Home or Overseas? An Analysis of Sourcing Strategies under Competition

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Motivated by the recent backshoring trend, this paper studies a sourcing game where competing firms may choose between efficient sourcing (e.g., sourcing from overseas) and responsive sourcing (e.g., sourcing from home country). Efficient sourcing usually provides a cost advantage, while responsive sourcing allows a firm to obtain more accurate demand information when making procurement decisions. By characterizing the equilibrium outcome, we find some interesting results driven by the strategic interaction between the firms. First, a firm may still use efficient sourcing in equilibrium even when the cost advantage associated with efficient sourcing does not exist. This is because the firm can dampen competition by reducing the correlation between her own demand information and the competitor’s. Second, a cost hike in efficient sourcing (e.g., the rising labor cost in Asia) may benefit all the firms in the industry. The reason is that the cost hike may alleviate competition by inducing a new equilibrium sourcing structure. This paper also sheds some light on the recent backshoring trend. First, our analysis indicates that more firms will shift from efficient sourcing to responsive sourcing in equilibrium (i.e., backshore) if the market size shrinks, the demand becomes more volatile, or the sourcing costs rise simultaneously. Second, a firm’s backshoring behavior reduces the competition on the cost dimension, but it also has an ambiguous informational impact on the other firms in the market. In particular, some firms may benefit from increased correlation of their demand information under Cournot competition with substitutable products. Overall, the backshoring behavior can be beneficial to all the firms sticking to their original sourcing strategies under certain conditions.

Keywords: Offshoring, backshoring, sourcing strategy, Cournot competition, supply responsiveness, information correlation.

1. Introduction

Driven by the pressure to control operating costs and focus on core competencies, the past few decades have witnessed a growing trend toward offshore manufacturing as firms move their production activities from developed countries to emerging economies (Wilson 2010 and Plunkett 2011). It has been reported that in 2002-2003, about a quarter to half of the manufacturing companies in Western Europe were involved in production offshoring (Dachs et al. 2006). As of 2008, more than 50% of U.S. companies had a corporate offshoring strategy (Minter 2009). The pros and cons of offshoring have been widely discussed in the business media and the research literature (see, e.g., Van Mieghem 2008). The most-cited advantage of offshoring is the cost savings due to less
expensive labor in emerging economies. In contrast, domestic manufacturing enables firms to react quickly to market changes, improve customer service, and reduce inventory levels. Therefore, a major trade-off is between cost and responsiveness (Farrell 2005, Anderson 2006, and Ryley 2010). The offshoring decision is debatable in many situations since it is not always clear whether the cost factor dominates the responsiveness factor (see, e.g., Goel et al. 2008 and De Treville and Trigeorgis 2010).

The debate on offshoring has intensified in recent years. More voices have been heard that are skeptical about the offshoring trend for various reasons. For example, sourcing costs from emerging economies have been rising rapidly. As of mid-2010, many Chinese firms were facing labor shortages and were forced to boost wages to attract qualified workers (Plunkett 2011). In addition, the global commodity price index has risen significantly (Ferreira and Prokopets 2009). This has led to higher transportation costs as well as production costs (higher oil prices and raw material costs). Because of the unexpected high supply chain costs in offshoring, many firms are considering domestic sourcing rather than international sourcing. Not surprisingly, “reshoring”, “onshoring”, and “backshoring” have frequently made headlines in the business press. Master Lock, the world’s largest manufacturer of padlocks and related security products, has been moving their production from Asia back to Milwaukee since 2010, motivated by economic conditions such as increasingly higher labor and logistics costs in Asia. President Barack Obama highlighted Master Lock in his State of the Union address and encouraged businesses to bring jobs back to the U.S. (Rovito 2012). More examples involved in the most recent backshoring discussion include Caterpillar, General Electric, and Apple (Bussey 2011 and Denning 2012).

The trade-off between cost and responsiveness exists in a firm’s supplier selection problem as well. Similar to offshoring, the practice of outsourcing, where firms move their production activities from in-house to third-party suppliers, has been widely adopted in the industry. Should a firm select an overseas supplier with low sourcing cost or a domestic supplier with short lead time? Clearly, the cost-responsiveness trade-off plays an important role both in offshoring and in outsourcing decisions. However, as the above industry observations demonstrate, such a trade-off can be quite involved and difficult to evaluate. Moreover, a globalizing economy has made the market environment more competitive and volatile. This has forced firms to rely more on effective sourcing strategies to maintain a competitive edge (Rohwedder and Johnson 2008). Firms at the crossroad of making offshoring/outourcing decisions should not simply follow the trend. To make an intelligent decision, a firm needs to understand the business environment as well as the competitor’s sourcing strategy.
Although the cost-responsiveness trade-off has been widely recognized in the literature, the majority of studies consider the trade-off from a single firm’s perspective. There are few papers that study firms’ strategic interactions in a competitive setting, and the driving forces underlying firms’ sourcing strategies have not been fully explored. The main purpose of this paper is to obtain a better understanding on this topic. We develop a game-theoretic model where two firms engage in quantity competition by selling substitutable products. There are two types of sourcing strategies from which the firms can choose. The first strategy is called efficient sourcing (e.g., overseas sourcing), under which the procurement price is low but the delivery lead time is long. The second is called responsive sourcing (e.g., domestic sourcing), under which the procurement price is high but the delivery lead time is short. The responsive sourcing firm may observe better market signals when making the procurement quantity decision. In other words, the benefit of responsive sourcing is captured by accurate market demand information. The firms engage in a two-stage game: They first choose their sourcing strategies; then they compete by determining the quantities they want to sell in the market.

With this model setup, we find that asymmetric equilibrium may exist in the sourcing game. This means that although the firms are symmetric, they may employ different sourcing strategies under competition. In particular, when the two sourcing modes are equally costly (i.e., there is no cost advantage associated with efficient sourcing), a firm may still choose efficient sourcing to differentiate her strategy. This is because by using efficient sourcing, the firm can reduce the correlation between her own demand information and the competitor’s, which in turn dampens the quantity competition between them.

Based on the characterized equilibrium, we conduct comparative statics analysis to examine whether the recent market changes (e.g., the shrinking market size due to economic recession, the increasing labor costs in emerging economies, and the rising global commodity prices) can be used to explain the backshoring trend mentioned above. It has been found that all else being equal, more firms will shift from efficient sourcing to responsive sourcing in equilibrium if the market size shrinks, the demand becomes more volatile, or the sourcing costs rise simultaneously. These findings corroborate the industry observations. In addition, we find that both firms can be better off when the cost of efficient sourcing increases. This implies that, contrary to our intuition, the rising labor and logistics costs in Asia may actually improve the profitability of all firms in the market.

We also investigate the strategic impact of the backshoring trend in a general setting with multiple firms. It has been found that backshoring reduces competition on the cost dimension,
which benefits the rest of the firms in the industry. However, it also has an informational effect due to the change in the information structure of the game. The informational effect is generally not monotonic. Specifically, we find that increasing the correlation of the firms’ demand information may benefit some of the firms. This is in contrast with the existing results in the literature showing that higher correlation is detrimental to the firms under Cournot competition with substitutable products. Overall, our analysis shows that the backshoring behavior may benefit the rest of the firms in the market under certain conditions.

The organization of the paper is as follows. Section 2 reviews the literature. Section 3 describes the model. Sections 4 to 6 analyze the model and derive the main insights. This paper concludes with Section 7. All proofs are given in Appendix A.

2. Literature Review

This paper studies firms’ sourcing decisions that take supply lead-time or responsiveness into consideration. Fast and flexible delivery performance is considered a key supplier selection criterion and has been extensively studied in the literature. Fisher (1997) proposes the notion of responsive supply chains (i.e., supply chains with flexible capacities and fast lead times) and efficient supply chains (i.e., supply chains that emphasize low production and logistics costs), which is similar to the two sourcing modes in this paper. There is a large group of papers that focus on how to secure sufficient and responsive supply by designing appropriate contracts and incentives for the suppliers. Cachon (2003) and Elmaghraby (2000) provide comprehensive reviews of the contracting and sourcing strategies in supply chain settings. The majority of studies in this category consider a single firm’s sourcing problem; in contrast, our paper analyzes the firms’ sourcing strategies in a competitive setting.

An effective strategy that can be used to increase the responsiveness of supply is the so-called quick response (QR). In quick response, lead time is shortened so that a firm may place a second order after observing early demand information. Caro and Martínez-de-Albéniz (2010) study quick response in a duopoly game: The firms make quick response related decisions in the first stage; then they engage in inventory competition in the second stage. Despite the similarity in the game structures, our paper differs from Caro and Martínez-de-Albéniz (2010) in several important aspects. First, in quick response, a firm has two replenishment opportunities, while in our model there is only a single ordering opportunity. Second, the second-stage game is formulated in different ways. Caro and Martínez-de-Albéniz (2010) consider inventory competition based on stockout substitution. Instead, we use Cournot competition to model the second-stage game, which allows endogenous
market price and is more amenable to analysis. Third, the purpose of our paper is to understand the role of information structure in firms’ sourcing decisions and the offshoring/backshoring phenomenon, while Caro and Martínez-de-Albéniz (2010) focus on the value of quick response under competition. Accordingly, these papers present distinctive, but complementary sets of results and managerial insights.

Our paper adopts the Cournot model and is related to papers that study the first-mover advantage in quantity competition. Gal-Or (1985) shows that in a Stackelberg game, the first mover has the first-mover advantage (disadvantage) when the reaction function of the follower is downwards (upwards) sloping. She uses quantity competition as an example to illustrate the first-mover advantage. Wang et al. (2013) study firms’ quick response choice in a competitive setting. They find that even when quick response is not more costly, firms may not choose to use it in equilibrium. The reason is that without using quick response, a firm can move first and enjoys the first-mover advantage by producing a large quantity to deter the follower. A key assumption made by these papers is that the first-mover’s output decision is observable or committable. However, we study a situation where the quantity decision of the efficient sourcing firm is not observable. As a result, the key driving force in our sourcing game is the information structure, rather than the first-mover advantage.

This paper involves information acquisition in a Cournot setting, which has been studied in the literature. Novshek and Sonnenschein (1982), Li et al. (1987), and Vives (1988) study information acquisition games in various settings and find that only symmetric equilibria exist. Similar to Daughety and Reinganum (1994), we show that asymmetric equilibrium may arise, but for a different reason. In our paper, the driving force is the correlation between firms’ information, while in Daughety and Reinganum (1994), the result is driven by the assumption that the follower can observe the leader’s decision and thus obtain free information.

There is a related literature on information sharing under Cournot competition with substitutable products; see Raith (1996) and Vives (1999) for reviews of this literature. In these information-sharing models, each firm is equipped with an exogenously given signal, and her decision is whether to share this signal. The majority of studies consider symmetric model settings and observe that information sharing will increase the correlation of the firms’ signals, which is detrimental to all firms. In our model setting, each firm chooses her sourcing strategy, which determines both the sourcing cost and demand signal. Due to these distinct features, our paper generates different results and managerial implications. For instance, we find that a higher signal correlation does not necessarily hurt all the firms when they have asymmetric locations in the sourcing game.
Similar information paradigms have been applied in the supply chain management literature to model information sharing/leaking/investment; see, for example, Li and Zhang (2008), Gal-Or et al. (2008), Anand and Goyal (2009), Shin and Tunca (2010), Taylor and Xiao (2010), and Ha et al. (2011). These studies have quite different model settings, and none of them considers the location-based supplier selection decision as in our paper.

There is a group of papers that study the benefit of outsourcing in competitive settings. In these papers, two firms compete in the same market, and each firm may choose to either insource or outsource the production of the product/service. Some of the representative studies include Cachon and Harker (2002), Liu and Tyagi (2011), and Feng and Lu (2012). These studies do not consider supply responsiveness when modeling the difference between insourcing and outsourcing. By contrast, our paper explicitly includes the responsiveness factor in firms’ decisions. We model the responsiveness by accurate demand information and find that the information structure plays a critical role in the firms’ sourcing strategies. In particular, we show that information correlation may serve as a new driving force underlying firms’ outsourcing decisions.

Resource flexibility and delayed differentiation have been widely acknowledged as effective tools to manage uncertain demand. Several papers examine these two strategies under competition; see, for example, Goyal and Netessine (2007) on capacity flexibility and Anand and Girotra (2007) on delayed differentiation. These studies focus on ex post flexibility, i.e., the flexibility to change production mix after demand is realized, while our paper is about ex ante flexibility, i.e., the opportunity to obtain accurate information when making the quantity decision before demand is realized. Moreover, we study firms’ strategic sourcing decisions in a single-product setting, while they focus on the value of resource flexibility in a multiple-product setting.

In summary, this paper contributes to the literature by obtaining a better understanding of how strategic interactions affect firms’ sourcing choices that involve the cost-responsiveness trade-off. In addition, the backshoring phenomenon is rather new, and there has been little research dedicated to exploring this industry trend. This paper also aims to shed some light on the driving forces behind such a trend and investigate its potential impact on market competition.

