal factor pricing model using a different liquidity factor and forward-looking estimates of expected excess returns instead of CDS spreads. Bühler and Trapp (2009) estimate a reduced-form model in which CDS spreads are affected by CDS- and bond-specific liquidity factors. In contrast, our focus is on the effect of exposure to market-wide CDS liquidity.

The paper proceeds as follows: Section 2 describes the construction of the CDS market illiquidity measure, and Section 3 analyzes its time-series properties. Asset pricing implications are discussed in Section 4, and Section 5 concludes.

2 Measuring CDS Market Illiquidity

This section presents the construction of the CDS market illiquidity measure and the tradable liquidity factor. Furthermore, it briefly describes credit indices, the replication argument on which index arbitrage is based, and the data used in the construction.

2.1 Credit Indices

Credit indices are standardized credit derivatives that provide insurance against any defaults among their constituents. They allow investors to gain or reduce credit risk exposure in certain segments of the market. Due to their widespread use and standardized terms, they
trade with lower cost and higher liquidity than most single-name CDSs or cash bonds.\(^8\)

Credit indices trade in OTC markets for maturities between one and ten years. The five-year maturity is typically the most liquid and is the focus of our empirical analysis.\(^9\)

Each credit index is a separate CDS contract with a specific maturity, fixed spread, and underlying basket of reference entities. Over the life of the contract, the seller of protection on the index provides default protection on each index constituent, with the notional amount of the contract divided evenly among the index constituents. In return, the seller of index protection earns the fixed spread. In case of default, the seller of index protection pays the loss-given-default and the notional amount of the contract is reduced accordingly. If the quoted level of the index differs from its fixed spread, counterparties initially exchange an upfront payment equal to the contract’s present value.

As a clarifying example, suppose that on September 21, 2007, an investor sells a 10 million USD notional amount of protection on the main North American investment grade credit index (CDX.NA.IG 9) with a maturity of five years and a fixed spread of 60 bps.\(^10\) On that date the index traded at 49.92 bps which translates into a 46,183.13 USD upfront charge for the seller of protection. Over the next three quarters he receives quarterly spread payments each being approximately equal to \(\frac{1}{4} \times 0.0060 \times 10,000,000 = 15,000\) USD (for the purpose of illustration, we abstract from the actual day-count convention). On September 7, 2008, Fannie Mae and Freddy Mac, both reference names of the CDX.NA.IG 9, were placed into conservatorship by their regulator. Creditors recovered 91.51 cents and 94 cents per dollar of senior unsecured debt issued by Fannie Mae and Freddy Mac, respectively. Thus, the seller of

\(^8\)For example, the Depository Trust & Clearing Corporation’s “Market Activity Report” for the three-month period from June 20, 2011, to September 19, 2011, shows that the average daily notional amount of trades is 29 million USD, on average, across single-name CDSs referencing corporate names that belong to the 1000 most actively traded single-name CDSs. In contrast, the average daily notional amount of untranched index transactions is approximately one billion USD.

\(^9\)Using CDS transaction data, Chen, Fleming, Jackson, Li, and Sarkar (2011) find that 84% of all index transactions are in the five-year maturity.

\(^10\)The number following the index name is referred to as the index’s series and uniquely identifies the underlying basket of reference names.
index protection compensates the losses incurred, paying 1/125 \times (1 - 0.9151) \times 10,000,000 + 1/125 \times (1 - 0.94) \times 10,000,000 = 11,592 \text{ USD}.^{11} \text{ Due to the credit events, the spread payment on September 20, 2008, is reduced to } 1/4 \times 123/125 \times 0.0060 \times 10,000,000 = 14,760 \text{ USD. Until expiry of the index on December 20, 2012, another two credit events occurred: first, the default of Washington Mutual on September 27, 2008, triggers a } 1/125 \times (1 - 0.57) \times 10,000,000 = 34,400 \text{ USD payout and reduces subsequent spread payments to } 1/4 \times 122/125 \times 0.0060 \times 10,000,000 = 14,640 \text{ USD. Second, the Chapter 11 filing of CIT Group on November 1, 2009, triggers a } 1/125 \times (1 - 0.68125) \times 10,000,000 = 25,500 \text{ USD payout and reduces successive spread payments to } 1/4 \times 121/125 \times 0.0060 \times 10,000,000 = 14,520 \text{ USD.}

Twice a year, on the so-called index roll dates in March and September, a new series of each credit index is launched, with the basket of reference entities revised according to credit rating and liquidity criteria. Entities that fail to maintain a rating within a specified range, due to either an upgrade or a downgrade, and entities whose CDS contracts have deteriorated significantly in terms of their liquidity are replaced by the most liquid reference names meeting the rating requirements. Liquidity is typically concentrated in the most recently launched series, which are referred to as the on-the-run series. Consequently, these are the subject of our empirical analysis.

In case of a credit event, a new version of the index series starts trading, with the entity that triggered the event having been removed from the index. Because triggered CDSs usually continue to trade in the market until their recovery values are determined, multiple versions of the same index series can trade at the same time. In such cases, we focus on the most liquid version.

All the credit indices considered in this paper are administrated by Markit. It sets the rules and procedures that govern the index revisions on the roll dates. In addition, it

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11 In this example, we assume that cash settlement, the standard settlement method of credit index transactions, applies. Furthermore, we ignore accrual payments on default and the fact that recovery values are determined in credit event auctions that usually do not take place on the default dates.
determines a group of licensed dealers, who actively make markets for credit indices. Based on their spread quotes, Markit computes index levels that are published on a daily basis.

The most liquid credit indices currently traded are the ones that belong to the CDX North American and iTraxx Europe families. The two index families differ in the region from which reference entities are eligible for inclusion, in the currency in which they trade, in the rules that govern the index revisions, and in some technicalities of the contract terms, e.g., documentation clauses. In the Internet Appendix, we briefly summarize index rules and provide additional information concerning the major indices of the CDX North American and iTraxx Europe families.

2.2 Index Replication

Investors can gain credit risk exposure either by selling protection on the index contract or by selling protection on a basket of single-name CDSs that replicates the cash flow of the index contract. Thus, an alternative index level can be implied from single-name CDS quotes on the index constituents. This gives rise to what we call the index-to-CDS basis, defined as the difference between the index level and the CDS-implied level. In perfect capital markets, index arbitrage will keep index-to-CDS bases close to zero.

Suppose that on date $t$ an investor wants to sell index protection with a five-year maturity, fixed spread $C$, and notional amount $A$. This involves an initial upfront payment equal to the contract’s present value. Instead of selling index protection, the investor can sell protection on the index constituents via single-name CDSs. In particular, to replicate the payments of the index contract, the investor must sell protection on each of the $I_t$ index constituents that, prior to the inception of trade, have not triggered a credit event. Each single-name

\[ \text{In addition, there will be an accrual payment. The seller of index protection is entitled to a full spread payment on the first payment date after inception of trade, regardless of the actual time of opening his position. Therefore, he has to compensate the buyer of protection for the fixed spread accrued between the last spread payment date and the inception of trade. We abstract from these accrual payments in our discussion of the index replication.} \]
CDS must have a five-year maturity, fixed spread $C$, and notional amount $A/I$, where $I$ denotes the number of reference entities at the launch of the index’s series. As for credit indices, upfront payments are necessary when trading single-name CDSs at off-par spreads. Hence, the investor faces cost equal to the aggregate amount of all upfront charges from the single-name CDS transactions.

Until the earlier of the maturity date and the first credit event by one of the remaining index constituents, the seller of index protection earns quarterly spread payments of $d/360 \times C \times I_t/I \times A$, while the seller of protection via single-name CDSs receives quarterly spread payments of $\sum_{i=1}^{I_t} d/360 \times C \times A/I$. Here, $I_t/I \times A$ is the index’s adjusted notional amount, and $d/360$ denotes the accrual time during a given quarter determined by ACT/360 day-count convention. Obviously both payment streams are identical.

In case that one of the remaining reference names, say $i^*$, defaults prior to maturity, the seller of index protection has to make a payment of $1/I \times (1 - R_{i^*}) \times A$, where $R_{i^*}$ is the recovery per dollar of notional on $i^*$’s debt. This payment coincides with the one that the seller of protection via single-name CDSs has to make.\(^{13}\)

Following the credit event, the notional amount of the index is adjusted to $(I_t - 1)/I \times A$ and quarterly spread payments earned by the seller of index protection decrease to $d/360 \times C \times (I_t - 1)/I \times A$. Because there is also one single-name CDS less in the basket, the seller of protection via single-name CDSs collects quarterly spread payments of $\sum_{i=1}^{I_t-1} d/360 \times C \times A/I$. Thus, payments coincide in this case as well.

Because the same reasoning applies to any possible credit event that may occur prior to maturity, it follows that the cash flows for the seller of index protection and the seller of protection via single-name CDSs are identical. The CDS-implied index level, $C_{t}^{CDS}$, is now obtained as that fixed spread on the single-name CDSs that makes the replicating basket

\(^{13}\)Upon default, both the seller of index protection and the seller of protection via single-name CDSs will receive an accrual payment. This payment compensates for the protection they provided on the defaulted reference name since the last spread payment date prior to the credit event.
have zero net present value. The index-to-CDS basis, $B_t$, of a credit index is then defined as $B_t = C_t^{IDX} - C_t^{CDS}$, where $C_t^{IDX}$ denotes the index level as of date $t$.

### 2.3 Data

The credit index data are obtained from Markit and comprise index levels, CDS-implied levels, and the corresponding upfront amounts for all credit indices administrated. In addition, the number of licensed dealers that submit spread quotes for the computation of the index level is reported. We conduct our analysis on ten credit indices that belong to either the CDX North American or the iTraxx Europe family, always focusing on the five-year maturity. For each index, we splice together on-the-run series to create continuous time series for the period from September 20, 2006, to February 1, 2012. Whenever multiple versions of the on-the-run series trade simultaneously, we choose the version with the largest number of contributing dealers.

The ten credit indices considered in the construction of the CDS market illiquidity measure can be briefly described as follows: the CDX.NA.IG and the iTraxx Eur (Main) are broad-based indices that reflect the credit risk of investment grade entities in North America and Europe, respectively. Both indices comprise 125 reference names. The thirty reference names with the widest five-year CDS spreads constitute the CDX.NA.IG.HVOL and iTraxx Eur HiVol sub-indices. The 25 financial sector reference names included in the iTraxx Eur form a separate sub-index, the iTraxx Eur Sr Finls. The iTraxx Eur Sub Finls comprises the same reference names as the iTraxx Eur Sr Finls, but with subordinated reference obligations. The CDX.NA.HY consists of 100 non-investment-grade entities domiciled in North America and its European counterpart, the iTraxx Eur Xover, consists of up to 50 non-investment-grade reference names. BB and B rated entities of the CDX.NA.HY form the CDX.NA.HY.BB and CDX.NA.HY.B sub-indices, respectively.

Descriptive statistics of the credit index data are reported in Panel A of Table 1. Both the
average and the standard deviation of index levels increase as we consider indices of reference names with progressively lower credit quality. The cross-sectional correlation between the average and the standard deviation is 0.98.

[Table 1 about here.]

The panels in Figure 1 display the time series of on-the-run index levels (thin black lines) for the main indices of the CDX North American and iTraxx Europe families. Figures for the sub-indices can be found in the Internet Appendix. Each of the indices increased shortly before the March 2008 roll date when Bear Stearns was on the brink of bankruptcy and, after a short period of relief, peaked in the aftermath of the September 2008 credit events of Fannie Mae, Freddy Mac, Lehman Brothers, and Washington Mutual. The indices of the iTraxx Europe family again sharply increased towards the end of the sample period as the European sovereign debt crisis worsened.

[Figure 1 about here.]

2.4 CDS Market Illiquidity Measure

In addition to index levels, the panels in Figure 1 also display CDS-implied levels (thick gray lines) and the corresponding index-to-CDS bases (light gray shaded areas). The panels reveal that nonzero index-to-CDS bases frequently arise. In particular, between the September 2008 index roll and the next index roll in March 2009, i.e., at the height of the financial crisis, bases are wide and very volatile. For instance, bases of the main indices of North American and European investment grade credit risk, the CDX.NA.IG and the iTraxx Eur, drop to -61.12 bps and -58.55 bps, respectively. These are extreme moves compared to the standard deviations of these bases which are 11.58 bps and 9.05 bps, respectively. Among indices of high yield credit risk, the widening of index-to-CDS bases is even more extreme. For
instance, bases of the CDX.NA.HY and the iTraxx Eur Xover reach -451.93 bps and -106.15 bps, respectively, at the height of the financial crisis.

Descriptive statistics of the index-to-CDS bases are reported in Panel B of Table 1. For both index families, bases of investment grade indices are negative, on average, while bases of indices referencing lower-quality credits are positive, on average. The standard deviations of bases are higher within the CDX North American family than within the iTraxx Europe family and, within both index families, standard deviations are highest for those indices referencing the names with the lowest credit quality. As explained below, we measure illiquidity from absolute values of index-to-CDS bases. We observe a strong relation between index levels and the volatilities of the absolute bases. The table shows that the time-series correlations between index levels and conditional volatilities of absolute bases range from 0.43 to 0.91 across indices.\textsuperscript{14} Furthermore, the cross-sectional correlation between average index levels and unconditional volatilities of absolute bases is 0.84.

To measure market-wide CDS illiquidity, we aggregate absolute values of index-to-CDS bases across indices. We use absolute values because positive and negative bases are equally informative about liquidity in the CDS market. Given the significant cross-sectional variation in the volatilities of absolute bases, taking an equally weighted average of absolute bases would cause variation in the CDS market illiquidity measure to be driven mostly by the bases of indices referencing lower-quality credits. One option would be to weight the absolute bases by the inverse of their conditional volatilities; however, this has the disadvantage that the sample period is shortened by the window over which the initial conditional volatilities are estimated. Instead, we opt to weight the absolute bases by the inverse of the index levels, exploiting the strong relation between the index levels and the volatilities of the absolute bases.

\textsuperscript{14}Conditional volatilities are inferred from GARCH(1,1) models for the conditional variances of the error terms in ARMA(1,1) specifications of the absolute value of index-to-CDS bases.
basses. Thus, the CDS market illiquidity measure, $CDSILLIQt$, is given by

$$CDSILLIQt = \sum_{i=1}^{n_t} w_{i,t} |B_{i,t}|,$$  \hspace{1cm} (1)

where $w_{i,t} = (1/C_{i,t}^{IDX}) / (\sum_{j=1}^{n_t} 1/C_{j,t}^{IDX})$ and $n_t$ is the number of indices with available data on date $t$.

Figure 2 shows the time series of the CDS market illiquidity measure at a weekly frequency. As can be seen from the figure, the illiquidity measure is very persistent with a 0.93 first-order autocorrelation. The measure suggests that liquidity was relatively high and stable until June 2007. The deterioration in liquidity towards the end of that month coincided with the high-profile collapse of two Bear Stearns structured credit hedge funds, which was followed by further turmoil in credit and funding markets.\textsuperscript{15} Liquidity deteriorated significantly in the aftermath of the Lehman Brothers default and the AIG bailout in September 2008, and the illiquidity measure peaked at 66 bps at the end of December 2008. Since then, liquidity has recovered substantially, but, within our sample period, did not reach pre-crisis levels.

[Figure 2 about here.]

2.5 Tradable Liquidity Factor

We construct a tradable liquidity factor based on index arbitrage strategies. For each index, we consider a trading strategy that profits from a narrowing of the index-to-CDS basis. If the index trades above its CDS-implied level, the strategy sells protection on the index contract and buys protection via the replicating basket of single-name CDSs. If the index trades below its CDS-implied level, the strategy is the reverse trade. As shown in Section 2.2, if

\textsuperscript{15}The two funds—the High Grade Structured Credit Strategies Fund, and the High Grade Structured Credit Strategies Enhanced Leverage Fund—were largely invested in collateralized debt obligations tied to subprime mortgages (see “Two Big Funds At Bear Stearns Face Shutdown”, Wall Street Journal, June 20, 2007).
held to the index’s maturity, this strategy is an arbitrage in a textbook sense. However, for a shorter holding period, the strategy is risky. For some index $i$, the holding period excess return on the strategy is given by

$$ \text{sgn}(B_{i,t-1}) \left( r_{IDX,i,t} - r_{CDS,i,t} \right), $$  

(2)

where $r_{IDX,i,t}$ and $r_{CDS,i,t}$ denote holding period excess returns from selling protection on the index contract and via its replicating basket of single-name CDSs, respectively, and $B_{i,t-1}$ is the index-to-CDS basis at the beginning of the holding period.\footnote{We compute holding period excess returns from upfront amounts on credit index contracts and their replicating baskets of single-name CDSs; for details see Appendix B.} Because excess returns on the strategy are positive when index-to-CDS bases narrow, excess returns should be negatively correlated with changes in the absolute basis.

