Understanding Gamer Retention in Social Games using Aggregate DAU and MAU data: A Bayesian Data Augmentation Approach

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Abstract

With an estimated market size of over $6 billion in 2011, "social" games (games played over social networks such as Facebook or Google+) have become increasingly popular recently. Understanding gamer retention and churn is important for game developers, as retention rate is a key input to gamer lifetime value. However, as individual-level data on gaming behavior are not publicly available, developers generally rely on only aggregate statistics such as DAU (daily active users) and MAU (monthly active users), and compute *ad hoc* metrics such as the DAU/MAU ratio to assess retention rate, often resulting in very inaccurate estimates.

I propose a Bayesian approach to estimate retention rates of social games using only aggregate DAU and MAU data. The proposed method is based on a BG/BB model (Fader et al. 2010) at the individual level in conjunction with a data augmentation approach to estimate the model parameters. After validating the performance of the proposed method through a simulation study, I apply the proposed approach to a sample of 379 social games. I find that the average 1-day and 7-day retention rates for new players across games are 59.0% and 10.5%, respectively. Further, my results suggest that the median break-even acquisition cost per gamer is about 13.1 cents. In addition, giving out a "daily bonus" or limiting the amount of time that gamers can play each day may increase 1-day retention rate by 6.3% and 6.9%, respectively.

Keywords: Social gaming, retention rate, churn, aggregate data, BG/BB model, data augmentation, Bayesian statistics, satiation, acquisition cost.

1. Introduction

With a market size of over $6 billion in 2011 (MacMillan and Stone 2011), social games, defined as games that are embedded in and played through social networks (e.g., Facebook, Google+, Tencent), have become a popular form of entertainment recently. Currently, more than 100 million consumers in the United States (PopCap Research 2011) and more than 200 million consumers worldwide (according to industry reports published by the Causal Gaming Association) engage in social gaming regularly. For instance, *Cityville*, a popular simulation game, obtained nearly 100 million monthly active users in just two months since its release, while *FarmVille*, another popular farm-simulation game, has over 50 million users (Winkler 2011).

Most practitioners in the gaming industry are keenly interested in understanding gamer retention. The level of retention of a social game is believed to be a strong driver of gamer engagement (Stark 2010), monetization (von Coelln 2009), growth of the customer base (Hyatt 2009), and ultimately the financial success or failure of a social game (Hyatt 2009). Furthermore, similar to academic researchers (Gupta et al. 2004), practitioners use the retention rate of new gamers as a key input into customer lifetime value (LTV) calculations (e.g., Chen 2009), which in turn drive their gamer acquisition and promotion strategies (e.g., how much should game developers spend on acquiring a new gamer?). In addition, a better understanding of the drivers of gamer churn would help developers design games that minimize satiation and increase retention rate, thereby making their customer base more valuable.

However, measuring the retention rate of a social game is challenging because individual-level data on gaming behavior are not publicly available. Most game developers have access to only aggregate DAU (Daily Active User) and MAU (Monthly Active User) measures released daily by Facebook and other social networks. On a given day, DAU refers to the number of unique users who play the game at least once on that day, while MAU refers to the number of unique users who play the game at least once during the last 30 days. To illustrate the format of the data, Figure 1 shows the DAU and MAU for the game *Bouncing Balls*, from 11/1/2010 (the release date of the game) to 10/13/2011, the end of my data collection period.

[Insert Figure 1 about here]

Estimating retention rates using only DAU and MAU data is difficult as both measures confound the activities of new users with that of repeat users, since "cohort" information is unavailable. Thus, most practitioners have to resort to *ad hoc* metrics such as the "DAU/MAU ratio" to estimate retention rates. Unsurprisingly, the resulting estimates are often highly inaccurate, thus practitioners have yet to agree on a consensus of what the average retention rate is across the industry: The estimated average retention rates reported in several industry reports range from as low as a 15% 1-day retention rate reported in Playnomics (2012) to as high as 53% at the *monthly* level (Farago 2012). Nor does the academic literature offer much guidance on this issue. With the exception of Fader et al. (2007), almost all research in the area of customer base analysis has focused exclusively on modeling retention and churn using individual-level data on consumer visit behavior (Fader et al. 2005, 2010; Fader and Hardie 2009; Schmittlein et al. 1987), and to the best of my knowledge, none has studied the issue of inferring retention rates using aggregate data that are in the form of DAU and MAU.