3. Model

Two firms (A and B) compete in the same market by selling substitutable products. Both firms need to decide where to locate their production facilities (i.e., offshoring or not). Alternatively, the firms source their product (or a critical input of the product) from external suppliers and need to make their supplier selection decisions. For ease of exposition, we consider the firms’ supplier
selection decisions rather than offshoring decisions. There are two types of suppliers: efficient and responsive. An efficient supplier incurs a low production cost, but the procurement lead time is long; a responsive supplier has a short lead time, but the production cost is high. In this paper, we focus on the interpretation that the efficient type refers to the suppliers located overseas, while the responsive suppliers are domestic ones; however, our model also applies to situations where both supplier types have similar geographical locations, but quote different lead times and prices. The firms may choose between the efficient sourcing mode (i.e., sourcing from an efficient supplier) and the responsive sourcing mode (i.e., sourcing from a responsive supplier). We focus on sole-sourcing strategies for the firms in this paper, i.e., sourcing from both types of suppliers is not a viable strategy. In the context of offshoring, it might be prohibitively costly to invest in production facilities at more than one location. It has also been reported that sole sourcing is commonly used in certain industries due to cost and long-term relationship considerations (see Lester 2002 for a discussion of sole sourcing in the automobile industry).

The life cycle of the products is relatively short compared to the procurement lead time. As a result, the firms have to make their procurement decisions before the selling season starts, when the market demand is still uncertain. We assume that the firms engage in Cournot (quantity) competition after choosing the sourcing strategies. The Cournot model is appropriate in our problem because we are interested in the firms’ procurement quantity decisions under demand uncertainty. We analyze a one-shot Cournot game in this paper for simplicity. The qualitative insights will be similar if the firms compete in multiple independent periods. Specifically, given the firms’ procurement quantities $Q_i$ ($i = A, B$), the market clearing price is $p = a - (Q_A + Q_B) + u$, where $a$ is the intercept of the inverse demand function, and $u$ is a random term that represents the firms’ common prior belief of the demand uncertainty with mean $E[u] = 0$ and variance $Var[u] = \eta$. Later we will refer to $a$ as the market size since $a$ is a measure of the expected market potential.

We denote the efficient supplier type by $S_l$ and the responsive supplier type by $S_s$, where the subscripts $l$ and $s$ stand for long and short lead times, respectively. Let $w_l$ and $w_s$ ($w_l \leq w_s$) be the exogenous procurement prices associated with the two types of suppliers. These procurement prices determine the cost differential between the sourcing modes. Sourcing from $S_s$ is a feasible strategy only if the profit margin $p - w_s = (a - w_s) - (Q_A + Q_B) + u$ is non-negative. We assume $a$ is greater than a lower bound $a$, where $a - w_s$ is large enough (relative to the standard deviation of $u$) so that the probability of negative profit margin is negligible.

Responsive sourcing allows a firm to make her quantity decision when she has a better understanding of the market demand. We model such an informational advantage as follows. If a firm
sources from $S_i$, then she will obtain a signal about future demand at time 1; if a firm sources from $S_k$, then she will obtain a demand signal at time 2, which is closer to the selling season. Let $x_{ti}$ ($t = 1, 2, i = A, B$) denote the signal of firm $i$ at time $t$. The signal is determined by the demand term $u$ and some random noises. We assume the demand signals satisfy the following conditions: First, both $E[u|x_{ti}]$ and $E[x_{t′j}|x_{ti}]$ are affine in $x_{ti}$; second, $E[x_{ti}|u] = u$, i.e., the signals are unbiased estimators of $u$. Let $m_t = E[Var[x_{ti}|u]]$, the expected variance of the signals conditional on $u$. We may view $m_t$ as the noise level in a signal and use $1/m_t$ as a measure of signal accuracy. Clearly, there is $m_1 > m_2$ because the time-2 signal should be more accurate than the time-1 signal. Moreover, the signal noises can be correlated. Let

$$\frac{E[Cov[x_{ti}, x_{t′j}|u]]}{\sqrt{E[Var[x_{ti}|u]]E[Var[x_{t′j}|u]]}} = \begin{cases} \rho, & \text{if } t = t′, \\ \rho_{12}, & \text{if } t \neq t′. \end{cases}$$

where $i \neq j$, $\rho$ is the coefficient of correlation of the signal noises from the same time, and $\rho_{12}$ is the coefficient of correlation of the signal noises from different times.\(^1\) Then, using the results in Ericson (1969) and Li (1985), we can derive the following useful expressions for analysis:

$$E[u|x_{ti}] = \left(\frac{\eta}{\eta + m_t}\right) x_{ti}, \quad \text{(1)}$$

$$E[x_{t′j}|x_{ti}] = \begin{cases} \left(\frac{\eta + \rho_m}{\eta + m_t}\right) x_{ti}, & \text{if } t = t′, \\ \left(\frac{\eta + \rho_{12} \sqrt{m_t m_{t′}}}{\eta + m_t}\right) x_{ti}, & \text{if } t \neq t′. \end{cases} \quad \text{(2)}$$

As an illustration of the above model, we present a special case based on the information setup used in the literature (see, e.g., Novshek and Sonnenschein 1982). Suppose each firm depends on an independent forecasting agency to generate her demand signal. At time $t$, there is a pool of sample observations $G_t = \{g_{t1}, g_{t2}, \cdots, g_{tn}\}$, where $g_{tk} = u + \varepsilon_{tk}$ and $\varepsilon_{tk}$ is a random noise that may affect the firms’ demand belief. The noise terms $\varepsilon_{tk}$ ($t = 1, 2$ and $k = 1, 2, \ldots, n$) are independent of each other and independent of $u$; in addition, they follow normal distribution and have mean zero and variance $\delta_i$ ($\delta_1 > \delta_2$). The signal $x_{ti}$ in the above model is generated at time $t$ based on $n_t$ sample observations taken by firm $i$’s agency. For example, if both firms source from $S_t$, then each firm’s signal is generated by $n_1$ ($n_1 \leq n$) samples from $G_1$. Since there may be overlapping samples, the noises in the firms’ signals $x_{1A}$ and $x_{1B}$ can be correlated with a coefficient $\rho \geq 0$. Similarly, if firm A sources from $S_l$ and firm B sources from $S_s$, then $x_{1A}$ is based on the $n_1$ samples from $G_1$ while $x_{2B}$ is based on $n_2$ samples from both $G_1$ and $G_2$. Again, depending on the number of overlapping samples, the noises in the firms’ signals $x_{1A}$ and $x_{2B}$ can also be correlated. We provide\(^1\) For simplicity, we assume that the coefficient of correlation $\rho$ is the same for time 1 and time 2. Relaxing this assumption will not affect the qualitative results.
Firms decide sourcing mode

Time 0
Demand signals $x_i$ ($i = A, B$)

Time 1
Efficient sourcing firm(s) places order
Responsive sourcing firm(s) places order

Time 2
Demand signals $x_j$ ($j = A, B$)

Time 3
Demand is realized
Firms sell products at market-clearing price

Figure 1 Timeline of Information and Decisions

a detailed description of this signal generating process in Appendix B. It can be shown that the correlation coefficient $\rho_{12}$ for cross-time signal noises satisfies $\rho_{12} \leq \sqrt{\frac{m_2}{m_1}}$, where the equality holds only when $x_{2B}$ contains all the $n_1$ samples based on which $x_{1B}$ would be generated. Based on this example, we will focus our analysis on $0 \leq \rho_{12} \leq \sqrt{\frac{m_2}{m_1}}$ in the rest of the paper.

We study the firms’ sourcing strategy equilibrium under Cournot competition in a two-stage game. The timing of the events is shown in Figure 1. First, at time 0, the firms simultaneously choose their sourcing modes (i.e., either sourcing from $S_l$ or $S_s$); second, at time 1, the firm(s) sourcing from $S_l$ receives her signal $x_{1i}$ and places an order; then, at time 2, the firm(s) sourcing from $S_s$ receives her signal $x_{2j}$ and places an order; finally, at time 3, the market demand is realized and the firms sell their products at the market-clearing price. The firms’ demand signals are private information. We emphasize that the quantity decision made by a firm at time 1 is not observable to the other firm. All the parameters are common knowledge except the firms’ private signals. Both firms are risk-neutral and aim at maximizing their expected profits. For concision, we will use profit instead of expected profit when no confusion will be caused.

Let $(S_l, S_l)$, $(S_l, S_s)$, $(S_s, S_l)$ and $(S_s, S_s)$ denote the four possible equilibrium structures of the sourcing game. We use superscripts $ll$, $ls$, $sl$, and $ss$ to refer to these sourcing structures with the first (second) letter denoting firm A’s (B’s) supplier. For instance, $\Pi^{ll}_{B}$ denotes the expected profit of firm B under the sourcing structure $(S_l, S_s)$ with firm B sourcing from supplier $S_s$. The above sourcing game can be characterized by the $2 \times 2$ matrix in Figure 2. For future comparison, we first analyze a base case of the model with deterministic demand. We use $Q^*_i$ to denote firm $i$’s ($i = A, B$) equilibrium quantity in the Cournot competition.

2 For tractability, we focus on the situation where the firms will not withhold the products after receiving more accurate demand information. That is, there is a negligible chance that the firms find it optimal to hold back some inventory from the market due to extreme demand realizations. Such an assumption has been used in Anand and Girotra (2007) and Goyal and Netessine (2007).

3 In our model setting, the quantity ordered at time 1 will not be delivered until time 3. So it is generally not possible for the other firm to observe or verify the quantity before the delivery time. Further, it can be costly for firms to credibly share information or make a commitment in practice (this may require enforcement by a third-party such as the court of law). Thus, we focus on the situation where the quantity decision at time 1 is not observable.
**Proposition 1.** Under deterministic demand (i.e., \( \eta = 0 \)), \((S_l, S_l)\) is the unique sourcing equilibrium where \( Q_A^* = Q_B^* = \frac{a-wl}{3} \) and \( \Pi_A^s = \Pi_B^s = \left( \frac{a-wl}{3} \right)^2 \).

Without demand uncertainty, both firms will choose to source from the low-cost supplier. The major benefit of responsive sourcing is to obtain more accurate demand information, which does not exist if there is no demand uncertainty. As a result, the competitive advantage rests solely on cost efficiency and hence \((S_l, S_l)\) is the only sourcing equilibrium. This result implies that for products with highly predictable demand, offshoring is still a useful strategy because firms compete mainly on cost. In the next two sections we will see how demand uncertainty may change the sourcing equilibrium.

### 4. Equilibrium Analysis

This section presents the equilibrium analysis of the sourcing game. Since there is incomplete information (the firms observe private signals), we apply the Bayesian Nash Equilibrium (BNE) concept to characterize the outcome of the game. The Bayesian game can be solved backward. First, we derive the equilibrium of the Cournot competition under each sourcing structure. This gives us the firms’ expected profits in the 2 × 2 matrix in Figure 2. Then, we compare the profits to determine the equilibrium of the sourcing game.

We present the analysis for sourcing structures \( ls, ss \), and \( ll \); the structure \( sl \) is essentially the same as \( ls \) by symmetry. We first consider the Cournot competition outcome under the sourcing structure \( ls \). In this case, firm A sources from \( S_l \), so she only obtains the signal \( x_{1A} \) when placing her order at time 1; firm B only obtains the signal \( x_{2B} \) when placing her order at time 2. When placing her order, firm A knows that firm B will obtain a signal \( x_{2B} \) at time 2. Although firm A cannot observe \( x_{2B} \), she can update her belief of \( x_{2B} \) based on \( x_{1A} \). When making her ordering decision at time 2, firm B does not know firm A’s exact order quantity placed at time 1. Therefore, even though the firms place their orders at different times, they essentially engage in a simultaneous-move game rather than a Stackelberg game. Proposition 2 shows there is a unique BNE of such a Cournot subgame with incomplete information.
Proposition 2. Consider the sourcing structure ls, where firm A obtains the signal $x_{1A}$ at time 1 and firm B obtains the signal $x_{2B}$ at time 2.

