Modeling Multi-Channel Advertising Attribution Across Competitors

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Abstract

The bursts and multiplicity of Internet advertising have made multi-channel attribution an immediate challenge for marketing practitioners. Existing attribution models predominantly focus on analyzing consumers' converting path with respect to one focal firm while largely overlooking the impact of their interactions with competing firms, leading to biased advertising effectiveness estimates. We address this problem by developing an integrated individual-level choice model that considers consumers' online visit and purchase decisions across all competitors within one industry. We specifically analyze the effects of multi-channel advertising on: (1) consumer choice of entry site, (2) consumer search decisions concerning the remaining websites that compete in the same industry, and (3) subsequent purchase at one of the searched websites. We quantify the impact of different digital advertising channels on consumers' decisions at different purchase funnel stages based on an individual-level click stream data for the online air ticket booking industry. We find that information stock collected through all advertising channels contributes significantly to consumers' visit and purchase decisions, among which search advertising is more effective in driving the choice of entry site while email advertising has a larger effect on visit decision concerning remaining websites and purchase decision. The own- and cross-marginal impacts of various ad channels on each firm vary widely across competitors, and this is true at all purchase funnel stages. We also show that neglecting competition may lead to underestimated advertising effects and worse predictions, by comparing the estimated advertising effectiveness and predictive performance of our proposed model with those of the common baseline model that only models consumers' binary purchase decisions on a focal firm.

Keywords: multi-channel attribution, competition, purchase funnel

1. Introduction

The advancement of digital technology has enabled firms to reach consumers through a variety of advertising channels such as search engines, email, referral sites, and display ads. The United States digital advertising market is estimated to reach US $72.09 billion by the end of 2016, surpassing the same year TV ad spending for the first time (emarketer.com, 2016).[1] Consumers have also become more shopping savvy than ever before as many of them visit multiple stores repeatedly while seeking the best deal before making a purchase (Park and Fader 2004). During this process it is highly likely that consumers are exposed to many ads posted by different advertisers through multiple digital channels at different points in their purchase journeys. It is therefore vital for firms to have a sound understanding of ad attribution and analytics regarding the effects of various types of online advertisement across multi-channels and multi-touch-points (MCMT hereafter).

Determining the value of digital advertising channels has attracted a growing interest recently among researchers in Information Systems (e.g. Ghose and Todri 2015, Xu et al. 2014) and Marketing (e.g. Li and Kannan 2014). The effectiveness of cross-channel advertising was initially assessed with simple heuristic attribution rules. The "last-touch" model, the linear model, the time decay model, and the position-based model are all examples of ad attribution models using heuristic rules (Tornquist and Tradewell 2012). However, these models often do not fully consider consumers' entire purchase funnel (Li and Kannan 2014). To address this

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[1] https://www.emarketer.com/Article/US-Digital-Ad-Spending-Surpass-TV-this-Year/1014469, last accessed on 12/13/2016.

issue, researchers have proposed a number of algorithmic attribution models in recent years as richer multi-channel data become available (e.g., Li and Kannan 2014, Abhishek et al. 2014, Xu et al. 2014). Industry leaders such as Visual IQ, Convertro and Google also provide their clients with advanced algorithmic-based models for understanding marketing performance across multiple channels (Forrester 2014). These data-driven models attempt to go beyond advertising's direct effect on conversion by considering a consumer's entire purchase journey. However, these models predominantly focus on consumer interactions with a single focal firm while leaving out the impact of competitive actions implemented by competing firms, which we view as a critical determinant of consumer conversion.

Our goal is to develop a new cross-channel attribution model that incorporates consumers' both searching and purchasing behaviors across multiple competing online stores within a competitive online shopping environment. The most important distinction between our model and other extant attribution models is that we consider consumers' online store choice decisions across multiple sellers, thus expanding the literature's single-seller scope (e.g. Li and Kannan 2014, Xu et al. 2014) to a multi-seller scope. Consumers are likely to be exposed to various types of digital ads from various competing sellers, and failing to account for consumers' interactions with these sellers may yield biased estimates of ad effectiveness. Advertising can exert two separate effects on a consumer's final purchase decision: converting the consumer from not buying into buying, and influencing the consumer in choosing where to buy. With a single-seller model, we are only able to capture the second effect, thus can underestimate the overall effect of advertising. Moreover, the problem (of not accounting for competitive advertising) exacerbates in a purchase funnel model because competitive advertising not only influences the final conversion stage, but also the earlier product information search and alternative evaluation stages. It is natural to expect that the marginal impact of different advertising channels on consumers' conversion probabilities varies across competitive firms and across different stages in a purchase funnel. Firms can benefit from having competitive intelligence by considering consumers' interactions with all other firms competing within the same industry.

Toward this goal, we propose an integrated two-stage choice model in which a consumer considers all available websites that offer the relevant product and then decide which ones to visit and which one to make the purchase from. We follow previous literature (e.g. Li and Kannan 2014, Abhishek et al. 2014) and adopt a funnel view of consumer online store choice decisions for two reasons. First, doing so allows us to capture the heterogeneity in consumers' consideration set which directly impacts their final purchase decisions. Ignoring this source of heterogeneity will lead to biased advertising effect estimates on purchase decisions (Goeree 2008). Second, this model structure differentiates between the impacts of various advertising channels on different purchase funnel stages, allowing us to separate the two advertising effects - the effect on a brand's inclusion within the consideration set and its effect on consumer purchase utility. The resulting inferences therefore provide guidelines for more efficient ad allocation along the purchase decision process.

We estimate this model using a unique individual-level panel dataset that records consumers' interactions in their purchase funnel with all competing websites within the online air booking industry through various online advertising channels including search engines, email, display ads, referral engines and direct channel. Our result shows that the effects of information stock gained through different channels are significant at both the visit stage and purchase stage. However, the size of these effects varies across channels and stages. Specifically, we find that information stock gained through direct channel is the most effective in driving both visit and purchase decisions, followed by search advertising and email advertising. Display ads and referral engines are less effective on either decision compared to the other channels. With our competitive model, we are able to compute the own- and cross-marginal impacts of various ad channels on each website. Our marginal impact analysis shows that the effectiveness of advertising channels varies widely across competitors, and this is true for all purchase stages considered in our model. The disparate effectiveness of the same advertising channel across competitors highlights the importance of ad content and message strategies in a competitive environment. A firm needs to determine its standing among the competitive pack and revise its strategy accordingly by benchmarking its own ad effectiveness against competitors. Finally, we highlight the importance and necessity of considering competition by comparing the estimated advertising effectiveness and predictive performance derived from the two models: our proposed model and a baseline model where we assume each firm only knows consumers' interactions within its own website. It turns out that the baseline model significantly underestimates the advertising effectiveness and performs worse than our proposed model in predicting purchase of a holdout sample.

To the best of our knowledge, this paper represents the first attempt in modeling multi-channel attribution across competitors. Our primary contribution is that we develop an integrated model that captures individual consumers' interactions with all competing firms at different stages along their online purchase journey. Our competition-centric analysis represents a significant enrichment over extant ad attribution models that typically take a single firm perspective, while neglecting competitive interactions. As companies are attaching more importance to deriving competitive intelligence in ad attribution, there is an increasing availability in cross-competitor ad clicks and browsing data provided by many marketing analytics companies. For example, both ACNielsen and ComScore constantly track representative consumers' web browsing and purchases through cookies installed on their browsers[2]. Companies can also form strategic alliance and share customer data with each other, e.g. the strategic data sharing agreement between Amazon and Salesforce in 2016[3]. In addition, the advance of the new digital technologies has also made it possible for companies to infer competitive intelligence through social media and online surveys[4]. In light of the increasing availability of data that tracks consumers' MCMT interactions with all competing firms, our study represents a timely precursor to what is to come in the imminent competitive analytics field. Our model will become more and more feasible as the information technology advances. The inferences from our model will provide companies critical competitive intelligence that helps guide their advertising allocation decisions in an online shopping environment that is

[2] These firms are even able to link consumers' behavior across multiple devices. According to their websites (e.g., http://www.comscore.com/Industries), many leading firms (including those in our analysis) have subscribed to these services.

[3] http://www.nasdaq.com/article/amazon-and-salesforce-extend-strategic-alliance-to-deliver-service-integrations-20161202-00256/amp, last accessed on 12/13/2016.

[4] Tracking customers' social media activities can also help firms to identify their customers' interactions with their competitors. In fact, SAS's social media analytics tool provides such analysis (https://www.sas.com/content/dam/SAS/pl_pl/doc/factsheet/sas-social-media-analytics.pdf) .

characterized by intense competition.

The rest of this paper is organized as follows. In §2 we review the related research and discuss how our study contributes to the literature. In §3 we set up the proposed integrated two-stage model. We then provide an overview of the data used for this study and show model-free evidence of the online shoppers' searching and purchasing patterns in §4. In §5 we present the estimation results and evaluate the effectiveness of different ad formats implied by these model estimates. In §6, we demonstrate the importance and necessity of modeling competition between websites by comparing our model with a baseline model which only considers consumers' interactions with a focal firm. Finally, we conclude with discussions and directions for future research in §7.

2. Literature Review

Information Systems and Marketing researchers have long been interested in measuring the effectiveness of different online advertising formats such as search advertising (Agarwal et al. 2011 & 2015, Ghose and Yang 2009) and display ads (e.g., Lewis and Reiley 2014, Rutz and Bucklin 2012, Goldfarb and Tucker 2011). Recently as rich multi-channel data becomes available, multi-channel ad attribution has gained special attention. Table 1 provides an overview of the extant research on multi-channel attribution, where we compare the studies along dimensions including the level of data aggregation, product category, ad channels investigated, the consideration of pre-purchase decisions, the consideration of competition and the modeling approach. Some notable recent papers in this stream of literature include Ghose and Todri (2015), Li and Kannan (2014), Xu et al. (2014), Abhishek et al. (2014), and Zantedeschi et al.

(2016). Ghose and Todri (2015) study how exposure to various types of display ads (e.g., retargeting, affiliate targeting advertising, etc.) and the duration of exposures affect consumers' active and passive search behaviors and conversion probability using a difference-in-difference (DID) matching estimator. Li and Kannan (2014) model a consumer's consideration of advertising channels, the visit through a given channel and purchase decision jointly; and estimate the carryover and spillover effects of advertising channels. They find that information stock gained through firm-initiated channels significantly increases a consumer's conversion probability, but the incremental impact of paid search channels is smaller than previously estimated. Abhishek et al. (2014) apply a Hidden Markov Model to uncover the impact of ads at different latent purchase stages. They show that display ads exert a positive impact on early stages but do not increase the conversion probability directly, while search ad effects show up throughout all stages. Xu et al. (2014) report similar findings using a Mutually Exciting Point Process Model in which ad clicks and purchases are modeled as different types of random points in continuous time and earlier points can affect the types of later points. Zantedeschi et al. (2016) conduct randomized field experiments to estimate the short- and long-term effects of multiple online and offline advertising formats on consumer purchases from a single firm.