We construct the tradable liquidity factor, $LIQ_t$, by aggregating the excess returns on the individual index arbitrage strategies using the same weighting scheme as for $CDSILLIQ_t$; that is,

$$ LIQ_t = \sum_{i=1}^{n_t} w_{i,t-1} \text{sgn}(B_{i,t-1}) \left( r_{IDX,i,t} - r_{CDS,i,t} \right), $$  

(3)

where the weights, $w_{i,t-1}$, are given in Section 2.4. We use one-week holding periods because the asset pricing model in Section 4 is estimated at a weekly frequency. Descriptive statistics of the returns on the individual index arbitrage trades are given in Panel A of Table 2. There is considerable variation in the means and standard deviations of excess returns and the annualized Sharpe ratios, using Lo’s (2002) correction for non-i.i.d. excess returns, are between 1.15 and 2.42.\footnote{Note that the Sharpe ratios are inflated because we neither take into account transaction costs nor margin requirements (for margin requirements of CDSs, see Rule 4240 of the Financial Industry Regulatory Authority).}

Figure 3 displays the time-series evolution of the tradable liquidity factor. Its correlation...
with changes in the CDS market illiquidity measure is -0.70. The factor’s annualized mean and standard deviation are 3.00% and 1.03%, respectively, and its annualized Sharpe ratio, using Lo’s (2002) correction for non-i.i.d. excess returns, is 2.38. The high Sharpe ratio of the factor is, in part, due to the moderate correlations between strategy excess returns, which are reported in Panel B of Table 2.

[Figure 3 about here.]

### 3 Time-Series Properties of CDS Market Illiquidity

This section explores the time-series properties of the CDS market illiquidity measure. In particular, we investigate its relation to alternative measures of CDS market illiquidity as well as measures of bond market illiquidity, cost and supply of capital, and overall market conditions. Exact definitions of all measures are provided in Appendix A and plots of their time-series dynamics can be found in the Internet Appendix.

#### 3.1 Explanatory Variables

**Alternative CDS Market Illiquidity Measures**

We consider three alternative measures of CDS market illiquidity. The first measure is the average bid-ask spread of single-name CDSs. When average bid-ask spreads are wider, index arbitrage is more expensive, and index levels can drift further away from CDS-implied levels before index arbitrage becomes profitable. The second measure is the absolute spread change per contributed quote, averaged across single-name CDSs. To the extent that volume can be proxied by the number of contributors, this captures the price impact of CDS trades much like the Amihud (2002) illiquidity measure. The third measure is the absolute change in the index level per contributed quote, averaged across on-the-run credit indices. This captures
the price impact of index trades. We expect higher CDS market illiquidity, the higher the price impact of trade for single-name and index contracts.

**Bond Market Illiquidity Measures**

We use several bond market illiquidity measures. First, as a measure of Treasury market illiquidity, we use that of Hu et al. (2013), which aggregates deviations of U.S. Treasury yields from the respective points on a fitted smooth yield curve. Hu et al. (2013) argue that deviations from fair values are wider whenever risk capital is scarcer, in which case we also expect less activity by index arbitrage traders and a wider CDS market illiquidity measure. Second, as a measure of corporate bond market illiquidity, we use Dick-Nielsen, Feldhütter, and Lando’s (2012) \( \lambda \), which captures the common component of several bond-specific illiquidity measures. In theory, CDS spreads and bond yields are linked by an approximate no-arbitrage relation, so we expect liquidity in the two markets to be correlated. Third, we use the average CDS-bond basis across U.S. investment grade bonds. As this measure captures deviations from an approximate no-arbitrage relation, it can be viewed as capturing illiquidity in corporate bond and CDS markets in broad terms and, therefore, it is expected to be correlated with our CDS market illiquidity measure.

**Capital Cost and Supply Measures**

As explained in Section 2.2, index arbitrage requires capital to make upfront payments and mark to market CDSs. Given that arbitrageurs are typically highly leveraged, the cost of short-term debt financing and its availability may be important determinants of illiquidity in the CDS market. We use the three-month LIBOR-OIS spread and the spread between three-month Agency MBS and Treasury general collateral repo rates to capture the costs associated with unsecured and secured financing, respectively. The LIBOR-OIS spread is a commonly used measure of risk in (unsecured) interbank markets (see, e.g.,
Filipović and Trolle (2013)), while the repo spread reflects differential collateral values of Agency MBS and Treasury securities (see, e.g., Bartolini, Hilton, Sundaresan, and Tonetti (2011)). We expect the CDS market illiquidity measure to widen when funding becomes more expensive. Another important determinant may be the amount of capital that hedge funds direct towards index arbitrage. This amount is likely to be correlated with hedge fund returns, which we approximate by the one-month return on the Hedge Fund Research Global Index. This index represents the overall hedge fund universe both in terms of strategies and geographical locations.

**Market Conditions**

Overall market conditions are captured by three measures: the return on the S&P 500 index, the yield spread between Baa- and Aaa-rated bonds, and the CBOE VIX index, which is an option-implied estimate of S&P 500 volatility. We expect the CDS market illiquidity measure to widen when overall market conditions deteriorate (see, e.g., He and Milbradt (2013) for a model relating default risk and corporate bond market illiquidity and Brunnermeier and Pedersen (2009) for a model relating market volatility and market illiquidity).

**3.2 Results**

For each group of explanatory variables, we separately run both univariate and multivariate ordinary least squares (OLS) regressions of monthly changes in the CDS market illiquidity measure on monthly changes in the explanatory variables (for explanatory variables that are returns, we do not use their first-differences in the regressions). We run regressions in first-differences to avoid spurious results due to persistence of the dependent and explanatory variables (unit root tests are available upon request). For those measures that are available at a daily frequency, we obtain the monthly time series by averaging daily observations within each month. Regression results are exhibited in Panels A to D of Table 3 and pairwise
correlations of monthly changes in the explanatory variables are reported in Table 4.

First, consider the alternative CDS market illiquidity measures (Panel A of Table 3). We find a significant relation of our illiquidity measure with bid-ask spreads and the price impact of credit index trades. This holds true in both univariate and multivariate regressions. The relation with the other price impact measure is insignificant. Collectively, the alternative CDS market illiquidity measures explain 45% of the time-series variation of our measure.

Second, consider the measures of bond market illiquidity (Panel B of Table 3). The Treasury bond market illiquidity measure is strongly significant in both univariate and multivariate regressions. Irrespective of which corporate bond market illiquidity measure we consider, we find a significant relation with our illiquidity measure. Together, the bond market illiquidity measures explain 48% of the time-series variation of the CDS market illiquidity measure.

Third, consider the measures of the cost and supply of capital (Panel C of Table 3). The unsecured funding cost is significant, while the secured one is only marginally significant. This indicates that index arbitrage traders primarily fund their trades in unsecured interbank markets, which is consistent with the fact that CDSs cannot be used as collateral in repo transactions. Moreover, hedge fund returns are significantly related to changes in the CDS market illiquidity measure suggesting that liquidity provision by hedge funds constitutes an important source of liquidity in the CDS market. Together, the measures of the cost and supply of capital explain 26% of the time-series variation of the CDS market illiquidity measure.

Finally, consider the measures of overall market conditions (Panel D of Table 3). In univariate regressions, changes in yield spreads and the VIX index are significantly related
to changes in the illiquidity measure. However, in a multivariate regression only changes in
yield spreads are (marginally) significant. Nevertheless, overall market conditions explain
37% of the time-series variation of the CDS market illiquidity measure.

As can be seen from Table 4, many of the explanatory variables are relatively highly cor-
related. Thus, a regression including all twelve variables will be subject to multicollinearity.
Indeed, unreported results show that most variables become insignificant in this regression
and some regression coefficients switch their sign (results are available upon request). Col-
lectively, all explanatory variables together explain 63% of the time-series variation of the
CDS market illiquidity measure.

4 Asset Pricing Implications

This section investigates whether liquidity risk is priced in the cross section of CDS returns.
We focus on liquidity risk defined as covariation between CDS returns and our tradable
liquidity factor. For assets in zero net supply, this notion of liquidity risk is theoretically
supported by the model of Kondor and Vayanos (2014) in which liquidity varies due to
capital-constrained liquidity providers and expected excess returns are exactly proportional
to the covariance between returns and market-wide liquidity.

4.1 Asset Pricing Model

For our analysis, we consider one-week excess returns from a CDS protection seller’s per-
spective and apply a parsimonious factor pricing model. Betas of one-week excess returns
on a CDS contract referencing entity $i$, $r_{i,t}^e$, are the slope coefficients, $\beta_i$, in the regression

$$r_{i,t}^e = \alpha_i + \beta_i^{DEF} DEF_t + \beta_i^{LIQ} LIQ_t + \epsilon_{i,t}. \tag{4}$$

20
where \(DEF_t\) denotes a default factor and \(LIQ_t\) is the tradable liquidity factor from Section 2.5. 
\(DEF_t\) is the realized excess return from selling protection on the CDX.NA.IG and iTraxx Eur indices, with equal weights on the two indices. Our benchmark model specification does not include a stock market factor. This is for parsimony and because of the strong correlation that usually exists between stock market and default factors and their corresponding betas. As one of our robustness checks, we include the excess return on a stock market index as an additional factor.

We compute excess returns assuming that CDS contracts are covered by collateral agreements and marked to market frequently. In this case the excess return on a CDS contract is the change in its mark-to-market value over the holding period relative to the initial amount of collateral that is posted. We assume that the initial collateral equals the notional amount of the contract resulting in an “unlevered” excess return. Details on the excess return computation are given in Appendix B.

In the cross section, the model relates expected excess returns to factor betas and factor prices of risk. Specifically, expected excess returns are given by

\[
E[r_{i,t}^e] = \beta_i^{DEF} \lambda_{DEF} + \beta_i^{LIQ} \lambda_{LIQ}, \tag{5}
\]

where \(\lambda_s\) denote factor prices of risk.

The factor pricing model is estimated in two steps. In the first step, we determine betas as OLS estimates, \(\widehat{\beta}_i\), of the slope coefficients in regression (4). In the second step, we estimate factor prices of risk via OLS from a cross-sectional regression of expected excess returns on first-step betas; that is,

\[
\widehat{E}[r_{i,t}^e] = \widehat{\beta}_i^{DEF} \lambda_{DEF} + \widehat{\beta}_i^{LIQ} \lambda_{LIQ} + u_i, \tag{6}
\]

where \(\widehat{E}[r_{i,t}^e]\) is an expected excess return estimate.
In the empirical asset pricing literature, expected excess returns are typically estimated by sample means of realized excess returns. Because of the rare occurrence of credit events and the extreme returns associated with them, sample means of realized excess returns are very imprecise estimates of expected excess returns on CDSs. Instead, we follow BDD in using CDS spreads and physical survival probabilities to construct forward-looking estimates of conditional expected one-week excess returns on CDSs, $\hat{E}_t [r_{i,t+1}]$. Unconditional expectations of excess returns are then estimated by the sample means of conditional expected excess returns. That is, $\hat{E}[r_{i,t}]$ in regression (6) is given by

$$\hat{E}[r_{i,t}] = \frac{1}{T} \sum_{s=1}^{T} \hat{E}_s [r_{i,s+1}]$$

(7)

where $T$ denotes the sample size.

The empirical setup necessitates several adjustments to the standard errors in regression (6). First, an errors-in-variables (EIV) adjustment arising from betas being estimated with errors in the first step. Second, an adjustment for heteroscedastic and autocorrelated errors. Third, an adjustment for potential model misspecification, arising from the possibility that, even in population, there is no combination of $\lambda$s such that Equation (5) is satisfied. To make these three adjustments, we use the approach of Kan et al. (2013). Adjusting standard errors for potential model misspecification allows one to draw inference on the relation between betas and expected excess returns in cases where betas do not explain the entire cross-sectional variation in expected excess returns. As shown in Kan et al. (2013), ignoring potential model misspecification typically leads to an overly positive assessment of the performance of an asset pricing model and the significance with which risk factors are priced (see also Gospodinov, Kan, and Robotti (2014) and the discussion in Ludvigson (2013)). Details of the standard error computation are provided in the Internet Appendix. For comparison, we also report results based on generalized method of moments standard...
errors, which adjust for EIV as well as heteroscedastic and autocorrelated errors but not for potential model misspecification.

4.2 Data and Portfolio Construction

Data

The daily data that we use in the construction of our sample come from Markit, Bloomberg, and Moody’s Analytics and extend from June 1, 2006, to February 1, 2012. From Markit, we collect five-year composite mid CDS spreads, the expected recovery rates, and the average rating by Moody’s and S&P for all companies domiciled in North America and Europe.\textsuperscript{18} We focus on CDS contracts written on senior unsecured debt and denominated in either EUR or USD.\textsuperscript{19} From Bloomberg, we obtain composite bid and ask CDS spreads, with the matching of CDS contracts from the two sources based on the reference entities’ six-digit Reference Entity Database (RED) codes and the currency denominations. From Moody’s Analytics, we obtain one-year and five-year EDFs for all public companies that are contained in the Markit database. Thus, our sample consists of North American and European reference names with data coverage by each of the three providers. Credit events in our sample are identified from settlement auctions of CDSs and we collect credit event data from the corresponding settlement protocols and auction results.\textsuperscript{20}

Because the key ingredients to our asset pricing tests, namely CDS excess returns and estimates of their conditional expectations, are inferred from mid CDS spreads, we filter

\textsuperscript{18}Ratings are adjusted for seniority of the reference obligation and subcategories are rounded to major rating categories.

\textsuperscript{19}We select those contract terms that, on a given date, were the market standard. That is, for EUR denominated contracts, we select the modified-modified restructuring clause and for USD denominated contracts referencing high yield names, we select the no restructuring clause. For USD denominated contracts referencing investment grade names, we select the modified restructuring clause prior to 2009 and the no restructuring clause thereafter, in order to account for a change in the market standard due to the implementation of the ISDA’s CDS ‘Big Bang’ Protocol.

\textsuperscript{20}Creditex and Markit administrate credit event auctions and publish auction results on \url{www.creditfixings.com}. Settlement protocols are published by the ISDA.
those for stale quotes. A quote is classified as stale, once it does not change over five or more consecutive trading days. In this case, only the spread quotation on the first of the consecutive days is retained in the sample, while the remaining ones are excluded.

From the collected data, we compute realized excess returns on CDSs over distinct one-week periods and weekly observations of conditional expected one-week excess returns; details of the numerical implementation are deferred to Appendix B. We also construct weekly series of bid-ask spreads and price impact measures. Due to a considerable number of missing bid and/or ask spreads, we use weekly averages of bid-ask spreads instead of end-of-period observations. The price impact measure is constructed as in Section 3, with the exception that we average absolute spread changes per contributed quote over one-week as opposed to one-month periods. Weekly observations are sampled on Wednesdays. We exclude all companies with less than fifty joint observations of conditional expected excess returns, realized excess returns, bid-ask spreads, and price impact measures. This leaves a sample of 666 companies, of which 426 are domiciled in North America and 240 in Europe, and a total of 144,163 joint observations.

**Portfolio Construction**

Because individual-asset beta estimates are usually very imprecise, we conduct our analysis on a set of 40 equally-weighted test portfolios rather than at the level of individual CDSs. Portfolios are rebalanced at a quarterly frequency and formed such that they exhibit variation across the default risk and liquidity dimensions.

The portfolio formation is as follows: on month-ends of March, June, September, and December of a given year, we first sort reference names from best to worst credit quality according to the average issuer credit rating over the previous quarter, and then group them into five credit rating categories: AAA–AA, A, BBB, BB, and B–CCC. Subsequently, we sort reference names within a given default risk group from most liquid to least liquid
either according to the average bid-ask spread over the previous quarter or according to the average price impact over the previous quarter. In both cases, we group reference names into illiquidity quartiles.