To fill this important gap in research, I propose a Bayesian approach, based on data augmentation (Tanner and Wong 1987), to estimate the retention rates of social games using only DAU and MAU data. Specifically, I start with a Beta-Geometric/Beta-Bernoulli (BG/BB) model (Fader et al. 2010) of play behavior at the individual level, in which each gamer has a certain probability of churning in each period, which are assumed heterogeneous across gamers. Similar to Chen and Yang (2007), I simulate a matrix of (latent) “play histories” and a matrix of latent “state information” for a representative sample of *R* players. Importantly, conditional on the augmented “play histories" and “state information” matrices, the model parameters governing churn behavior can be easily sampled using a Gibbs sampler. Thus, I develop a MCMC procedure that alternates between sampling the individual-level model parameters and the augmented “play histories” and “state information” matrices to estimate retention rates.

After validating that the proposed methodology is able to accurately recover retention rates through a simulation study, I apply the proposed model to a dataset comprising of 379 social games that are released between 2009 and 2011. I find that the 1-day retention rates across games range from 18.2% to 97.1%, with 7-day retention rates ranging from 0.01% to 84.4%. The overall average 1-day retention rate for new gamers across games is about 59% (with an average 7-day retention rate of 10.5%). Using the posterior estimates of retention rates, I show that the median break-even acquisition cost is around 13.1 cents per gamer. Next, I explore the relationship between retention rates, game genre, and other game mechanics. My results suggest that games that belong to the “strategy” genre have, on average, higher churn rates than others. In addition, offering a daily incentive to gamers is associated with an increase of 1-day retention rate by 6.3%, while “punishing” gamers for not coming back regularly does not appear to affect retention rate. Next, limiting the amount of time or actions that players can spend in the game each day is associated with an increases of 1-day retention rate by 6.9%; this finding is consistent with recent behavioral research on satiation (Galak et al. 2013), which suggests that satiation can be reduced by slowing the rate of consumption.

In summary, this paper makes both important methodological and substantive contributions. Methodological, the current research tailors the Bayesian data augmentation approach to the new context of estimating retention rates using aggregate usage data (DAU and MAU). This is a particularly important extension because such data are commonly reported in many digital and "big data" settings such as visits to websites (Google Analytics) and consumers' engagement with social media. Substantively, this research is the first to provide model-based estimates of retention rates for social games, and hence compute per-gamer break-even acquisition costs. Further, by linking estimated retention rates to game characteristics, certain genres and game mechanics (daily bonus, limited energy) are found to be significantly associated with higher retention rates.

The remainder of this paper is organized as follows. In Section 2, I briefly describe the current industry practice of using the DAU/MAU ratio to assess retention, and review methods from the previous literature that infer individual-level parameters using aggregate data. Section 3 describes the proposed Bayesian data augmentation approach. Next, Section 4 validates the proposed approach using a simulation study. Section 5 applies the propose method to a sample of 379 social game titles, and also relates genre and game mechanics to estimated retention rates. Finally, Section 6 concludes with directions for future research.

2. Background and literature review

2.1. Current industry practice: The DAU/MAU ratio

Due to the lack of individual-level data that are needed to compute retention rate, most practitioners rely on a metric known as the DAU/MAU ratio to gauge the level of retention of a social game. Termed as the "social game sticky factor" (von Coelln 2009), the DAU/MAU ratio is defined as the average of $\frac{DAU(t)}{MAU(t)}$ over time. For instance, the popular Facebook game *Scrabble* has a DAU/MAU ratio of around 0.30, while *Bejeweled Blitz*, another popular Facebook game, has a DAU/MAU ratio of 0.27. The DAU/MAU ratio is widely interpreted among practitioners as the ability of a game to retain its users, and specifically as a "retention probability" at the daily level (Hyatt 2009; von Coelln 2009). A higher DAU/MAU ratio is believed to be predictive of the success of a social game (Hyatt 2009). More specifically, some practitioners claim that a DAU/MAU ratio of 0.15 is considered the "tipping point" to sustain growth (Lovell 2011; von Coelln 2009), and that a DAU/MAU ratio of around 0.2 to 0.3 for an "extended period of time" is necessary for the ultimate success of a game (Barnes 2010; Lovell 2011).

Despite its widespread acceptance and usage in the social gaming industry, some practitioners have begun to question whether the DAU/MAU ratio can indeed be directly interpreted as a 1-day retention rate as discussed above. By deriving a simple counterexample, Stark (2010) shows that in some cases, the DAU/MAU ratio "tells [the analyst] nothing whatsoever about whether any particular user ever comes back to the game." The main reason is that, as discussed before, the aggregate DAU and MAU data are the additive result of the behavior of new and returning users (Stark 2010), and hence it is unclear how the DAU/MAU ratio should be interpreted in the absence of a formal probability model of adoption, retention, and play behavior. Consistent with the hypothesis of Stark (2010), in Section 4, I demonstrate, through a set of simulation studies, that interpreting DAU/MAU ratio directly as a measure of 1-day retention rate can be potentially misleading.