Most of the above studies suggest that the effect size of ad channels may vary across different stages along the conversion funnel, emphasizing the importance of modeling multiple stages in consumer's purchase funnel. We accordingly adopt the same funnel view and model a consumer's website visit decisions in addition to her final purchase choice. However, the key distinction of our model from these previous studies lies in our explicit consideration of cross-competitor interactions. Table 1 shows that all extant studies rely on data from a single focal firm despite the fact that consumers' website visit and purchase decisions are clearly influenced by their interactions with competing firms. Disregarding these interactions will lead to inefficient and biased estimates of advertising channels' effectiveness and therefore suboptimal ad budget allocation of firms. This oversight in prior research motivates our study.

The richness of our model comes not only from the more complete data (which include consumers' interactions with all firms) we have access to, but also from the more accurate identification of the advertising effects in the presence of competition. This is a non-trivial task. By the same token, although we also develop a multi-stage model, our "funnel" view differs from Li and Kannan (2014) and Ghose and Todri (2016) in that they modeled consumers' visit and purchase decisions concerning a single focal firm through various channels, whereas we are modeling consumers' visit and purchase decisions at different funnel stages across competing firms. Our model framework is therefore closer to the traditional AIDA (Awareness, Interest, Decision and Action) model where consumers consider and evaluate multiple alternatives before a purchase decision is made (Kerin et al. 2014). Abhishek et al. (2014) modeled three latent states and allowed advertising to affect transitions between latent states and conversion probability within a latent state, whereas we are modeling the observed visit and purchase decisions. Although their model has many merits, it is not quite feasible computationally to model the latent states in a competitive setting where consumers' states with each alternative can vary, leading to exponential growth in terms of computation requirement where more than ten (competing) alternatives and multiple states within each alternative need to be modeled.

Table 1. Summary of Research on Multi-channel Attribution

	Individual /aggregate	Data: product category	Ad channels	Funnel View	Dynamic Effect	Competitive Effect	Methodology
Ghose and Todri 2015	Individual	Concealed	1,2,4	Yes	No	No	DID Matching
Shao and Li 2011	Individual	Software	3,5,6,7	No	No	No	Bagged Logit
Li and Kannan 2014	Individual	Hospitality	1,2,4,5,6	Yes	No	No	Hierarchical Bayes model
Xu et al. 2014	Individual	Electronics	3,6,7	Yes	No	No	Mutually exciting point process model
Anderl et al. 2013	Individual	Fashion, luggage, travel	1,2,4,5,6,7	No	No	No	Graph-Based data mining
Anderl 2013	Individual	Fashion	1,2,4,5,6,7	No	No	No	Proportional hazard
Abhishek et al. 2014	Individual	Automobile	3,6	Yes	No	No	Dynamic Hidden Markov model
Zantedeschi et al. 2016	Individual	Specialty retailer	5,8	No	Yes	No	Bayesian Tobit model
Wiesel et al. 2011	Aggregate	Furniture	1,5,7	Yes	No	No	VAR
Haan et al. 2014	Aggregate	5 categories	3,4,5,6,7,8	Yes	No	No	Structural VAR
Demirci et al. 2014	Aggregate	4 categories	1,2,5,6,7,8	No	No	No	Bayesian VAR
Raman et al. 2012	Analytical	NR	2 channels	No	No	No	Time-varying Nerlove-Arrow
Our paper	Individual	Air travel	3,4,5,6	Yes	Yes	Yes	Three-Stage Choice Model

Note: 1) Ad channels: 1- search: paid, 2- search: organic, 3- search: general, 4- referral, 5- email, 6- display, 7- other online, 8- offline 2) NR = Not Relevant

3. Empirical Setting

We obtained a click-stream data set collected by a leading media measurement and analytics US company in 2010. This company manages a large-scale consumer panel that is representative of the U.S. population. The company recruits panelists to install a software meter so that all their PC-based Internet activities across all web entities will be recorded non-intrusively. The click stream data is organized into user sessions which are defined as visits to a sequence of specific websites consecutively without a break of more than 30 minutes. Each session tracks a panelist's ID, domain name, date and time of visit, referral domain name (from which advertising channel can be inferred), browsing activity measured by the number of pages viewed and duration in minutes, as well as the purchase activity including product category, product description, quantity and price paid if a transaction occurs.

The analysis is confined to data that pertain to online flight booking service. We choose this category among more than 60 product categories for the following reasons. First, travel is the largest eCommerce category, comprising of 36% of all B2C eCommerce sales in 2012 (eMarketer, 2013)[5], so we will have a sufficiently large sample to ensure reliable estimation. Second, air tickets tend to be non-differentiated search goods which reduce the possible confounding effect resulting from product-level heterogeneity. The price differentiation across online agents for a given trip at a given time point is also minimal; price is therefore unlikely to play a major role in a consumer's decision regarding which website to purchase from. Third, air tickets are relatively expensive, averaging US $450 per ticket (Li et al. 2014). Since the prices of air tickets are extremely dynamic, savvy consumers act strategically with extensive searches and patience to wait for discounted tickets (Li et al. 2014). Advertising plays an important role in

[5] https://www.emarketer.com/Article/Slow-Steady-Continued-Gains-US-Digital-Travel-Sales/1009909, last accessed on 12/13/2016.

informing and reminding consumers in such an environment, making it an appealing test-bed for our model. These features ensure that consumers will be highly subject to ad exposures and engage in substantial searches for air travel, making this category an ideal setting for our research.

We identify the set of relevant websites from the panelists' purchase records. A website is included in the analysis if at least one purchase in the air travel category occurred on the website during our entire observational period[6]. These websites can be grouped into two types: 1) major airlines' official websites, including AA.com, Airtran.com, Alaskaair.com, Continental.com, Delta.com, Jetblueairways.com, Nwa.com, Southwest.com, Spiritair.com, United.com, and Usairways.com; 2) major online travel agencies (OTA hereafter), including Expedia.com, Priceline.com, Orbitz.com, Travelocity.com, Cheaptickets.com, Hotwire.com, Americanexpress-travel.com, Sabresonicweb.com, and Wwte1.com. We exclude Southwest.com from our analysis because their tickets are sold only via the company's direct website but not through OTAs. Consumers who purchase from Southwest.com may have completely different consideration set (only consists of Southwest.com) than others (consists of all available websites). Due to computational considerations (to alleviate the curse of dimensionality), we opted to consolidate some smaller airlines and OTAs when constructing the full choice set. For airlines, we separate the top three airlines, Delta Airlines, American Airlines and United Airlines from the rest because they were the top three largest major airlines in the US in year 2010 and were visited more frequently in our data sample than any single remaining airline website as shown in Table 2[7]. We accordingly combine the remaining airline websites into one "other airlines" option. By

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[6] There might be other websites that also sell air tickets excluded from our analysis. However, since no air-travel ticket purchase took place on those websites during the entire sample period these websites are likely to be very small players in this category. The exclusion of these websites should therefore not significantly alter the results.

[7] In 2010, Delta , United and American Airlines enplaned 162.6, 145.6 and 104.5 billion passengers respectively,

the same token, we group Hotwire.com, Americanexpress-travel.com, Sabresonicweb.com, and Wwte1.com into one "other OTAs" option because they represent a much smaller market share and are visited and purchased from less frequently compared to the top five OTAs, also shown in Table 2.

Table 2 List of Websites in the Air Travel Category

j	Domain Name	# of Occasions Visited	# of Occasions as Entry Site	# of Purchases
1	Delta.com	2,206	881	234
2	AA.com	2,190	875	190
3	United.com	1,047	336	88
4	Other Airlines	4,530	2,365	745
5	Expedia.com	4,379	1,748	431
6	Priceline.com	3,017	1,038	223
7	Orbitz.com	3,012	857	246
8	Travelocity.com	2,813	857	216
9	Cheaptickets.com	1,658	623	167
10	Other OTAs	1,483	386	50
Sum		26,335*	9,966	2,590

*This is the number of purchase occasions during which the website was visited at least once.

Toward a purchase, a consumer may need to search several websites and visit each website in her choice set multiple times before a purchase decision is made. These visits can occur across different sessions. Therefore, one important yet challenging task when processing data is to identify and group related sessions pertaining to a purchase occasion. There is no universal rule for grouping such sessions. We use a seven-day window policy which is also adopted by De los Santos et al. (2012) because one week is considered sufficient to capture all relevant visits for a purchase on Internet.[8] Another problem we are facing is that all the OTAs also sell other

ranking 1, 2 and 4 in the US. The 5th largest airline was US Airways, enplaned only 59.8 million passengers. The 3rd largest airline was Southwest Airlines.

[8] The validity of the 7-day window policy is verified though our personal communication with a Sabre executive who is in charge of the data analytics division. According to him, majority of customers start searching for a flight

products such as hotel rooms, car rentals, vacation packages and cruises in addition to air tickets. Yet we do not directly observe what product(s) the consumer is searching information for unless a purchase is made; for non-purchasing sessions, we do not have data on the specific contents of a visit such as what the consumer saw at a website.

Since we include non-purchase sessions in our analysis, we need to preprocess the data with caution to determine which search sessions are relevant to flight booking. We take several steps to ensure a clean data preprocessing. We first assume that if a flight ticket is purchased at the end of a seven-day window, then all preceding site-sessions are relevant for searching flight-related information. Second, if no purchase is made in the end, we eliminate irrelevant sessions and purchase occasions as follows: 1) if the session is generated within seven days *after* a fight ticket is booked then the session is considered irrelevant and thus deleted. This is because very few consumers would purchase different tickets within a short time frame and so these visits occurred after the purchase are more likely for after-purchase reassurance and confirmation purposes, rather than looking for information about a new purchase; 2) if less than 10 pages are browsed in total across all websites combined during the entire 7-day window, we assume it is too short for any serious product-related search to take place and thus such sessions are dropped; 3) if any website outside of the set of relevant websites as we have defined is visited and at the same time no airline website (which only sold flight tickets in 2010) is visited during a seven-day window, we deem such a session to be irrelevant because the consumer is more likely to be searching for other products, not flight tickets. The above steps yield a data sample containing 18,936 sessions, out of which 4,897 occasions ended with a flight ticket purchase. We then use the first three

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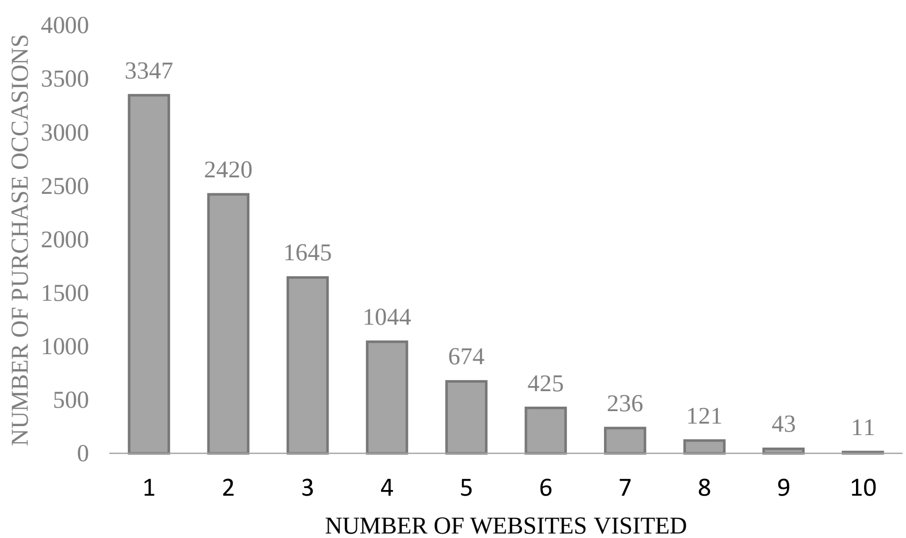
ticket three days before making the purchase. Saber is the leading online flight reservation platform in the US that powers OTAs and airlines' ticket booking. We have also conducted robustness checks by varying the window length to 15 days and 30 days and re-estimated the model. We find that our results are qualitatively unchanged. These results are available from the authors upon request.

months (January to March 2010) to construct the historical variables for each individual user, and the next six months (April to September 2010) as the estimation sample to calibrate the model, and the last three months (October to December 2010) as the out-of-sample to conduct predictive analysis. Our final estimation sample consists of 9,966 purchase occasions with 2,590 of them ended up with a purchase. Table 2 presents the complete list of websites included in our analysis and their respective frequencies of being searched, being chosen as the entry website as well as being the website to make the purchase in the estimation sample.