Because the first quarter of data is used for portfolio formation, this procedure yields portfolio time series from October 11, 2006, to February 1, 2012.\textsuperscript{21} During this period, we find two weeks where only a small number of North American reference names have quoted bid-ask spreads.\textsuperscript{22} We exclude the corresponding portfolio observations from the analysis, leaving a total of 276 one-week periods during the sample period.

4.3 Results

Descriptive Statistics

Table 5 displays descriptive statistics for the two factors. During our sample period, the average realized excess return on the default factor is positive, but not statistically significant. In contrast, the average realized excess return on the liquidity factor is positive and significant, with a Newey and West (1987) $t$-statistic of 5.42. The two factors are virtually uncorrelated, with a correlation coefficient of 0.07.

[Table 5 about here.]

Table 6 displays descriptive statistics for the 20 portfolios formed by first sorting CDS contracts according to credit ratings and then according to bid-ask spreads. Descriptive statistics for the remaining 20 portfolios are provided in the Internet Appendix. Sample means of expected excess returns are positive across portfolios and strongly significant with

\textsuperscript{21}Initial portfolio constituents are selected based on data up to and including September 29, 2006, and the actual formation takes place on the following Wednesday; i.e., October 4, 2006. Obviously, the first return is then observed a week later; i.e., October 11, 2006. On any rebalancing date the portfolio formation proceeds equivalently.

\textsuperscript{22}In the week from December 20, 2007, to December 26, 2007, there is a total of 28 bid-ask spread observations for eleven North American reference names. Similarly, in the week from May 22, 2008, to May 28, 2008, there is a total of 38 observations for 18 reference names.
Newey and West (1987) $t$-statistics between 4.13 and 11.14. This reflects the fact that risk neutral default probabilities, on average, exceed physical default probabilities. Expected excess returns tend to increase with portfolio illiquidity and deteriorating credit quality. For instance, among the bid-ask-spread-sorted portfolios, we observe a difference of 5.45% per year in the expected excess return between a portfolio consisting of the most illiquid low-credit-quality CDSs (B–CCCQ4) and a portfolio consisting of the most liquid high-credit-quality CDSs (AAA–AAQ1). Sample means of realized excess returns are not significantly different from zero, underscoring the importance of using forward-looking information when estimating expected excess returns. Like expected excess returns, the standard deviations of realized excess returns increase with portfolio illiquidity and deteriorating credit quality, and the resulting unconditional and forward-looking annualized Sharpe ratios lie in a reasonable range from 0.13 to 0.32.

[Table 6 about here.]

**Regression Results**

First-step regression results are displayed in Table 7. For ease of interpretation, instead of reporting the raw beta estimates, we report the product of the beta estimates and the standard deviations of the respective factors. Consequently, the table shows weekly realized portfolio excess returns (in bps) in response to a one standard deviation shock to each of the factors.

[Table 7 about here.]

Default risk betas are positive and statistically significant throughout portfolios and almost monotonically increasing along both the liquidity and credit quality dimensions. Default risk is economically important with a one standard deviation shock to the default factor having an impact on portfolio excess returns between 10 bps and 199 bps.
Liquidity risk betas are positive throughout portfolios and statistically significant at the five percent level for 28 out of the 40 portfolios. These betas also tend to increase along the liquidity and credit quality dimensions. However, especially along the liquidity dimension there are exceptions indicating that portfolios with higher bid-ask spreads or price impact measures do not necessarily exhibit larger liquidity risk. Liquidity risk is also economically important with a one standard deviation shock to the liquidity factor having an impact on portfolio excess returns between 3 bps and 99 bps. The positive liquidity risk betas imply that sellers of credit protection systematically realize negative excess returns when liquidity in the CDS market deteriorates. Unreported adjusted $R^2$s of the regressions range from 18% to 76% across portfolios.\(^{23}\)

Second-step regression results for alternative specifications of the cross-sectional regression (6) are displayed in Table 8. The table shows regression coefficients with $t$-statistics that account for EIV as well as heteroscedastic and autocorrelated errors reported in parentheses, and $t$-statistics that in addition account for potential model misspecification reported in brackets. The table also shows cross-sectional $R^2$s with 95% confidence intervals in brackets. These confidence intervals are computed along the lines of Kan et al. (2013), with details deferred to the Internet Appendix.

[Table 8 about here.]

Factors carry significant prices of risk in one- and two-factor models regardless of whether standard errors are adjusted for potential model misspecification or not. For instance, in the two-factor specification with an imposed zero-intercept restriction (specification 3), the most conservative $t$-statistics are 5.72 and 2.42 for the default and liquidity factors, respectively. Imposing the zero-intercept restriction is inconsequential as intercepts are small in magnitude and not statistically significant (see specifications 4–6). Cross-sectional $R^2$s are substantial

\(^{23}\)Unreported results for nested one-factor specifications of regression (4) show that, on its own, each of the two factors constitutes a significant explanatory variable of CDS portfolio excess returns.
across model specifications, which is, in part, a consequence of using less-noisy and forward-looking information when estimating expected excess returns. For instance, our preferred model specification (specification 3; henceforth, the benchmark model specification) has a cross-sectional $R^2$ of 0.97, and a specification test with null hypothesis $H_0 : R^2 = 1$ cannot be rejected (the $p$-value is 0.16). Figure 4 shows the test portfolios’ model-implied expected excess returns for the benchmark model specification. In terms of expected excess returns, the model seems to fit the data quite well.

[Figure 4 about here.]

**Economic Importance**

To assess the economic importance of the risk factors, we use the benchmark model specification to decompose the annualized expected excess return on each test portfolio into default and liquidity risk premia (defined as $52 \times \hat{\beta}_F \times \hat{\lambda}_F, F \in \{DEF, LIQ\}$). We summarize the results in two ways. First, we consider the contributions of these components to the difference in the expected excess return between the two extreme (bid-ask-spread-sorted) portfolios B–CCCQ4 and AAA–AAQ1. We use the term *expected return differential* to refer to this difference. Second, we consider the contributions to the average expected excess return across test portfolios.

The first column of Table 9 shows the contributions to the expected return differential and, in brackets, the contributions to the average expected excess return. Default and liquidity risk contribute 3.33% and 2.66% per year, respectively, to the expected return differential. In case of the average expected excess return, default and liquidity risk contribute 1.11% and 0.52% per year, respectively. That liquidity risk is economically important is in contrast to the findings in BDD and is addressed in more detail below.

[Table 9 about here.]
As an alternative illustration of economic importance, we decompose the CDS spreads instead of the expected excess returns, again using the benchmark model specification. The CDS spread of each test portfolio is decomposed into components due to default and liquidity risk, as well as an additional component reflecting the expected default loss (computational details are deferred to Appendix C). Figure 5 displays the resulting decomposition for the 20 bid-ask-spread-sorted portfolios. The corresponding figure for the 20 price-impact-sorted portfolios can be found in the Internet Appendix.

[Figure 5 about here.]

We consider first the extreme (bid-ask-spread-sorted) portfolios. For the AAA-AAQ1 portfolio, the average CDS spread is 45 bps of which expected default losses and the default risk premium account for 7 bps (17%) and 26 bps (57%), respectively, while the liquidity risk premium accounts for 9 bps (20%) (the remaining 3 bps is a pricing error). At the other end of the spectrum is the B-CCCQ4 portfolio with an average CDS spread of 1710 bps of which expected default losses and the default risk premium account for 830 bps (49%) and 540 bps (32%), respectively, while the liquidity risk premium accounts for 408 bps (24%) (here the pricing error is -67 bps). Averaging the relative contributions across test portfolios, we find that expected default losses and the default risk premium account for 32% and 54% of the spread, respectively, while the liquidity risk premium accounts for 21%.

That expected default losses only account for a relatively small fraction of observed credit spreads, in particular for highly rated firms, is well known; see, e.g., Elton, Gruber, Agrawal, and Mann (2001) and Driessen (2005) for corporate bond yield spreads, and Berndt, Douglas, Duffie, Ferguson, and Schranz (2005) for CDS spreads. There is some disagreement concerning the size of the default risk premium with some papers using structural credit risk models to argue that default risk premia are small (again, in particular for highly rated firms; see, e.g., Huang and Huang (2012)), while other papers, mainly using reduced-form
credit risk models argue that default risk premia are sizable. Our analysis indicates that the default risk premium is the largest component of CDS spreads. Most importantly, however, we find evidence for a sizable liquidity risk premium.

4.4 Robustness Checks

We conduct a range of robustness checks; we control for the level of illiquidity, account for an alternative notion of liquidity risk suggested by BDD, consider an alternative construction of our CDS market liquidity factor, and include additional risk factors in the asset pricing model. For each robustness check, the result of the cross-sectional regression is reported in Table 10 and the decomposition of expected excess returns is reported in Table 9. Because our two measures of economic importance typically give similar results, we only comment on the expected return differential.

[Table 10 about here.]

Level of Illiquidity

For assets that cannot be sold short, the effect of the level of illiquidity on expected excess returns is positive (see, e.g., Amihud and Mendelson (1986) and Acharya and Pedersen (2005)). For assets that can be sold short, including derivatives contracts in zero net supply, the effect of the level of illiquidity on expected excess returns is either positive, negative, or nonexistent and crucially depends on the endowments, investment horizons, and risk attitudes of investors (see BDD). In the case of CDSs, BDD estimate that the effect is positive and economically important. Therefore, we control for the illiquidity level of CDS contracts when assessing liquidity risk. Specifically, we separately add bid-ask spreads and the levels of the price impact measure as portfolio characteristics to the second-step regression (see specifications 1 and 2 in Tables 9 and 10, respectively). However, in the presence of
the liquidity risk betas, neither bid-ask spreads nor the levels of the price impact measure are significantly related to expected excess returns. As such, it appears that our notion of liquidity risk largely subsumes the effect of the level of illiquidity. The statistical significance and economic importance of liquidity risk are very similar to the benchmark case.

Alternative Notion of Liquidity Risk

BDD also investigate liquidity risk in CDS markets, but use a very different notion of liquidity risk, namely covariation between innovations to transaction costs on individual CDS contracts and systematic default risk. Their reasoning is that when transaction costs covary negatively with systematic default risk (which is the case empirically), CDS contracts are less effective hedges as it is more costly to unwind the hedge positions in states with high aggregate default risk. This, they argue, should reduce the demand for CDS protection and lower the expected excess returns earned by sellers of credit protection. Empirically, however, they find that the liquidity risk premium is economically negligible. Nonetheless, we control for this alternative notion of liquidity risk by adding the corresponding betas to the second-step regression (see specification 3 in Tables 9 and 10). These alternative liquidity risk betas have the expected negative sign but are insignificant for most test portfolios. The associated price of risk is statistically insignificant and, as in BDD, economically negligible. In contrast, the statistical significance and economic importance of our notion of liquidity risk are virtually unchanged relative to the benchmark case.

24 Specifically, we follow BDD in estimating single-factor betas of bid-ask spread innovations with respect to the default factor. We use BDD’s time-series model of liquidity to compute bid-ask spread innovations and orthogonalize innovations for stock market returns (i.e., returns on the “nonhedge” asset in BDD’s terminology).

25 A possible reason for the lack of importance of BDD’s notion of liquidity risk may be that the majority of CDS contracts are marked to market very frequently. This implies that protection buyers realize gains when aggregate default risk increases without having to unwind their positions and incur transaction costs.
Alternative Construction of CDS Market Liquidity Factor

We consider an alternative construction of the tradable liquidity factor in which the excess returns on the individual index arbitrage strategies are weighted by the inverse of their conditional volatilities (see specification 4 in Tables 9 and 10). This is akin to the construction of the time series momentum factor in Moskowitz, Ooi, and Pedersen (2012) (see Appendix D for details). The resulting factor is very highly correlated with our original liquidity factor and we obtain results that are similar to those in the benchmark case. If anything, liquidity risk increases in economic importance, contributing 2.81% per year to the expected return differential.

Additional Factors

*Treasury market illiquidity factor.* Hu et al. (2013) find that exposure to their “Noise” measure, which captures Treasury market illiquidity, is priced in the cross section of returns on hedge funds and currency carry trades—assets/strategies that are particularly sensitive to liquidity. Consequently, we add the residual of an AR(2) specification of their “Noise” measure as an additional factor (see specification 5 in Tables 9 and 10). 17 test portfolios load significantly on this factor, all with a negative sign, and its price of risk is statistically significant when ignoring potential model misspecification and has the expected negative sign. The price of CDS market liquidity risk remains statistically significant when ignoring potential model misspecification, but its magnitude is reduced somewhat. This is consistent with the analysis in Section 3, which identified the “Noise” measure as the variable that is most strongly related to variation in CDS market illiquidity. In terms of economic importance, the Treasury market illiquidity factor contributes 1.19% per year to the expected return differential, while the contribution of the CDS market liquidity factor is 1.74%.

*Corporate bond market illiquidity factor.* Several papers including Lin et al. (2011) and
Acharya, Amihud, and Bharath (2013) find that exposure to market-wide corporate bond liquidity is priced in the cross section of corporate bond returns. Given the relation between CDS spreads and corporate bond yields, such exposure may also be priced in the cross section of CDS returns. Therefore, we add a corporate bond market illiquidity factor (see specification 6 in Tables 9 and 10). We measure corporate bond market illiquidity by aggregating bond-specific Amihud (2002) illiquidity measures and obtain an illiquidity factor as the residual of an AR(2) specification of the market-wide illiquidity measure (see Appendix D for details). At a monthly frequency, the resulting corporate bond market illiquidity factor is highly correlated with innovations to Dick-Nielsen et al.’s (2012) $\lambda$. 13 test portfolios load significantly on the corporate bond market illiquidity factor, all with a negative sign. However, the price of corporate bond market liquidity risk is statistically insignificant and not economically important, while the statistical significance and economic importance of CDS market liquidity risk are very similar to the benchmark case.

Stock market illiquidity factor. Both Acharya et al. (2013) and Bongaerts et al. (2012) show that exposure to aggregate liquidity in the stock market is priced in the cross section of corporate bond returns. We add a stock market illiquidity factor to investigate if this also holds true for CDS returns (see specification 7 in Tables 9 and 10). The stock market illiquidity factor is constructed along the same lines as the corporate bond market illiquidity factor by aggregating stock-specific Amihud (2002) illiquidity measures (see Appendix D for details). 24 test portfolios load significantly on the stock market illiquidity factor, all with a negative sign. However, the price of stock market liquidity risk is statistically insignificant and of limited economic importance. The CDS market liquidity factor remains statistically

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26 Dick-Nielsen et al.’s (2012) $\lambda$ is only available at a monthly frequency, which is why we construct our own corporate bond market illiquidity factor.

27 Readily available measures of stock market illiquidity such as those of Pástor and Stambaugh (2003) and Sadka (2006) are only available at a monthly frequency, which is why we construct our own stock market illiquidity factor.
significant when ignoring potential model misspecification and contributes even more to the expected return differential (3.18% per year) than in the benchmark case.

Balance sheet constraints of financial intermediaries. The limits to arbitrage literature predicts an impact of financial intermediaries’ risk bearing capacity on asset prices. Adrian, Etula, and Muir (2013) capture such effects by a stochastic discount factor that depends on the leverage of broker-dealers and show that their model prices portfolios of stocks and bonds remarkably well. Given its institutional nature and the central role played by broker-dealers, the CDS market is presumably an even better testing ground for the effects of financial intermediaries’ balance sheet constraints on asset prices. We add Adrian et al.’s (2013) factor mimicking portfolio for broker-dealer leverage to the model (see specification 8 in Tables 9 and 10). The construction of this factor is detailed in Appendix D. It appears that the liquidity factor largely subsumes the effect of broker-dealer leverage; only ten test portfolios load significantly on the leverage factor and, while it has a statistically significant price of risk, its economic importance is limited. The liquidity factor remains statistically significant and economically important, contributing 2.14% per year to the expected return differential.

Stock market factor. We control for stock market risk by including a factor which is the excess return on an equally weighted portfolio of the S&P 500 and EURO STOXX 50 indices (see specification 9 in Tables 9 and 10). 16 test portfolios load significantly on the stock market factor, all but one with a positive sign, and its price of risk is statistically significant when ignoring potential model misspecification and has the expected positive sign. Economically, the factor contributes 2.23% per year to the expected return differential, and reduces the effect of default risk substantially to 1.95%.\footnote{While the market price of default risk is largely unaffected, the spread in default risk betas across test portfolios shrinks substantially when the stock market factor is included in the model.} The liquidity factor remains statistically significant when ignoring potential model misspecification, but its contribution to the expected
return differential is reduced somewhat to 1.86% per year.