Because of the limitation of the DAU/MAU ratio in assessing retention rates, it is perhaps unsurprising that practitioners disagree sharply on what the average retention rate is across social games. At one end of the spectrum, Playnomics (2012) claims that, on average, only about 15% of players return after their first day, and only 5% return after the first week. Similarly, Tan (2012) states that even a good game generally has a 1-day retention rate of only around 35%. In sharp contrast to the above estimates, Doshi (2010) claims that a 7-day retention rate of 35% is "pretty good" and a 7-day retention rate of around 55% is "excellent". At the other end of the spectrum, Duryee (2012) cites a study by Flurry (a social gaming analytics company) that estimates that the average *monthly* retention rate for social games is around 47%, with a *quarterly* retention rate of 30%.

Given that customer lifetime value computations (which are important for computing per-gamer break-even acquisition cost, as will be discussed later) are highly sensitive to estimated retention rates (Chen 2009), a better and more formal methodology is needed to assess retention rates using only aggregate DAU and MAU data. This is closely related to previous research on estimating individual-level behavioral parameters using aggregate market-level data, which I briefly review in the next section.

2.2. Inferring individual-level behavioral parameters from aggregate data

Several approaches have been proposed in the previous academic literature to make inference about individual-level behavioral parameters (e.g., individual brand preferences) when only aggregate data (e.g., brand market shares) are available. One approach is to "integrate out" the individual-level heterogeneity, either analytically or numerically, to obtain the (marginalized) likelihood of the aggregate data. For instance, Fader et al. (2007) analytically derive the likelihood of "data summaries" in the form of cross-sectional histograms when the individual-level behavior follows a Pareto/NBD model (Schmittlein et al. 1987), resulting in a closed-form likelihood function. In the context of estimating a random coefficient logit model using aggregate data, Jiang et al. (2009) and Park and Gupta (2009) approximate the likelihood function of the aggregate data using numerical integration procedures (e.g., Robert and Casella 2004). Once the likelihood function of the aggregate data is evaluated, one can then proceed with classical maximum likelihood estimation (Fader et al. 2007; Park and Gupta 2009) or Bayesian estimation (Jiang et al. 2009).

In some cases when the likelihood of the aggregate data cannot be derive analytically and is difficult to evaluate numerically, an alternative approach based on data augmentation (Tanner and Wong 1987) can be employed. The basic idea is to augment the aggregate data by simulating the (latent) behavior of a large set of R "representative" consumers, in a way that is consistent with the observed aggregate data and the assumed model (Chen and Yang 2007; Musalem et al. 2008, 2009). For instance, Chen and Yang (2007) utilize a data augmentation approach to assess the impact of purchase history on current brand choice using aggregate market share data. In a similar vein, Musalem et al. (2008) develop a data augmentation approach, where they simulate the brand choices and coupon usage behaviors for a representative set of consumers, to understand the effect of coupon distribution and redemption using aggregate market-level data. In this paper, I further tailor the data augmentation approach to estimate retention rate using aggregate DAU and MAU data.

3. Estimating retention rate with aggregate DAU and MAU data

I now develop the proposed Bayesian approach to estimate retention rates using only aggregate DAU and MAU data. Section 3.1 describes a parsimonious individual-level model of gamer churn and play behavior that is built upon the BG/BB model (Fader et al. 2010). Next, Section 3.2 discusses the data augmentation approach used to simulate individual-level play histories and latent state information. Section 3.3 outlines how the model parameters are calibrated using an MCMC procedure.

3.1. Individual-level model of gamer churn and play behavior

The following notations are used throughout this paper: Subscript *i* $(i = 1,2,...,I)$ indexes social game titles; M_i denotes the market potential (i.e., number of potential players) for game *i*, and subscript *j* ($j = 1,2,...,M_i$) indexes players in the *i*-th game. Subscript *t* ($t = 1,2,...,T_i$) indexes the number of days since the release date of the *i*-th game, where *t* = 1 on the release date of the game.