(i) There is a unique BNE $(Q^*_A(x_{1A}), Q^*_B(x_{2B}))$ in the Cournot competition, where

$$Q^*_A(x_{1A}) = \left( \frac{a}{3} - \frac{2}{3} w_l + \frac{1}{3} w_s \right) + \frac{\eta \left( 2m_2 + \eta - \sqrt{m_1 m_2 \rho_{12}} \right)}{4(m_1 + \eta) (m_2 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_{12}})^2} x_{1A},$$

$$Q^*_B(x_{2B}) = \left( \frac{a}{3} - \frac{2}{3} w_s + \frac{1}{3} w_l \right) + \frac{\eta \left( 2m_1 + \eta - \sqrt{m_1 m_2 \rho_{12}} \right)}{4(m_1 + \eta) (m_2 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_{12}})^2} x_{2B}.$$

(ii) The firms’ profits in equilibrium are given by

$$\Pi^*_A = \frac{1}{9} (a - 2w_l + w_s)^2 + \frac{\eta^2 \left( 2m_2 + \eta - \sqrt{m_1 m_2 \rho_{12}} \right)^2}{4(m_1 + \eta) (m_2 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_{12}})^2} (m_1 + \eta),$$

$$\Pi^*_B = \frac{1}{9} (a - 2w_s + w_l)^2 + \frac{\eta^2 \left( 2m_1 + \eta - \sqrt{m_1 m_2 \rho_{12}} \right)^2}{4(m_1 + \eta) (m_2 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_{12}})^2} (m_2 + \eta).$$

We emphasize a couple of observations from this proposition. First, a firm’s equilibrium strategy $Q^*_i$ is a function of the signal the firm obtains. Both $Q^*_i$ and $\Pi^*_i$ consist of two terms, each corresponding to an exclusive set of parameters. The first term represents the profit under no demand uncertainty, which only depends on the sourcing costs ($w_l, w_s$) and market size ($a$). Such a term reflects the cost efficiency of the sourcing mode, and thus is referred to as an efficiency term hereafter.

The second term denotes the additional profit due to demand uncertainty, which is referred to as information term because it only consists of the informational parameters ($\eta, m_t, \rho_{12}$). From Equations (5) and (6), we can see that compared to firm B’s profit, firm A’s equilibrium profit has a larger efficiency term but a smaller information term. This illustrates the trade-off between cost and responsiveness/information. It can also be shown that each firm’s profit increases in the accuracy of her own signal and decreases in the accuracy of the competitor’s signal. Another finding is that the firms’ profits are increasing in $\eta$, i.e., under Cournot competition, they actually prefer higher demand variability. In particular, both firms can make more profit with demand uncertainty ($\eta > 0$) than without demand uncertainty ($\eta = 0$).

Proposition 3 characterizes the Cournot competition outcome under the sourcing structures ll and ss.

Proposition 3. Consider the sourcing structure jj ($j = l, s$), where firm A obtains the signal $x_{1A}$ and firm B obtains the signal $x_{1B}$ at time $t$ ($t = 1$ for $j = l$, $t = 2$ for $j = s$).

(i) There is a unique BNE $(Q^*_A(x_{1A}), Q^*_B(x_{1B}))$ in the Cournot competition, where

$$Q^*_A(x_{1A}) = \frac{a - w_j}{3} + \frac{\eta}{m_t (2 + \rho) + 3\eta} x_{1A},
Q^*_B(x_{1B}) = \frac{a - w_j}{3} + \frac{\eta}{m_t (2 + \rho) + 3\eta} x_{1B}.$$
(ii) The firms’ profits in equilibrium are given by

\[ \Pi_A^j = \Pi_B^j = \frac{(a - w_j)^2}{9} + \frac{\eta^2 (m_t + \eta)}{(m_t (2 + \rho) + 3\eta)^2}. \]  

Again, the firms’ profits are increasing in demand uncertainty. In addition, it can be readily shown that the firms’ profits are increasing in the signal accuracy (measured by $1/m_t$). Decreasing $m_t$ directly benefits a firm, but meanwhile it also benefits the competitor, which indirectly hurts the firm. Our result shows that the direct benefit always dominates the indirect effect, i.e., the firms’ profits are always increasing in the signal accuracy $1/m_t$. This means that if there is a better forecasting technology that can improve both firms’ signals, then both firms will benefit from the technology.

Now we are ready to characterize the equilibrium of the sourcing game. The results will be presented in the next two subsections. We separate the discussion for $w_l = w_s$ and $w_l < w_s$ because both the analysis and insights are quite different in these two cases.

### 4.1. Sourcing Equilibrium with $w_l = w_s$

We first analyze the benchmark case where the sourcing costs from $S_l$ and $S_s$ are equal, i.e., $w_l = w_s$. Although $S_l$ is not more efficient than $S_s$, we still refer to $S_l$ as the “efficient” supplier for consistency. The sourcing game is defined by the $2 \times 2$ matrix in Figure 2. Since sourcing from $S_l$ is strictly dominated by sourcing from $S_s$ (the trade-off between cost and responsiveness no longer exists), intuitively, no firm would choose $S_l$ in this case. However, our analysis shows that such an intuition is not always true.

**Proposition 4.** Consider the sourcing game with $w_l = w_s$ and $w_l < w_s$. There exists a $\xi$ ($0 < \xi < 1$) such that $(S_l, S_s)$ is the unique equilibrium if $m_2 > \xi m_1$, $(S_s, S_s)$ is the unique equilibrium if $m_2 < \xi m_1$, and both $(S_l, S_s)$ and $(S_s, S_s)$ are equilibria if $m_2 = \xi m_1$.

Proposition 4 states that if the gap between $m_1$ and $m_2$ is less than a threshold, then one of the two firms will choose $S_l$ in equilibrium. This suggests that in a competitive setting, it might be optimal for a firm to source from the long lead time supplier, even though the supplier has no cost advantage at all. In fact, it can be shown that this may happen even when there is a cost disadvantage associated with $S_l$. Why would a firm choose a strictly dominated sourcing strategy? The explanation is as follows. Suppose firm B has already chosen responsive sourcing. When choosing her sourcing strategy, firm A needs to compare the profit functions $\Pi_A^s$ in (8) and $\Pi_A^s$ in (5). Given $w_l = w_s$, we know the efficiency terms in these two profit functions are equal. So firm A’s optimal strategy hinges upon the comparison of the information terms. By
sourcing from \( S_s \), firm A benefits from observing a more accurate demand signal. Call this accuracy effect. However, both firms sourcing from \( S_s \) implies a higher correlation of the firms’ signals (the correlation between \( x_{2A} \) and \( x_{2B} \) is higher than that between \( x_{1A} \) and \( x_{2B} \)). Consider a special case with \( m_1 = m_2 \), i.e., sourcing from \( S_s \) offers no advantage on signal accuracy. Then we can see that the coefficient of \( x_{1A} \) in (3) is larger than the coefficient of \( x_{2A} \) in (7), which means firm A’s output quantity is less variable in the \( ss \) sourcing structure than in the \( ls \) structure. In other words, increased correlation will intensify competition, making firm A’s order quantity less responsive to observed signal and resulting in a lower profit. Call this correlation effect. Consequently, firm A will choose \( S_s \) only if the accuracy effect dominates the correlation effect, which happens when \( x_{2A} \) is sufficiently more accurate than \( x_{1A} \) (i.e., the gap between \( m_1 \) and \( m_2 \) is sufficiently large).

It is noteworthy that with \( w_l = w_s \), the firms’ sourcing decisions boil down to choosing which signal to observe. This resembles the information acquisition game studied in the literature; see, for example, Novshek and Sonnenschein (1982), Li et al. (1987), and Vives (1988). It has been reported in the literature that only symmetric equilibria exist in general. However, we find that asymmetric equilibria may arise in such an information acquisition game. This difference is because those studies only consider situations with independent signal noises, and thus the correlation effect in our paper is absent. As an exception, Daughety and Reinganum (1994) identify an asymmetric equilibrium in which one firm acquires a perfect demand signal while the other chooses to be uninformed. Since perfect signals are considered, there is no correlation effect in Daughety and Reinganum (1994) either. However, it has been assumed that the leader’s quantity decision is observable to the follower so that the follower can perfectly infer the demand information from the leader’s quantity decision. This provides incentives for a firm not to acquire any information early on because she may obtain the same information later at no cost by observing the leader’s action. Therefore, the driving force underlying the asymmetric equilibrium in Daughety and Reinganum (1994) is different from that in our paper.

The above analysis shows that firms may choose differentiated sourcing strategies to reduce competition. It identifies a new explanation of outsourcing: Under certain competitive settings, the aforementioned correlation effect may incentivize a firm to outsource from a long lead time supplier even if the supplier has no cost advantage. So far the role of the correlation effect has received little attention in the sourcing literature. This is mainly because when studying the trade-off between sourcing cost and responsiveness, the majority of the literature focuses on single-firm settings. In a single-firm setting, the firm only needs to consider the accuracy of demand information. However, Proposition 4 shows that in a competitive setting, the correlation of information may also influence
firms’ strategic interactions and lead to interesting game outcomes. In Section 5, we consider multiple-firm settings and present more results about the impact of information correlation. To our knowledge, this paper is the first to develop a theory about the correlation effect on firms’ competitive sourcing strategies.

From anecdotal industry evidences, managers seem to understand the value of market information when making their sourcing decisions. Many firms actually emphasize that by being closer to the market, they can obtain more accurate information and respond more quickly to market changes. In the academic literature, it is well known that correlation of information plays an important role in Cournot competition (Vives 1999). Our research suggests that managers should not overlook the correlation effect when devising sourcing strategies. For example, following the competitors’ footprint to either offshore or backshore would clearly change the information structure of the competition, and firms need to take both the accuracy and correlation effects into account.

Based on this work, one may naturally ask whether the theoretical prescriptions are consistent with firms’ actual behavior in making sourcing decisions. For instance, how does the information structure, especially the correlation effect, drive firms’ sourcing decisions in practice? This is an important empirical question that deserves further research attention.

4.2. Sourcing Equilibrium with \( w_l < w_s \)

We proceed to derive the firms’ sourcing equilibrium for \( w_l < w_s \). Again, the sourcing game is given by the 2 × 2 matrix in Figure 2. Define the following notation:

\[
\Gamma_A = \left( \frac{\eta^2 (m_2 + \eta)}{(m_2 (2 + \rho) + 3\eta)^2} - \frac{\eta^2 (2m_2 + \eta - \sqrt{m_1 m_2 \rho_{12}}) (m_1 + \eta)}{(4(m_1 + \eta) (m_2 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_{12}})^2)^2} \right),
\]

\[
\Gamma_B = \left( \frac{\eta^2 (2m_1 + \eta - \sqrt{m_1 m_2 \rho_{12}}) (m_2 + \eta)}{(4(m_1 + \eta) (m_2 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_{12}})^2)^2} - \frac{\eta^2 (m_1 + \eta)}{(m_1 (2 + \rho) + 3\eta)^2} \right),
\]

\[
T_1 = w_l + \frac{9}{4} \left( \frac{\Gamma_A}{w_s - w_l} \right) \quad \text{and} \quad T_2 = w_s + \frac{9}{4} \left( \frac{\Gamma_B}{w_s - w_l} \right).
\]

To simplify the exposition, when one firm sources from \( S_l \) and the other from \( S_s \) in equilibrium, we assume without losing generality that firm A sources from \( S_l \). Then, \( \Gamma_A \) is the difference of firm A’s information terms under the ss and the ls structures (see Propositions 3 and 2); \( \Gamma_B \) is the difference of firm B’s information terms under the ls and the ll structures. It can be shown \( \Gamma_A < \Gamma_B \) and \( T_1 < T_2 \).

**Proposition 5.** The equilibria of the sourcing game are given as follows.\(^4\)

\(^4\) A necessary condition for equilibrium \((S_s, S_s)\) is \( a \leq T_1 = w_l + \frac{9}{4} \left( \frac{\Gamma_A}{w_s - w_l} \right) \). However, the non-negative constraint requires \( a \geq \frac{w_s - w_l}{3} \). Therefore, \((S_s, S_s)\) is an equilibrium only when \( T_1 \) is sufficiently large (e.g., when \( w_s - w_l \) is small or \( \Gamma_A \) is large enough); otherwise \((S_s, S_s)\) cannot be an equilibrium. Similarly, \((S_l, S_s)\) cannot be an equilibrium if \( T_2 \leq \frac{w_s - w_l}{3} \).
(i) If \( a < T_1 \), then \((S_s, S_s)\) is the unique Nash equilibrium;
(ii) If \( a = T_1 \), then both \((S_s, S_s)\) and \((S_l, S_s)\) are Nash equilibria;
(iii) If \( T_1 < a < T_2 \), then \((S_l, S_s)\) is the unique Nash equilibrium;
(iv) If \( a = T_2 \), then both \((S_l, S_l)\) and \((S_l, S_s)\) are Nash equilibria;
(v) If \( a > T_2 \), then \((S_l, S_l)\) is the unique Nash equilibrium.

In contrast with the deterministic demand case, Proposition 5 shows there is a larger set of possible equilibria in the presence of demand uncertainty. The equilibrium is not necessarily unique. When \( T_1 \leq a \leq T_2 \), the symmetric sourcing game has an asymmetric equilibrium, \((S_l, S_s)\). The equilibria characterized in Proposition 5 enable us to investigate how different problem parameters affect the outcome of the sourcing game. First, we consider the impact of the demand uncertainty. Extensive numerical analysis shows that both \( T_1 \) and \( T_2 \) increase in \( \eta \), i.e., both threshold values will be larger as the market demand becomes more variable. The increase in \( T_2 \) means that \((S_l, S_l)\) will be less likely to happen because it is the equilibrium of the sourcing game only if \( a \geq T_2 \). In addition, the increase in \( T_1 \) means that \((S_s, S_s)\) will more likely be an equilibrium outcome. Thus, we conclude that a more volatile market demand drives more firms to source from \( S_s \).