Volatility factor. Several papers find that volatility is an important risk factor in asset markets (see, e.g., Ang, Hodrick, Xing, and Zhang (2006) and Bongaerts et al. (2012) for evidence from stock and corporate bond markets, respectively). Therefore, we include a volatility factor which is the residual of an AR(2) specification of the VIX index (see specification 10 in Tables 9 and 10). Only ten test portfolios load significantly on this factor, mostly with a negative sign, and, while it has a statistically significant price of risk, it is of limited economic importance. The liquidity factor remains statistically significant and economically important, contributing 2.86% per year to the expected return differential.

5 Conclusion

We analyze whether liquidity risk is priced in the cross section of returns on CDS contracts. First, we construct a model-independent measure of CDS market illiquidity by aggregating deviations of credit index levels from their no-arbitrage values implied by the index constituents’ CDS spreads. The measure captures illiquidity in broad terms and correlates not only with measures of bid-ask spreads and the price impact of trades in the CDS market but also with measures of illiquidity in the corporate bond and Treasury markets as well as measures of the cost and supply of capital. Second, we construct a tradable liquidity factor, based on index arbitrage strategies, that is highly negatively correlated with innovations to the CDS market illiquidity measure. Third, we define liquidity risk as covariation between CDS returns and the liquidity factor and show that liquidity risk is both statistically and economically important for the pricing of CDSs. In particular, liquidity risk increases CDS spreads and the expected excess returns earned by sellers of credit protection.
Appendices

A Time-Series Properties: Explanatory Variables

Bid-Ask. Bid and ask quotes for EUR or USD denominated senior five-year CDS contracts come from Bloomberg. Contract-specific bid-ask spreads are monthly averages of daily bid-ask spreads, which are calculated whenever more than five nonnegative daily bid-ask spread observations are available within the month. For each month, Bid-Ask is the average of contract-specific bid-ask spreads.

ILLIQ$^{CDS}$. Single-name CDS data for the construction of ILLIQ$^{CDS}$ come from Markit. For each reference name $i$, the ILLIQ$^{CDS}$ measure is the monthly average of absolute spread changes divided by the number of contributors to the spread quotation on date $t$, $Depth_{i,t}$. That is,

$$ILLIQ^{CDS}_{i,m} = \frac{1}{n_{i,m}} \sum_{t=1}^{n_{i,m}} \frac{|C_{i,t} - C_{i,t-1}|}{Depth_{i,t}},$$

(A.1)

where $n_{i,m}$ is the number of consecutive spread changes in month $m$ and $C_{i,t}$ is the five-year par spread. For each month, ILLIQ$^{CDS}$ is the average of ILLIQ$^{CDS}_{i,m}$ across those reference names with $n_{i,m} > 5$.

ILLIQ$^{IDX}$. Data for the construction of ILLIQ$^{IDX}$ are those described in Section 2.3. For each credit index $i$, the ILLIQ$^{IDX}$ measure is the monthly average of absolute changes in the level of the five-year on-the-run series divided by the number of contributors to the computation of the index quotation on date $t$, $Depth^{IDX}_{i,t}$. That is,

$$ILLIQ^{IDX}_{i,m} = \frac{1}{n_{i,m}} \sum_{t=1}^{n_{i,m}} \frac{|C^{IDX}_{i,t} - C^{IDX}_{i,t-1}|}{Depth^{IDX}_{i,t}},$$

(A.2)
where \( n_{i,m} \) is the number of consecutive index level changes in month \( m \) and \( C_{i,t}^{IDX} \) is defined as in Section 2.2. For each month, \( ILLIQ^{IDX} \) is the average value of \( ILLIQ_{i,m}^{IDX} \) across credit indices.

\textit{Noise}. Data come from Jun Pan’s website \url{http://www.mit.edu/~junpan}. \textit{Noise} is the monthly average of daily observations.

\( \lambda \). The monthly time series of Dick-Nielsen et al.’s (2012) \( \lambda \) corporate bond illiquidity measure comes from Peter Feldhütter’s website \url{http://feldhutter.com}.

\textit{CDS-Bond}. The time series of the average CDS-bond basis across U.S. investment grade bonds comes from J.P. Morgan. \textit{CDS-Bond} is the monthly average of daily observations of the CDS-bond basis.

\textit{LIB-OIS}. USD LIBOR and OIS data come from Bloomberg. \textit{LIB-OIS} is the monthly average of daily spread observations between three-month LIBOR and OIS rates.

\textit{Repo}. Repo rates come from Bloomberg. \textit{Repo} is the monthly average of daily spread observations between three-month Agency MBS and Treasury general collateral repo rates.

\( R_{HFRX} \). The monthly time series of the Hedge Fund Research Global Index comes from Bloomberg. \( R_{HFRX} \) is the simple one-month return on the index.

\( R_{SPX} \). S&P 500 index levels come from Bloomberg. \( R_{SPX} \) is the simple one-month return on the index.

VIX. VIX data come from Bloomberg. VIX is the monthly average of daily index levels.

B Excess Return Computation

This Appendix describes the computation of expected and realized excess returns on a CDS trading at par as well as a portfolio composed of such CDSs. It also describes the computation of realized excess returns on a credit index and its replicating portfolio.

Realized CDS Excess Return

When computing realized excess returns, we assume that contracts are marked to market using the ISDA CDS Standard Model, which is the market standard for determining mark-to-market payments in credit derivatives transactions.\textsuperscript{29} Consider a CDS contract referencing entity \(i\) with a notional amount of one dollar and fixed spread \(C\).\textsuperscript{30} At date \(t\), the present

\textsuperscript{29}The ISDA CDS Standard Model is a reduced form model which assumes that (i) credit events occur randomly and independently across reference names at the first jump times of homogeneous Poisson processes with constant intensities, (ii) interest rates evolve independent of the occurrence of credit events, and (iii) in case that a credit event occurs, creditors recover a constant fraction of the reference obligation’s par value.

\textsuperscript{30}We follow market standard in assuming that CDS contracts are standardized with respect to their spread payment dates and maturities. Payment dates of standardized contracts fall on the 20th of March, June, September, and December of each year, and the maturity date is the first payment date that follows the trade date by the term of the contract. CDS contracts trade with standardized payment dates and maturities since 2003 (O’Kane (2008)).
The present value of the contract from the perspective of the protection seller is

\[ PV_t(C; C_{i,t}, R^*_i) = \text{Prem}_t(C; C_{i,t}, R^*_i) - \text{Prot}_t(C_{i,t}, R^*_i), \]  

(B.1)

where \( C_{i,t} \) denotes the date-\( t \) par spread and \( R^*_i \) denotes the expected recovery rate on (senior unsecured) debt issued by entity \( i \). The first term on the right hand side of Equation (B.1) is the date-\( t \) present value of the premium leg

\[ \text{Prem}_t(C; C_{i,t}, R^*_i) = C \times PVBP_t(C_{i,t}, R^*_i), \]  

(B.2)

where

\[ PVBP_t(C_{i,t}, R^*_i) = \sum_{j=1}^{J} \left( \frac{(t_j - t_{j-1})}{360} D(t, t_j) S_i(t, t_j) - \int_{t \vee t_{j-1}}^{t_j} \frac{(u - t_{j-1})}{360} D(t, u) dS_i(t, u) \right) \]  

(B.3)

is the date-\( t \) present value of a risky annuity with payment dates \( t < t_1 < \cdots < t_J \) (\( t_0 \leq t \) being the start date of the CDS contract and \( t_J \) being its maturity), \( D(t, t_j) \) is the date-\( t \) discount factor applicable to a risk free cash flow on date \( t_j \), and \( S_i(t, t_j) \) is the date-\( t \) risk neutral survival probability of entity \( i \) up to date \( t_j \).\(^{31}\) The second term of Equation (B.1) is the date-\( t \) present value of the protection leg \(^{32}\)

\[ \text{Prot}_t(C_{i,t}, R^*_i) = - \int_{t}^{t_J} (1 - R^*_i) D(t, u) dS_i(t, u). \]  

(B.4)

The present value of the contract can be decomposed into an accrual amount, \( C \times (t - t_0)/360 \), and a residual upfront amount. The par spread is defined such that the upfront amount is

\(^{31}\)The integral term in Equation (B.3) is the present value of the accrual amount on default.

\(^{32}\)Note that both \( \text{Prem}_t(C; C_{i,t}, R^*_i) \) and \( \text{Prot}_t(C_{i,t}, R^*_i) \) depend on the par spread, \( C_{i,t} \), and the expected recovery rate, \( R^*_i \), via the survival probabilities, \( S_i(t, t_j) \).
zero, which means that (B.1) can be equivalently expressed as

$$PV_t(C; C_{i,t}, R^*_i) = (C - C_{i,t}) \left( PVBP_t(C_{i,t}, R^*_i) - \frac{t - t_0}{360} \right) + C \frac{t - t_0}{360}. \quad (B.5)$$

We compute the excess return from $t$ to $t'$ from the protection seller’s perspective. At date $t$, we assume that the protection seller posts initial collateral equal to the notional of the contract.\textsuperscript{33} This collateral earns the risk-free rate. In addition, the protection seller pays the protection buyer the present value of the contract, but under standard margining rules, this amount is immediately refunded to the protection seller as variation margin.\textsuperscript{34} Moreover, to simplify matters, we assume that the contract is initiated at the par spread.\textsuperscript{35}

If there is no credit event between dates $t$ and $t'$, the contract is marked to market at date $t'$ and the protection seller receives an amount equal to the change in the present value of the contract from $t$ to $t'$. In this case, the excess return is

$$r^e_{i,t,t'} = -(C_{i,t'} - C_{i,t}) \left( PVBP_{t'}(C_{i,t'}, R^*_i) - \frac{t' - t}{360} \right) + C_{i,t} \frac{t' - t}{360} \quad (B.6)$$
on the notional amount of the contract.\textsuperscript{36}

If there is a credit event between dates $t$ and $t'$, the excess return is

$$r^e_{i,t,t'} = -(1 - R_i) + C_{i,t} \frac{\tau_i - t}{360}, \quad (B.7)$$

\textsuperscript{33}This makes the return “unlevered” and is economically similar to how Berndt and Obreja (2010), BDD, Bao and Pan (2013) compute CDS returns.
\textsuperscript{34}The interest that is paid on variation margin varies with contract terms. For simplicity, we assume that it is zero.
\textsuperscript{35}Alternatively, we could assume that contracts are traded with upfront amounts and fixed spreads (as we do below when computing realized excess returns on credit indices), which is the convention for trading standardized single-name contracts since the implementation of the ISDA’s “Big Bang” protocol. Assuming that contracts are traded at their par spreads has the advantage that those quotations are available throughout the sample period.
\textsuperscript{36}We choose to write Equation (B.6) with a “$-$” sign preceding the spread change to emphasize that credit protection sellers incur mark-to-market losses when spreads widen.
where $R_i$ is the actual recovery rate and the second term of Equation (B.7) is the accrual amount on default (where $\tau_i$ is the credit event date).

We use Markit five-year mid spreads and the corresponding expected recovery rates to construct one-week realized excess returns (we denote by $r^e_{i,t}$ the realized excess return over a one-week period ending on date $t$).\(^{37}\) Risk free discount factors are bootstrapped from the term structure of LIBOR/swap rates. For each reference name that triggered a credit event, we compute the realized excess return over the one-week period that contains the credit event date, using the actual recovery rate determined in the credit event auction. In case of failure to pay and restructuring credit events, we resume return computations from the first week following the credit event auctions and delete all intermediate data. Our sample includes a total of 22 credit events among 21 different reference names and losses per dollar of notional range from 23.38% for the Governor and Company of the Bank of Ireland to 98.75% for Landsbanki.\(^{38}\)

**Expected CDS Excess Return**

We follow BDD in defining the date-$t$ conditional expected excess return over the life of a five-year CDS contract as

$$
\hat{E}_t[r^e_{i,t,t_j}] = C_{i,t} PVBPP_{i,t} - EL^P_{i,t},
$$

(B.8)

where

$$
PVBPP_{i,t} = \sum_{j=1}^{J} \left( \frac{(t_j - t_{j-1})}{360} D(t, t_j)P_i(t, t_j) - \int_{t\vee t_{j-1}}^{t_j} \frac{(u - t_{j-1})}{360} D(t, u) dP_i(t, u) \right) - \frac{t - t_0}{360},
$$

(B.9)

\(^{37}\)Expected recovery rates are set to 40% whenever they are not available in the Markit database.\(^{38}\) Two restructuring credit events occurred for Irish Life & Permanent in 2011.
and

\[ EL_{i,t}^P = - \int_t^{t_J} (1 - R_{i,t}^*) D(t, u) dP_i(t, u), \]  

(B.10)

in which the physical survival probability of entity \( i \), \( P_i(t, u) \), integrates payoffs instead of the risk neutral survival probability.\(^3\) Physical survival probabilities are extracted from Moody’s KMV one-year and five-year EDFs through

\[ P_i(t, t + 1Y) = 1 - EDF_{1Y_{i,t}} \quad \text{and} \quad P_i(t, t + 5Y) = (1 - EDF_{5Y_{i,t}})^5 \]  

(B.11)

and intermediate values are obtained by interpolation based on the assumption of piecewise constant instantaneous physical default intensities. Conditional expected excess returns for a holding period shorter than five years are obtained by assuming that returns scale proportionally with time-to-maturity. In particular,

\[ \widehat{E}_t[r_{i,t+1}^e] = \frac{7}{t_J - t} \widehat{E}_t[r_{i,t,t_J}^e]. \]  

(B.12)

We emphasize that the accuracy of conditional expected excess returns depends on EDFs being an appropriate measure of conditional default probabilities. As shown in Duffie, Saita, and Wang (2007), there exist alternative specifications of conditional default probabilities that have marginally higher predictive power than EDFs. However, EDFs have the advantages that they are readily available for reference names in our sample, are widely used in the industry (and, as such, are part of the information set of most market participants), and are unbiased estimates (in-sample) of average default rates. Compared to credit ratings, EDFs have the advantage of adjusting faster to new information and consequently have better predictive power. Recent studies including Korablev and Qu (2009) and Crossen and Zhang

\(^3\)By using the same expected recovery rate under the risk neutral and physical probability measure, we implicitly assume that there is no recovery risk premium.
confirm the performance of EDFs for predicting defaults during both the financial crisis and the pre-crisis period.