At the heart of the proposed methodology is a Hidden Markov Model (HMM) (Netzer et al. 2008) of individual gamer's churn and play behavior that is built upon the BG/BB model proposed by Fader et al. (2010). The following model description focuses on the *i*-th game. On day *t*, each gamer *j* of game *i* is assumed to be in one of three latent states $s_{ijt} \in \{$ U ("Unaware"), A ("Active"), or D ("Dead")$\}$. Before the game is released, all gamers are assumed to be in the "Unaware" state, i.e., $s_{ij0} = U \ \forall j$. Then given her current latent state, the stochastic process by which a gamer transitions between the three latent states is specified as follows.

First, a gamer *j* who is "Unaware" of the game on day *t-1* (i.e., $s_{ij(t-1)} = U$) may become "Active" at the beginning of day *t* with adoption probability π_{it} . On the first day that she becomes "Active", it is assumed that the gamer will play the game on that day. Further, the adoption probability π_{it} in each period is allowed to be time-varying (presumably due to variations in advertising and promotional activities), with its temporal variations captured by a

Beta distribution, where $\pi_{it} \sim Beta(a_i^{(\pi)}, b_i^{(\pi)})$.[1] Next, on each day, an "Active" player may become satiated and quit the game (i.e., becomes "Dead") with probability θ_{ij}, where the individual-level churn probability θ_{ij} is allowed to be heterogeneous across gamers. Following the Beta-Geometric model in Fader et al. (2010), the heterogeneity of θ_{ij} is captured by another Beta distribution where $\theta_{ij} \sim Beta(a_i^{(\theta)}, b_i^{(\theta)})$. Once "Dead", a gamer is assumed to remain in the "Dead" state indefinitely; i.e., gamers may not transition from a "Dead" state back to "Unaware" or "Active" states (Fader et al. 2010; Schmittlein et al. 1987). Thus, the way a gamer stochastically transitions between three latent states (U, A, D) can be summarized by the following state transition matrix:

$$\Pr(s_{ijt} \mid s_{ij(t-1)}) = \begin{array}{cc} & s_{ijt} \\ & \begin{array}{ccc} U & A & D \end{array} \\ s_{ij(t-1)} \begin{array}{c} U \\ A \\ D \end{array} & \begin{pmatrix} 1-\pi_{it} & \pi_{it} & 0 \\ 0 & 1-\theta_{ij} & \theta_{ij} \\ 0 & 0 & 1 \end{pmatrix} \end{array} \qquad [1]$$

where, as discussed earlier,

$$\pi_{it} \sim Beta(a_i^{(\pi)}, b_i^{(\pi)}), \qquad [2]$$

and $$\theta_{ij} \sim Beta(a_i^{(\theta)}, b_i^{(\theta)}). \qquad [3]$$

Given the latent state (s_{ijt}) that the gamer is in on each day, her play behavior is modeled as follows. Let y_{ijt} denote whether player *j* (of game *i*) plays the game on day *t*, where $y_{ijt} = 1$ if the gamer plays the game, and 0 otherwise. Following the BG/BB model (Fader et al. 2010), the proposed model assumes that a player may only play the game if she is in the "Active" state, i.e.,

[1] Note that an i.i.d. specification is used here to maintain computational tractability, as will be discussed in Section 3.3 and Appendix I. An alternative specification that allows the adoption probability to be dependent of the number of current adopters (e.g., Bass 1969) or the proportion of current adopters to the number of potential adopters would lead to more intensive computations. I leave such extensions for future research.

$P(y_{ijt} = 1 | s_{ijt} \neq A) = 0$. Further, if a player is in the "Active" state on day *t* (i.e., $s_{ijt} = A$), she plays the game with probability ϕ_{ij} (except for the first "Active" day when she will definitely play, as discussed earlier), which is also allowed to be heterogeneous across gamers according to a Beta distribution, where $\phi_{ij} \sim Beta(a_i^{(\phi)}, b_i^{(\phi)})$. Thus, the "play" part of the model follows closely from the Beta-Bernoulli specification in Fader et al. (2010). Following the same assumption in Fader et al. (2010) and Schmittlein et al. (1987), the individual-level churn parameter θ_{ij} and play parameter ϕ_{ij} are assumed to be a priori independent.[2]

To summarize, the following set of equations describe the individual-level model of gaming behavior:

$$P(y_{ijt} = 1 | s_{ijt} \neq A) = 0\,, \tag{4}$$

$$P(y_{ijt} = 1 | s_{ijt} = A) = \phi_{ij}\,, \tag{5}$$

$$\phi_{ij} \sim Beta(a_i^{(\phi)}, b_i^{(\phi)})\,. \tag{6}$$

Thus, at the individual level, Equation [1]—[6] is a straightforward extension of the Beta-Geometric/Beta-Bernoulli (BG/BB) model proposed by Fader et al. (2010), with the addition of the "Unaware" state to capture new player acquisition. The introduction of an "Unaware" state allows the proposed model to later interface with the aggregate DAU and MAU data where cohort information is unavailable. The BG/BB model and its continuous analogue, the Pareto/NBD model, have been applied to various customer base analytic settings and have been found to generally fit the data quite well, both in- and out- of sample (e.g., Fader et al. 2010; Fader and Hardie 2009; Schweidel and Knox 2013).