Second, the market size plays a critical role in determining the equilibrium of the sourcing game. When the market is relatively small \((a < T_1)\), both firms will adopt the responsive sourcing mode; when the market size is intermediate \((T_1 < a < T_2)\), the firms will diversify their sourcing strategies; when the market is relatively large \((a > T_2)\), both firms will adopt the efficient sourcing mode. That is, all else being equal, as the market size shrinks, firms are more likely to use responsive sourcing in equilibrium. This result suggests that different competitive weapons might have different values depending on the market condition: In a small niche market, firms should give higher priority to responsiveness when choosing their suppliers; however, in a large mass market focusing on efficiency and cost reduction through offshoring would be a more effective strategy. There is a two-fold explanation for this observation. When the market size is large, the firms' selling quantities are large too, which implies that a low procurement cost can bring in more benefits. Additionally, recall that \( a \) is the expected market potential, so a smaller \( a \) implies a more variable market (the coefficient of variation of the demand increases as \( a \) decreases).

We also investigate the impact of decreasing the market size while keeping the coefficient of variation \((\sqrt{\eta}/a)\) constant. In this case, the impact is a combination of the effect of only decreasing \( a \) and the effect of only decreasing \( \eta \), both of which have been discussed above. The result can happen either way (i.e., either more firms source from \( S_s \) or more firms source from \( S_l \)), depending on which effect is dominant. It is found that if \( w_s - w_l \) is greater (less) than a market-size-dependent
threshold, the effect of only decreasing \( a (\eta) \) dominates and more (less) firms will source from \( S_s \). This is because as \( a \) decreases, the benefit of efficient sourcing decreases, and the decreasing speed is faster for larger \( w_s - w_l \); as the demand uncertainty \( \eta \) decreases, the benefit of responsive sourcing also decreases. Therefore, when \( w_s - w_l \) is large enough, the attractiveness of efficient sourcing decreases faster than that of responsive sourcing, which drives more firms to source from \( S_s \).

Third, it is worth noting how the sourcing equilibrium varies with the sourcing costs. What will happen if \( w_l \) and \( w_s \) rise simultaneously? This question is motivated by the observation that the rising commodity prices in the global market would affect a supplier’s cost regardless of her location (imagine the production requires a certain input available only in the global commodity market). Hence the sourcing costs for both the efficient supplier (e.g., located in Asia) and the responsive supplier (e.g., located in the U.S.) may inflate by the same amount at the same time. Suppose \( w_l \) and \( w_s \) increase simultaneously while \( w_s - w_l \) is held constant. Under this condition, it is clear that both \( T_1 \) and \( T_2 \) increase in the sourcing costs. The increase in \( T_1 \) and \( T_2 \) will make \((S_l, S_l)\) less likely and \((S_s, S_s)\) more likely to occur. Thus, even if the cost differential remains constant, a universal cost increase will drive more firms to source from \( S_s \). This is because the cost differential as a fraction of the sourcing costs (i.e., the relative cost advantage of efficient sourcing) will decrease as the sourcing costs rise. Such a finding corroborates the recent “backshoring” trend, which has been taking place when the raw materials prices in the global market increase rapidly.

So far we have discussed the impact of parameter changes on the equilibrium sourcing structure. Based on Proposition 5, we may also study the impact of parameter changes on the firms’ performances. We focus on the cost increase at \( S_l \) because the rising labor and logistics costs in emerging economies have come under the spotlight in recent years. A natural question to ask is: How does such a cost increase affect firms’ profits? To answer this question, define

\[
\bar{\tau} = -a + 2w_s - w_l + \sqrt{(a - w_l)^2 - 9\Gamma_B}, \quad \bar{\tau}_1 = \frac{a + w_s}{2} - w_l - \frac{1}{2}\sqrt{(a - w_s)^2 + 9\Gamma_A},
\]

\[
\bar{\tau}_2 = \frac{a + w_s}{2} - w_l - \frac{1}{2}\sqrt{(a - w_l)^2 + \frac{9\eta^2 (m_1 + \eta)}{(m_1 (2 + \rho) + 3\eta)^2}} - \frac{9\eta^2 (2m_2 + \eta - \sqrt{m_1 m_2 \rho_2})^2}{(4(m_1 + \eta)(m_2 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_2})^2)^2}.
\]

**Proposition 6.** Suppose \( a > T_2 = w_s + \frac{9}{4} \left(\frac{\Gamma_B}{w_s - w_l}\right) \) so that \((S_l, S_l)\) is the sourcing equilibrium. Then an increase of \( w_l \) by \( \tau > 0 \), where \( \bar{\tau} < \tau < \min\{\bar{\tau}_1, \bar{\tau}_2\} \), will shift the sourcing equilibrium from \((S_l, S_l)\) to \((S_l, S_s)\) and make both firms better off.

Interestingly, Proposition 6 suggests that a cost hike in emerging economies may benefit both firms in the industry. To see an example, consider \( a = 25, w_s = 0.5, w_l = 0.4, \eta = 10, m_1 = 10, m_2 = \)
1, \( \rho_{12} = 0 \), and \( \rho = 0.5 \). Under these parameters, there is \( a > T_2 = 22.2 \), so \((S_l, S_l)\) is the sourcing equilibrium, under which the firms’ profits are \( \Pi_{A}^{l} = \Pi_{B}^{l} = 67.90 \). Now, suppose \( w_l \) increases by 7.5\%, or 0.03. Then it can be shown that the new sourcing equilibrium is \((S_l, S_s)\), under which \( \Pi_{A}^{ls} = 67.93 \) and \( \Pi_{B}^{ls} = 67.94 \). A few points are worth emphasizing about Proposition 6. First, a premise for this result is that the cost increase must lead to a different sourcing equilibrium. From firm A’s (the firm remaining at \( S_l \)) perspective, firm B backshoring alleviates the competition on the cost dimension (firm B now faces a higher cost \( w_s \)) and reduces the signal correlation. For firm B, backshoring is beneficial due to improved signal accuracy as well as reduced correlation. Therefore, both firms may benefit from the backshoring trend driven by cost hikes at \( S_l \).\(^5\) Second, the result requires differential costs (i.e., \( w_s > w_l \)); however, it is not necessary for both the accuracy and correlation effects to be present. Such a result may still hold even without the accuracy effect (i.e., under \( m_1 = m_2 \)) or without the correlation effect (i.e., under \( \rho = 0 \)). Next, Proposition 6 requires \( \bar{\tau} < \tau < \min\{\bar{\tau}_1, \bar{\tau}_2\} \). That is, the cost increase must be large enough to make a firm benefit from backshoring; at the same time, the cost increase is not too large so that only one firm backshores (by \( \tau < \bar{\tau}_1 \)) and the efficient sourcing firm can still be better off (by \( \tau < \bar{\tau}_2 \)). In fact, a large cost increase (\( \tau > \bar{\tau}_1 \)) may also make the firms better off by driving both of them to backshore; this happens when \( \bar{\tau}_1 < \bar{\tau}_2 \), which is equivalent to \( \Pi_{i}^{ss} > \Pi_{i}^{l}, i = A, B \). Note that these observations do not necessarily happen for any initial equilibrium \((S_l, S_l)\). In particular, there might be \( \bar{\tau} > \bar{\tau}_2 \) and \( \bar{\tau}_1 > \bar{\tau}_2 \) when \( w_s, m_2, \) or \( \rho_{12} \) is large enough; in this case, at least one firm will be worse off no matter how many firms backshore. Lastly, it can be shown that increasing \( w_s \) could make both firms better off if it causes the equilibrium to shift from \((S_s, S_s)\) to either \((S_l, S_s)\) or \((S_l, S_l)\); such a result is analogous to the above discussion and therefore omitted.

5. Multiple Firms

This section extends the basic model in the previous sections to \( N \geq 2 \) firms. The purpose is to check the robustness of results and derive additional insights in a more general setting. Let \((K_l, K_s)\) denote the sourcing structure with \( K_l \) firms sourcing from \( S_l \) and \( K_s \) firms sourcing from \( S_s \). The total number of firms \( N \) is kept constant, so \((K_l, K_s)\) can also be written as \((K_l, N - K_l)\) or \((N - K_s, K_s)\). With \( N > 2 \), it is no longer appropriate to use subscripts \( A, B \) for the firms. Instead,
we use subscripts \(l, s\) to stand for the firms sourcing from \(S_l\) and \(S_s\), respectively. For instance, 
\(\Pi_i(K_l, K_s)\) \((i = 1, \ldots, K_s)\) is the profit for the \(i^{th}\) firm that sources from \(S_l\) under the structure \((K_l, K_s)\). Suppose the \(i^{th}\) firm that sources from \(S_l\) obtains a signal \(x_{it}\), \(i = 1, \ldots, K_l\); the \(j^{th}\) firm that sources from \(S_s\) obtains a signal \(x_{jt}\), \(j = 1, \ldots, K_s\). Recall that \(E[Cov[x_{it}, x_{tj} | u]] = \rho m_t\) for \(t' = t\) and \(E[Cov[x_{it}, x_{tj} | u]] = \rho_{12} \sqrt{m_t m_{t'}}\) for \(t' \neq t\). Define

\[
A_i(K_l, K_s) = \frac{\eta (m_1((K_l - 1) \rho + 2) + \eta - K_l \sqrt{m_1 m_2 \rho_{12}})}{(m_1 ((K_l - 1) \rho + 2) + (K_l + 1) \eta)(m_2 ((K_s - 1) \rho + 2) + (K_s + 1) \eta) - K_l K_s (\eta + \sqrt{m_1 m_2 \rho_{12}})^2},
\]

\[
A_s(K_l, K_s) = \frac{\eta (m_1((K_s - 1) \rho + 2) + \eta - K_s \sqrt{m_1 m_2 \rho_{12}})}{(m_1 ((K_l - 1) \rho + 2) + (K_l + 1) \eta)(m_2 ((K_s - 1) \rho + 2) + (K_s + 1) \eta) - K_l K_s (\eta + \sqrt{m_1 m_2 \rho_{12}})^2}.
\]

It can be shown that for a given sourcing structure \((K_l, K_s)\), there is a unique equilibrium in the Cournot subgame, where the firms’ profits are given by

\[
\Pi_i(K_l, K_s) = \Pi_i(K_l, K_s) = \left[ \frac{a + K_s w_l - (K_s + 1) w_t}{N + 1} \right]^2 + [A_i(K_l, K_s)]^2 (m_1 + \eta), \quad i = 1, \ldots, K_l \quad (9)
\]

\[
\Pi_s(K_l, K_s) = \Pi_s(K_l, K_s) = \left[ \frac{a + K_l w_t - (K_l + 1) w_s}{N + 1} \right]^2 + [A_s(K_l, K_s)]^2 (m_2 + \eta), \quad j = 1, \ldots, K_s \quad (10)
\]

Next we characterize the equilibrium conditions for the sourcing game. For ease of exposition, we focus on the case \(w_l < w_s\). Define a sequence of thresholds as follows: \(T_0 = -\infty\), \(T_{N+1} = +\infty\), and

\[
T_{K_l} = \frac{(N + 1)^2}{2N(w_s - w_l)} \left[ [A_s(K_l - 1, K_s + 1)]^2 (m_2 + \eta) - [A_s(K_l, K_s)]^2 (m_1 + \eta) \right] - \frac{1}{2} [(K_s - K_l) (w_s - w_l) - 2w_l], \quad \text{for } K_l = 1, \ldots, N.
\]

Then we have the following proposition.

**Proposition 7.** Consider the sourcing game with \(N \geq 2\) and \(w_l < w_s\). \((K_l, K_s)\) is the sourcing equilibrium if and only if \(T_{K_l} \leq a \leq T_{K_l+1}\).

Similar equilibrium conditions can be established for the case \(w_l = w_s\). Through extensive numerical experiments we find that \(T_{K_l}\) is strictly increasing in \(K_l\). This implies that there is a unique sourcing equilibrium unless \(a\) is exactly equal to some threshold \(T_{K_l}\). In that case, both \((K_l, K_s)\) and \((K_l - 1, K_s + 1)\) are equilibria. When \(N = 2\), the proposition reduces to Proposition 5. It has also been found that the comparative statics results in the two-firm setting still hold here. That is, more firms switch to responsive sourcing in equilibrium if the market size shrinks, the demand becomes more volatile, or the sourcing costs rise simultaneously.