We observe that consumers generally only search a small number of websites before making purchases. Figure 1 plots the histogram for the number of websites visited during a purchase occasion, with 33.58% of consumers visiting only one website. We also observe that 1,528 transactions are made on the first website visited during the seven-day window, representing 58.99% of all transactions taking place in our estimation sample. These statistics lend support to the importance of modeling entry site choice as a distinct decision in the purchase funnel for the online air booking industry.

Figure 1 Frequency Distribution of the Number of Websites Visited



We identified three types of marketing channels: 1) search engines in which the consumer enters a keyword and redirects to the website by clicking a search result[9]; 2) display ads or referral where a third-party website runs banner (or video) ads or provide referral links (for example, referral engines such as TripAdvisor.com and Kayak.com)[10]; 3) email engines from which the consumer is directed to the website by clicking a link embedded in the email. In case that the referral domain name is absent when a consumer types the URL directly into the browser or locate the site through bookmarks, we label the referring channel for this session as "direct". The top five referral domains in each channel are presented in Table 3. We observe website visits from all referral domains at the individual level, and for every user we construct ad stock by weighting ad clicks by the number of pages viewed. We present summary statistics regarding the number of visits to each website through a specific channel in a purchase occasion in Table 4. Most consumers visit the websites directly. Among advertising channels, search engine is the most popular one, followed by display/referral ads. We also notice significant differences across websites in terms of different ad channel usage, suggesting the presence of large variations in ad budgets and allocations across competitors in this industry.

Table 3 Top Redirecting Websites in Each Marketing Channel

Rank	Search Engine	Display/Referral Engine	Email
1	google.com	kayak.com	live.com
2	yahoo.com	tripadvisor.com	comcast.net
3	aol.com	bookingbuddy.com	conduit.com
4	bing.com	lowfares.com	verizon.net
5	mywebsearch.com	doubleclick.net	juno.com

[9] We cannot differentiate between organic versus sponsored searches due to data limitation though.
[10] We do not further differentiate display ads versus referral engines because many referral engines also run display ads.

Table 4 Channel Usage (Conditional on Visit)

Alternatives	N Obs	Variable	Mean	Std Dev	Min	Max
Delta.com	2206	Search	0.65	1.25	0	17
		Display/Referral	0.08	0.32	0	4
		Email	0.06	0.36	0	9
		Direct	1.41	2.48	0	37
AA.com	2190	Search	0.52	0.90	0	14
		Display/Referral	0.10	0.33	0	3
		Email	0.06	0.30	0	4
		Direct	1.31	1.89	0	25
United.com	1047	Search	0.50	0.80	0	8
		Display/Referral	0.15	0.47	0	4
		Email	0.03	0.18	0	3
		Direct	1.24	2.33	0	28
Other Airlines	4530	Search	0.81	1.50	0	21
		Display/Referral	0.15	0.50	0	8
		Email	0.07	0.32	0	4
		Direct	1.65	2.72	0	46
Expedia.com	4379	Search	0.44	0.93	0	17
		Display/Referral	0.24	0.63	0	9
		Email	0.05	0.31	0	6
		Direct	1.52	2.73	0	45
Priceline.com	3017	Search	0.38	0.77	0	9
		Display/Referral	0.24	0.55	0	6
		Email	0.05	0.28	0	5
		Direct	1.23	1.47	0	17
Orbitz.com	3012	Search	0.32	0.79	0	20
		Display/Referral	0.35	0.78	0	13
		Email	0.02	0.17	0	4
		Direct	1.19	1.57	0	20
Travelocity.com	2813	Search	0.36	0.79	0	13
		Display/Referral	0.13	0.38	0	3
		Email	0.03	0.20	0	4
		Direct	1.31	1.43	0	18
Cheaptickets.com	1658	Search	0.44	0.89	0	15
		Display/Referral	0.12	0.38	0	3
		Email	0.03	0.23	0	5
		Direct	1.17	1.31	0	12
Other OTAs	1483	Search	0.31	0.71	0	9
		Display/Referral	0.28	0.56	0	4
		Email	0.04	0.27	0	4
		Direct	1.03	1.21	0	10

4. The Model

Our model adopts the purchase funnel view in the context of online shopping of high-involvement products within a competitive environment. The funnel view is grounded in the information processing theory describing the path through which consumers make their purchase decisions, from perceiving a need to post-purchase behaviors (Bettman et al., 1998). A consumer's decision process is metaphorically referred to as a funnel because the number of alternatives considered diminishes as she moves along the purchase journey.

The purchase funnel view can be applied to both product (or brand) choices and store (or website) choices. We apply it to the second scenario and study how online multi-channel advertising affects consumers' decision to visit and purchase from available websites. We specifically model a two-stage choice process in which consumers first select a subset of relevant websites to search for product information and then choose a single website from the subset to make the purchase or choose the outside option of no purchase.

We allow the effect of the information stock (accumulated through different advertising channels) of a consumer to vary across different funnel stages. Previous literature on multichannel marketing supports this assumption. For example, Abhishek et al. (2014) show that display ads tend to be more effective during the early stage in a purchase funnel; while the impact of email, search, and referral is more pronounced in later stages. Li and Kannan (2014) find that consumer-initiated channels (paid search, referral and direct) are more effective in reducing consumer search cost in early stages, while firm-initiated channels (email and display) contribute more at later stages. However, in contrast to most extant ad attribution models that focus on choices within a focal website only (for example, in-store product choices in Abhishek et al. 2014 and ad channel choices in Li and Kannan 2014), our primary interest is to model consumer choices across competing websites at different stages of a purchase funnel. Next we describe the details of each stage.

4.1 Stage 1: The Website Visit Decision

The objective of the Stage 1 model is to capture and represent consumer search process in terms of: 1) at which website to start the information search and 2) what other websites (if any) to visit after the initial search. A consumer's decision to visit a specific website depends on her perceived benefits derived from the visit relative to the costs incurred due to the visit. The perceived benefits depend on her intrinsic preference for the website and her goodwill towards the website built through various online and offline advertising channels. The costs are associated with the effort required to find the necessary product information and make purchase on the website (Shugan 1980). Hence, consumer *i*'s perceived utility of visiting website *j* can be expressed as:

$$U_{ijt} = \overline{U}_{ijt} + \varepsilon_{ijt} = \alpha_{ij} + G_{ijt} - S_{ijt} + \varepsilon_{ijt} \qquad (1)$$

$$\text{where } G_{ijt} = \sum_{q=1}^{Q} \alpha_q \sum_{h=1}^{m_{ijt}} d_{ijqh} \times (1-\lambda_1)^{(m_{ijt}-h)} \qquad (2)$$

$$S_{ijt} = \alpha_v v_{ij,t-1} + \alpha_s s_{ijt} + \sum_{c=1}^{C} \alpha_c E_{ijct} \qquad (3)$$

$$E_{ijct} = \sum_{h=1}^{m_{ijt}} g_{ijch} \times (1-\lambda_2)^{(m_{ijt}-h)} \qquad (4)$$

In the above equations, α_{ij} captures consumer *i*'s intrinsic net preference for visiting website *j* and can vary over people but stay constant over choice situations for each person (Erdem and Keane 1996), and ε_{ijt} follows a generalized extreme value distribution. The term G_{ijt} detailed in Equation 2 represents the consumer's goodwill or ad-stock accumulated towards the website through exposures to various firm-initiated advertising formats (Nerlove and Arrow 1962). Ad exposures to channel q in each period h are approximated using the website's national and local ad spending in that period, d_{ijqh}. We will provide details on the ad expenditure data in Section 4.3. The informational effect of pervious exposures decays at a monthly discount rate λ_1, according to the elapsed number of months (m_{ijt}-h). The information channel-specific goodwill can take many possible functional forms; we adopt a parsimonious one that assumes a linear functional form, computed as a linear function of the decayed effects of a website's investment in various advertising channels. However, we must note that which functional form to use is nonessential to our model and our model can be readily adapted to incorporate other specifications for the latent goodwill accumulation process such as a nonlinear decay model.

The term S_{ijt} detailed in Equation 3 represents consumer i's cost of visiting website j. The cost is non-zero, so an intercept is needed in the cost function. However, we would not be able to separately identify the cost intercept and the website-specific preference. So, the term α_{ij} in Equation 1 denotes the composite intercept of consumer i with respect to site j, and represents i's "net preference" on j. Search cost usually reduces as a consumer gains familiarity with the website through previous visits and purchases (Li and Kannan 2014), exhibited as reduced time in finding relevant information and a faster checkout process (Shugan 1980). The dummy term $v_{ij,t-1}$ indexes lagged visit, indicating whether or not the consumer visited the same website in the preceding purchase occasion. The term s_{ijt} represents the logarithm of the consumer's cumulative spending on the website. The term E_{ijct} captures a channel-specific information stock accumulated through navigating the website in the previous purchase occasions, which is calculated according to Equation 4. The term g_{ijch} denotes the logarithm of the number of pages viewed on website j coming through channel c in purchase occasion h. We then compute the channel-specific information stock as the sum of decayed page-views at a monthly discount rate of λ_2, according to the elapsed number of months (m_{ijt}-h). When coefficients α_v, α_s and α_c are negative, their associated variables reduce the cost of visiting the website.

The decision regarding which website to visit first is critical. The search theory asserts that a consumer begins her search process with the highest-utility alternative (Weitzman 1979). The conditional probability of observing website F_{it} being chosen as the entry site by consumer *i* at time *t* given the individual-specific preference is therefore expressed as:

$$P\left(F_{it}\,|\alpha_i\right)=\prod_{j=1}^{J}\left[\frac{e^{\alpha_{ij}+G_{ijt}-S_{ijt}}}{\sum_{k=1}^{J}e^{\alpha_{ik}+G_{ikt}-S_{ikt}}}\right]^{f_{ijt}}\left[\frac{1}{\sum_{k=1}^{J}e^{\alpha_{ij}+G_{ikt}-S_{ikt}}}\right]^{1-f_{ijt}} \qquad (5)$$

where f_{ijt} =1 if website *j* is chosen as the entry site and f_{ijt} = 0 otherwise.