[2] In Appendix III, I conduct a robustness check to assess the extent to which estimated retention rates are affected when this independence assumption is violated. The results suggest that the model estimates are fairly robust to moderate correlations between θ_{ij} and ϕ_{ij}.

Once the above individual-level model (Equation [1]—[6]) is calibrated, estimates for 1-day and 7-day retention rates for new gamers can be obtained directly from the model parameters $a_i^{(\theta)}$ and $b_i^{(\theta)}$. Specifically, the 1-day retention rates for new players of game *i* is:

$$\text{1-day retention rate of game } i = E(1-\theta_{ij}) = 1 - \frac{a_i^{(\theta)}}{a_i^{(\theta)} + b_i^{(\theta)}}, \qquad [7]$$

while the 7-day retention rate of game *i* can be compute numerically as follows:

$$\text{7-day retention rate of game } i = E\left[(1-\theta_{ij})^7\right] = \int (1-\theta)^7 Beta(\theta \mid a_i^{(\theta)}, b_i^{(\theta)}) d\theta, \qquad [8]$$

where the integral in Equation [8] can be easily evaluated numerically using standard Monte Carlo integration methods (Robert and Casella 2004).

3.2. Augmenting the aggregate data with latent play histories and state information

The remaining methodological challenge at this point is to calibrate the proposed individual-level model of play behavior (defined by Equation [1]—[6]) using only aggregate DAU and MAU data. As stated in the introduction, my primary goal is to estimate the "churn" parameters $a_i^{(\theta)}$ and $b_i^{(\theta)}$ for each game. While it is straightforward to calibrate the aforementioned individual-level model given individual-level data on play behavior (see Fader et al. 2010), it is impossible to analytically derive the marginalized likelihood of the observed aggregate DAU and MAU data given model parameters. Thus, the estimation procedure proposed below relies on a data augmentation approach (Tanner and Wong 1987), which has been implemented previously in the marketing literature by Chen and Yang (2007) and Musalem et al. (2009), as reviewed in Section 2.2.

Similar to the previous literature (Chen and Yang 2007; Musalem et al. 2009; Park and Gupta 2009), the proposed estimation procedure augments the aggregate data for each game *i* with latent "play histories" (from time $t = 1$ to T_i) and latent state information for a random, representative sample of R players, where $R << M_i$. Thus, for each game i, the augmented data are comprised of two R x T_i matrices: (i) a "play histories" matrix $Y_{(i)}$, where $y_{(i)jt} = 1$ if the j-th gamer in the sample plays the game on day t, and 0 otherwise, and (ii) a latent "state information" matrix $S_{(i)}$, where $s_{(i)jt}$ denotes the latent state ({U, A, D}) that the j-th player is in on day t.

Note that, technically, one can potentially augment the data "fully" by setting $R = M_i$, a proposal that is also considered in Chen and Yang (2007). However, Chen and Yang (2007) state that having R that is too close to M_i may lead to poor "mixing" performance in the MCMC sampler and hence inefficient posterior sampling. In addition, in the context of social gaming, setting $R = M_i$ is impractical given that many social games have up to millions of players. First, there is not enough computer memory to store and process the full latent play histories and state information matrices. [3] Further, even if the two latent matrices can fit into computer memory, the computational burden of handling two full M_i x T_i matrices in MCMC computations will still be prohibitively expensive (Musalem et al. 2009). Thus, throughout this paper I set R = 10,000; as will be validated in Section 5, the model is able to fit the observed data patterns very well.