\(^6\)For \(w_l = w_s\), the thresholds will change to \(T_{K_l} = a + [A_s(K_l - 1, K_s + 1)]^2 (m_2 + \eta) - [A_s(K_l, K_s)]^2 (m_1 + \eta)\), for \(K_l = 1, \ldots, N\).
5.1. Correlation Effect

In the two-firm setting, we find that firms may want to diversify their sourcing strategies due to the negative correlation effect. The correlation effect has been studied in the literature on information sharing under Cournot competition with substitutable products; see Vives (1999) for a comprehensive review. It has been observed that increasing the correlation of firms’ information induces less responsive output strategies and thus hurts the firms’ profits. Does this result hold in our sourcing problem setting? To address this question, we proceed to examine the impact of \( \rho \) on the firms’ equilibrium profits. Define

\[
\Upsilon = (\eta + \sqrt{m_1 m_2 \rho_{12}}) \left[ \sqrt{m_1 m_2 \rho_{12}} (K_l m_2 (K_s - 1) - K_s m_1 (K_l - 1)) + m_1 (K_l - 1) (2 m_2 + \eta) - m_2 (K_s - 1) (2 m_1 + \eta) \right].
\]

**Proposition 8.** Under the sourcing structure \((K_l, K_s)\), the responsive sourcing firms’ profits increase in \( \rho \) when

\[
(m_1 ((K_l - 1) \rho + 2) + \eta - K_l \sqrt{m_1 m_2 \rho_{12}})^2 m_2 (K_s - 1) \leq K_l \Upsilon,
\]

and the efficient sourcing firms’ profits increase in \( \rho \) when

\[
(m_2 ((K_s - 1) \rho + 2) + \eta - K_s \sqrt{m_1 m_2 \rho_{12}})^2 m_1 (K_l - 1) \leq -K_s \Upsilon.
\]

Proposition 8 states that given a fixed sourcing structure, a higher \( \rho \) may improve certain firms’ profits. One can easily construct examples where either the responsive or efficient sourcing firms’ equilibrium profits increase in \( \rho \). Consider \( N = 8, a = 17, w_s = 0.5, w_l = 0.4, \eta = 10, m_1 = 25, m_2 = 1, \) and \( \rho_{12} = 0 \). In this example, \((K_l = 5, K_s = 3)\) is the sourcing equilibrium regardless of the \( \rho \) value. It can be readily shown that \( A_s \) increases while \( A_l \) decreases in \( \rho \), which implies that the responsive sourcing firms will benefit from a higher correlation. Therefore, in our sourcing problem, higher signal correlation does not necessarily hurt all firms. We offer the following explanation for the discrepancy between our finding and the observation in the literature. In our model setting, the impact of a higher correlation can be decomposed into two effects. The first effect causes the firms to use more correlated output strategies, which always hurts the firms when they source from the same location. This is the effect that has been identified in the literature.

The second effect arises due to the asymmetry of the firms. Such an effect has not been fully explored in the literature as most existing studies consider symmetric firms.\(^7\) In the sourcing

\(^7\) There are studies in the information sharing literature that consider asymmetric settings (e.g., Novshek and Sonnenschein 1982). However, they do not identify the positive correlation effect because it is difficult to separate the correlation effect from the accuracy effect caused by information sharing.
problem, however, the firms can be asymmetric if they choose to source from different locations. There are two aspects of such an asymmetry: First, the firms’ signals may have different accuracy levels (i.e., \( m_1 > m_2 \)); second, the congestion levels may be different at the two locations (i.e., \( K_l \neq K_s \)). Consider the first aspect of asymmetry by fixing \( K_l \) and \( K_s \). In this case, we find that if \( m_1 - m_2 \) is sufficiently large (the accuracy asymmetry is significant enough), then increasing \( \rho \) always benefits the responsive sourcing firms. As \( m_1 - m_2 \) decreases to a certain range, then the responsive sourcing firms benefit from increasing \( \rho \) only when \( \rho \) is less than a threshold. To further explain, suppose \( m_1 - m_2 \) is sufficiently large and \( K_l = K_s \). Increasing \( \rho \) will change the correlations among the firms at \( S_l \) and among the firms at \( S_s \) simultaneously. The correlation change drives firms at each location to react less strongly to signals. Since the firms at \( S_s \) have much more accurate demand information (they already have a good estimate of the other responsive sourcing firms’ quantity decisions), an increased correlation \( \rho \) will have a smaller impact on the firms at \( S_s \) than on the firms at \( S_l \). That is, the firms at \( S_s \) will adopt relatively more responsive output strategies and benefit from an increased correlation.

Analogous observations have been obtained about the second aspect of asymmetry. That is, if \( K_l \) is sufficiently larger than \( K_s \), then increasing \( \rho \) will always benefit the responsive sourcing firms; if the difference between \( K_l \) and \( K_s \) decreases to a certain range, then the responsive sourcing firms benefit from increasing \( \rho \) only when \( \rho \) is less than a threshold. These results indicate that a change in the correlation structure may influence the firms’ output strategies differently depending on their sourcing locations. In particular, a higher \( \rho \) tends to have a greater negative impact on the firms either with less accurate demand information or in a more congested location.

### 5.2. Impact of Backshoring

The recent backshoring trend means that some firms are switching from \( S_l \) to \( S_s \). For a given market condition, how does such behavior affect other firms’ performances? This is a practical question because managers may wish to evaluate the impact of a change in their competitor’s sourcing strategy. We investigate the impact of backshoring on sourcing competition in this subsection.

First, the backshoring firm’s sourcing cost will increase, which affects the efficiency terms in all firms’ profits. Analysis shows that the efficiency terms in firms’ profits are increasing in \( K_s \) (\( K_s \) increases by 1 when a firm backshores). This means that more firms backshoring decreases the competition intensity on the sourcing cost dimension, and thus all firms are better off without considering the information term. It can also be shown that the expected procurement quantity of a firm sourcing from \( S_l \) is always greater than that of a firm sourcing from \( S_s \): \( E(Q_{l_i}) - E(Q_{s_j}) = w_s - w_l > 0 \). This is consistent with the empirical finding in Jain et al. (2013) that global sourcing
tends to increase firms’ inventory investments. Further, the total expected output \( \sum_{i=1}^{K_l} E(Q_{li}) + \sum_{j=1}^{K_s} E(Q_{sj}) \) equals \( \frac{K_l(w_s-w_l)}{N+1} + \frac{N}{N+1}(a-w_s) \), which is increasing in \( K_l \). This explains why the efficiency terms in both types of firms’ profit functions are decreasing in \( K_l \) but increasing in \( K_s \).

Second, the backshoring behavior has a two-fold informational impact on market competition. On one hand, it intensifies competition because the backshoring firm will obtain more accurate demand information; on the other hand, it alters the correlation structure of the signals in the Cournot subgame. It is straightforward to show that the first impact is always negative for the rest of the firms. However, the second impact caused by correlation change is more complex. In particular, the change may benefit certain firms. For illustration, consider an example where \( N = 4 \), \( \eta = 30 \), \( m_1 = m_2 = 10 \), \( \rho_{12} = 0 \), and \( \rho = 0.5 \). Suppose one firm backshoring shifts the sourcing structure from \( (K_l = 1, K_s = 3) \) to \( (K_l = 0, K_s = 4) \); then it can be shown that \( A_s \) (which determines the information term \( I_s \)) improves from 0.1613 to 0.1622. In this example, backshoring (weakly) increases the signal correlation between any two firms in the game, which makes all the responsive sourcing firms better off. This is in line with the finding in Section 5.1 that increased signal correlation may be beneficial to some firms. Due to the above countervailing forces, the information terms in firms’ profits may not be monotone in \( K_s \). For brevity reasons, we present the details of the relationship between backshoring and firms’ information terms in Appendix A (see Lemma 1).

In summary, backshoring improves the efficiency term in other firms’ profits, but it may or may not increase the information term. The next result is about the overall impact of backshoring on other firms’ performances for a given market condition.

**Proposition 9.** If \( \rho_{12} < \sqrt{m_2/m_1} \rho \), then there exist thresholds \( N' \) and \( K'_s \) such that any firm backshoring will benefit all other firms if \( N > N' \) and \( K_s > K'_s \).

According to Proposition 9, when the market is highly competitive (\( N \) is large), all the firms sticking to their original sourcing modes will benefit from any firm backshoring as long as the number of firms sourcing from \( S_s \) exceeds a threshold. This indicates that backshoring could be favorable from the industry’s perspective under the sufficient conditions provided above.\(^8\)

6. Concluding Remarks

Motivated by the boom of offshoring/outsourcing in the past few decades and also the recent new trend of backshoring, this paper develops a game-theoretic model to study firms’ competitive choice.

\(^8\) We have also studied the impact of a parameter change that may drive the backshoring behavior. In this case, we compare the firms’ equilibrium profits before and after the parameter change. Proposition 6 shows an example where an increase in sourcing cost might cause backshoring and benefit all the firms. Similar examples can be readily obtained by changing other parameters such as demand variability.
between efficient sourcing and responsive sourcing. The main findings and managerial insights from this paper can be summarized as follows.

First, we find that a firm may still choose efficient sourcing in equilibrium even when there is no cost advantage associated with it. By doing so, the firm can reduce the correlation between her own information and the competitor’s and thus dampen the competition. This finding highlights the role of information in firms’ sourcing strategies, which also provides a new explanation for outsourcing. That is, in certain competitive settings, firms may outsource to a long lead time supplier in order to reduce competition rather than to take advantage of low sourcing cost.

Second, we conduct comparative statics analysis to investigate how the equilibrium outcome depends on the problem parameters. Our result suggests that more volatile demand, shrinking market size, and rising global commodity prices are three possible factors that contribute to the recent backshoring trend. Interestingly, all else being equal, a cost hike in efficient sourcing may benefit all firms in the industry. In other words, the rising labor and logistics costs in emerging economies may improve all firms’ profits. The reason is that the cost increase may drive a firm to backshore, resulting in a new equilibrium sourcing structure that can alleviate market competition. This result, though unexpected, may bode well for the U.S. government, which is now calling for the insourcing of jobs as overseas costs rise rapidly.

Third, we extend the basic model to multiple firms. It can be shown that due to the asymmetry of the firms, increasing the information correlation may benefit some firms in our sourcing problem. This differs from the findings in most existing studies where the correlation effect is found to be negative under Cournot competition with substitutable products. Further, we examine the impact of backshoring on firms’ performances. It has been found that backshoring will reduce competition on the cost dimension, while its impact on the informational dimension is ambiguous. Overall, we show that the backshoring trend might benefit all the firms sticking to their original sourcing modes under certain conditions.

This work can be extended in several directions. The firms are restricted to sole sourcing in the current paper. One potential direction for future research is to allow firms to use both responsive and efficient sourcing simultaneously. In addition, this paper focuses on Cournot (quantity) competition. Whether the results continue to hold under other competition modes (e.g., price competition) remains an open question. Next, the sourcing costs are exogenously given in our model setting. Endogenizing the suppliers’ pricing decisions is also a promising research direction. Finally, our paper is among the first to identify the role of information structure (e.g., the correlation effect) in sourcing competition. It would be interesting to empirically investigate how the information structure actually drives firms’ competitive sourcing strategies in practice.
References


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**Appendix A: Proofs of Propositions**

**Proof of Proposition 1.** The proof is straightforward and therefore omitted.

**Proof of Propositions 2 and 3.** We provide a proof for the general case with $K_i$ firms sourcing from $S_i$ and $K_s$ firms sourcing from $S_s$. The total number of firms is $N = K_i + K_s$. The proof covers Propositions 2 and 3 as special cases.

Let $x_{i1}$ be the signal for the $i$th firm sourcing from $S_i (i = 1, \ldots, K_i)$, and $x_{j2}$ be the signal for the $j$th firm sourcing from $S_s (j = 1, \ldots, K_s)$. Then for $i \neq j$, $E[\text{Cov}[x_{i1}, x_{j2} | u]] = \rho_i \rho_j$ for $t' = t$ and $E[\text{Cov}[x_{i1}, x_{j2} | u]] = \rho_{i2} \sqrt{\text{Var}(x_{i1})} \sqrt{\text{Var}(x_{j2})}$ for $t' \neq t$.