Ideally we would like to model all the subsequent website visits of the consumer one by one sequentially. However, the decision of which website to visit subsequently may depend on the consumer's interactions with the prior website(s). Such dependency can be complicated and arbitrary that can render the model intractable. Moreover Figure 2 shows that the number of observations for later visits (3^{rd}, 4^{th}, 5^{th}, etc.) is relatively small in our estimation sample, prohibiting reliable estimates even if one would like to factor in such dependencies. In compromise, we simplify the search decision for the remaining websites within all available websites as a number of independent binary choices. A consumer will only visit a website if the perceived value of that visit is positive. Given the entry site F_{it} , and her individual net preferences, the conditional probability of observing a consumer's choice set V_{it}, a combination of any number of websites out of *J* websites, is given as:

$$P\left(V_{it}\left|F_{it},\beta_i\right.\right)=\prod_{\substack{j=1,\\ j\neq F_{it}}}^{J}\left[\frac{e^{\beta_{ij}+G_{ijt}-S_{ijt}}}{1+e^{\beta_{ij}+G_{ijt}-S_{ijt}}}\right]^{v_{ijt}}\left[\frac{1}{1+e^{\beta_{ij}+G_{ijt}-S_{ijt}}}\right]^{1-v_{ijt}} \quad (6)$$

where v_{ijt} = 1 if website j is visited in the current purchase occasion and v_{ijt} = 0 otherwise.

4.2 Stage 2: The Purchase Decision

The purchase stage can be conceptualized as an option to purchase from a set of websites a consumer has already visited, V_{it}. Since we directly observe the entire search process made by each and every consumer, we are able to capture their respective choice set without any uncertainty. The consumer can also exercise the option of not buying if none of her searched alternatives yields a utility exceeding her purchasing threshold (which is normalized to zero). The utility of purchasing from website $j \in V_{it}$ can be written as:

$$W_{ijt}=\overline{W}_{ijt}+\zeta_{ijt}=\gamma_{ij}+\sum_{c=1}^{C}\gamma_c g_{ijct}+z_{ijt}\gamma+\zeta_{ijt} \quad (7)$$

$$\begin{aligned} z_{ijt}\gamma &= \gamma_{nn}\times FLN_{jt}+\gamma_{lc}\times FLL_{ijt}+\gamma_f\times FST_{ijt}+\gamma_l\times LST_{ijt}+\gamma_{np}\times LNP_{it}+\gamma_p\times LP_{ijt} \\ &\quad +\gamma_v\times LV_{ijt}+\gamma_{cs}\times CS_{ijt}+\gamma_{cp}\times CP_{ijt} \end{aligned} \quad (8)$$

where γ_{ij} captures consumer i's intrinsic net preference for purchasing from website j and can vary over people but being constant over choice situations for each person (Erdem and Keane 1996), and ζ_{ijt} follows a generalized extreme value distribution. The term g_{ijct} is the cumulative informational stock derived from channel c. It is operationalized as the logarithm of the number of pages viewed through channel c in the current purchase occasion. These channel-specific information stock terms are likely to be correlated with the error term ζ_{ijt} because the unobserved factors that drive purchase decision may also lead to more page views. We will discuss how we handle such endogeneity issues next in Section 4.3. The vector z_{ijt} includes other covariates that may affect consumer *i*'s purchase utility, which we specify in Equation 8[11]. The coefficients γ_{nn} and γ_{lc} capture the effect of website *j*'s offline advertising spending at the national and local level (where consumer *i* resided) at purchase occasion *t*, denoted by FLN_{lt} and FLL_{ilt}, respectively. The coefficients γ_f and γ_l capture the effect of being the first and last website visited during the current occasion, represented by two dummy variables FST_{ilt} and LST_{ilt}, respectively. The coefficients γ_{np}, γ_p and γ_v measure the effect of visit and purchase decisions in the last purchase occasion. LNP_{it} is a dummy variable that indicates whether the last purchase occasion ended with a purchase, LP_{ijt} is a dummy variable that indexes whether the purchase made in the last purchase occasion was on website *j*, and LV_{ijt} is a dummy variable indicating whether website *j* was visited in the last purchase occasion. Finally, the coefficients γ_{cs} and γ_{cp} capture the effects of past spending and browsing history with website *j*, represented by two continuous stock variables CS_{ijt}, the cumulative amount of money spent on website *j*, and CP_{ijt}, the cumulative number of pages browsed on websites *j*.

The consumer also has the option of not buying and the utility of this outside option is normalized to 0. Taken together, the probability of observing website B_{it} being chosen as the conversion site by consumer *i* at time *t* conditional on entry site, choice set and individual-specific preferences is as follows:

[11] Some other factors may also affect consumer purchase decision, including but not limited to the product availability (not every carrier/time/stop combination is available at all websites), price information (for non-purchase sessions as well as non-chosen alternative for purchase sessions), and how far in advance the ticket is being purchase. We acknowledge these as a potential limitation of our study. These concerns can be addressed if screen-shot data for webpages visited is available. However, these variables and advertising are not likely to be correlated with each other, as advertising mainly serves as reminder in the airline industry. Omitting these variables is thus unlikely to affect the estimates for multi-channel ad stock.

$$P\left(B_{it}\left|F_{it},V_{it},\gamma_i\right.\right)=\prod_{j\in V_{it}}\left[\frac{e^{\gamma_{ij}+\sum_{c=1}^{C}\gamma_c g_{ijct}+z_{ijt}\gamma}}{1+\sum_{k\in V_{it}}e^{\gamma_{ik}+\sum_{c=1}^{C}\gamma_c g_{ikct}+z_{ikt}\gamma}}\right]^{b_{ijt}}\left[\frac{1}{1+\sum_{k\in V_{it}}e^{\gamma_{ik}+\sum_{c=1}^{C}\gamma_c g_{ikct}+z_{ikt}\gamma}}\right]^{1-b_{ijt}} \tag{9}$$

where b_{ijt} = 1 if the consumer converts on website j and b_{ijt} = 0 otherwise.

4.3 Endogeneity and the Instruments

Endogeneity in the context of measuring advertising effectiveness is a well-known concern for studies using observational data (Rossi 2014). In our case, the channel-specific informational stocks are likely to be correlated with the error term in the purchase stage and thus pose challenge to identification. Field experiments are ultimate arbitrator for causal inferences (Johnson et al. 2015). However, in our situation where many competing firms are involved, conducting a large-scale experiment is close to impossible because of the difficulty in controlling all competing firms' advertising actions. Alternatively, we adopt the control function approach in discrete choice models popularized by Petrin and Train (2010). The idea behind this approach is to partition the variation in the endogenous variable into two parts: one that is exogenous and the other that is potentially correlated with the error term. Then by including some extra instrument variables that contain the second part in the utility function, we can condition out the variation in the unobserved factor that is not independent of the endogenous variable. This IV approach has been proven to be effective in dealing with endogeneity when randomized experiment is not available, with the premise that the instruments need to be valid and strong (Rossi 2014)[12] as weak instruments may cause more problems than they solve. We elaborate below how our

[12] Validity means that the instruments must be correlated with the endogenous variable, but does not affect the purchase utility in a direct way. Strength means that the instruments must be able to explain a large enough portion of the variation in the endogenous variable.

proposed instruments help mitigate the endogeneity concerns coming from multiple potential sources.

The first potential source of endogeneity is *customer selection*, which happens when these consumers who are exposed to and interact with advertising are systematically different than the others who are not (Manchanda et al. 2004). This can occur for any marketing channels in our context, even for the direct channel. For example, consumers who are more loyal to a website are more likely to accumulate information by visiting the website directly more frequently than other people and to make purchases on this website eventually. This could also happen for firm-initiated marketing channels such as search advertising, email campaigns and display ads if the companies select customers to target for a specific campaign based on certain information such as past browsing or purchase history, responsiveness to ad campaigns, or demographics. If this is the case, the estimates for advertising effectiveness will be biased upward because these selected customers have a higher propensity to convert inherently. The second potential endogeneity source can arise when the unobserved ad impressions simultaneously drive advertising click and purchase intention. The third potential endogeneity source can come from the temporal simultaneity between advertising and unobserved demand shocks (Zantedeschi et al. 2016). It can happen at both the market level and the individual level. At the market level, firms may increase advertising spending when there is an external demand shock unobservable to researchers. As a result, consumers are exposed to more ads leading to simultaneous increases both in terms of ad clicks and purchases. The individual level demand shock can occur due to factors such as a sudden change in her income, a large bonus etc.

We devised several instrument variables to alleviate the aforementioned endogeneity concerns. We first construct an instrument variable using the consumer’s lagged channel-specific information stock. We expect the cost of visiting through a specific channel to decrease with information stock accumulated through the channel in the past, and thus, the consumer may visit the website and accumulate more information through the same channel in the current purchase occasion as well. However, this lagged variable should not be systematically co-determined by the future purchase intention.

We further acquired an external data source from the Kantar Media Database (KMD) to address endogeneity. This database tracks all the companies in our analysis on their monthly spending on offline advertising channels including TV, newspaper and radio, as well as two major online advertising channels, search advertising and display ads at both the national and local DMA (Designated Market Area) level. This supply-side ad data enables us to directly control ad impression. The ad spending reflects potential unobserved demand shocks and inclusion of ad spending would directly control for the increase in ad impression due to unobserved random shocks.

We then construct several additional instrument variables using the KMD data. For each firm in a DMA in a month, we compute its competitors' online ad spending on the focal channel (search engine and display/referral) and use this as the IV. Exogeneity of this IV is warranted because it is hard to imagine that competitors would adjust their ad spending according to the searching and purchasing behavior of the focal firm's consumers observed in our data. Furthermore, when a random demand shock occurs, it is reasonable to expect all other competitors to respond by adjusting their ad spending accordingly, but this activity should not be systematically co-determined by the consumer's purchase utility on our focal website.

At the individual level, a consumer may increase the likelihood to click on ads (e.g. to seek deal information actively) and purchase simultaneously due to a random personal shock, e.g.

due to a windfall or a sudden increase in income. To control for this type of possibility, we use each individual's number of clicks on ads of the same channel for other product categories (car rental and hotel) as instrument. We argue that when random personal shock occurs (e.g. income increases), the consumer's interest in ads for other product categories also changes, but this should not be directly affected by her purchase intention on the focal product category.

4.4 Likelihood Function

Overall, the joint likelihood function in Equation 10 takes into account the selection of entry site, the formation of the choice set, and purchase decision. It is conditional on the individual-specific preferences for various websites in the two stages. We assume the individual website-specific intercepts α_{ij}, β_{ij}, γ_{ij} follow a normal distribution specified below:

$$\begin{pmatrix} \alpha_{ij} \\ \beta_{ij} \\ \gamma_{ij} \end{pmatrix} \sim N\left(\begin{bmatrix} \bar{\alpha}_j \\ \bar{\beta}_j \\ \bar{\lambda}_j \end{bmatrix}, \Omega_j\right), \text{ where } j = 1, 2, \ldots, J$$

Ω_j is the variance-covariance matrix for the website-specific intercepts. We assume that the correlation between intercepts of different stages remains the same across websites, but allow the variance of each intercept to vary by website and stage to reduce the number of parameters to be estimated[13]. We estimated the model using the Simulated Maximum Likelihood approach. The details for the estimation procedure are provided in Appendix A.

$$L(B|\theta) = \prod_{i=1}^{N}\prod_{t=1}^{T_i} P(F_{it} = j|\alpha_i) \times P(V_{it}|F_{it}, \beta_i) \times P(B_{it}|F_{it}, V_{it}, \gamma_i) \qquad (10)$$

5. Model Results

5.1 Stage 1: The Website Visit Decision

[13] Due to dimensionality concern, we do not allow for correlation across websites, such as allowing intercepts for expedia.com to be correlated with intercepts for orbitz.com, to maintain model tractability.