Given that the augmented "play histories" $Y_{(i)}$ and "state information" $S_{(i)}$ matrices are defined on a sample of R players, I need to relate summary statistics computed with $Y_{(i)}$ to the observed DAU and MAU time series. Let $\hat{d}_{it}$ and $\hat{m}_{it}$ be the sample-based estimates of DAU and MAU, which are computed by "scaling" the summary statistics in the latent play histories of R players to the entire population of M_i potential players. That is:

$$\hat{d}_{it} = \frac{M_i}{R} \sum_j y_{(i)jt} \qquad [9]$$

[3] Consider M = 100 million (as discussed in the introduction, the social game *CityVille* has an MAU of 100 million, which sets a lower bound for its potential number of players M_i). Augmenting a year of play history of each player would require simulating 2 * 365 * 100 million = 7.3 x 10^{10} entries. Assuming that each entry takes only 1 byte of memory to store, the two latent matrices alone would require 8,000 terabytes of memory to store and process.

and $\hat{m}_{it} = \frac{M_i}{R}\sum_j z_{(i)jt}$, where $z_{(i)jt} = \sum_{s=\max(0,t-29)}^{t} y_{(i)js}$. [10]

Note that $z_{(i)jt}$ takes the value of 1 if the *j*-gamer plays the game at least once during the last 30 days, and the value of 0 otherwise. If $R = M_i$ and there is no measurement error in the DAU and MAU data, the sample statistics ($\hat{d}_{it}, \hat{m}_{it}$) should correspond exactly to the observed DAU and MAU data, i.e., $\hat{d}_{it} = d_{it}$ and $\hat{m}_{it} = m_{it}$. However, given that, as discussed earlier, $R \ll M_i$ and in addition there are some known measurement/recording errors in observed DAU and MAU data,[4] the corresponding (scaled) summary statistics from $Y_{(i)}$ are only assumed to be only approximately equal to the observed DAU and MAU time series as follows:

$$\log(\hat{d}_{it}) = \log(d_{it}) + \varepsilon_{it}^{d}, \qquad \varepsilon_{it}^{d} \sim N(0, \sigma_d^2) \qquad [11]$$

$$\log(\hat{m}_{it}) = \log(m_{it}) + \varepsilon_{it}^{m}, \qquad \varepsilon_{it}^{m} \sim N(0, \sigma_m^2) \qquad [12]$$

where ε_{it}^{d} and ε_{it}^{m} represent "fudge factors" (e.g., Lehmann 1971) that capture the deviations of the (scaled) sample-based estimates of DAU and MAU from their observed counterparts, due to a combination of sampling error and measurement error. Note that Equation [11]—[12] differs from the specification in previous literature on data augmentation (Musalem et al. 2008, 2009), where the sample statistics are often assumed to the exactly equal to observed aggregate statistics. Throughout this paper, I set $\sigma_d = \sigma_m = 0.05$, which allows for (roughly) a 5% error between the sample estimates and the observed DAU and MAU.[5]

3.3. Model calibration

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[4] According to Appdata (the data provider), there are some minor non-specific measurement errors in the DAU and MAU datat. For instance, in several cases I notice that the DAU and MAU values on the release date of a game are not the same (when they should be equal by definition), which Appdata ascribes to server-side recording error. Such measurement errors are assumed to occur at random.

[5] Robustness checks on other values of σ_d, σ_m are conducted. The results are the substantively unchanged and are available upon request.

Similar to the assumption made in the previous literature (Chen and Yang 2007; Musalem et al. 2008, 2009), the market potential for each game (M_i) is assumed known and thus have to be selected before calibrating the model. Similar to the approach taken by Chen and Yang (2007), I use a heuristic approach to choose M_i. Specifically, I use the following estimator of market potential:

$$\hat{M}_i = m_{i(1)} + m_{i(31)} + m_{i(61)} + \cdots + m_{i(361)} . \qquad [13]$$

That is, a rough estimator for the market potential is computed by summing together the MAU values at 30-day intervals. The intuition is as follows. Assuming that most gamers churn within 30 days and that most potential players have joined within a year after game is released, Equation [13] provides a reasonable approximation for the market potential M_i. Of course, Equation [13] will tend to over-estimate M_i if the retention rate is high (as some gamers who are active on day 1 still have not churned on day 31). However, in the simulation studies described in Section 4, I find that this "ballpark" estimator for $\hat{M}_i$ appears to work fairly well, and that the estimation of retention rates is rather insensitive to estimation error of $\hat{M}_i$.