Suppose the firms sourcing from $S_i$ adopt strategies $Q_{i}(x_{i1}), i = 1, \ldots, K_i$, and the firms sourcing from $S_s$ adopt strategies $Q_{sj}(x_{j2}), j = 1, \ldots, K_s$. The expected profit of firm $i$ sourcing from $S_i$ conditional on signal $x_{i1}$ is

$$\pi_i(x_{i1}) = Q_{i} E \left[ a - \left( Q_{i} + \sum_{j=1}^{K_i} Q_{sj} + \sum_{j=1}^{K_s} Q_{sj} \right) + u - w_i \right] x_{i1}.$$ 

The first-order condition yields

$$2Q_{i} = a - E \left[ \sum_{j=1, j \neq i}^{K_i} Q_{sj} x_{i1} \right] - E \left[ \sum_{j=1}^{K_s} Q_{sj} x_{i1} \right] + E[|u|x_{i1}] - w_i.$$ 

(11)

Similarly, the expected profit of firm $j$ sourcing from $S_s$ conditional on signal $x_{j2}$ is

$$\pi_s(x_{j2}) = Q_{sj} E \left[ a - \left( Q_{sj} + \sum_{i=1}^{K_i} Q_{i} + \sum_{i=1}^{K_s} Q_{si} \right) + u - w_s \right] x_{j2},$$

which leads to

$$2Q_{sj} = a - E \left[ \sum_{i=1}^{K_i} Q_{i} x_{j2} \right] - E \left[ \sum_{i=1, i \neq j}^{K_s} Q_{si} x_{j2} \right] + E[|u|x_{j2}] - w_s.$$ 

(12)

Define

$$\tilde{Q}_{i} = A_{i} x_{i1} + B_{i} w_t + C_{i} w_s + D_{i}, i \leq K_i,$$

$$\tilde{Q}_{sj} = A_{sj} x_{j2} + B_{sj} w_s + C_{sj} w_t + D_{sj}, j \leq K_s.$$ 

Let

$$g_i(x_{i1}) = Q_{i} - \tilde{Q}_{i} = Q_{i} - (A_{i} x_{i1} + B_{i} w_t + C_{i} w_s + D_{i}), i \leq K_i,$$

$$g_j(x_{j2}) = Q_{sj} - \tilde{Q}_{sj} = Q_{sj} - (A_{sj} x_{j2} + B_{sj} w_s + C_{sj} w_t + D_{sj}), j \leq K_s.$$
Then Equations (11) and (12) become

\[ g_i(x_{1i}) = -\sum_{j=1}^{K_i} E[g_i(x_{1j})|x_{1i}] - \sum_{j=1}^{K_i} E[g_j(x_{2j})|x_{1i}], \quad i \leq K_i, \]  
\[ g_j(x_{2j}) = -\sum_{i=1}^{K_s} E[g_j(x_{2i})|x_{2j}] - \sum_{i=1}^{K_s} E[g_i(x_{1i})|x_{2j}], \quad j \leq K_s, \]  

with the following eight simultaneous conditions:

\[-2A_{ii} - \sum_{j=1,j\neq i}^{K_i} A_{ij} \frac{\eta + \rho M_1}{\eta + M_1} = -\frac{\eta}{\eta + m_1} + \sum_{j=1}^{K_i} A_{ij} \frac{\eta + \rho M_2}{\eta + M_2}, \quad i = 1, \ldots, K_i, \]  
\[-2B_{ii} = \sum_{j=1,j\neq i}^{K_i} B_{ij} = 1 + \sum_{j=1}^{K_i} C_{ij}, \quad i = 1, \ldots, K_i, \]  
\[-2C_{ii} = \sum_{j=1,j\neq i}^{K_i} C_{ij}, \quad i = 1, \ldots, K_i, \]  
\[-2D_{ii} = \sum_{j=1,j\neq i}^{K_i} D_{ij} = -a + \sum_{j=1}^{K_i} D_{ij}, \quad i = 1, \ldots, K_i, \]  
\[-2A_{ij} - \sum_{k=1, k \neq j}^{K_s} A_{kj} \frac{\eta + \rho M_2}{\eta + M_2} = -\frac{\eta}{\eta + m_2} + \sum_{k=1}^{K_s} A_{kj} \frac{\eta + \rho M_1}{\eta + M_1}, \quad j = 1, \ldots, K_s, \]  
\[-2B_{ij} = \sum_{k=1, k \neq j}^{K_s} B_{kj} = 1 + \sum_{k=1}^{K_s} C_{ki}, \quad j = 1, \ldots, K_s, \]  
\[-2C_{ij} = \sum_{k=1, k \neq j}^{K_s} C_{ki}, \quad j = 1, \ldots, K_s, \]  
\[-2D_{ij} = \sum_{k=1, k \neq j}^{K_s} D_{ki} = -a + \sum_{k=1}^{K_s} D_{ki}, \quad j = 1, \ldots, K_s. \]  

Note that the system of equations is derived by applying the results:

\[ E[u|x_{1i}] = \left( \frac{\eta}{\eta + m_1} \right) x_{1i}, \]  
\[ E[x_{1t'}|x_{1i}] = \begin{cases} \left( \frac{\eta + \rho M_2}{\eta + M_2} \right) x_{1i}, & \text{if } t = t', \\ \left( \frac{\eta + \rho M_1}{\eta + M_1} \right) x_{1i}, & \text{if } t \neq t', \end{cases} \]

where \( i \neq j \). Such results can be derived by using Ericson (1969), Lemma 3 in Li (1985), the law of total variance, and the law of total covariance.

Multiplying Equation (13) by \( g_i(x_{1i}) \) and taking expectations, we obtain

\[ E \left[ (g_i(x_{1i}))^2 \right] = -E \left[ \sum_{j=1}^{K_i} g_i(x_{1j}) g_i(x_{1i}) + \sum_{j=1}^{K_i} g_j(x_{2j}) g_i(x_{1i}) \right], \quad i = 1, \ldots, K_i. \]  

Multiplying Equation (14) by \( g_j(x_{2j}) \) and taking expectations, we obtain

\[ E \left[ (g_j(x_{2j}))^2 \right] = -E \left[ \sum_{i=1}^{K_s} g_j(x_{2i}) g_j(x_{2j}) + \sum_{i=1}^{K_s} g_i(x_{1i}) g_j(x_{2j}) \right], \quad j = 1, \ldots, K_s. \]  

Summing Equations (16) for \( i = 1, \ldots, K_i \) and (17) for \( j = 1, \ldots, K_s \) yields

\[ E \left[ \sum_{i=1}^{K_i} (g_i(x_{1i}))^2 \right] + \sum_{j=1}^{K_s} (g_j(x_{2j}))^2 = -E \left[ \sum_{j=1}^{K_i} g_j(x_{1i}) + \sum_{j=1}^{K_s} g_j(x_{2j}) \right]^2. \]
Since the left-hand side of Equation (18) is non-negative, and the right-hand side is non-positive, the equality implies $E \left[ \sum_{i=1}^{K_1} (g_i(x_1))^2 + \sum_{j=1}^{K_2} (g_j(x_2))^2 \right] = 0$, which further implies $g_i(x_1) = g_j(x_2) = 0$, for $i = 1, \ldots, K_1$ and $j = 1, \ldots, K_2$. That is,

$$Q_{ti} = \tilde{Q}_{ti} = A_t x_i + B_t w_i + C_t w_s + D_t, \quad i \leq K_t,$$

$$Q_{sj} = \tilde{Q}_{sj} = A_s x_j + B_s w_s + C_s w_t + D_s, \quad j \leq K_s.$$

Therefore, the Cournot competition under the sourcing structure $(K_t, K_s)$ has a unique equilibrium in the linear form. Solving the system of equations in (15) leads to

$$A_t = A_{ti} = \frac{\eta (m_2 ((K_t - 1) \rho + 2) + \eta - K_s \sqrt{m_1 m_2 \rho_1})}{(m_1 ((K_t - 1) \rho + 2) + (K_t - 1) \eta) (m_2 ((K_t - 1) \rho + 2) + (K_t - 1) \eta) - K_t \eta ((\eta + \sqrt{m_1 m_2 \rho_1})^2),}

A_s = A_{sj} = \frac{\eta (m_2 ((K_s - 1) \rho + 2) + \eta - K_t \sqrt{m_1 m_2 \rho_1})}{(m_1 ((K_t - 1) \rho + 2) + (K_t - 1) \eta) (m_2 ((K_t - 1) \rho + 2) + (K_t - 1) \eta) - K_t \eta ((\eta + \sqrt{m_1 m_2 \rho_1})^2),}

B_t = -\frac{1 + K_t}{1 + \eta}, B_s = -\frac{1 + \eta}{1 + \eta}, C_t = \frac{K_t}{1 + \eta}, C_s = \frac{K_t}{1 + \eta},

D_t = D_s = \frac{\eta (m_2 + \eta - \sqrt{m_1 m_2 \rho_1})}{(m_1 + \eta) (m_2 + \eta) - ((\eta + \sqrt{m_1 m_2 \rho_1})^2) - \frac{m_1 + \eta}{(m_1 + \eta)}},

\begin{align*}
\Pi_{hl} &= \frac{\eta^2 (m_2 + \eta - \sqrt{m_1 m_2 \rho_1})^2}{(m_1 + \eta) (m_2 + \eta) - ((\eta + \sqrt{m_1 m_2 \rho_1})^2) - \frac{m_1 + \eta}{(m_1 + \eta)}}
> \eta^2
\end{align*}

where the first inequality is by the fact that $\frac{m_2 + \eta}{(m_1 + \eta)(m_2 + \eta) - ((\eta + \sqrt{m_1 m_2 \rho_1})^2)}$ decreases in $m_2$ for $y < 4 (m_1 + \eta) (m_2 + \eta)$, and the second inequality is by the fact that $2 (m_1 + \eta) + \eta + \sqrt{m_1 m_2 \rho_1} - (m_1 + \eta) = 2 (m_1 + \eta) + \eta + \sqrt{m_1 m_2 \rho_1}$ decreases in $m_2$ for $y < 4 (m_1 + \eta) (m_2 + \eta)$.

Next, we examine $\Pi_{hl}^* - \Pi_{hl}^{**}$. It can be shown $\Pi_{hl}^* - \Pi_{hl}^{**} = \frac{\eta^2 (m_2 + \eta - \sqrt{m_1 m_2 \rho_1})^2 (m_2 + \eta)}{4 (m_1 + \eta)(m_2 + \eta) - ((\eta + \sqrt{m_1 m_2 \rho_1})^2) - \frac{m_1 + \eta}{(m_1 + \eta)}} - \frac{\eta^2 (m_2 + \eta)}{(m_1 + \eta)(m_2 + \eta) - ((\eta + \sqrt{m_1 m_2 \rho_1})^2) - \frac{m_1 + \eta}{(m_1 + \eta)}}$.

The sign of $\Pi_{hl}^* - \Pi_{hl}^{**}$ is determined by $F(m_2) = \frac{\eta^2 (m_2 + \eta - \sqrt{m_1 m_2 \rho_1})^2 (m_2 + \eta)}{4 (m_1 + \eta)(m_2 + \eta) - ((\eta + \sqrt{m_1 m_2 \rho_1})^2) - \frac{m_1 + \eta}{(m_1 + \eta)}} - \frac{\eta^2 (m_2 + \eta)}{(m_1 + \eta)(m_2 + \eta) - ((\eta + \sqrt{m_1 m_2 \rho_1})^2) - \frac{m_1 + \eta}{(m_1 + \eta)}}$.

It is straightforward to show $F < 0$ for $m_2 = 0$, and $F > 0$ for $m_2 = m_1$ and $\rho_1 < \sqrt{m_2/m_1} = \rho$. Since $F$ is
a continuous function of $m_2$, by the intermediate value theorem, there exists at least one $m_2 \in (0, m_1)$ such that $F(m_2) = 0$. In fact we can prove that such an $m_2$ is unique.

Let $\Xi = \frac{(m_2 + \eta \sqrt{m_1 m_2})^2}{4(m_1 + \eta)(m_2 + \eta) - (m_1 + \eta)^2 - m_2 (2 + \rho)}$. Then $F(m_2) = 2 \Xi^2 - \frac{m_2 + \rho}{m_1 + \eta}$. In the following, we show that given $F(m_2) \geq 0$ (i.e., $\Xi \geq \sqrt{\frac{m_2 + \eta}{m_1 + \eta}}$), $F'(m_2) > 0$ holds. This implies that as $m_2$ increases, once $F(m_2)$ becomes positive, it will stay positive, so there is only one $m_2 \in (0, m_1)$ satisfying $F(m_2) = 0$. Note that

$$F'(m_2) = 2 \Xi \left[ 2 - \frac{1}{2} \frac{\sqrt{m_1 m_2 \rho_2}}{m_2 + \eta - \sqrt{m_1 m_2 \rho_2}} + \frac{2 + \rho}{m_2 (2 + \rho) + 3 \eta} - \frac{4(m_1 + \eta) - (\eta \sqrt{m_1 m_2 \rho_2})}{4(m_1 + \eta)(m_2 + \eta) - (\eta \sqrt{m_1 m_2 \rho_2})^2} \right] - \frac{1}{m_1 + \eta}.$$  

We can write

$$4(m_1 + \eta) - (\eta + \sqrt{m_1 m_2 \rho_2}) \sqrt{m_1/m_2 \rho_1} = \frac{2 - \left( \eta + \sqrt{m_1 m_2 \rho_2} \right) \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)} - \frac{2 - x \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)}.$$  

where $x = \frac{2 - \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)}$. By the condition $m_2 \rho > \sqrt{m_1 m_2 \rho_2}$ (i.e., $\rho_1 < \rho \sqrt{m_1/m_2}$), it can be shown that

$$\frac{2 - \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)} = \frac{2 - \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)} + \frac{1}{2} \frac{2 - \sqrt{m_1/m_2 \rho_1}}{m_2 + \eta} = \frac{2 - \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)} + \frac{1}{2} \frac{2 - \sqrt{m_1/m_2 \rho_1}}{m_2 + \eta}.$$  

which implies

$$F'(m_2) \geq 2 \Xi \left[ 2 - \frac{\sqrt{m_1/m_2 \rho_1}}{2m_2 + \eta - \sqrt{m_1 m_2 \rho_2}} + \frac{2 + \rho}{m_2 (2 + \rho) + 3 \eta} - \frac{2 - \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)} - \frac{2 - \sqrt{m_1/m_2 \rho_1}}{2(m_2 + \eta)} \right] - \frac{1}{m_1 + \eta}.$$  

Since $\Xi \geq \sqrt{\frac{m_2 + \eta}{m_1 + \eta}}$, we know $F'(m_2) > 0$. Up to now, we have shown $F'(m_2) > 0$ if $F(m_2) \geq 0$, which implies there is only one $m_2 \in (0, m_1)$ satisfying $F(m_2) = 0$. Then there is a unique $\xi \in (0, 1)$, such that $\Pi^u \xi < \Pi^a \xi$ for $m_2/m_1 < \xi$ and $\Pi^a \xi > \Pi^u \xi$ for $m_2/m_1 > \xi$. Thus, for $m_2 > \xi m_1$, firm A will source from $S_t$ and $(S_t, S_s)$ is the equilibrium; for $m_2 < \xi m_1$, firm A will source from $S_s$ and $(S_s, S_s)$ is the equilibrium; for $m_2 = \xi m_1$, both $(S_s, S_s)$ and $(S_s, S_s)$ are equilibria.