**Portfolio Excess Returns**

Because we consider equally weighted portfolios of reference names, the realized excess return on a portfolio \( p \) of five-year CDS contracts over the one-week period from \( t - 1 \) to \( t \) is the average realized excess return on the \( n_{p,t-1} \) CDS contracts that constitute portfolio \( p \) on date \( t - 1 \); that is,

\[
 r_{p,t}^e = \frac{1}{n_{p,t-1}} \sum_{i \in I_{p,t-1}} r_{i,t}^e, \tag{B.13}
\]

where \( I_{p,t-1} \) denotes the set of reference names. Similarly, the date-\( t \) conditional expected excess return on the portfolio is the average conditional expected excess return on the CDS contracts that constitute the portfolio; that is,

\[
 \hat{E}_t[r_{p,t,t,j}^e] = \frac{1}{n_{p,t}} \sum_{i \in I_{p,t}} \hat{E}_t[r_{i,t,t,j}^e] = C_{p,t} PVBP_{p,t}^P - EL_P, \tag{B.14}
\]

where the portfolio level quantities in Equation (B.14) are defined as

\[
 C_{p,t} = \frac{1}{n_{p,t}} \sum_{i \in I_{p,t}} C_{i,t}, \tag{B.15}
\]

\[
 EL_P^P = \frac{1}{n_{p,t}} \sum_{i \in I_{p,t}} EL_P, \tag{B.16}
\]

\[
 PVBP_{p,t}^P = \frac{1}{n_{p,t}} \sum_{i \in I_{p,t}} \frac{C_{i,t}}{C_{p,t}} PVBP_{i,t}^P. \tag{B.17}
\]

**Realized Credit Index Excess Return**

Finally, consider a five-year credit index contract with one dollar notional amount. The contract trades with fixed spread \( C \) and date-\( t \) upfront amount, \( UF_{i,t}^{IDX} (C) \), that is received
by the seller of credit protection. As in the case of single-name CDS contracts, we compute “unlevered” realized excess returns on credit index contracts, \( r_{i,i,t,t'}^{IDX} \), assuming that contracts are covered by collateral agreements and standard margining rules apply; that is,

\[
r_{i,i,t,t'}^{IDX} = - \left( U_P^{IDX}_{i,t'}(C) - U_P^{IDX}_{i,t}(C) \right) + \frac{I_t}{I} C \frac{t' - t}{360} - \frac{1}{I} (L_{i,t'} - L_{i,t}), \tag{B.18}
\]

where \( L_{i,t} \) is the cumulative loss due to credit events among index constituents on date \( t \).\(^{40}\)

Replacing the upfront amounts in Equation (B.18) with those on the replicating basket of single-name CDSs gives the “unlevered” realized excess return on the replicating basket, \( r_{i,i,t,t'}^{CDS} \). Hence, realized credit index excess returns can be readily computed from Markit’s credit index data described in Section 2.3 (as before we denote by \( r_{i,t}^{IDX} \) and \( r_{i,t}^{CDS} \), respectively, the realized excess returns over a one-week period ending on date \( t \)). Whenever an index roll date, \( t_{roll} \), falls between dates \( t \) and \( t' \), the realized excess return is obtained by first computing the realized excess return on series \( S_i \) of index \( i \) between \( t \) and \( t_{roll} \) and then adding to it the realized excess return on series \( S_i + 1 \) over \( t_{roll} \) to \( t' \).

### C Model-Implied CDS Spread Decomposition

From Equation (B.8), the five-year CDS spreads can be expressed in terms of conditional expected excess returns and expected default losses; that is,

\[
C_{i,t} = EL_{i,t}^{P} + \hat{E}_t[r_{i,t,t,t}'^{e}].
\tag{C.1}
\]

\(^{40}\)As in case of CDS excess returns, losses accumulated in \( L_{i,t} \) are given as one minus the recovery value determined in a credit event auction. The expression in Equation (B.18) presumes that all defaults between dates \( t \) and \( t' \) occur exactly on date \( t' \). In our implementation, we account for the fact that defaults may happen in between \( t \) and \( t' \) and adjust the accrual term in Equation (B.18) accordingly.
Because conditional instead of unconditional expected excess returns determine CDS spreads, we estimate the following conditional version of Equation (6)

\[
\hat{E}_t[r_{i,t+1}^e] = \hat{\beta}_i^{DEF} \lambda_{DEF,t} + \hat{\beta}_i^{LIQ} \lambda_{LIQ,t} + u_{i,t}.
\] (C.2)

Because betas in Equation (C.2) are fixed, sample means of factor price of risk estimates in the conditional model coincide with those in the unconditional model; that is

\[
\lambda_{DEF} = \frac{1}{T} \sum_{t=1}^{T} \lambda_{DEF,t}, \tag{C.3}
\]

\[
\lambda_{LIQ} = \frac{1}{T} \sum_{t=1}^{T} \lambda_{LIQ,t}. \tag{C.4}
\]

From Equation (B.12), the conditional expected one-week excess return in Equation (C.2) can be converted to a five-year holding period by multiplication with \((t_J - t)/7\). Replacing \(\hat{E}_t[r_{i,t,t_J}^e]\) in Equation (C.1) with the resulting expression gives the following decomposition of reference entity \(i\)'s five-year CDS spread on date \(t\):

\[
C_{i,t} = \frac{EL_{i,t}^P}{PVBP_{i,t}^P} + \frac{(t_J - t)\hat{\beta}_i^{DEF} \lambda_{DEF,t}}{7 \times PVBP_{i,t}^P} + \frac{(t_J - t)\hat{\beta}_i^{LIQ} \lambda_{LIQ,t}}{7 \times PVBP_{i,t}^P} + \frac{(t_J - t)u_{i,t}}{7 \times PVBP_{i,t}^P}. \tag{C.5}
\]

The first term on the right hand side of Equation (C.5) is the expected default loss, the second and third terms are the default and liquidity risk premia, respectively, and the last term is the pricing error. The components of our model-implied CDS spread decomposition are the sample means of the terms in Equation (C.5). Note that the same decomposition holds for portfolio level CDS spreads using the respective expressions in Appendix B.
Conditional volatility weighting. The construction of our tradable liquidity factor is similar to that of Moskowitz et al.’s (2012) time series momentum factor in that it aggregates signed returns. To account for the considerable cross-sectional variation in volatilities across assets, Moskowitz et al. (2012) scale returns by their conditional volatilities. We construct a tradable liquidity factor in a similar way with weights inversely proportional to conditional volatilities; that is,

\[ LIQ_{CVW}^t = \sum_{i=1}^{n_t} w_{CVW, i, t} \cdot \text{sgn} \left( B_{i, t-1} \right) \left( r_{IDX, i, t} - r_{CDS, i, t} \right), \tag{D.1} \]

where \( w_{CVW, i, t} = \frac{1}{\sigma_{i,t}} \cdot \frac{1}{\sum_{j=1}^{n_t} 1/\sigma_{j,t}} \) and \( \sigma_{i,t}^2 \) is an estimate of annualized conditional variance of \( r_{IDX, i, t} - r_{CDS, i, t} \) that is obtained from daily returns as in Equation (1) of Moskowitz et al. (2012). Because we use the first six-month period to estimate the conditional volatilities for the computation of the alternative liquidity factor’s first observation, its time series consists of 252 weekly observations from March 28, 2007, to February 1, 2012. The alternative liquidity factor has a correlation of 0.97 with the benchmark liquidity factor indicating that our index-level-based weighting scheme is effectively a weighting by conditional volatilities.

Corporate bond market illiquidity factor. We use transaction data from the Financial Industry Regulatory Authority’s Trade Reporting and Compliance Engine (TRACE) to construct the corporate bond market illiquidity factor. In particular, we obtain transaction data for plain-vanilla fixed-rate bullet bonds issued by U.S. corporations. The data are filtered for erroneous transactions using Dick-Nielsen’s (2009) methodology and, as in Dick-Nielsen et al. (2012), transactions with par volume below 100,000 USD are discarded. Bond-specific Amihud (2002) illiquidity measures are obtained each day by averaging absolute returns of consecutive transactions per million dollar of par volume traded. These are converted to a weekly frequency by taking the within-week median of daily measures. Each week the
market-wide measure is obtained as the weighted average (by amount issued) of bond-specific measures. The corporate bond market illiquidity factor is then given as the residual of an AR(2) specification of the market-wide illiquidity measure. When converting bond-specific Amihud (2002) illiquidity measures to a monthly rather than a weekly frequency, the resulting corporate bond illiquidity measure has a correlation of 0.93 in levels and 0.79 in first differences with Dick-Nielsen et al.’s (2012) λ.

Stock market illiquidity factor. To construct the stock market illiquidity factor, we obtain price, return, and volume data for NYSE- and AMEX-traded ordinary common shares of U.S. companies from the Center of Research in Security Prices’ daily stock file. Individual-stock Amihud (2002) illiquidity measures are given as weekly averages of absolute one-day returns per million dollar of daily trading volume. By construction the individual-stock measures are very noisy and outliers may have a nonnegligible impact when aggregating them into a market-wide illiquidity measure. We, therefore, follow Korajczyk and Sadka (2008) and “Winsorize” the individual-stock measures for a given week at the 1st and 99th percentiles of their distribution. Each week the market-wide illiquidity measure is obtained as the cross-sectional mean of “Winsorized” individual-stock measures, and the stock market illiquidity factor is the residual of an AR(2) specification of the market-wide measure.

Balance sheet constraints of financial intermediaries. The leverage factor mimicking portfolio is a tradable stock portfolio that is maximally correlated with Adrian et al.’s (2013) nontradable leverage factor (the seasonally adjusted log change in the leverage ratio of the aggregate broker-dealer balance sheet). The mimicking portfolio is composed of Fama and French (1993) portfolios (two-way sort on size and book-to-market into six portfolios) and the momentum factor. We construct the leverage factor mimicking portfolio as the weighted average of one-week excess returns on the components, using the weights provided in Adrian
et al. (2013).

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Tang, Dragon Yongjun, and Hong Yan, 2007, Liquidity and credit default swap spreads, Working paper, University of Hong Kong.
Figure 1: Credit Index Levels, CDS-Implied Index Levels, and Index-to-CDS Bases.
The figure displays daily observations of credit index levels of the five-year on-the-run series
(thin black lines, left hand scales), CDS-implied index levels (thick gray lines, left hand
scales), and index-to-CDS bases (light gray shaded areas, right hand scales) from September
20, 2006, to February 1, 2012. Index levels and bases are in basis points and dashed vertical
lines correspond to index roll dates.
Figure 2: CDS Market Illiquidity Measure.
The figure displays the CDS market illiquidity measure (in basis points). The time series consists of 281 weekly observations from September 20, 2006, to February 1, 2012. Dotted vertical lines correspond to (from left to right) the collapse of two Bear Stearns structured credit hedge funds on June 20, 2007, the Bear Stearns near-bankruptcy on March 17, 2008, and the default of Lehman Brothers on September 15, 2008.
Figure 3: Liquidity Factor.
The figure displays one-week excess returns (in %) on the tradable liquidity factor. The time series consists of 279 weekly observations from October 4, 2006, to February 1, 2012. Dotted vertical lines correspond to (from left to right) the collapse of two Bear Stearns structured credit hedge funds on June 20, 2007, the Bear Stearns near-bankruptcy on March 17, 2008, and the default of Lehman Brothers on September 15, 2008.
Figure 4: Model-Implied Expected Excess Returns Vs. Expected Excess Returns.
The figure displays a scatterplot of model-implied expected excess returns on the test portfolios (vertical axis in % per year) vs. expected excess returns on the test portfolios (horizontal axis in % per year). The benchmark model specification is used to infer model-implied expected excess returns. The 45-degree line is shown in light gray.
Figure 5: CDS Spread Decomposition.
The figure displays a model-implied decomposition of five-year CDS spreads (in % per year) at the test portfolio level for the bid-ask-spread-sorted portfolios. CDS spreads are decomposed into expected default losses, factor risk premia, and pricing errors implied by the benchmark model specification. The horizontal axis displays portfolio identifiers.
### Panel A: Credit Index Levels

<table>
<thead>
<tr>
<th></th>
<th>CDX North American</th>
<th>iTraxx Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IG</td>
<td>IG.HVOL</td>
</tr>
<tr>
<td>Mean</td>
<td>106.48</td>
<td>221.62</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>49.66</td>
<td>125.87</td>
</tr>
<tr>
<td>Minimum</td>
<td>28.88</td>
<td>67.38</td>
</tr>
<tr>
<td>Maximum</td>
<td>279.67</td>
<td>682.82</td>
</tr>
<tr>
<td>N</td>
<td>1338</td>
<td>1342</td>
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</table>

### Panel B: Index-to-CDS Bases

<table>
<thead>
<tr>
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<th>iTraxx Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IG</td>
<td>IG.HVOL</td>
</tr>
<tr>
<td>Mean</td>
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<td>-6.00</td>
</tr>
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<td>Standard Deviation</td>
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<tr>
<td>Maximum</td>
<td>12.17</td>
<td>29.47</td>
</tr>
<tr>
<td>Corr($C_t^{IDX}$, $\sigma_t(</td>
<td>B</td>
<td>)$)</td>
</tr>
</tbody>
</table>

Table 1: Descriptive Statistics.

The table displays descriptive statistics of credit index levels and index-to-CDS bases. Panel A provides descriptive statistics of credit index levels of the five-year on-the-run series. Panel B provides descriptive statistics of the corresponding index-to-CDS bases. Mean, standard deviation, minimum, and maximum are in basis points, and Corr($C_t^{IDX}$, $\sigma_t(|B|)$) denotes the time-series correlation between the index level and the conditional volatility of the index-to-CDS basis’ absolute value. For each index, the conditional volatility is inferred from a GARCH(1,1) model for the conditional variance of the error term in an ARMA(1,1) specification of the absolute value of the index-to-CDS basis. Missing observations are neglected in the computation of descriptive statistics and time-series correlations. The sample period is from September 20, 2006, to February 1, 2012. $N$ denotes the number of daily observations.
Panel A: Return Descriptive Statistics

<table>
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<tr>
<th>CDX North American</th>
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<th>HY.BB</th>
<th>HY.B</th>
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<tbody>
<tr>
<td>Mean</td>
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<td>8.48</td>
<td>12.78</td>
<td>11.42</td>
<td>13.32</td>
</tr>
<tr>
<td>Standard Deviation</td>
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<td>35.93</td>
<td>73.11</td>
<td>95.07</td>
<td>71.68</td>
</tr>
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<td>1.48</td>
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<td>1.85</td>
</tr>
<tr>
<td>Skewness</td>
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<td>0.26</td>
<td>1.56</td>
<td>1.43</td>
</tr>
<tr>
<td>Kurtosis</td>
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<td>9.14</td>
<td>5.99</td>
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<td>8.01</td>
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<tr>
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<td>0.01</td>
<td>-0.06</td>
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<td>N</td>
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<td>267</td>
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<table>
<thead>
<tr>
<th>iTraxx Europe</th>
<th>Main</th>
<th>HiVol</th>
<th>Sr Finls</th>
<th>Sub Finls</th>
<th>Xover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
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<td>4.56</td>
<td>3.13</td>
<td>10.17</td>
<td>12.31</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>16.30</td>
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<td>18.18</td>
<td>33.22</td>
<td>44.71</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
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<td>1.61</td>
<td>1.22</td>
<td>1.15</td>
</tr>
<tr>
<td>Skewness</td>
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<tr>
<td>N</td>
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Panel B: Pairwise Correlations

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<th>HY.B</th>
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<td>IG</td>
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<td>0.14</td>
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</tr>
<tr>
<td>IG.HVOL</td>
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<td>0.04</td>
<td>0.08</td>
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<td></td>
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<tr>
<td>HY</td>
<td>0.45</td>
<td>0.60</td>
<td></td>
<td></td>
<td></td>
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<td>HY.BB</td>
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<th>iTraxx Europe</th>
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<th>HiVol</th>
<th>Sr Finls</th>
<th>Sub Finls</th>
<th>Xover</th>
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</thead>
<tbody>
<tr>
<td>Main</td>
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<td>0.27</td>
<td>0.08</td>
<td>0.28</td>
<td></td>
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</tr>
<tr>
<td>Sr Finls</td>
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<td>0.00</td>
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<td></td>
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</tr>
<tr>
<td>Sub Finls</td>
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Table 2: Descriptive Statistics of Index Arbitrage Returns.