Table 5 presents parameter estimates for the first stage in the purchase funnel - website visit decision. This stage is composed of two sub-decisions: 1) choosing an entry site from the full choice set of all relevant websites, and 2) deciding whether or not to visit other remaining websites. The estimates for the choice of entry site are presented in the first column of Table 5. We observe that Delta.com and Cheaptickets.com are more likely to be chosen as the entry site compared to the other alternatives, all else being equal. We also find that there is a significant variation in preference for the same website across consumers for all websites except United.com and Cheaptickets.com. The coefficients for cumulative ad spending on offline channels at both the national level and the local DMA level are positive and significant, indicating that offline advertising is still important in generating awareness and goodwill for online consumers. Cumulative ad spending on the local-level display ads also significantly increase the website's likelihood of being chosen as the entry site, but the effects of ad spending on the national-level display ads and search ads are not significant. This reflects the difficulty of converting ad spending into exposure, even more so at the national level.

Our results show that the coefficients for lag visit, cumulative amount of money spent on the website and channel-specific lag information stock are all negative and significant, indicating that they can increase visit utility by reducing search costs. These results are not surprising as they suggest that the more experience a consumer accumulates with a website, the more likely she will choose it to be the entry site. The cognitive lock-in effects explain it well (Johnson et al. 2003). When the amount of experience with the website increases via repeated visits and purchases, the consumer becomes locked in due to the reduction in costs associated with navigating or understanding the website.

Table 5 Model Estimates: Visit Decision

Parameter	Entry Site		Remaining Sites	
	Mean	**Std Dev**	**Mean**	**Std Dev**
Delta.com	**0.68**	**0.69**	**-1.38**	**1.37**
AA.com	-0.02	**0.53**	**-1.94**	**1.27**
United.com	0.52	0.07	**-1.94**	0.03
Other Airlines	**0.69**	**1.14**	-0.65	**1.46**
Expedia.com	0.07	**1.19**	-0.37	**2.14**
Priceline.com	-0.02	**1.15**	-1.01	**1.89**
Orbitz.com	0.18	**0.73**	-1.11	**1.95**
Travelocity.com	-0.11	**0.76**	**-1.34**	**1.85**
Cheaptickets.com	**1.57**	0.01	**-1.38**	**1.14**
Other OTAs	baseline	baseline	**-1.67**	**1.25**
Cum Ad spending: National Offline	**0.11**		0.01	
Cum Ad spending: Local Offline	**0.07**		0.01	
Cum Ad spending: National Display	0.00		**0.05**	
Cum Ad spending: Local Display	**0.06**		**0.09**	
Cum Ad spending: Search Ads	0.03		**-0.21**	
Lag Visit *j*	**-1.11**		**-1.36**	
Cumulative Spending[14]	**-0.07**		**-0.14**	
Lag Info Stock: Search	**-0.33**		**-0.29**	
Lag Info Stock: Display/Referral	**-0.05**		**-0.10**	
Lag Info Stock: Email	**-0.21**		**-0.31**	
Lag Info Stock: Direct	**-0.56**		**-0.46**	
Log Likelihood	-15,672		-46,865	
AIC	31,401		93,793	

More specifically, our results indicate that information stock accumulated via direct channel (-0.56) is the most effective in reducing cost of visit, followed by information stock accumulated via search engines (-0.33). Although email is the least popular channel according to Table 4, information stock accumulated via this channel is still quite effective in making a website the

[14] Since the monthly decay rate is not a parameter of central interest, we conducted a grid search from 0.6 to 0.9 rather than estimating it directly in the Simulated Maximum Likelihood Estimator with other parameters. It is set to 0.7 during the estimation because 0.7 yields the best log-likelihood.

entry site. This could be due to the fact that email subscribers are more loyal to the website compared to those who visit the website via display ads or referral engines. Our result indicates that e-mail is more effective than display ads or referral engines in influencing consumers to choose a website to enter.

Next, we discuss the estimation results for the visit decision beyond the entry site. The estimates for website-specific intercepts represent individual consumers' net preference for visiting the website. In general consumers have a low probability of visiting other websites beyond the entry site, and this is particularly true for the airline companies' own online booking websites and small OTAs. Again, we observe significant variations across individuals in their preferences for visiting a website (standard deviations are statistically significant varying from 1.14 to 2.14, except for United.com). Offline channel spending does not significantly influence consumers' decisions to visit the remaining websites. The coefficients for lag visit, cumulative amount of money spent on the website and channel-specific lag information stock are found to be significant and negative, indicating that they can lead to lower search costs. Direct channel (-0.46) turns out to be the most effective channel in enticing consumers to visit beyond the entry site, followed by email (-0.31) and search ads (-0.29).

5.2 Stage 2: Purchase Decision

5.2.1 Control Function Results

We first report the first stage regression results in Table 6, which details the effects of the instruments on the information stock for the four channels (the endogenous variables). The R-squares of these four regression models (ranging from 0.76 to 0.91) are high, indicating that our instruments explain a great deal of variations in the endogenous variables. We then employ the J-statistic of Hansen (1982) to test exogeneity of these instruments. The statistic is distributed as chi-square with the degree of freedom equal to the number of over-identifying restriction. The result shows that the Hansen's J of our model is $\chi^2(6) = 9.17$ (p=.164), which implies exogeneity of these instruments. The estimates for the lagged information stock for all four channels are positive and significant, meaning that the individual's past interaction with the same channel is a good candidate to account for *customer selection*. Firm spending on search ads significantly increases a consumer's information stock through search engines. This is unsurprising – higher spending leads to higher ranks in sponsored ads and in turn higher consumer click-through rate (Ghose and Yang 2009). Offline ad spending also increases information stock for the four channels significantly as expected, making it a strong instrument to control for the temporal simultaneity between advertising and unobserved demand shocks.

We then include the four residuals generated in the first-stage regression as control functions in the purchase utility function. The estimates for these four control functions are positive and significant (ranging from 0.73 to 2.45), suggesting that controlling for endogeneity is necessary in estimating the effectiveness of multi-channel advertising.

Table 6 Model Estimates: Control Function

	Search Ads	Display /Referral	Email	Direct
Intercept	**-0.01**	0.00	0.00	**-0.03**
Channel-specific browsing history	**0.02**	**0.01**	**0.01**	**0.03**
Channel-specific ad spending (local)	-----	0.00	-----	-----
Channel-specific ad spending (national)	**0.13**	0.00	-----	-----
Offline ad spending (local)	0.00	**0.01**	**-0.01**	0.00
Offline ad spending (national)	**0.04**	**0.11**	**0.21**	**0.13**
Competitor's channel-specific ad spending (local)	-----	0.00	-----	-----
Competitor's channel-specific ad spending (national)	0.03	**0.08**	-----	-----
Channel-specific ad clicks on other categories	**-0.02**	**-0.06**	0.00	**-0.04**
R-square	0.88	0.88	0.91	0.76
F	8,892.2	8,232.5	12,820.7	4,163.5

5.2.2 Model Estimates for Purchase Decision

We present the parameter estimates for the purchase decision of our proposed model in the first and second columns of Table 7. First, we notice that among all the competing websites, Cheapticket.com, being a relatively niche OTA, is most preferred conditional on it being searched. Its net preference is the least negative (baseline is the no purchase option). Among the airlines' official websites, United.com is the most preferred. This indicates that once the consumers have visited United.com, their conversion rate is higher compared to other airline websites, everything else being equal. There is significant variation in these website-specific intercepts, indicating high level of heterogeneity across consumers in their preferences for these websites.

The coefficients for the information stocks gained from all four channels are positive and significant, suggesting that all advertising channels are important in generating sales. However, among the four channels, the direct channel (0.56) is the most effective, followed by search ads (0.41) and email ads (0.39). We discuss the implications of these estimates in detail using marginal impact analysis in the next section.

We find that the sequence in website visits also matters. Being either the first or the last website visited during the visit stage significantly increases conversion probability compared to those visited in-between. This finding shows that while consumers usually start with their favorable website, they are also likely to convert on the last-visited website. In addition, we find that consumers exhibit strong inertia when making online purchases. A prior online flight purchase increases the likelihood of a current purchase, especially if the prior purchase is made on the same website. And the more money a consumer has spent on booking flights at a website in the past, the higher her conversion probability will be at the website. Lag visit also increases conversion probability significantly, suggesting a carryover effect on future purchases when the site was included in a consumer's choice set.

Table 7 Model Estimates: Purchase Decision

Parameter	Proposed Model		Baseline Model	
	Mean	Std Dev	Mean	Std Dev
Delta.com	**-7.46**	**1.90**	**-8.23**	**2.25**
AA.com	**-9.04**	**3.07**	**-9.87**	**3.51**
United.com	**-7.08**	**1.76**	**-7.85**	**2.05**
Other Airlines	**-7.70**	**3.29**	**-8.32**	**3.64**
Expedia.com	**-7.76**	**1.77**	**-8.40**	**2.14**
Priceline.com	**-7.88**	**1.73**	**-8.63**	**2.18**
Orbitz.com	**-7.38**	**1.39**	**-8.07**	**1.81**
Travelocity.com	**-8.28**	**1.99**	**-9.16**	**2.53**
Cheaptickets.com	**-5.51**	**1.36**	**-6.53**	**1.62**
Other OTAs	**-10.73**	**3.71**	**-11.86**	**4.28**
Info Stock: Search	**0.41**		**0.44**	
Info Stock: Display/Referral	**0.18**		0.11	
Info Stock: Email	**0.39**		**0.42**	
Info Stock: Direct	**0.56**		**0.59**	
Ad spending: National Offline	0.19		0.14	
Ad spending: Local Offline	**0.04**		**0.04**	
Entry Site	**1.23**		**1.58**	
Last Site	**1.38**		**1.76**	
Lag No Purchase	**-0.59**		**-0.55**	
Lag Purchase on *j*	**0.75**		**0.88**	
Lag Visit on *j*	**0.59**		**0.63**	
Cumulative Spending	**0.07**		**0.08**	
Cumulative Page	**-0.38**		**-0.38**	
Log-likelihood	5,444		5,650	
AIC	10,962		12,080	

5.3 Advertising Elasticity of Visit and Demand

While parameter estimates show the relative significance of different channels, it is often more informative to know the extent to which visit and purchase probabilities change in response to a change in information stock. We quantify the impact of information stock of different channels on visit and purchase probability using a marginal impact analysis. The marginal impact is measured as the change in visit and purchase probabilities for each website given a change of the same magnitude in information stock collected through a specific channel for a specific website. For example, a marginal impact analysis can inform us how probabilities of being chosen as the entry site change for each website if a consumer has collected more information about Expedia.com by browsing more webpages under the influence of search advertising in the past. This marginal impact analysis therefore enables us to compare the effects of ad stock for the same website across different channels and for the same channel across different websites.