**Proof of Proposition 5.** The equilibrium of the game is determined by comparing the profit functions in the $2 \times 2$ matrix in Figure 2. For $(S_t, S_t)$ to be an equilibrium, we need $\Pi^u_B \geq \Pi^u_A$ and $\Pi^a_B \geq \Pi^a_A$. Since $\Pi^u_B = \Pi^u_A$ and $\Pi^a_B = \Pi^a_A$, we know $\Pi^u_B \geq \Pi^u_B$ if and only if $\Pi^a_B \geq \Pi^a_A$. Since

$$\Pi^u_B - \Pi^a_B = \frac{1}{9} (a - w_t)^2 - \frac{1}{9} (a - 2w_s + w_t)^2 - \Gamma_B = \frac{1}{9} (w_t - w_s - \Gamma_B),$$

we know $\Pi^u_B \geq \Pi^a_B$ if and only if $a \geq w_s + \frac{9}{4} \frac{\Gamma_B}{w_t - w_s} = T_2$. Thus, $(S_t, S_t)$ is an equilibrium if $a \geq T_2$.

For $(S_t, S_s)$ to be an equilibrium, we need $\Pi^u_B \geq \Pi^a_B$ and $\Pi^a_A \geq \Pi^a_A$. By Equation (19), $\Pi^u_B \geq \Pi^a_B$ if and only if $a \leq T_2$. Since

$$\Pi^a_A - \Pi^a_A = \frac{1}{9} (a - 2w_t + w_s)^2 - \frac{1}{9} (a - w_s)^2 - \Gamma_A = \frac{1}{9} (w_s - w_t - \Gamma_A),$$

we know $\Pi^a_A \geq \Pi^a_A$ if and only if $a \geq w_t + \frac{9}{4} \frac{\Gamma_A}{w_t - w_s} = T_1$. Thus, $(S_t, S_s)$ is an equilibrium if $T_1 \leq a \leq T_2$. 


For \((S_s, S_s)\) to be an equilibrium, we need \(\Pi^s_{l^*} \leq \Pi^s_{B^*}\) and \(\Pi^s_{A^*} \leq \Pi^s_{B^*}\). Since \(\Pi^s_{B^*} = \Pi^s_{A^*}\) and \(\Pi^s_{B^*} = \Pi^s_{A^*}\), we know \(\Pi^s_{l^*} \leq \Pi^s_{B^*}\) if and only if \(\Pi^s_{A^*} \leq \Pi^s_{B^*}\). By Equation (20), \(\Pi^s_{A^*} \geq \Pi^s_{1}\) if and only if \(\alpha \leq T_1\). Thus, \((S_s, S_s)\) is an equilibrium if \(\alpha \leq T_1\). Combining the above three cases completes the proof.

**Proof of Proposition 6.** Since \(a > w_s + \frac{9}{4} \frac{\Gamma_B}{a - w_s} = T_2\), the current equilibrium is \((S_s, S_s)\). Suppose the value of \(w_s\) increases by \(\tau\). Then, we use the superscript \(\tau\) to denote the thresholds and expected profits after the cost increase, e.g., \(T_2^\tau = w_s + \frac{9}{4} \frac{\Gamma_B}{a - w_s + \tau}\). Next, we show that \(\tau > \bar{\tau}\) implies \(a < T_2^\tau\), i.e.,

\[
\tau > w_s - w_l - \frac{9}{4} \frac{\Gamma_B}{a - w_s}.
\]

Note that 
\[
\frac{(a - w_s)^2 - 9\Gamma_B}{2} - (a - w_s - \frac{9}{4} \frac{\Gamma_B}{a - w_s})^2 = (w_s - w_l)(2a - w_s - w_l) - \frac{81}{16} (a - w_s)\tau - \frac{9}{2} \Gamma_B.
\]

Thus, \(\tau > w_s - w_l - \frac{9}{4} \frac{\Gamma_B}{a - w_s}\) and \(\tau > \bar{\tau}\) imply \(\tau > w_s - w_l - \frac{9}{4} \frac{\Gamma_B}{a - w_s}\).

It is straightforward to show that \(\tau < \bar{\tau}_{1}\) implies \(a > T_1^\tau\). Then, \(\tau < \bar{\tau} < \bar{\tau}_{1}\) implies \(T_1^\tau < a < T_{1^*}\) and the new sourcing equilibrium is \((S_s, S_s)\). Without loss of generality, suppose firm B switches from efficient sourcing to responsive sourcing. Then, firm B's expected profit changes by \(\Pi^A_{B^*} - \Pi^A_{l^*} = \frac{1}{8} (a - 2w_s + w_l + \tau) - \frac{1}{8} (a - w_s)^2 + \Gamma_B\). Firm A still sources from \(S_s\) and firm A's expected profit changes by \(\Pi^A_{B^*} - \Pi^A_{l^*} = \frac{1}{8} (a - 2w_s + w_l + \tau) + w_s)^2 + \frac{\eta^2(2m_2 + \eta - \sqrt{m_1 m_2 \rho_1})^2 (m_1 + \eta)}{(m_1 + \eta)(m_2 + \eta)} - \frac{\eta^2(2m_2 + \eta - \sqrt{m_1 m_2 \rho_1})^2 (m_1 + \eta)}{(m_1 + \eta)(m_2 + \eta)}\). Firm A switches from \(S_s\) to \((S_s, S_s)\) and both firms are better off.

**Proof of Proposition 7.** \((K_t, K_s)\) is an equilibrium sourcing structure if and only if \(\Pi_t(K_t, K_s) \geq \Pi_s(K_t - 1, K_s + 1)\) and \(\Pi_s(K_t, K_s) \geq \Pi_t(K_t + 1, K_s - 1)\), where the first condition guarantees the efficient sourcing firms will not deviate while the second condition guarantees the responsive sourcing firms will not deviate. It is straightforward to show that \(\Pi_t(K_t, K_s) \geq \Pi_s(K_t - 1, K_s + 1)\) is equivalent to \(a \geq T_{K_t}\), and \(\Pi_s(K_t, K_s) \geq \Pi_t(K_t + 1, K_s - 1)\) is equivalent to \(a \leq T_{K_s + 1}\). Thus, \((K_t, K_s)\) is an equilibrium if and only if \(T_{K_t} \leq a \leq T_{K_s + 1}\). Note that when \(a = T_{K_t}\), both \(T_{K_t - 1} \leq a \leq T_{K_t}\) and \(T_{K_s - 1} \leq a \leq T_{K_t + 1}\) hold, i.e., both \((K_t - 1, K_s + 1)\) and \((K_t, K_s)\) are equilibria.

**Proof of Proposition 8.** We can write \(A_s\) as \(\eta/A_s\), where

\[
A_s = \frac{m_2((K_t - 1) \rho + 2) + \eta + K_s (2\eta + \sqrt{m_1 m_2 \rho_1})}{(K_t - 1)(K_t - 1) \rho + 2 - K_s m_1 ((K_t - 1) \rho + 2) - K_s K_t \eta \rho + K_t \eta (\eta + \sqrt{m_1 m_2 \rho_1})}
\]

Then \(A_s > 0\) always holds, and \(\frac{dA_s}{d\rho} = m_2(K_t - 1) + \frac{-K_t}{(m_1((K_t - 1) \rho + 2) + \eta + K_t \sqrt{m_1 m_2 \rho_1})} \eta, \) \(A_s\) increases in \(\rho\) if and only if \(\frac{dA_s}{d\rho} < 0\), which is equivalent to \(m_1((K_t - 1) \rho + 2) + \eta - K_t \sqrt{m_1 m_2 \rho_1}) \leq K_t \eta, \)

Similarly, we can write \(A_t\) as \(\eta/A_t\), where

\[
A_t = \frac{m_1((K_t - 1) \rho + 2) + \eta + K_t (2\eta + \sqrt{m_1 m_2 \rho_1})}{m_1((K_t - 1) \rho + 2) + \eta - K_t \sqrt{m_1 m_2 \rho_1})}
\]
where

\[ (-K_1 m_2 ((K_s - 1) \rho + 2) + K_s m_1 ((K_s - 1) \rho + 2) + K_s \eta - K_s \eta) \left( \eta + \sqrt{m_1 m_2 \rho_{12}} \right) \]

\[ m_2 ((K_s - 1) \rho + 2) + \eta - K_s \sqrt{m_1 m_2 \rho_{12}}. \]

Then \( \Lambda_s > 0 \) always holds, and \( \frac{d \Lambda_s}{d \rho} = m_1 (K_s - 1) + \frac{K_s \eta}{m_2 ((K_s - 1) \rho + 2)} - K_s \sqrt{m_1 m_2 \rho_{12}} \). \( \Lambda_s \) increases in \( \rho \) if and only if \( \frac{d \Lambda_s}{d \rho} \leq 0 \), which is equivalent to \( \left( m_2 ((K_s - 1) \rho + 2) + \eta - K_s \sqrt{m_1 m_2 \rho_{12}} \right)^2 m_1 (K_s - 1) \leq -K_s \eta. \]

To prove Proposition 9, we first present Lemma 1. Define \( \Psi = m_1 (m_2 - \sqrt{m_1 m_2 \rho_{12}}) (2 - \rho) + \eta \left( m_2 - m_1 \right) (2 - \rho) + m_1 \rho - \sqrt{m_1 m_2 \rho_{12}} \).

**Lemma 1.** Consider the sourcing game with \( N \geq 2 \) firms. Let \( H_j(K_s, K_s) \) and \( I_j(K_s, K_s) \) be the efficiency and information terms in \( \Pi_j(K_s, K_s) \) (\( j = s, l \)), respectively.

(i) Both \( H_j(K_s, K_s) \) and \( I_j(K_s, K_s) \) increase in \( K_s \).

(ii) If \( \rho_{12} < \sqrt{m_2/m_1} \rho \), then there exists a threshold \( \hat{N} \) such that \( I_j(K_s, K_s) \) always decreases in \( K_s \) for \( N < \hat{N} \), and \( I_j(K_s, K_s) \) first decreases and then increases in \( K_s \) for \( N > \hat{N} \). If \( \rho_{12} = \sqrt{m_2/m_1} \rho \), then \( I_j(K_s, K_s) \) always decreases in \( K_s \).

(iii) If \( \Psi \leq 0 \), then there exists a threshold \( \hat{N} \) such that \( I_j(K_s, K_s) \) always decreases in \( K_s \) for \( N \leq \hat{N} \), and \( I_j(K_s, K_s) \) first decreases and then increases in \( K_s \) for \( N > \hat{N} \).

**Proof of Lemma 1.** The profit functions can be written as

\[ \Pi_j(K_s, K_s) = H_j(K_s, K_s) + I_j(K_s, K_s), \]

where

\[ H_j(K_s, K_s) = \frac{(a + K_s \omega_j - (K_s + 1) \omega_j)}{N + 1} \]

\[ I_j(K_s, K_s) = \frac{(a + K_s \omega_j - (K_s + 1) \omega_j)}{N + 1} \]

(i) The proof is straightforward.

(ii) The sensitivity analysis of \( I_j(K_s, K_s) \) and \( I_j(K_s, K_s) \) with respect to \( K_s \) is equivalent to that of \( A_s \) and \( A_s \). By observation, the numerator of \( A_s \) strictly decreases in \( K_s \). Let the denominator of \( A_s \) be \( \Theta \). It can be shown that \( \frac{d \Theta}{d K_s} \) decreases in \( K_s \), and \( \frac{d \Theta}{d K_s} \mid_{K_s = N/2} \geq 0 \). Thus \( \Theta \) increases in \( K_s \) for \( K_s \leq N/2 \). Combining the analysis for the numerator and denominator of \( A_s \), we know that \( A_s \) strictly decreases in \( K_s \) for \( K_s \leq N/2 \).