Next, I choose the values of hyperparameters ($a_i^{(\phi)}, b_i^{(\phi)}$), which describe the heterogeneity distribution of ϕ_{ij} across gamers, based on the information provided in a supplemental industry survey on social gaming behavior (PopCap 2011). Note that I do not sample the hyperparameters ($a_i^{(\phi)}, b_i^{(\phi)}$) jointly along with other model parameters to avoid identification problems: In preliminary simulation studies where ($a_i^{(\phi)}, b_i^{(\phi)}$) are sampled jointly with other parameters, the estimation procedure is unable to accurately recover the "true" churn parameters ($a_i^{(\theta)}, b_i^{(\theta)}$) of interest. In the aforementioned survey of social gaming behavior (PopCap 2011), 70% of social gamers indicate that they engage in social gaming "once a day or more", 25% indicate that they play "2-3 times a week", and 4% indicate that they play "once a week or less". Based on this survey information, I set $(a_i^{(\phi)} = 0.76, b_i^{(\phi)} = 0.12)$ to match the summary statistics presented above.[6] I have also conducted robustness checks with respect to this assumption in Appendix II.

To complete the model specification, I specify weakly informative, $N(0,1000^2)$ priors on the hyperparameters $\left(a_i^{(\pi)}, b_i^{(\pi)}\right)$ and $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$. Given $\hat{M}_i$ and ($a_i^{(\phi)}, b_i^{(\phi)}$), and conditional on the augmented play histories matrix $Y_{(i)}$ and latent state information matrix $S_{(i)}$, it is straightforward to sample from the posterior distributions of the adoption parameters (π_{it}), individual-level churn (θ_{ij}), play (ϕ_{ij}) parameters, and the hyperparameters ($a_i^{(\theta)}, b_i^{(\theta)}$) using a Gibbs sampler (Casella and George 1992). Next, I sample the augmented play histories matrix $Y_{(i)}$ and latent state information matrix $S_{(i)}$ row-by-row using an independent Metropolis-Hastings sampler (Gelman et al. 2003). Specifically, in each iteration, for each representative gamer j I simulate her "proposed play history" using the HMM specified in Equation [1]—[6] based on the current draw of θ_{ij} and ϕ_{ij} . I then compute the likelihood of the proposed play history given Equation [11] and [12], and accept/reject the new draw based on the Metropolis-Hastings acceptance probability (Gelman et al. 2003). Other computational details, which are all based on standard Bayesian conjugate computations conditional on augmented $Y_{(i)}$ and $S_{(i)}$, are described in Appendix I. I code up the above MCMC procedure in C++, using the GSL library, and run the code for 4,000 iterations for each game. The first 2,000 iterations are discarded as burn-in samples, leaving the last 2,000 draws to summarize the posterior distribution (Gelman et al.

[6] More specifically, these statistics are assumed to come from a $x \sim BetaBinomial(n = 7, a_i^{(\phi)}, b_i^{(\phi)})$ distribution. I choose ($a_i^{(\phi)} = 0.76, b_i^{(\phi)} = 0.12$) so that the conditions $\Pr(x = 7) = 0.7$ (i.e., 70% of social gamers play "once a day or more") and $\Pr(x \le 1) = 0.06$ (i.e., 6% of social gamers play "once a week or less") are satisfied.

2003). Standard diagnostics confirm that convergence has been reached. The full C++ code and posterior draws are available upon request.

4. Simulation studies

To ensure that the proposed estimation procedure can accurately recover retention rates from aggregate DAU and MAU data, I conduct a series of simulation studies using different sets of known model parameters. Specifically, for the following simulations, the market potential M is set to be 100,000,[7] while the length of the data collection period is set to T = 300 days, which roughly corresponds to the average number of observations in the actual data. As described in Section 3.3, I set the hyperparameters governing gamers' "play" behavior as $a^{(\phi)} = 0.76$ and $b^{(\phi)} = 0.12$. The hyperparameters governing the adoption probabilities are set as $a^{(\pi)} = 2$ and $b^{(\pi)} = 98$, so that across time, the mean adoption rate on each day is 2/(2+98) = 0.02. I then vary the hyperparameters governing the retention rate from ($a^{(\theta)} = 1, b^{(\theta)} = 19$), ($a^{(\theta)} = 2, b^{(\theta)} = 18$), …., to ($a^{(\theta)} = 19, b^{(\theta)} = 1$), which corresponds 1-day retention rates of 19 / (1+19) = 95%, 90%, …, 5%, respectively. The range of 1-day retention rates above covers the range of estimated retention rates in the actual dataset (to be discussed in Section 5).

I conduct two separate sets of simulations: (i) with $\hat{M}$ set at the true value $\hat{M}$ = 100,000, and (ii) with $\hat{M}$ selected using the "ballpark" estimator specified in Equation [13]. Due to spatial constraint, I only present the result where $\hat{M}$ is chosen using Equation [13]. The results of the first specification are very similar and are available upon request. For each of the 19 simulated datasets (each simulated using different parameters), I run the MCMC estimation procedure as described in Section 3 to calibrate the model, and record the posterior distribution of the

[7] I have also conducted another set of simulation studies where M is set to be 1,000,000. The results are substantially unchanged and are available upon request.