We need to simulate the change in choice probabilities because the net preferences for different websites are heterogeneous across consumers and are drawn from a normal distribution. The simulation follows six steps, which is explained in detail in Appendix B. The resulting average changes across consumers for the choice of entry site, visit probability of remaining websites, and purchase probability of visited websites are reported in Table 8 to 10.

Table 8 tabulates the own- and cross-impacts on the entry site choice as a result of 10 additional pages of a focal site browsed in the last month through additional ad clicks resulted from the four different channels. The own marginal impacts show that in general information stock collected through direct channel is most effective in raising the probability of being chosen as the entry website, followed closely by search ads. Display/referral ads and email ads also increase the choice probability but the effect is small compared to the other two channels. Every website sees a boost in choice probability due to additional information stock, but this effect size varies significantly across websites. Ad stock is most effective for Expedia.com. For example, increased information stock through search ads due to 10 more pages browsed can increase Expedia.com's probability of being chosen as the entry site by 5.86%, but the same amount of change in information stock only increases United.com's choice probability by 1.77%.

[Insert Table 8 Here]

The off-diagonal entries of Table 8 show the cross-impact of change in information stock for the website in the row on choice probability for the website in the column. This effect size varies by channels. For example, if the consumer reaches Expedia.com through search engine, Priceline.com's probability of being chosen as the entry site will decrease by 0.76%. However, the same probability will only decrease by 0.12% under the influence of display/referral ads.

Table 9 shows the changes in visit probability given an increase in information stock through different channels by websites. Again, information stock collected through direct channel is most valuable in driving up the visit probability. However, information stock gathered through email ads is also very effective, compared to search ads and display/referral ads. For example, for AA.com, if the consumer browsed 10 more webpages in the last month through email ad, the probability that she will visit the website in this purchase occasion increases by 7.54%, more than triple the change in visit probability as a result of display/referral ads. Across websites, Expedia.com is most sensitive to the change in information stock through a channel, followed by other airlines' direct websites, Priceline.com and Orbitz.com.

[Insert Table 9 Here]

Table 10 tabulates the change in purchase probabilities for every website as a result of 10 more pages browsed through additional ad clicks resulted from different channels on a focal website. The diagonal entries record the own-marginal impacts for each website. In general email ads have a greater impact on conversion probability than any other advertising formats. For example, the change in information stock gathered through email ads increases conversion probability by 4.16% for expedia.com, higher than the 3.23% increase due to more information stock through search ads or 1.79% increase due to more information stock through display/referral ads. While the high effectiveness of search engine advertising or direct visits is expected because they are consumer-initiated, the larger impact of email ads might be attributed to the higher level of consumer loyalty. Consumers who subscribed to a website's email list are often more loyal than those who are not, so they may also become more responsive to email campaigns. Moreover, if a consumer happens to receive an email ad when she has a purchase need, the coincidence will also increase her purchase probability on the website.

[Insert Table 10 Here]

However, even though every website enjoys a boost in conversion probability due to additional ad clicks, this effect size is not uniform across websites. Advertising is most effective for Expedia.com, Orbitz.com, but has barely any impact on small OTAs' conversion probability. The disparate effectiveness of the same advertising channel across competitors suggests the importance of ad content and message strategies. Firms can use these figures as the starting point toward a diagnostic process to improve its own ad messaging and copy strategies. By benchmarking its own ad effectiveness with a competitor's, a firm can determine its standing among the competitive pack.

The off-diagonal entries in Table 10 show the cross-marginal impacts of advertising on competitors' conversion probabilities. Each off-diagonal entry in Table 10 indicates the change in conversion probability for the column website due to more information stock on the row website.

Increasing information stock draws disproportionately from the website's competitors. For example, an increase in information stock gathered through email ads for Expedia.com will decrease the purchase probability for Priceline.com, Orbitz.com and Travelocity.com by more than 0.33%, but will only decrease the purchase probabilities for the other websites by less than 0.2%. This indicates that the competition between these OTAS is more intense. Moreover, the cross-impact of ad stock varies across channels. For example, an increase in information stock through Expedia.com's email ads (-0.33%) imposes greater damage versus an increase in information stock through its search ads (-0.25%) to the conversion probability of Oritz.com.

6. Predictive Analysis and Comparisons

In order to demonstrate the importance and necessity of modeling competition between firms, we compare our competitive analytics approach with a baseline model. The baseline model mimics the scenario where the company only knows its customers' interactions with itself, but not with their competitors. In this case, a consumer's purchase decision is modeled as a binary choice between buying and not buying on a visited website (in contrast to a multinomial choice from all the visited websites in the proposed model). Our comparison is carried out on the purchase stage with a focus on their predictive performances in terms of predicting purchases in the out-of-sample.

We first present the parameter estimates and in-sample model fit statistics of the baseline model in the third and fourth columns of Table 7. It is clear that our proposed model outperforms the baseline model in terms of model fitness as measured by the log-likelihood and the AIC criterion.

We then compare the predictive performance of these two models on a holdout sample. The holdout sample is composed of 3,576 purchase occasions made by the same group of consumers for a three month period between October 2010 and December 2010. Each consumer on average visited 2.41 websites in a purchase occasion, and 950 out of these 3,576 occasions ended up with a purchase, consistent with our estimation sample. The predicted purchase probability is simulated using the same method described in Appendix B.

We use two criteria to evaluate the predictive performance. The first one is the true positive (TP) rate. It is defined as the percentage of actual positive events that are predicted as "positive" by the model. We label the predicted outcome as "positive" if the probability exceeds 0.1, which is close to the actual purchase probability (950/(2.41*3,576)=0.11). The result shows that the TP rate of our proposed model is 63.37%, significantly higher than that of the alternative model, which is only 43.89%. As is shown in Table 11, the TP rate of our proposed model is 4.76% to 30.99% higher than that of the baseline model. The proposed model predicts the purchase probability for OTAs especially well, with the TP rates all above 70%.

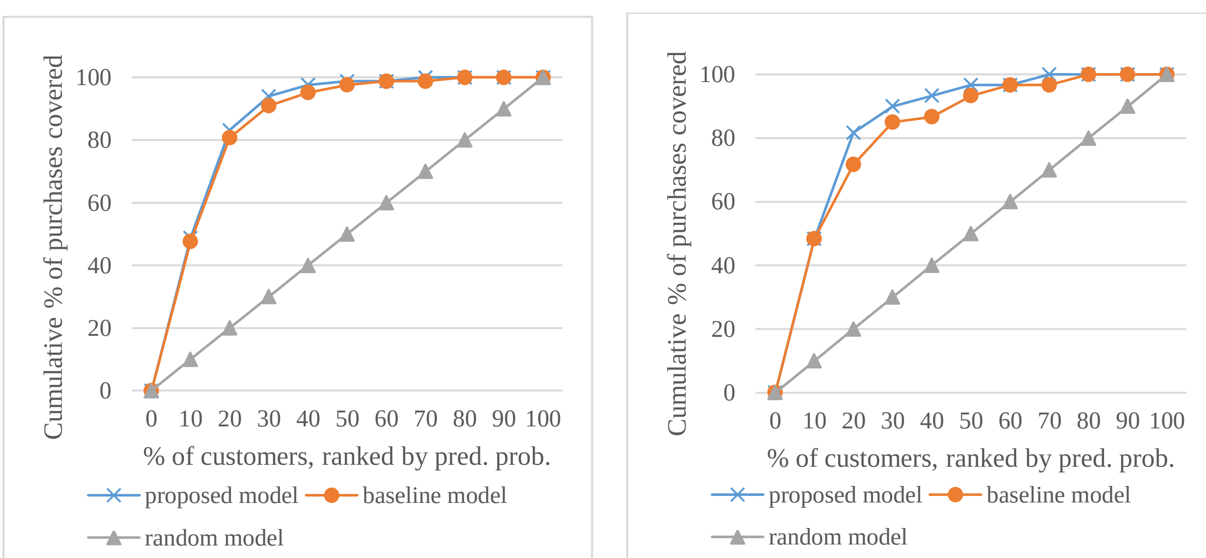
Table 11 TP Rate Comparison

	Actual # of purchases	TP rate of proposed model	TP rate of baseline model
Delta.com	90	74.44%	55.56%
AA.com	105	25.71%	14.29%
United.com	37	67.57%	54.05%
Other Airlines	278	52.88%	37.77%
Expedia.com	166	80.72%	59.64%
Priceline.com	68	77.94%	47.06%
Orbitz.com	71	83.10%	52.11%
Travelocity.com	54	72.22%	48.15%
Cheaptickets.com	60	83.33%	55.00%
Other OTAs	21	4.76%	0.00%
Average		63.37%	43.89%

The second criterion we employ is lift, which is a measure of the effectiveness of a predictive model calculated as the ratio between the results obtained with and without the predictive model. Cumulative gains and lift charts are often used to visualize model performance. We rank the observations based on the purchase probability and count the cumulative number of actual positive events picked up in each percentile and plotted the lift charts over every ten percentiles. Figure 2 illustrates the lift chart for two representative websites, Expedia.com and Cheaptickets.com for Airline and OTA sites respectively. Both the proposed model and the baseline model capture the actual purchases faster than a random model represented by the 45-degree line. However, our proposed model outperforms the comparison model as the lift line for our model is consistently above the line for the baseline model. We also find that our proposed model performs especially well for websites with smaller market share, such as Cheaptickets.com. The gap between the two lines is much larger.

Figure 2 Lift charts for Expedia.com and Cheaptickets.com

Expedia.com Cheaptickets.com



Lastly we compare the two models in terms of the marginal impacts of the estimated coefficients. The results indicate that information stock collected through display/referral ads does not contribute to the purchase probability significantly, which is an obvious departure from the insights accrued from our proposed model. The marginal impacts of the other advertising channels for the baseline model are reported in Table 12. By comparing Table 12 and the diagonal entries in Table 10, we can see that the baseline model underestimates the impact of an increase in information stock on the website's purchase probability. For example, the purchase probability of Orbitz.com will increase 4.71% due to an increase in information stock through search advertising as predicted by the proposed model, but the baseline model entails that the purchase probability will only increase by 3.93%. This demonstrates that failing to factor in competition can lead to biased estimation of the ad effects.

[Insert Table 12 Here]

7. Conclusions, Limitations, and Future Research

Practitioners have long been calling for better methods to solve the multi-channel attribution problem. Although recent research has made strides when estimating the effects of prior advertising touches, the current ad attribution models still suffer from one fundamental drawback. That is, they focus on analyzing the converting path with respect to one focal firm while largely overlooking the impact of competitors' advertising, leading to biased ad effectiveness estimates. The problem exacerbates in a purchase funnel model because competing advertising not only influences the final conversion stage, but also the earlier product information search and alternative evaluation stage.

We overcome these deficiencies and improvise the current multi-channel attribution models by accounting for competition in a consumer's purchase funnel, estimating both the direct and indirect effect of ad-clicks prior to conversion. Specifically, we develop an integrated multi-stage choice model in order to measure the advertising effectiveness on a consumer's online search and purchase decisions in a multi-channel, multi-touch-point, multi-competitor environment using individual-level advertising response data. Our model considers: (1) consumer choice of entry site, (2) consumer search decisions concerning the remaining websites that compete in the same industry and (3) subsequent purchases at one of the websites searched. We estimate this model on a unique panel data set covering both search and purchase decisions across multiple websites in the online air ticket booking industry. We then use these parameter estimates to compute the direct and indirect marginal impacts of online advertising channels and predict a consumer's future purchasing behavior based on observed variables. We further compare our proposed competition-centric model against a baseline model using data from a single-firm only. Our model outperforms the baseline model in terms of in-sample model fit as well as out-of-sample predictive performance, necessitating modeling competition in multichannel ad attribution.