To further understand the sensitivity of \( A_s \) with respect to \( K_s \), we take derivative of \( A_s \) and find that the sign of \( \frac{d A_s}{d K_s} \) is determined by \( W_s = C_{s1} K_s^2 + C_{s2} K_s + C_{s3} \), where

\[ C_{s1} = - (m_1 \rho - \sqrt{m_1 m_2 \rho_{12}}) \eta (m_1 \rho + m_2 \rho - 2 \sqrt{m_1 m_2 \rho_{12}} - m_1 m_2 (\rho^2 - \rho_{12}^2)) < 0, \]

\[ C_{s2} = 2 \eta^2 (m_1 \rho + m_2 \rho - 2 \sqrt{m_1 m_2 \rho_{12}}) + 2 \eta \rho (2 m_1^2 + 2 m_1 m_2) - 4 m_1 \sqrt{m_1 m_2 \rho_{12}} \]

\[ + (N - 1) (m_1 \rho - \sqrt{m_1 m_2 \rho_{12}})^2 + N (m_1 \rho - \sqrt{m_1 m_2 \rho_{12}}) (m_2 \rho - \sqrt{m_1 m_2 \rho_{12}}) \]

\[ + 2 (\rho^2 - \rho_{12}^2) m_1 m_2 (m_1 (2 + (N - 1) \rho) - N \sqrt{m_1 m_2 \rho_{12}}), \]
and $C_{33}$ is a long expression of $N$. Thus $W_s$ is a concave function of $K_s$. The above analysis implies $W_s < 0$ for $K_s \leq N/2$. Based on the expression of $W_s$, $W_s$ is maximized at $K_s = -C_{22}/(2C_{11}) > 0$. It can be shown that $\frac{dW_s}{dK_s}|_{K_s=N} > 0$, i.e., $K_s > N$. Thus, $W_s$ increases in $K_s$ for $K_s \leq N$. Now we analyze $W_s$ at $K_s = N$. $W_s|_{K_s=N} = (m_1(2 - \rho) + \eta) \left( (2 - \rho) m_2 + \eta \right) \left( \frac{2 \eta (1 - \rho) (m_1 - m_2)}{m_2 - \sqrt{m_1 m_2}} \right) + N (m_1(2 - \rho) + \eta) (m_2 - \sqrt{m_1 m_2}) \left( \frac{2 \eta (1 - \rho) (m_1 - m_2)}{m_2 - \sqrt{m_1 m_2}} \right)$, where the coefficient of $N$ is non-negative and the constant term is negative.

For $m_2 \rho > \sqrt{m_1 m_2}$, if $N > \frac{\sqrt{m_1 m_2}}{2 (m_2 - \sqrt{m_1 m_2}) (\sqrt{m_1 m_2} + \eta)} \equiv \tilde{N}$, $W_s|_{K_s=N} > 0$, which implies $W_s$ is first negative and then positive. That is, $A_t$ first decreases and then increases in $K_s$. If $N \leq \tilde{N}$, $W_s|_{K_s=N} \leq 0$, which implies $W_s < 0$ for $K_s < N$; that is, $A_t$ always decreases in $K_s$.

For $m_2 \rho = \sqrt{m_1 m_2}$, $W_s|_{K_s=N} < 0$, i.e., $A_t$ always decreases in $K_s$.

(iii) We take derivative of $A_t$ with respect to $K_s$ and find that the sign of $\frac{dA_t}{dK_s}$ is determined by $W_t = C_{11} K_s^2 + C_2 K_s + C_3$, where $C_{11} = (\rho^2 - \rho_1^2) m_1 m_2 (m_2 - \sqrt{m_1 m_2}) + \eta (m_2 - \sqrt{m_1 m_2}) \geq 0$ with equality holds when $m_2 \rho = \sqrt{m_1 m_2}$.

For $m_2 \rho > \sqrt{m_1 m_2}$, $W_t|_{K_s=N} < 0$, i.e., $A_t$ always decreases in $K_s$. If $N > \tilde{N}$, $C_{33} < 0$. That is, $W_t|_{K_s=N} < 0$. Then, we examine $W_t$ at $K_s = N$. We have $W_t|_{K_s=N} = C_{N1} N^2 + C_{N2} N + C_{N3}$, where $C_{N1} = (\rho^2 - \rho_1^2) m_1 m_2 (m_2 - \sqrt{m_1 m_2}) + \eta (m_2 - \sqrt{m_1 m_2}) \geq 0$ with equality holds when $m_2 \rho = \sqrt{m_1 m_2}$.

For $m_2 \rho = \sqrt{m_1 m_2}$, $W_t|_{K_s=N}$ is a convex function of $K_s$ and increases in $K_s$. If $N > \tilde{N}$, $C_{N3} < 0$. That is, $A_t$ always increases in $K_s$. If $N < \tilde{N}$, $C_{N3} > 0$. That is, $A_t$ always decreases in $K_s$. For $N > \tilde{N}$, $A_t$ first decreases and then increases in $K_s$. If $N < \tilde{N}$, $A_t$ always increases in $K_s$. For $N > \tilde{N}$, $W_t|_{K_s=N} < 0$, and thus $A_t$ first decreases and then increases in $K_s$. If $N \leq \tilde{N}$, $W_t|_{K_s=N} < 0$ holds. Let $\tilde{N}$ denote the root of $W_t|_{K_s=N} = 0$ when $C_{N1} = 0$ and the larger root of $W_t|_{K_s=N} = 0$ when $C_{N1} > 0$, then $W_t|_{K_s=N} \leq 0$ for $N \leq \tilde{N}$ and $W_t|_{K_s=N} > 0$ for $N > \tilde{N}$, which implies $A_t$ always decreases in $K_s$ for $N \leq \tilde{N}$, and $A_t$ first decreases and then increases in $K_s$ for $N > \tilde{N}$.

If $\Psi \geq 0$ (i.e., $C_{N3}, \tilde{N} \leq 0$), then $N > \tilde{N}$ always holds, i.e., $W_t|_{K_s=N} < 0$. Then, we examine $W_t$ at $K_s = N$. We have $W_t|_{K_s=N} = C_{N1} N^2 + C_{N2} N + C_{N3}$, where $C_{N1} = (\rho^2 - \rho_1^2) m_1 m_2 (m_2 - \sqrt{m_1 m_2}) + \eta (m_2 - \sqrt{m_1 m_2}) \geq 0$ with equality holds when $m_2 \rho = \sqrt{m_1 m_2}$.

If $C_{N2} = (\rho^2 - \rho_1^2) m_1 m_2 (m_2 - \sqrt{m_1 m_2}) + \eta (m_2 - \sqrt{m_1 m_2}) \geq 0$, and

If $C_{N3} = (\rho^2 - \rho_1^2) m_1 m_2 (m_2 - \sqrt{m_1 m_2}) + \eta (m_2 - \sqrt{m_1 m_2}) \geq 0$, and

If $\Psi \leq 0$ (i.e., $C_{N3}, \tilde{N} \leq 0$), then $N < \tilde{N}$ always holds, i.e., $W_t|_{K_s=N} > 0$ holds. Let $\tilde{N}$ denote the root of $W_t|_{K_s=N} = 0$ when $C_{N1} = 0$ and the larger root of $W_t|_{K_s=N} = 0$ when $C_{N1} > 0$, then $W_t|_{K_s=N} \leq 0$ for $N \leq \tilde{N}$ and $W_t|_{K_s=N} > 0$ for $N > \tilde{N}$, which implies $A_t$ always decreases in $K_s$ for $N \leq \tilde{N}$, and $A_t$ first decreases and then increases in $K_s$ for $N > \tilde{N}$.

If $\Psi > 0$ (i.e., $C_{N3}, \tilde{N} > 0$), then $W_t|_{K_s=N} > 0$. For $N \leq \tilde{N}$, $A_t$ always increases in $K_s$. For $N > \tilde{N}$, $W_t|_{K_s=N} < 0$, and thus $A_t$ first decreases and then increases in $K_s$.

**Proof of Proposition 9.** According to Lemma 1, all efficiency terms $H_i$ and $H_s$ increase in $K_s$. In addition, for $\rho_1 < \sqrt{m_2/m_1}$, if we take $N' = \max[N, \tilde{N}]$, then for $N > N'$, all information terms $I_i$ and $I_s$ increase in $K_s$ for $K_s$ greater than a threshold, say $K'_s$. Together we know that all firms' profits increase in $K_s$ for $N > N'$ and $K_s > K'_s$. This implies that backshoring by any firm will benefit all other firms in the market if $N > N'$ and $K_s > K'_s$. ■
Appendix B: Proof of $\rho_{12} \leq \sqrt{m_2/m_1}\rho$

In this appendix, we show that in the special case described in Section 3, the correlation coefficient $\rho_{12}$ for cross-time signal noises satisfies $\rho_{12} \leq \sqrt{m_2/m_1}\rho$, where $\rho$ is the correlation coefficient for the signal noises at the same time.

First, we consider the correlation when both firms source from $S_i$ (the case when both firms source from $S_s$ is similar). Suppose firm A’s agency obtains a sample set $G_{1A} = \{g_1^{1A}, g_2^{1A}, \cdots, g_{n_1}^{1A}\} \subseteq G_1$. Then, firm A’s signal $x_{1A}$ is the agency’s best estimate of $u$ that maximizes the likelihood function based on the $n_1$ samples: $x_{1A} = \frac{1}{n_1} \sum_{i=1}^{n_1} g_i^{1A}$. Suppose firm B’s agency receives a sample set $G_{1B} = \{g_1^{1B}, g_2^{1B}, \cdots, g_{n_2}^{1B}\} \subseteq G_1$. Then, firm B’s signal is $x_{1B} = \frac{1}{n_2} \sum_{i=1}^{n_2} g_i^{1B}$, the best estimate of $u$ based on $G_{1B}$. It is clear that the variance of the signal noise is $m_1 = \frac{\Delta}{n_1}$, where $\Delta$ is the variance of the sample noises in $G_1$. Assume that there are $\zeta$ common samples in $G_{1A}$ and $G_{1B}$. Then we have $m_1 \rho = \text{Cov}[x_{1A} - u, x_{1B} - u] = \text{Cov} \left[ \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1A,i} - \frac{1}{n_2} \sum_{i=1}^{n_2} x_{1B,i} \right] = \frac{\zeta \Delta}{n_1} = \frac{\zeta}{n_1} m_1$. So the coefficient of correlation between the noises in the firms’ signals is given by

$$\rho = \frac{\zeta}{n_1}. \tag{22}$$

Next, we consider the case when firm A sources from $S_i$ and firm B sources from $S_s$. Again suppose firm A’s agency gets a signal $x_{1A} = \frac{1}{n_1} \sum_{i=1}^{n_1} g_i^{1A}$ based on $G_{1A}$, while firm B’s agency gets a signal $x_{2B}$ based on the sample set $G_{2B} = \{g_1^{2B}, g_2^{2B}, \cdots, g_{n_2}^{2B}\}$, where $g_i^{2B}$ are the samples from $G_{1B}$ and $g_i^{\prime 2B}$ are the samples from $G_2$ ($n' \leq n_1$, $n'' \leq n$, and $n' + n'' = n_2$). Then $x_{2B} = \frac{\sum_{i=1}^{n_2} g_i^{2B} + \sum_{i=1}^{n'} g_i^{\prime 2B}}{n_2 + n'}$ is the best estimate of $u$ based on these $n' + n''$ samples in $G_{2B}$, and the variance of the signal noise is

$$m_2 = \frac{1}{\frac{n'}{\tau_1} + \frac{n''}{\tau_2}}. \tag{23}$$

Assume that there are $\zeta'$ common samples in $G_{1A}$ and $G_{2B}$. There must be $\zeta' \leq \zeta$ since the common samples can only come from $G_{1B}$, where there are at most $\zeta$ common samples with $G_{1A}$. It can be shown that $\text{Cov}[x_{1A} - u, x_{2B} - u] = \text{Cov} \left[ \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1A,i} - \frac{\sum_{i=1}^{n'} g_i^{\prime 2B}}{n_2 + n'} \right] = \frac{\zeta' \Delta}{n_1 (\frac{n'}{\tau_1} + \frac{n''}{\tau_2})} = \frac{\zeta'}{n_1 (\frac{n'}{\tau_1} + \frac{n''}{\tau_2})}$. That is,

$$\rho_{12} \sqrt{m_1 m_2} = \frac{\zeta'}{n_1 (\frac{n'}{\tau_1} + \frac{n''}{\tau_2})}. \tag{24}$$

Given $\zeta' \leq \zeta$, Equations (22), (23), and (24) lead to $\rho_{12} \sqrt{m_1 m_2} \leq m_2 \rho$. Finally, note that the equality holds when $\zeta' = \zeta$, i.e., firm B uses all the common samples if she would collect at time 1 only. ■