The table displays descriptive statistics of one-week excess returns on the trading strategies underlying the construction of the tradable liquidity factor. Mean and standard deviation are in basis points per week, the Sharpe ratio is annualized using Lo’s (2002) correction for non-i.i.d. excess returns, and $\rho_1$ denotes first-order autocorrelation. Missing observations are neglected in the computation of descriptive statistics. The sample period is from October 4, 2006, to February 1, 2012. N denotes the number of weekly observations.
<table>
<thead>
<tr>
<th>Panel A: CDS Market Illiquidity Measures</th>
<th>Panel B: Bond Market Illiquidity Measures</th>
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<tbody>
<tr>
<td>Intercept</td>
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<td>0.05</td>
<td>0.07</td>
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<td>[0.07]</td>
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<td>∆Bid-Ask</td>
<td>∆Noise</td>
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<tr>
<td>1.11</td>
<td>2.36</td>
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<td>[5.64]</td>
<td>[7.30]</td>
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<tr>
<td>∆ILLIQ&lt;sub&gt;CDS&lt;/sub&gt;</td>
<td>∆λ</td>
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<tr>
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<tr>
<td>∆ILLIQ&lt;sub IDX&lt;/sub&gt;</td>
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<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
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<tr>
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<td>0.45</td>
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<tr>
<td>0.06</td>
<td>0.04</td>
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<tr>
<td>0.17</td>
<td>0.31</td>
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<tr>
<td>0.45</td>
<td>0.48</td>
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<table>
<thead>
<tr>
<th>Panel C: Capital Cost and Supply Measures</th>
<th>Panel D: Market Conditions</th>
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<tr>
<td>Intercept</td>
<td>Intercept</td>
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<tr>
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<td>[0.21]</td>
</tr>
<tr>
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<td>0.00</td>
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<tr>
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<td>[-0.00]</td>
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<td>∆SPX</td>
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<td>[1.80]</td>
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<td>[1.80]</td>
<td>[-1.45]</td>
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<tr>
<td>∆Repo</td>
<td>∆Default</td>
</tr>
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<td>13.44</td>
</tr>
<tr>
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<td>[3.10]</td>
</tr>
<tr>
<td>-3.03</td>
<td>11.66</td>
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<td>R&lt;sup&gt;HFRX&lt;/sup&gt;</td>
<td>∆VIX</td>
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<tr>
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<td>0.12</td>
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<tr>
<td>[-2.44]</td>
<td>[0.76]</td>
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<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>0.12</td>
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<tr>
<td>0.07</td>
<td>0.37</td>
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<tr>
<td>0.26</td>
<td>0.18</td>
</tr>
<tr>
<td>0.26</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 3: Time-Series Properties of CDS Market Illiquidity Measure.
The table displays results from regressing monthly changes in the CDS market illiquidity measure on monthly changes in
the average bid-ask spread of single-name CDSs (Bid-Ask), the average absolute spread change per quote contributed
across single-name CDSs (ILLIQ<sub>CDS</sub>), the average absolute change in the index level per quote contributed across on-
the-run credit indices (ILLIQ<sub>IDX</sub>), the Hu, Pan, and Wang (2013) “Noise” measure (Noise), Dick-Nielsen, Feldhütter,
and Lando’s (2012) λ corporate bond illiquidity measure, the average CDS-bond basis across U.S. investment grade bonds
(CDS-Bond), the spread between three-month LIBOR and OIS rates (LIB-OIS), the spread between three-month Agency
MBS and Treasury general collateral repo rates (Repo), the one-month return on the Hedge Fund Research Global Index
(R<sup>HFRX</sup>), because R<sup>HFRX</sup> is a return, its level is the explanatory variable in the regressions), the one-month return on the
S&amp;P 500 index (R<sup>SPX</sup>), because R<sup>SPX</sup> is a return, its level is the explanatory variable in the regressions), the yield spread
between Baa- and Aaa-rated bonds (Default), and the VIX index (VIX). Reported are intercepts and slope coefficients, their
respective t-statistics in brackets, and adjusted R<sup>2</sup>s. t-statistics are based on Newey and West (1987) heteroscedasticity
and autocorrelation consistent standard errors with three lags. Bid-Ask, ILLIQ<sub>CDS</sub>, ILLIQ<sub>IDX</sub>, Noise, and the CDS market
illiquidity measure are in basis points. CDS-Bond, LIB-OIS, Repo, R<sup>HFRX</sup>, R<sup>SPX</sup>, Default, and VIX are in %. λ is in index
points. The time series consist of 64 monthly observations from October 2006 to January 2012.
Table 4: Correlations Between Explanatory Variables.
The table displays the correlation of monthly changes in the explanatory variables of the time-series properties regressions. The explanatory variables are: the average bid-ask spread of single-name CDSs (Bid-Ask), the average absolute spread change per quote contributed across single-name CDSs (ILLIQ\textsuperscript{CDS}), the average absolute change in the index level per quote contributed across on-the-run credit indices (ILLIQ\textsuperscript{IDX}), the Hu, Pan, and Wang (2013) “Noise” measure (Noise), Dick-Nielsen, Feldhüter, and Lando’s (2012) $\lambda$ corporate bond illiquidity measure, the average CDS-bond basis across U.S. investment grade bonds (CDS-Bond), the spread between three-month LIBOR and OIS rates (LIB-OIS), the spread between three-month Agency MBS and Treasury general collateral repo rates (Repo), the one-month return on the Hedge Fund Research Global Index ($R_{\text{HFRX}}$; because $R_{\text{HFRX}}$ is a return, correlations with its level are reported), the one-month return on the S&P 500 index ($R_{\text{SPX}}$; because $R_{\text{SPX}}$ is a return, correlations with its level are reported), the yield spread between Baa- and Aaa-rated bonds (Default), and the VIX index (VIX). Bid-Ask, ILLIQ\textsuperscript{CDS}, ILLIQ\textsuperscript{IDX}, Noise, and the CDS market illiquidity measure are in basis points. CDS-Bond, LIB-OIS, Repo, $R_{\text{HFRX}}$, $R_{\text{SPX}}$, Default, and VIX are in %. $\lambda$ is in index points. The time series consist of 64 monthly observations from October 2006 to January 2012.

| ΔBid-Ask  | 0.52 | 0.40 | 0.72 | 0.16 | -0.64 | 0.27 | 0.38 | -0.62 | -0.48 | 0.77 | 0.57 |
| ΔILLIQ\textsuperscript{CDS} | 0.51 | 0.28 | 0.34 | -0.45 | 0.24 | 0.11 | 0.11 | -0.42 | -0.30 | 0.38 | 0.41 |
| ΔILLIQ\textsuperscript{IDX} | 0.21 | 0.64 | -0.26 | 0.33 | 0.25 | 0.52 | 0.36 | 0.25 | 0.25 | 0.61 |
| ΔNoise | 0.15 | -0.63 | 0.31 | 0.50 | 0.60 | 0.46 | 0.65 | 0.54 |
| Δ$\lambda$ | -0.17 | 0.49 | 0.34 | -0.55 | -0.30 | 0.18 | 0.53 |
| ΔCDS-Bond | -0.54 | -0.46 | 0.55 | 0.25 | -0.80 | 0.49 | 0.49 |
| ΔLIB-OIS | 0.75 | -0.49 | -0.22 | 0.44 | 0.61 |
| ΔRepo | -0.51 | -0.23 | 0.41 | 0.68 |
| $R_{\text{HFRX}}$ | 0.72 | -0.61 | 0.76 |
| $R_{\text{SPX}}$ | -0.35 | -0.62 |
| ΔDefault | 0.52 |
Table 5: Descriptive Statistics of Factors.
The table displays descriptive statistics for the default and liquidity factors. Mean and standard deviation are in basis points per week, the Sharpe ratio is annualized using Lo’s (2002) correction for non-i.i.d. excess returns, and $\rho_1$ denotes first-order autocorrelation. Factor time series consist of 276 weekly observations from October 11, 2006, to February 1, 2012.
<table>
<thead>
<tr>
<th>Rating</th>
<th>Bid-Ask Spread</th>
<th>Realized Excess Returns</th>
<th>Bid-Ask Spread</th>
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<tbody>
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<td></td>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
</tr>
<tr>
<td>AAA–AA</td>
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<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>[6.75]</td>
<td>[7.54]</td>
<td>[6.22]</td>
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<tr>
<td>A</td>
<td>0.38</td>
<td>0.44</td>
<td>0.53</td>
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<tr>
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<td>[7.70]</td>
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<td>[5.81]</td>
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<td>[6.42]</td>
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<td>1.75</td>
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<td></td>
<td>[9.89]</td>
<td>[11.14]</td>
<td>[6.92]</td>
</tr>
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</table>

Table 6: Descriptive Statistics of Bid-Ask-Spread-Sorted Portfolios.
The table displays descriptive statistics for the 20 test portfolios formed by first sorting CDS contracts according to credit ratings and then according to bid-ask spreads. The upper part of the table reports sample means of conditional expected excess returns (in % per year) and realized excess returns (in % per year). In brackets are $t$-statistics based on Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors with 24 lags. The lower part of the table reports sample means of average five-year CDS spreads across portfolio constituents (in % per year) and standard deviations of realized excess returns (in % per year). Portfolio time series consist of 276 weekly observations from October 11, 2006, to February 1, 2012.
### Panel A: Bid-Ask-Spread-Sorted Portfolios

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<th>LIQ</th>
</tr>
</thead>
<tbody>
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<td>Bid-Ask Spread</td>
<td>Bid-Ask Spread</td>
</tr>
<tr>
<td></td>
<td>Q1</td>
<td>Q2</td>
</tr>
<tr>
<td>AAA–AA</td>
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<td>19.74</td>
</tr>
<tr>
<td></td>
<td>[14.24]</td>
<td>[9.69]</td>
</tr>
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<td>A</td>
<td>17.02</td>
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<td>[10.68]</td>
<td>[18.74]</td>
</tr>
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<td>27.69</td>
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<tr>
<td></td>
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<td>[20.62]</td>
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<td>61.67</td>
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<td></td>
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<td>[11.42]</td>
</tr>
<tr>
<td>B–CCC</td>
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<tr>
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</tr>
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</table>

### Panel B: Price-Impact-Sorted Portfolios

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<th>LIQ</th>
</tr>
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<td></td>
<td>[13.26]</td>
<td>[16.90]</td>
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<tr>
<td></td>
<td>[8.14]</td>
<td>[11.42]</td>
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</table>

Table 7: Results of Time-Series Regressions.
The table displays first-step regression results at the level of individual test portfolios. Reported are beta estimates times the standard deviation of the respective factors; i.e., the weekly realized portfolio excess returns (in basis points) in response to a one standard deviation shock to the factors. In brackets are $t$-statistics based on Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors with 24 lags. Time series consist of 276 weekly observations from October 11, 2006, to February 1, 2012.
Table 8: Results of Cross-Sectional Regressions.

The table displays results of several specifications of the second-step regression. Specifications of \( \hat{E}[r_{e,t}] = \lambda_0 + \hat{\beta}_{DEF}\lambda_{DEF} + \hat{\beta}_{LIQ}\lambda_{LIQ} + u_i \) are estimated from expected excess returns and beta estimates inferred from time series that consist of 276 weekly observations from October 11, 2006, to February 1, 2012. Reported are factor price of risk estimates (in basis points), \( t \)-statistics based on asymptotic generalized method of moments standard errors that account for error-in-variables problems (in parenthesis), \( t \)-statistics based on Kan, Robotti, and Shanken’s (2013) asymptotic standard errors that account for error-in-variables problems and potential model misspecification (in brackets), cross-sectional \( R^2 \)s, and their 95% confidence intervals. Standard errors are heteroscedasticity and autocorrelation consistent through the use of Newey and West’s (1987) method with 24 lags.

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<th>3</th>
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<td>0.49</td>
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<td></td>
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<td>(-0.42)</td>
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<tr>
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<tr>
<td></td>
<td>(4.70)</td>
<td>(8.25)</td>
<td>(3.59)</td>
<td>(4.36)</td>
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<td>[5.72]</td>
<td>[3.70]</td>
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<tr>
<td>( \lambda_{LIQ} )</td>
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<td>0.72</td>
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<tr>
<td></td>
<td>(3.86)</td>
<td>(3.49)</td>
<td>(4.16)</td>
<td>(4.16)</td>
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<tr>
<td></td>
<td>[4.19]</td>
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<td>[4.74]</td>
<td>[2.43]</td>
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<tr>
<td>( R^2 )</td>
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<td>0.97</td>
<td>0.95</td>
<td>0.92</td>
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<td>[0.76,1.00]</td>
<td>[0.94,1.00]</td>
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Table 9: Decompositions of Expected Excess Returns.
The table displays model-implied decompositions of expected excess returns for the benchmark model specification and the robustness checks. Annualized expected excess returns are decomposed into factor risk premia/contributions of characteristics. Reported are the contributions of these components (in % per year) to the difference in the expected excess return between the bid-ask-spread-sorted B–CCCQ4 and AAA–AAQ1 portfolios and, in brackets, the contributions of these components to the average expected excess return across the 40 test portfolios. Specification identifiers are given in the second row of the table.

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<th>BDD</th>
<th>CVW</th>
<th>NOISE</th>
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<td>1.55</td>
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Table 10: Robustness of Cross-Sectional Regressions.
The table displays results of a series of robustness checks. Specifications of \( \hat{E}[r_{i,t}] = \hat{\beta}_i^{\text{DEF}} \lambda_{\text{DEF}} + \hat{\beta}_i^{\text{LIQ}} \lambda_{\text{LIQ}} + \hat{\beta}_i^X \lambda_X + u_i \) (specifications 1 and 2) and \( \hat{E}[r_{i,t}] = \hat{\beta}_i^{\text{DEF}} \lambda_{\text{DEF}} + \hat{\beta}_i^{\text{LIQ}} \lambda_{\text{LIQ}} + \hat{\beta}_i^X \lambda_X + u_i \) (specifications 3–10) are estimated from expected excess returns, beta estimates, and sample means of characteristics inferred from time series that consist of 276 weekly observations from October 11, 2006, to February 1, 2012. Specification identifiers are given in the second row of the table. Reported are factor price of risk estimates (in basis points), \( t \)-statistics based on asymptotic generalized method of moments standard errors that account for error-in-variables problems (in parenthesis), \( t \)-statistics based on Kan, Robotti, and Shanken’s (2013) asymptotic standard errors that account for error-in-variables problems and potential model misspecification (in brackets), cross-sectional \( R^2 \)s, and their 95% confidence intervals. Standard errors are heteroscedasticity and autocorrelation consistent through the use of Newey and West’s (1987) method with 24 lags.
Internet Appendix to
"Liquidity Risk in Credit Default Swap Markets"

Benjamin Junge†  Anders B. Trolle‡

Content

Appendix E describes the computation of standard errors of factor price of risk estimates and cross-sectional $R^2$s under potential model misspecification. Appendix F contains additional figures and tables.

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‡École Polytechnique Fédérale de Lausanne and Swiss Finance Institute. E-mail: anders.trolle@epfl.ch
E Standard Error Computation

Standard Errors of Factor Price of Risk Estimates

We describe the standard error computation for a general $K$-dimensional vector of factors, $f_t = [f_{1,t}, \ldots, f_{K,t}]'$, and the most general case that we consider in the paper, namely the case of a cross-sectional regression with an intercept, a characteristic, and an additional univariate beta. In this case, the counterparts of Equations (4) and (5) in vector notation are

$$r_t = \alpha + \beta f_t + \epsilon_t,$$

(E.1)

and

$$\mu_\xi = 1_N \lambda_0 + \mu_c \lambda_c + \beta \lambda + \beta_k^* \lambda_k = X \gamma,$$

(E.2)

where $r_t = [r_{1,t}^e, \ldots, r_{N,t}^e]'$ is the $N$-dimensional vector of realized excess returns, $\alpha$ denotes the $N$-dimensional vector of regression intercepts, $\beta$ denotes the $N \times K$ matrix of factor betas, $\epsilon_t = [\epsilon_{1,t}, \ldots, \epsilon_{N,t}]'$ is the $N$-dimensional vector of mean zero error terms, $\mu_\xi$ denotes the mean of the $N$-dimensional vector of conditional expected excess returns, $\xi_t = [\xi_{1,t}, \ldots, \xi_{N,t}]'$ with $\xi_{i,t} = \hat{E}_t[r_{i,t+1}^e]$, $\mu_c$ denotes the mean of the $N$-dimensional vector of characteristics, $c_t = [c_{1,t}, \ldots, c_{N,t}]'$, $\beta_k^*$ denotes the $N$-dimensional vector of univariate betas of $\bar{y}_t$ with respect to the $k$-th factor, $f_{k,t}$ with $1 \leq k \leq K$, $y_t$ is an $N$-dimensional vector of exogenous variables, $y_t = [y_{1,t}, \ldots, y_{N,t}]'$, $\bar{y}_t = y_t - \beta_{yg}^* g_t$ is the $N$-dimensional vector of exogenous variables orthogonalized with respect to an additional factor $g_t$, the $N \times (K+3)$ matrix $X$ and the $(K+3)$-dimensional vector $\gamma$ are defined by $X = [1_N, \mu_c, \beta, \beta_k^*]$ and $\gamma = [\lambda_0, \lambda_c, \lambda', \lambda_k]'$, respectively, and $1_N$ denotes an $N$-dimensional vector of ones. Note that in contrast to the standard two-pass cross-sectional regression method, there is a distinction between expected excess returns, $\mu_\xi$, and the mean of realized excess returns, $\mu_r$.