Our findings offer managers new insights for evaluating advertising effectiveness. First, we find that in the online air travel industry, information stock collected through search advertising is the most effective ad format in increasing the visit probability. This is consistent with the findings in the prior literature that search ads are the most important channel under all attribution methodologies (Abhishek et al. 2014). The finding also echoes the industry belief that search engine ads are the king of online advertising. However, our result regarding the strong direct effect of email ads is contrary to the conventional belief that email ads are less useful in sealing the deal. This difference demonstrates the strength of our approach because we consider the entire purchase journey and reward all ad clicks prior to conversion, whereas most rule-based models adopted by practitioners only assign weights to touches that directly lead to a purchase; such approaches under-estimate the effect of assisting touches prior to purchase.

Second, we are able to quantify the channel- and stage-specific marginal ad impacts based on our model estimates. Some websites may find that advertising has small effect on improving its conversion probability directly, but this may not be the case for its competitors. These websites should accordingly benchmark against their competitors and focus more on ad designs. For example, firms may need to choose better keywords and improve their search ad rankings; they may also make display ads more attractive by matching the webpage content or targeting more interested online users. We also find that ad clicks through a specific channel can have either a direct impact on the conversion probability, an indirect impact through its influence on consumers' search decisions during the next purchase session, or both. Additional information stock accrued through a specific channel may still be beneficial because they increase the likelihood of these websites being included in the consumer's consideration set during the next purchase session and indirectly increase the future conversion probability, even if they have a small effect in leading to a higher direct conversion. Firms must consider both the direct and indirect effects of advertising when allocating their budgets.

Multi-channel attribution is a rapidly growing new research area and many aspects await future investigation. Our model spawns at least five interesting directions for further investigation. First, one of the main limitations of our research is that we do not observe the content of webpages visited by the consumers. If page content data is available, future research can take advantage of such data by incorporating product attribute information such as prices and product features into ad attribution models. Second, since we do not observe consumers' ad exposure in our data we cannot model their click-through decisions. It will be a very interesting extension to further model consumers' ad click decisions together with their purchase funnel decisions. Third, another limitation of our estimation sample is that the data was collected in 2010 and several popular ad channels firms in use today were not as developed at the time. This model can therefore be readily extended to incorporate the effects of more recent ad channels such as social media, mobile ads, etc. Fourth, marketing practitioners are particularly interested in determining how online and offline advertising interact, as well as the spillover effect of online advertising on offline sales and vice versa. Future research may answer this question by incorporating offline advertising and purchase decisions into the model. Finally, our model can be applied to other product categories in order to test these findings regarding the direct and indirect effects of advertising. Different industries have their own characteristics and may differ widely in channels they employ to reach out to customers.

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Appendix A: Procedures of Simulated Maximum Likelihood estimation

We assume that the overall attractiveness of visiting or purchasing from website j (website-specific intercepts in Equation 1, 2, and 3) can vary over people but being constant over choice situations for each person (Erdem and Keane 1996). The individual website-specific intercepts α_{ij}, β_{ij}, γ_{ij} are specified to follow normal distribution:

$$\begin{pmatrix} \alpha_{ij} \\ \beta_{ij} \\ \gamma_{ij} \end{pmatrix} \sim N\left(\begin{bmatrix} \overline{\alpha}_j \\ \overline{\beta}_j \\ \overline{\lambda}_j \end{bmatrix}, \Omega_j \right) \text{, where } j = 1, 2, \ldots, J$$

The probability conditional on θ_i is

$$L_i(\theta) = \prod_{t=1}^{T_i} L_{it}(\theta) = \prod_{t=1}^{T_i} P\left(F_{it} = j \middle| \alpha_i\right) \times P\left(V_{it} = C_{it} \middle| F_{it}, \beta_i\right) \times P\left(B_{it} = k \middle| F_{it}, V_{it}, \gamma_i\right)$$

The unconditional choice probability is therefore the integral of $L_i(\theta)$ over all possible values of θ_i:

$$P_{it} = \int L_{it}(\theta) f(\theta) d\theta$$

1. Draw θ_i^r from its distribution.

A set of random variables $\eta_i^r = \left(\eta_{j1}^r, \eta_{j2}^r, \eta_{j3}^r\right)$, j = 1, 2, …, J are drawn from i.i.d. standard normal distribution. Then we compute the website-specific intercepts for each person as follows:

$$\alpha_{ij}^r = \overline{\alpha}_j + \sigma_{\alpha_j} \eta_{j1}^r$$

$$\beta_{ij}^r = \overline{\beta}_j + \sigma_{\beta_j} \left(c_{21_j} \eta_{j1}^r + c_{22_j} \eta_{j2}^r\right)$$

$$\gamma_{ij}^r = \overline{\gamma}_j + \sigma_{\gamma_j} \left(c_{31_j} \eta_{j1}^r + c_{32_j} \eta_{j2}^r + c_{33_j} \eta_{j3}^r\right)$$

The benefit of this specification is that the correlation between intercepts of different stages remains the same across websites, thus significantly reducing the number of parameters to be estimated. With this specification, the variance and covariance matrix can be written as follows:

$$\Omega_j = \begin{bmatrix} \sigma_{\alpha_j}^2 & \sigma_{\alpha_j}\sigma_{\beta_j}c_{21} & \sigma_{\alpha_j}\sigma_{\gamma_j}c_{31} \\ \sigma_{\alpha_j}\sigma_{\beta_j}c_{21} & \sigma_{\beta_j}^2\left(c_{21}^2 + c_{22}^2\right) & \sigma_{\beta_j}\sigma_{\gamma_j}\left(c_{21}c_{31} + c_{22}c_{32}\right) \\ \sigma_{\alpha_j}\sigma_{\gamma_j}c_{31} & \sigma_{\beta_j}\sigma_{\gamma_j}\left(c_{21}c_{31} + c_{22}c_{32}\right) & \sigma_{\gamma_j}^2\left(c_{31}^2 + c_{32}^2 + c_{33}^2\right) \end{bmatrix}$$

2. $L_{it}(\theta)$ is calculated for each period, and the product of these $L_{it}(\theta)$'s is taken:

$$L_i^r(\theta) = \prod_{t=1}^{T_i} L_{it}^r(\theta) = \prod_{t=1}^{T_i} P\left(F_{it} = j \middle| \alpha_i^r\right) \times P\left(V_{it} = C_{it} \middle| F_{it}, \beta_i^r\right) \times P\left(B_{it} = k \middle| F_{it}, V_{it}, \gamma_i^r\right)$$

3. Repeat 1 and 2 for many time, and the results are averaged: $P_i = \frac{1}{R}\sum_{r=1}^{R} L_i^r(\theta)$.

4. Calculate the simulated log-likelihood: $SLL = \sum_{i=1}^{N} P_i$.

Appendix B: Procedures of Simulated Choice Probabilities

We will use the choice of entry site due to a change in information stock through search engine for Expedia.com as an example to illustrate. In step 1, we update the corresponding variable in the dataset, for example by adding 10 pages browsed a month ago through search advertising for Expedia.com to each individual and then re-computing the information stock through search advertising using Equation 4. In step 2, we make a draw of net preferences for each website as described in step 1 in Appendix A. In step 3, we calculate the probability of being chosen as the entry website for each website using the updated data and the net preferences drawn in step 2. Repeat step 2 and 3 for many times and in step 4, we calculate the simulated probabilities by taking a weighted average of the results in step 4. The weight is given as $w_{it}^{r} = \frac{P\left(F_{it} \mid \alpha^{r}\right)}{\sum_{r} P\left(F_{it} \mid \alpha^{r}\right)}$ where F_{it} is the actual chosen entry-site for individual i in purchase occasion t. In step 5, we follow step 2 to 4 to calculate the original choice probabilities without the change in ad stock. In the last step, we calculate the differences in choice probabilities for each website.

Table 8 Impacts of Ad Stock on Choice of Entry Site

Search Ads

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	3.17%	-0.31%	-0.14%	-0.67%	-0.56%	-0.37%	-0.34%	-0.32%	-0.27%	-0.20%
AA	-0.30%	3.40%	-0.15%	-0.70%	-0.61%	-0.41%	-0.37%	-0.37%	-0.27%	-0.22%
United	-0.16%	-0.18%	1.77%	-0.34%	-0.31%	-0.18%	-0.19%	-0.18%	-0.14%	-0.11%
Other Airlines	-0.55%	-0.60%	-0.25%	5.41%	-1.14%	-0.73%	-0.65%	-0.64%	-0.49%	-0.36%
Expedia	-0.53%	-0.59%	-0.25%	-1.27%	5.86%	-0.76%	-0.76%	-0.76%	-0.53%	-0.40%
Priceline	-0.38%	-0.44%	-0.17%	-0.89%	-0.85%	4.43%	-0.53%	-0.51%	-0.36%	-0.29%
Orbitz	-0.37%	-0.42%	-0.18%	-0.86%	-0.91%	-0.56%	4.45%	-0.50%	-0.37%	-0.29%
Travelocity	-0.34%	-0.40%	-0.17%	-0.81%	-0.87%	-0.53%	-0.49%	4.21%	-0.34%	-0.27%
Cheaptickets	-0.29%	-0.31%	-0.13%	-0.64%	-0.63%	-0.39%	-0.38%	-0.35%	3.32%	-0.21%
Other OTAs	-0.24%	-0.27%	-0.12%	-0.52%	-0.54%	-0.34%	-0.32%	-0.31%	-0.24%	2.90%

Display Ads/Referral Engines

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	0.49%	-0.05%	-0.02%	-0.11%	-0.09%	-0.06%	-0.05%	-0.05%	-0.04%	-0.03%
AA	-0.05%	0.54%	-0.02%	-0.12%	-0.10%	-0.06%	-0.06%	-0.06%	-0.04%	-0.03%
United	-0.02%	-0.02%	0.23%	-0.04%	-0.04%	-0.02%	-0.02%	-0.02%	-0.02%	-0.01%
Other Airlines	-0.10%	-0.11%	-0.04%	0.97%	-0.21%	-0.13%	-0.12%	-0.11%	-0.09%	-0.06%
Expedia	-0.08%	-0.09%	-0.04%	-0.20%	0.90%	-0.12%	-0.12%	-0.12%	-0.08%	-0.06%
Priceline	-0.05%	-0.06%	-0.02%	-0.13%	-0.12%	0.62%	-0.08%	-0.07%	-0.05%	-0.04%
Orbitz	-0.05%	-0.06%	-0.02%	-0.12%	-0.12%	-0.08%	0.60%	-0.07%	-0.05%	-0.04%
Travelocity	-0.05%	-0.06%	-0.02%	-0.12%	-0.13%	-0.08%	-0.07%	0.61%	-0.05%	-0.04%
Cheaptickets	-0.04%	-0.04%	-0.02%	-0.09%	-0.09%	-0.06%	-0.06%	-0.05%	0.48%	-0.03%
Other OTAs	-0.03%	-0.03%	-0.01%	-0.06%	-0.07%	-0.04%	-0.04%	-0.04%	-0.03%	0.37%