Moreover, we define the $d = (K + 1 + 4N)$-dimensional vector $Y_t = [f_t', g_t, r_t', c_t', y_t', \xi_t']$
and denote its mean and covariance matrix by \( \mu = [\mu'_f, \mu_g, \mu'_r, \mu'_c, \mu'_y, \mu'_\xi]' \) and \( V \), respectively. In what follows, we will use the following convenient partition of \( V \),

\[
V = \begin{bmatrix}
V_f & V'_{gf} & V'_{rf} & V'_{cf} & V'_{yf} & V'_{\xi f} \\
V_{gf} & V_g & V'_{rg} & V'_{cg} & V'_{yg} & V'_{\xi g} \\
V_{rf} & V_{rg} & V_r & V'_{cr} & V'_{yr} & V'_{\xi r} \\
V_{cf} & V_{cg} & V_{cr} & V_c & V'_{yc} & V'_{\xi c} \\
V_{yf} & V_{yg} & V_{yr} & V_{yc} & V_y & V'_{\xi y} \\
V_{\xi f} & V_{\xi g} & V_{\xi r} & V_{\xi c} & V_{\xi y} & V_{\xi}
\end{bmatrix},
\]

and express factor betas and univariate betas in terms of the elements of \( V \). The matrix of factor betas is given by \( \beta = V_{rf} V_{f}^{-1} \) and the vector of univariate betas of \( y_t \) with respect to the \( k \)-th factor is given by

\[
\beta^*_k = \beta^*_{yf} f_k = (\beta^*_{yf} - \beta^*_{yg} \beta^*_{gf}) f_k = V_{gf} D^{-1} f_k - V_{yg} V_{g}^{-1} V_{gf} D^{-1} f_k,
\]

where \( D = \text{diag}(V_f) \), \( \beta^*_{yf} = V_{gf} D^{-1} \) denotes the \( N \times K \) matrix of univariate betas of \( y_t \) with respect to \( f_t \), \( \beta^*_{yg} = V_{yg} V_{g}^{-1} \) denotes the \( N \)-dimensional vector of univariate betas of \( y_t \) with respect to \( g_t \), \( \beta^*_{gf} = V_{gf} D^{-1} \) denotes the \( 1 \times K \) matrix of univariate betas of \( g_t \) with respect to \( f_t \), and \( f_k \) denotes the \( K \)-dimensional unit vector whose \( k \)-th element is nonzero. As in Kan, Robotti, and Shanken (2013), we assume that \( Y_t \) is stationary and ergodic with finite fourth moment.

Under a potentially misspecified model, there is no \( \gamma \) such that Equation (E.2) is satisfied and \( \gamma \) is chosen to minimize the sum of squared population pricing errors, \( e = \mu_\xi - X\gamma \); that is,

\[
\gamma = \underset{\delta}{\text{argmin}} (\mu_\xi - X\delta)'(\mu_\xi - X\delta) = (X'X)^{-1} X'\mu_\xi.
\]
Note that with $e$ defined as above, $\gamma$ satisfies the first-order conditions

$$X'e = 0_{K+3} \iff 1_N' e = 0, \quad \mu' e = 0, \quad \beta' e = 0_K, \quad \text{and} \quad \beta^* e = 0, \quad (E.5)$$

where $0_m$ denotes an $m$-dimensional vector of zeros. From the final expression in Equation (E.4) an estimate of $\gamma$ can be obtained by replacing population moments with their sample counterparts; that is,

$$\hat{\gamma} = (\hat{X}' \hat{X})^{-1} \hat{X}' \hat{\mu}_\xi, \quad (E.6)$$

where $\hat{X} = [1_N, \hat{\mu}_c, \hat{\beta}, \hat{\beta}_k], \hat{\beta}$ and $\hat{\beta}_k^*$ are given by $\hat{V}_{f} \hat{V}_{f}^{-1}$ and $\hat{V}_{yf} \hat{D}^{-1} t_k - \hat{V}_{gg}^{-1} \hat{V}_{gf} \hat{D}^{-1} t_k$, respectively, and $\hat{\mu}_s$ and $\hat{V}_s$ are the corresponding elements of

$$\hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} Y_t, \quad (E.7)$$

and

$$\hat{V} = \frac{1}{T} \sum_{t=1}^{T} (Y_t - \hat{\mu})(Y_t - \hat{\mu})', \quad (E.8)$$

respectively.

Note that $\hat{\theta} = [\hat{\mu}', \text{vec}(\hat{V})']'$ is the method of moments estimator of $\theta = [\mu', \text{vec}(V)']'$. Under the above assumptions,$^1$

$$\sqrt{T}(\hat{\theta} - \theta) \overset{d}{\to} N(0_{d(1+d)}, S_0), \quad (E.9)$$

where $S_0 = \sum_{j=-\infty}^{\infty} E[\psi(Y_t; \theta)\psi(Y_{t+j}; \theta)']$ and $\psi(Y_t; \theta)$ is the moment function,

$$\psi(Y_t; \theta) = [(Y_t - \mu)', \text{vec}((Y_t - \mu)(Y_t - \mu)' - V)']'. \quad (E.10)$$

$^1$As noted by Kan et al. (2013), $S_0$ is a singular matrix. This is due to the fact that $\hat{V}$ is symmetric, i.e., it contains linearly dependent elements. One could alternatively consider the parameter vector $\theta = [\mu', \text{vech}(V)']'$, in which case the covariance matrix of the limiting normal distribution would be nonsingular.
Since \( \gamma \) is a smooth function of \( \theta \), an application of the delta method yields

\[
\sqrt{T}(\hat{\gamma} - \gamma) \xrightarrow{d} \ N(0_{K+3}, (\partial \gamma / \partial \theta')S_0(\partial \gamma / \partial \theta')).
\]  (E.11)

Using the expression for \( S_0 \) from above, the asymptotic covariance matrix of \( \hat{\gamma} \), i.e., \( (\partial \gamma / \partial \theta')'S_0(\partial \gamma / \partial \theta') \), becomes

\[
\sum_{j=-\infty}^{\infty} E[h_t h_{t+j}],
\]

with

\[
h_t = (\partial \gamma / \partial \theta')\psi(Y_t; \theta).
\]  (E.12)

In order to find an explicit expression for \( h_t \) it remains to compute \( \partial \gamma / \partial \theta' \). Using the above partition of \( \theta \), we have

\[
\frac{\partial \gamma}{\partial \theta} = \left[ \frac{\partial \gamma}{\partial \mu'}, \frac{\partial \gamma}{\partial \vec{\beta}} \right]' = \left[ \frac{\partial \gamma}{\partial \mu'}(Y_t - \mu) + \frac{\partial \gamma}{\partial \vec{\beta}}(Y_t - \mu)' - V \right].
\]  (E.13)

With \( H = (X'X)^{-1} \) and \( A = HX' \), the Jacobian matrices \( \partial \gamma / \partial \mu' \) and \( \partial \gamma / \partial \vec{\beta}' \) are given by

\[
\frac{\partial \gamma}{\partial \mu'} = \left[ 0_{(K+3)\times(K+1+N)}, \frac{\partial \gamma}{\partial \vec{X}} \frac{\partial \vec{X}}{\partial \mu'_c}, 0_{(K+3)\times N}, A \right],
\]  (E.14)

and

\[
\frac{\partial \gamma}{\partial \vec{\beta}'} = (H \otimes e') - (\gamma' \otimes A).
\]  (E.15)

Now, note that \( \vec{\beta} = \left[ 1_N', \mu'_c, \text{vec}(\beta)', \beta^{\star'}_k \right]' \). Thus,

\[
\frac{\partial \vec{\beta}'}{\partial \mu'_c} = \left[ 0_N \times N, I_N, 0_{N \times N \cdot (K+1)} \right]' = ([0, 1, 0_{K+1}']' \otimes I_N),
\]  (E.16)
where $I_N$ denotes the $N$-dimensional identity matrix, and, consequently,

$$
\frac{\partial \gamma}{\partial \mu}(Y_t - \mu) = A(\xi_t - \mu) - A(c_t - \mu_c)\lambda_c + H[0, \ c'_t, \ 0'_{K+1}]'.
$$

(E.17)

Similarly,

$$
\frac{\partial \text{vec}(X)}{\partial \text{vec}(V)'} = \left[0_{d^2 \times 2N}, \left(\frac{\partial \text{vec}(\beta)}{\partial \text{vec}(V)'}\right)', \left(\frac{\partial \beta^*_k}{\partial \text{vec}(V)'}\right)\right]'.
$$

(E.18)

For the remaining expressions, we get

$$
\frac{\partial \text{vec}(\beta)}{\partial \text{vec}(V)'} = (V_f^{-1} \otimes I_N)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'} - (V_f^{-1} \otimes \beta)\frac{\partial \text{vec}(V_f)}{\partial \text{vec}(V)'}
$$

(E.19)

and

$$
\frac{\partial \beta^*_k}{\partial \text{vec}(V)'} = (t_k' D^{-1} \otimes I_N)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'} - (t_k' \beta^*_{gf}' V_g^{-1} \otimes I_N)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'}
$$

$$
+ (t_k' \beta^*_{gf} V_g^{-1} \otimes \beta_{yg}^*)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'} - (t_k' D^{-1} \otimes \beta_{yg}^*)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'}
$$

(E.20)

$$
- (t_k' D^{-1} \otimes \beta_{gf}^*)\Theta\frac{\partial \text{vec}(V_f)}{\partial \text{vec}(V)'}
$$

where $\Theta$ is a $K^2 \times K^2$ matrix such that $\text{vec}(D) = \Theta \text{vec}(V_f)$. Thus, the Jacobian matrix $\partial \text{vec}(X)/\partial \text{vec}(V)'$ can be expressed as the following sum of Kronecker products

$$
\frac{\partial \text{vec}(X)}{\partial \text{vec}(V)'} = ([0_{K \times 2}, \ V_f^{-1}, \ 0_K]' \otimes I_N)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'} - ([0_{K \times 2}, \ V_f^{-1}, \ 0_K]' \otimes \beta)\frac{\partial \text{vec}(V_f)}{\partial \text{vec}(V)'}
$$

$$
+ ([0_{K \times (K+2)}, \ D^{-1} t_k]' \otimes I_N)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'} - ([0_{K \times 2}, \ V_f^{-1} \beta_{gf} t_k]' \otimes I_N)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'}
$$

$$
+ ([0_{K \times 2}, \ V_g^{-1} \beta_{gf}^* t_k]' \otimes \beta_{yg}^*)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'} - ([0_{K \times (K+2)}, \ D^{-1} t_k]' \otimes \beta_{yg}^*)\frac{\partial \text{vec}(V_g)}{\partial \text{vec}(V)'}
$$

$$
- ([0_{K \times (K+2)}, \ D^{-1} t_k]' \otimes \beta_{gf}^*)\Theta\frac{\partial \text{vec}(V_f)}{\partial \text{vec}(V)'}.
$$

(E.21)
Moreover,

\[
\frac{\partial \text{vec}(V_{rf})}{\partial \text{vec}(V)''} = ([I_K, 0_{K \times (4N+1)}] \otimes [0_{N \times (K+1)}, I_N, 0_{N \times 3N}]),
\]

(E.22)

\[
\frac{\partial \text{vec}(V_f)}{\partial \text{vec}(V)'} = ([I_K, 0_{K \times (4N+1)}] \otimes [I_K, 0_{K \times (4N+1)}]),
\]

(E.23)

\[
\frac{\partial \text{vec}(V_{gf})}{\partial \text{vec}(V)'} = ([I_K, 0_{K \times (4N+1)}] \otimes [0_{N \times (K+2N)}, I_N, 0_{N \times N}]),
\]

(E.24)

\[
\frac{\partial V_{gg}}{\partial \text{vec}(V)'} = ([0_{K}, 1, 0'_{4N}] \otimes [0_{N \times (K+1+2N)}, I_N, 0_{N \times N}]),
\]

(E.25)

\[
\frac{\partial V_g}{\partial \text{vec}(V)'} = ([0_{K}, 1, 0'_{4N}] \otimes [0'_{K}, 1, 0'_{4N}]),
\]

(E.26)

\[
\frac{\partial \text{vec}(V_{ff})}{\partial \text{vec}(V)'} = ([I_K, 0_{K \times (4N+1)}] \otimes [0'_{K}, 1, 0'_{4N}]).
\]

(E.27)

Substituting the expressions in Equations (E.15) and (E.21) into Equation (E.14), and using Equations (E.22)–(E.27) as well as the first-order conditions yields

\[
\frac{\partial \gamma}{\partial \text{vec}(V)'} \text{vec}((Y_t - \mu)(Y_t - \mu)' - V) = H \tilde{z}_t
\]

\[
+ A \{ \beta(f_t - \mu_f)(f_t - \mu_f)' - (r_t - \mu_r)(f_t - \mu_f)' \} V_f^{-1} \lambda
\]

\[
+ A \{ \beta_{gf}^* D_t - (y_t - \mu_y)(f_t - \mu_f)' + \beta_{gg}^*(g_t - \mu_g)(f_t - \mu_f)' \} D^{-1} t_k \lambda_k
\]

\[
- A \{ \beta_{gg}^*(g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g) \} V_g^{-1} \beta_{gf}^* t_k \lambda_k,
\]

where

\[
\tilde{z}_t = [0'_{2}, u_t(f_t - \mu_f)' V_f^{-1}, v_t(f_t - \mu_f)' D^{-1} t_k - \epsilon' \beta_{gg}^*(g_t - \mu_g)(f_t - \mu_f)' D^{-1} t_k \ldots
\]

\[
+ \epsilon' \{ \beta_{gg}^*(g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g) \} V_g^{-1} \beta_{gf}^* t_k \lambda_k - \epsilon' \beta_{gf}^* D_t D^{-1} t_k)' ,
\]

\[
u_t = \epsilon'(r_t - \mu_r), \quad v_t = \epsilon'(y_t - \mu_y), \quad \text{and} \quad D_t = \text{diag}((f_t - \mu_f)(f_t - \mu_f)').
\]

Finally, adding up the terms in Equations (E.17) and (E.28), \( h_t \) can be explicitly expressed
\[
\begin{align*}
    h_t &= (\gamma_t - \gamma) - A(c_t - \mu_c)\lambda_c + A\{\beta(f_t - \mu_f)(f_t - \mu_f)' - (r_t - \mu_r)(f_t - \mu_f)\}' V_f^{-1} \lambda \\
    &+ A\{\beta^*_{gf} D_t - (y_t - \mu_y)(f_t - \mu_f)\} D^{-1} t_k \lambda_k \tag{E.30} \\
    &- A\{\beta^*_{gg} (g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g)\} V_g^{-1} \beta^*_{gf} t_k \lambda_k \\
    &- A\beta^*_{gg} \{\beta^*_{gf} D_t - (g_t - \mu_g)(f_t - \mu_f)'\} D^{-1} t_k \lambda_k + Hz_t,
\end{align*}
\]

where \(\gamma_t = A\xi_t\) and

\[
    z_t = [0, c'e', u_t(f_t - \mu_f)'V_f^{-1}, v_t(f_t - \mu_f)'D^{-1} t_k - e'\beta^*_{gg} (g_t - \mu_g)(f_t - \mu_f)'D^{-1} t_k \ldots \\
    + e'\beta^*_{gg} (g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g)\} V_g^{-1} \beta^*_{gf} t_k - e'\beta^*_{gf} D_t D^{-1} t_k]' \tag{E.31}
\]

Applying the Newey and West (1987) method, a heteroscedasticity and autocorrelation consistent estimator for the asymptotic covariance matrix of \(\hat{\gamma}\) is given by

\[
    \frac{1}{T} \sum_{t=1}^{T} \hat{h}_t \hat{h}_t' + \frac{1}{T} \sum_{l=1}^{m} \sum_{t=l+1}^{T} \left(1 - \frac{l}{m+1}\right) \left(\hat{h}_t \hat{h}_t'_{t-l} + \hat{h}_{t-l} \hat{h}_t'\right), \tag{E.32}
\]

where \(\hat{h}_t\) is given by Equation (E.30) with population parameters replaced by their sample estimates. In particular, \(\hat{e} = \hat{\mu}_\xi - \hat{X}\hat{\gamma}\). The finite sample approximation of \(\hat{\gamma}\)'s covariance matrix is then obtained as \(1/T\) times the estimate of the asymptotic covariance matrix.