Email Ads

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	2.29%	-0.22%	-0.10%	-0.50%	-0.41%	-0.26%	-0.25%	-0.23%	-0.18%	-0.14%
AA	-0.22%	2.52%	-0.11%	-0.55%	-0.46%	-0.30%	-0.28%	-0.26%	-0.19%	-0.15%
United	-0.10%	-0.12%	1.18%	-0.22%	-0.21%	-0.12%	-0.12%	-0.12%	-0.09%	-0.07%
Other Airlines	-0.46%	-0.52%	-0.19%	4.44%	-0.94%	-0.59%	-0.54%	-0.52%	-0.39%	-0.28%
Expedia	-0.39%	-0.44%	-0.19%	-0.98%	4.49%	-0.60%	-0.60%	-0.59%	-0.40%	-0.31%
Priceline	-0.26%	-0.30%	-0.11%	-0.63%	-0.62%	3.14%	-0.38%	-0.37%	-0.26%	-0.20%
Orbitz	-0.25%	-0.29%	-0.12%	-0.59%	-0.64%	-0.40%	3.09%	-0.35%	-0.26%	-0.19%
Travelocity	-0.23%	-0.27%	-0.11%	-0.57%	-0.63%	-0.37%	-0.34%	2.93%	-0.23%	-0.18%
Cheaptickets	-0.19%	-0.20%	-0.09%	-0.44%	-0.43%	-0.27%	-0.26%	-0.24%	2.27%	-0.14%
Other OTAs	-0.15%	-0.17%	-0.07%	-0.33%	-0.35%	-0.22%	-0.20%	-0.19%	-0.15%	1.83%

Direct

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	5.24%	-0.51%	-0.21%	-1.05%	-0.94%	-0.61%	-0.59%	-0.54%	-0.44%	-0.35%
AA	-0.47%	5.21%	-0.22%	-1.02%	-0.96%	-0.59%	-0.60%	-0.55%	-0.45%	-0.35%
United	-0.26%	-0.29%	3.11%	-0.56%	-0.54%	-0.34%	-0.33%	-0.32%	-0.26%	-0.21%
Other Airlines	-0.78%	-0.87%	-0.35%	8.10%	-1.70%	-1.11%	-1.00%	-0.96%	-0.75%	-0.57%
Expedia	-0.74%	-0.83%	-0.36%	-1.75%	8.02%	-1.01%	-1.01%	-1.01%	-0.75%	-0.57%
Priceline	-0.54%	-0.58%	-0.25%	-1.29%	-1.16%	6.20%	-0.73%	-0.71%	-0.54%	-0.41%
Orbitz	-0.56%	-0.62%	-0.27%	-1.24%	-1.27%	-0.80%	6.47%	-0.72%	-0.57%	-0.43%
Travelocity	-0.51%	-0.57%	-0.25%	-1.18%	-1.24%	-0.76%	-0.70%	6.14%	-0.53%	-0.41%
Cheaptickets	-0.48%	-0.53%	-0.23%	-1.08%	-1.04%	-0.66%	-0.64%	-0.61%	5.67%	-0.38%
Other OTAs	-0.43%	-0.47%	-0.21%	-0.95%	-0.94%	-0.59%	-0.57%	-0.55%	-0.44%	5.16%

Table 9 Impacts of Ad Stock on Search Probability of the Remaining Websites

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Search	5.89%	5.42%	4.86%	7.94%	8.62%	7.33%	7.61%	6.97%	5.51%	5.11%
Display/Referral	2.14%	2.01%	1.60%	3.08%	3.00%	2.35%	2.40%	2.40%	1.89%	1.58%
Email	7.54%	7.10%	5.87%	10.31%	10.79%	8.98%	9.21%	8.57%	6.82%	6.06%
Direct	9.29%	8.10%	8.19%	12.24%	11.54%	9.60%	10.18%	9.47%	8.63%	7.80%

Table 10 Impacts of Ad Stock on Purchase Probability

Search Ads

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	3.23%	-0.06%	-0.08%	-0.10%	-0.14%	-0.14%	-0.14%	-0.10%	-0.08%	-0.02%
AA	-0.06%	3.25%	-0.05%	-0.12%	-0.14%	-0.20%	-0.10%	-0.08%	-0.06%	-0.01%
United	-0.11%	-0.04%	2.82%	-0.09%	-0.11%	-0.07%	-0.07%	-0.20%	-0.02%	0.00%
Other Airlines	-0.08%	-0.12%	-0.09%	4.48%	-0.21%	-0.17%	-0.17%	-0.14%	-0.09%	-0.01%
Expedia	-0.14%	-0.17%	-0.12%	-0.28%	5.12%	-0.25%	-0.29%	-0.24%	-0.15%	-0.03%
Priceline	-0.15%	-0.23%	-0.11%	-0.18%	-0.27%	4.13%	-0.24%	-0.18%	-0.15%	-0.03%
Orbitz	-0.15%	-0.15%	-0.09%	-0.23%	-0.32%	-0.26%	4.71%	-0.20%	-0.20%	-0.04%
Travelocity	-0.13%	-0.11%	-0.22%	-0.15%	-0.30%	-0.18%	-0.21%	4.14%	-0.17%	-0.02%
Cheaptickets	-0.08%	-0.06%	-0.03%	-0.09%	-0.15%	-0.17%	-0.21%	-0.17%	3.65%	-0.01%
Other OTAs	-0.04%	-0.01%	-0.01%	-0.01%	-0.03%	-0.02%	-0.03%	-0.02%	-0.01%	0.77%

Display Ads/Referral Engines

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	1.79%	-0.03%	-0.05%	-0.05%	-0.07%	-0.06%	-0.07%	-0.08%	-0.07%	-0.01%
AA	-0.03%	1.78%	-0.03%	-0.07%	-0.08%	-0.10%	-0.06%	-0.05%	-0.02%	-0.01%
United	-0.04%	-0.03%	1.50%	-0.04%	-0.07%	-0.06%	-0.03%	-0.09%	-0.01%	0.00%
Other Airlines	-0.05%	-0.07%	-0.04%	2.50%	-0.12%	-0.08%	-0.08%	-0.06%	-0.05%	0.00%
Expedia	-0.07%	-0.07%	-0.07%	-0.12%	2.73%	-0.12%	-0.14%	-0.14%	-0.07%	-0.02%
Priceline	-0.06%	-0.10%	-0.03%	-0.08%	-0.12%	2.02%	-0.11%	-0.09%	-0.07%	-0.02%
Orbitz	-0.06%	-0.06%	-0.03%	-0.09%	-0.13%	-0.10%	2.10%	-0.09%	-0.09%	-0.01%
Travelocity	-0.07%	-0.05%	-0.09%	-0.07%	-0.14%	-0.09%	-0.10%	2.05%	-0.07%	-0.01%
Cheaptickets	-0.07%	-0.03%	-0.01%	-0.05%	-0.08%	-0.07%	-0.10%	-0.08%	1.90%	0.00%
Other OTAs	-0.01%	0.00%	0.00%	0.00%	-0.02%	-0.02%	-0.02%	-0.01%	0.00%	0.33%

Email Ads

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	4.16%	-0.08%	-0.10%	-0.14%	-0.19%	-0.17%	-0.17%	-0.18%	-0.16%	-0.02%
AA	-0.08%	4.20%	-0.07%	-0.17%	-0.21%	-0.25%	-0.16%	-0.10%	-0.08%	-0.01%
United	-0.12%	-0.08%	3.72%	-0.10%	-0.17%	-0.14%	-0.09%	-0.21%	-0.04%	0.00%
Other Airlines	-0.13%	-0.18%	-0.10%	5.82%	-0.28%	-0.21%	-0.22%	-0.17%	-0.13%	-0.01%
Expedia	-0.19%	-0.20%	-0.15%	-0.29%	6.42%	-0.33%	-0.37%	-0.34%	-0.19%	-0.04%
Priceline	-0.17%	-0.26%	-0.12%	-0.23%	-0.34%	4.98%	-0.26%	-0.23%	-0.16%	-0.05%
Orbitz	-0.17%	-0.16%	-0.09%	-0.24%	-0.38%	-0.29%	5.43%	-0.24%	-0.26%	-0.04%
Travelocity	-0.19%	-0.11%	-0.20%	-0.18%	-0.37%	-0.21%	-0.24%	4.85%	-0.19%	-0.02%
Cheaptickets	-0.17%	-0.08%	-0.03%	-0.12%	-0.20%	-0.19%	-0.27%	-0.19%	4.45%	-0.01%
Other OTAs	-0.04%	-0.02%	-0.01%	-0.01%	-0.06%	-0.05%	-0.04%	-0.02%	-0.01%	0.91%

Direct

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc -ity	Cheaptic -kets	Other OTAs
Delta	2.60%	-0.05%	-0.06%	-0.08%	-0.14%	-0.10%	-0.09%	-0.15%	-0.09%	-0.01%
AA	-0.04%	2.46%	-0.03%	-0.12%	-0.15%	-0.11%	-0.11%	-0.06%	-0.04%	-0.01%
United	-0.05%	-0.08%	2.49%	-0.06%	-0.13%	-0.07%	-0.06%	-0.07%	-0.03%	0.00%
Other Airlines	-0.08%	-0.12%	-0.05%	3.93%	-0.19%	-0.12%	-0.15%	-0.11%	-0.08%	-0.01%
Expedia	-0.15%	-0.11%	-0.13%	-0.17%	4.35%	-0.17%	-0.21%	-0.17%	-0.12%	-0.03%
Priceline	-0.08%	-0.12%	-0.06%	-0.12%	-0.17%	3.08%	-0.10%	-0.12%	-0.10%	-0.02%
Orbitz	-0.13%	-0.11%	-0.07%	-0.15%	-0.25%	-0.19%	3.68%	-0.14%	-0.15%	-0.03%
Travelocity	-0.14%	-0.05%	-0.05%	-0.11%	-0.21%	-0.10%	-0.12%	2.86%	-0.09%	-0.01%
Cheaptickets	-0.09%	-0.07%	-0.01%	-0.06%	-0.11%	-0.08%	-0.17%	-0.08%	2.50%	-0.01%
Other OTAs	-0.01%	-0.01%	-0.01%	-0.01%	-0.03%	-0.02%	-0.03%	-0.01%	-0.01%	0.51%

Table 12 Impacts of Ad Stock on Search Probability of the Remaining Websites (Baseline Model)

	Delta	AA	United	Other Airlines	Expedia	Priceline	Orbitz	Traveloc-ity	Cheaptic-kets	Other OTAs
Search	3.07%	2.38%	2.69%	4.07%	4.28%	3.25%	3.93%	3.25%	3.59%	0.44%
Display/Referral	0.93%	0.69%	0.77%	1.23%	1.27%	0.87%	0.96%	0.86%	1.02%	0.10%
Email	4.12%	3.19%	3.64%	5.45%	5.62%	4.11%	4.74%	3.95%	4.54%	0.57%
Direct	2.38%	1.62%	2.27%	3.39%	3.38%	2.23%	2.82%	1.99%	2.41%	0.26%