Based on Equation (E.30) it is straightforward to break down asymptotic variation of \(\hat{\gamma}\) into three components. The first one, \(\gamma_t - \gamma\), is variation of \(\hat{\gamma}\) in case that the model is correctly specified and estimated using population values, i.e., there is no error associated with the estimation of the characteristic, \(\mu_c\), and betas, \(\beta\) and \(\beta^*_k\). It should be noted that this is the only source of variation that is taken into account by the Fama and MacBeth (1973) method. The second source of variation are errors-in-variables (EIV). The second term of Equation (E.30) captures variation associated with the estimation of the characteristic, \(\mu_c\),
the third term of Equation (E.30) captures variation associated with the estimation of factor betas, \( \beta \), the fourth, fifth, and sixth terms of Equation (E.30) capture variation associated with the estimation of the univariate betas, \( \beta_{yf}, \beta_{yg}^*, \) and \( \beta_{gf}^* \), respectively. Variation from the first two sources is, e.g., accounted for by generalized-method-of-moments-based inference. The third source of variation is due to potential model misspecification and captured by \( H z_t \).

Note that this term vanishes when the model is correctly specified, i.e., when \( e = \mu_t - X \gamma = 0_N \). Thus, setting \( e = 0_N \) gives the asymptotic variance of \( \hat{\gamma} \) in a generalized method of moments estimation of \( \mu_c, \beta, \beta_{yg}^* \), and \( \gamma \). As mentioned above, this asymptotic variance takes into account EIV but ignores potential model misspecification.

In case that the intercept is restricted to zero, \( \gamma = [\lambda_c, \lambda', \lambda_k]' \), \( X = [\mu_c, \beta, \beta_k^*] \), and \( h_t \) is given by Equation (E.30), where

\[
{z_t} = \left[ c^t e, u_t(f_t - \mu_f)'V_f^{-1}, v_t(f_t - \mu_f)'D^{-1}t_k - e^t \beta_{yg}^*(g_t - \mu_g)(f_t - \mu_f)'D^{-1}t_k \ldots \right]_{E(33)}
\]

\[
\quad \quad \quad { + e}^t \{ \beta_{yg}^*(g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g) \} V_g^{-1} \beta_{gf}^* t_k - e^t \beta_{gf}^* D t D^{-1}t_k \}
\]

and \( A, H, \) and \( e \) are defined as above. In case that the model specification does not include the characteristic, \( \gamma = [\lambda_0, \lambda', \lambda_k]' \), \( X = [1_N, \beta, \beta_k^*] \), and \( h_t \) is given by

\[
{h_t} = (\gamma_t - \gamma) + A \{ \beta(f_t - \mu_f)(f_t - \mu_f)' - (r_t - \mu_r)(f_t - \mu_f)' \} V_f^{-1} \lambda
\]

\[
\quad \quad \quad + A \{ \beta_{gf}^* D_t - (y_t - \mu_y)(f_t - \mu_f)' \} D^{-1}t_k \lambda_k
\]

\[
\quad \quad \quad - A \{ \beta_{yg}^*(g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g) \} V_g^{-1} \beta_{gf}^* t_k \lambda_k
\]

\[
\quad \quad \quad - A \beta_{yg}^* \{ \beta_{gf}^* D_t - (g_t - \mu_g)(f_t - \mu_f)' \} D^{-1}t_k \lambda_k + H z_t,
\]

where

\[
{z_t} = \left[ 0, u_t(f_t - \mu_f)'V_f^{-1}, v_t(f_t - \mu_f)'D^{-1}t_k - e^t \beta_{yg}^*(g_t - \mu_g)(f_t - \mu_f)'D^{-1}t_k \ldots \right]_{E(35)}
\]

\[
\quad \quad \quad { + e}^t \{ \beta_{yg}^*(g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g) \} V_g^{-1} \beta_{gf}^* t_k - e^t \beta_{gf}^* D t D^{-1}t_k \}.
\]
and $A$, $H$, and $e$ are defined as above. Finally, in case that the model specification does not include the univariate beta, $\gamma = [\lambda_0, \lambda_c, \lambda]'$, $X = [1_N, \mu_c, \beta]$, and $h_t$ is given by

$$h_t = (\gamma_t - \gamma) - A(c_t - \mu_c)\lambda_c + A\{\beta(f_t - \mu_f)(f_t - \mu_f)' - (r_t - \mu_r)(f_t - \mu_f)\}' V_f^{-1} \lambda + Hz_t,$$

(E.36)

where $z_t = [0, e'c_t, u_t(f_t - \mu_f)'V_f^{-1}]'$, and $A$, $H$, and $e$ are defined as above.

**Standard Error of the Cross-Sectional $R^2$**

The standard error computation of the cross-sectional $R^2$ is based on the same principle as that of the factor price of risk estimates. Again, we derive standard errors for the most general case that we consider in the paper and we discuss less general cases at the end of this section.

Let $\rho^2$ denote the population value of the $R^2$; that is,

$$\rho^2 = 1 - \frac{Q}{Q_0} = 1 - \frac{e'e}{e_0'e_0},$$

(E.37)

where $e_0 = (I_N - (1/N)1_N1_N')\mu_\xi$ are population deviations of expected excess returns from their cross-sectional average. Replacing population values in Equation (E.37) by their sample estimates, obviously, gives the $R^2$.

Assume that $0 < \rho^2 < 1$, i.e., the model is neither correctly specified nor is it misspecified and unable to explain any cross-sectional variation in expected excess returns. As in the previous section, $\rho^2$ is a smooth function of $\theta$ and an application of the delta method yields

$$\sqrt{T}(R^2 - \rho^2) \xrightarrow{d} \text{N}(0, (\partial \rho^2/\partial \theta')S_0(\partial \rho^2/\partial \theta')'),$$

(E.38)

where $S_0$ is defined as in the previous section, $(\partial \rho^2/\partial \theta')S_0(\partial \rho^2/\partial \theta')' = \sum_{j=-\infty}^{\infty} E[\eta_t\eta_{t+j}]$, and
\( \eta_t = (\partial \rho^2 / \partial \theta') \psi(Y_t; \theta) \). Thus, it remains to compute \( \partial \rho^2 / \partial \theta' \) in order to obtain an explicit expression for \( \eta_t \). Using the above partition of \( \theta \), we have \( \partial \rho^2 / \partial \theta' = [\partial \rho^2 / \partial \mu', \partial \rho^2 / \partial \text{vec}(V)'] \) and

\[
\eta_t = \frac{\partial \rho^2}{\partial \theta'} \psi(Y_t; \theta) = \frac{\partial \rho^2}{\partial \mu'} (Y_t - \mu) + \frac{\partial \rho^2}{\partial \text{vec}(V)'} \text{vec}((Y_t - \mu)(Y_t - \mu)' - V). \quad (E.39)
\]

The Jacobian matrices \( \partial \rho^2 / \partial \mu' \) and \( \partial \rho^2 / \partial \text{vec}(V)' \) are given by

\[
\frac{\partial \rho^2}{\partial \mu'} = \begin{bmatrix} 0'_{k+1+N}, \frac{\partial \rho^2}{\partial \text{vec}(X)'} \frac{\partial \text{vec}(X)}{\partial \mu'_c}, 0'_N, \frac{2}{Q_0} \{(1 - \rho^2) e'_0 - e'\} \end{bmatrix}, \quad (E.40)
\]

and

\[
\frac{\partial \rho^2}{\partial \text{vec}(V)'} = \frac{\partial \rho^2}{\partial \text{vec}(X)'} \frac{\partial \text{vec}(X)}{\partial \text{vec}(V)'}, \quad (E.41)
\]

respectively, with

\[
\frac{\partial \rho^2}{\partial \text{vec}(X)'} = -\frac{2}{Q_0} (\gamma' \otimes e'). \quad (E.42)
\]

Replacing \( \partial \text{vec}(X) / \partial \mu'_c \) and \( \partial \text{vec}(X) / \partial \text{vec}(V) \) by the expressions derived in the previous section and making use of the first-order conditions yields

\[
\eta_t = \frac{2}{Q_0} \{(1 - \rho^2) e'_0 - e'\} \{\xi_t - \mu_x\} + e' c_t \lambda_c + u_t (f_t - \mu_f)^' V_f^{-1} \lambda
\]

\[
- e' \{\beta_{gf} D_t - (y_t - \mu_y)(f_t - \mu_f)'\} D^{-1} t_k \lambda_k
\]

\[
+ e' \{\beta_{gg} (g_t - \mu_g)^2 - (y_t - \mu_y)(g_t - \mu_g)\} V_g^{-1} \beta_{gf}^* t_k \lambda_k
\]

\[
+ e' \beta_{gg} \{\beta_{gf} D_t - (g_t - \mu_g)(f_t - \mu_f)'\} D^{-1} t_k \lambda_k\}, \quad (E.43)
\]

where, as before, \( u_t = e'(r_t - \mu_r) \) and \( D_t = \text{diag}((f_t - \mu_f)(f_t - \mu_f)') \). As in the previous section, the Newey and West (1987) method applied to \( \eta_t \)’s sample analog, \( \hat{\eta}_t \), gives a heteroscedasticity and autocorrelation consistent estimate of the asymptotic variance of the \( R^2 \).
In case that the intercept is restricted to zero, the expression of $\eta_t$ for the standard error computation does not change.\textsuperscript{2} The expressions for $\eta_t$ in case that the model specification does not include the characteristic or the univariate beta can be obtained from Equation (E.43) by setting the respective parameters, i.e., $\lambda_c$ or $\lambda_k$, equal to zero.

**F Additional Figures and Tables**

Figure 1 displays on-the-run index levels, CDS-implied index levels, and index-to-CDS bases for the sub-indices of the CDX North American and iTraxx Europe families.

[Figure 1 about here.]

Figure 2 depicts monthly time series of the explanatory variables of the time-series properties regressions against the monthly time series of the CDS market illiquidity measure.

[Figure 2 about here.]

Figure 3 displays the model-implied CDS spread decomposition for the 20 price-impact-sorted portfolios.

[Figure 3 about here.]

Table 1 summarizes index rules for the main indices of the CDX North American and iTraxx Europe credit index families.

[Table 1 about here.]

Table 2 displays descriptive statistics for the 20 price-impact-sorted portfolios.

[Table 2 about here.]

\textsuperscript{2}Note that we do not redefine $\rho^2$ in case that the intercept is restricted to zero. Therefore, $\rho^2$ is not necessarily nonnegative. Nevertheless, $\rho^2 \leq 1$ and $\rho^2 = 1$ if and only if $e = 0_N$. 

11
References (Internet Appendix)


Figure 1: Credit Index Levels, CDS-Implied Index Levels, and Index-to-CDS Bases.
The figure displays daily observations of credit index levels of the five-year on-the-run series
(thin black lines, left hand scales), CDS-implied index levels (thick gray lines, left hand
scales), and index-to-CDS bases (light gray shaded areas, right hand scales) from September
20, 2006, to February 1, 2012. Index levels and bases are in basis points and dashed vertical
lines correspond to index roll dates.
Figure 2: Explanatory Variables vs. CDS Market Illiquidity Measure.
The figure displays monthly observations of the explanatory variables in the time-series properties regressions (thin black lines, left hand scales) and the CDS market illiquidity measure (thick gray lines, right hand scales). The explanatory variables are: the average bid-ask spread of single-name CDSs (Bid-Ask), the average absolute spread change per quote contributed across single-name CDSs (ILLIQ\textsubscript{CDS}), the average absolute change in the index level per quote contributed across on-the-run credit indices (ILLIQ\textsubscript{IDX}), the Hu, Pan, and Wang (2013) “Noise” measure (Noise), Dick-Nielsen, Feldhüter, and Lando’s (2012) \( \lambda \) corporate bond illiquidity measure, the average CDS-bond basis across U.S. investment grade bonds (CDS-Bond), the spread between three-month LIBOR and OIS rates (LIB-OIS), the spread between three-month Agency MBS and Treasury general collateral repo rates (Repo), the one-month return on the Hedge Fund Research Global Index (\( R_{HFRX} \); because \( R_{HFRX} \) is a return, the level of the index, HFRX, is displayed in the figure), the one-month return on the S&P 500 index (\( R_{SPX} \); because \( R_{SPX} \) is a return, the level of the index, SPX, is displayed in the figure), the yield spread between Baa- and Aaa-rated bonds (Default), and the VIX index (VIX). Bid-Ask, ILLIQ\textsubscript{CDS}, ILLIQ\textsubscript{IDX}, Noise, and the CDS market illiquidity measure are in basis points. CDS-Bond, LIB-OIS, Repo, Default, and VIX are in %. \( \lambda \), HFRX, and SPX are in index points. The time series consist of 64 monthly observations from October 2006 to January 2012.
Figure 3: CDS Spread Decomposition.
The figure displays a model-implied decomposition of five-year CDS spreads (in % per year) at the test portfolio level for the price-impact-sorted portfolios. CDS spreads are decomposed into expected default losses, factor risk premia, and pricing errors implied by the benchmark model specification. The horizontal axis displays portfolio identifiers.
### CDX North American iTraxx Europe

<table>
<thead>
<tr>
<th>Main index (No. of constituents)</th>
<th>CDX.NA.IG (125)</th>
<th>CDX.NA.HY (100)</th>
<th>CDX.NA.HY.B (≤100)</th>
<th>iTraxx Eur (125)</th>
<th>iTraxx Eur HiVol (30)</th>
<th>iTraxx Eur Sr Finls (25)</th>
<th>iTraxx Eur Sub Finls (25)</th>
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</thead>
<tbody>
<tr>
<td>Sub-indices (No. of constituents)</td>
<td>CDX.NA.IG HVOL (30)</td>
<td>CDX.NA.HY BB (≤100)</td>
<td>CDX.NA.HY.B (≤100)</td>
<td>Corporate &amp; Financial</td>
<td>High yield or not rated</td>
<td>Corporate &amp; Financial</td>
<td>Investment grade</td>
</tr>
</tbody>
</table>

### Eligible reference names (ref. names)
- **Corporate & Financial**
  - North America: Investment grade
  - Europe: High yield or not rated

### Domicile of eligible ref. names
- **North America**
- **Europe**

### Rating of eligible ref. names
- **Investment grade**
- **High yield or not rated**

### Index roll dates
- March and September
  - 20th

### Inclusion & exclusion (main index)
- Eligible ref. names that are not members of the current index series and that rank among the most liquid 20% in terms of market risk activity in the Depository Trust & Clearing Corporation’s (DTCC’s) Trade Information Warehouse (TIW) over the six-month period preceding an index roll date are to be included in the next index series. Eligible ref. names are members of the current index series and that rank among the most illiquid 30% in terms of market risk activity in the DTCC’s TIW over the six-month period preceding an index roll date are excluded from the new series along with ref. names that are members of the current index series but no longer meet eligibility criteria.

### Inclusion (sub-indices)
- The 30 eligible ref. names included in the main index with the widest average CDS spread over the past 90 calendar days, as measured from six days prior to the index roll date, constitute the HVOL sub-index.

### Currency of index contract
- USD

### Tier of index contract
- Senior unsecured debt

### Documentation clause of index contract
- No restructuring

### Maturity dates of index contract
- June and December

### Quotation of index contract
- Spread

### Table 1: Summary of Index Rules.

<p>| Market makers of the index are not eligible for inclusion. |
| Tier for the Sub Finls sub-index is subordinated or lower Tier 2 debt. |
| Contract maturities of the Sr Finls and Sub Finls sub-indices are 5 and 10 years. |</p>
<table>
<thead>
<tr>
<th>Rating</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Price Impact</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Price Impact</th>
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<td>0.44</td>
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<td>-0.36</td>
<td>-0.86</td>
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<td>[4.43]</td>
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<td>[-0.33]</td>
<td>[0.19]</td>
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</tr>
<tr>
<td>BB</td>
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<td>1.84</td>
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<td>3.07</td>
<td>-0.87</td>
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Table 2: Descriptive Statistics of Price-Impact-Sorted Portfolios.

The table displays descriptive statistics for the 20 test portfolios formed by first sorting CDS contracts according to credit ratings and then according to price impact. The upper part of the table reports sample means of conditional expected excess returns (in % per year) and realized excess returns (in % per year). In brackets are t-statistics based on Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors with 24 lags. The lower part of the table reports sample means of average five-year CDS spreads across portfolio constituents (in % per year) and standard deviations of realized excess returns (in % per year). Portfolio time series consist of 276 weekly observations from October 11, 2006, to February 1, 2012.