($a^{(\theta)}, b^{(\theta)}$) in each case. The estimated 1-day retention rates $1-\left(\frac{a^{(\theta)}}{a^{(\theta)}+b^{(\theta)}}\right)$, along with the corresponding 95% posterior intervals, are shown in Table 1. Figure 2 plots the estimated 1-day retention rates against the true 1-day retention rates.

[Insert Table 1 and Figure 2 about here]

As can be seen, the proposed model is able to recover the "true" retention rates fairly well from the aggregate DAU and MAU data, across all true values of retention rates from 0.05 to 0.95. Except in cases when the true 1-day retention rates are extremely low (≤10%), almost all the "true" values lie within their corresponding 95% posterior intervals, indicating that the estimation approach proposed here is able to accurately recover retention rates of social games from only aggregate DAU and MAU data. Overall, the RMSE (root mean squared error) of the estimate 1-day retention rates from true 1-day retention rates is 0.013, indicating an excellent fit.

Looking at the results in Table 1 more closely, I notice that the "ballpark" estimator $\hat{M}$ do not always reflect the market potential M accurately. In particular, when the true 1-day retention rate is high, $\hat{M}$ grossly over-estimates M because it "double-counts" the same gamer from one month to another, as discussed earlier. For instance, when the true 1-day retention rate is 0.95, $\hat{M}$ = 256,529, which over-estimates the true market potential by more than 250%. In such case, the mean adoption rate $\left(\frac{a^{(\pi)}}{a^{(\pi)}+b^{(\pi)}}\right)$ is also grossly under-estimated, as can be seen in Table 1. However, interestingly the estimates of 1-day retention rate are fairly insensitive to estimation errors of $\hat{M}$ and the adoption rate. Even when the true 1-day retention rate is 0.95 (where both $\hat{M}$ and the adoption rate are grossly mis-estimated as discussed above), the retention rate is estimated to be 0.945, with a 95% posterior interval of (0.938, 0.952) that covers the true value. I speculate that this apparent robustness of retention rate estimation to estimation errors of $\hat{M}$ and the adoption rate is due to the fact that the proposed model relies only on a representative sample of R gamers. If $\hat{M}$ is over-estimated, the model simply adjusts the "scaling" factor of $\frac{\hat{M}}{R}$ in Equation [9]—[10], so that each gamer in the sample "represents" more gamers in the population.

Next, I compare the performance of the proposed model with the DAU/MAU ratio currently used by many practitioners as a proxy for retention. As discussed earlier in Section 2.1, the average ratio between DAU and MAU is widely believed to measure how "sticky" (von Coelln 2009) a social game is, and is often interpreted as a measure of retention rate at the daily level (Hyatt 2009; von Coelln 2009). Thus, I compute the following metric and compare it vis-à-vis the estimates from the proposed model:

$$\text{Average } DAU / MAU \text{ ratio} = \frac{1}{T_i} \sum_{t=1}^{T_i} \left(\frac{d_{it}}{m_{it}} \right) \qquad [14]$$

As can be seen in the last column of Table 1, the average DAU/MAU ratio is a very poor estimator for the 1-day retention rate, with an overall RMSE of 0.425, which is more than 32 times larger than the RMSE of the model-based estimate stated earlier (0.013). This result clearly suggests that practitioners cannot rely on the average DAU/MAU ratio as an accurate estimator of the 1-day retention rate. As a result, using the average DAU/MAU ratio to estimate retention rate as an input to CLV models would grossly miscalculate the break-even acquisition cost per gamer.

Before applying the proposed methodology to actual data, note that one of the key limitations of the proposed model is that the hyperparameters that govern gamers' play behavior ($a_i^{(\phi)}, b_i^{(\phi)}$) are chosen based on a supplementary survey (as discussed in Section 3.3), and are assumed to be constant across different games. Estimated retention rates from the proposed model can be biased if the actual ($a_i^{(\phi)}, b_i^{(\phi)}$) hyperparameters for a certain game are different from the assumed values. To assess the sensitivity of estimated retention rates to mis-specification errors of ($a_i^{(\phi)}, b_i^{(\phi)}$), I conduct two sets of robustness checks in Appendix II where ($a^{(\phi)}, b^{(\phi)}$) are systematically varied. The results there suggest that the accuracies of estimated retention rates are reasonably insensitive to minor misspecification errors (around $\pm$ 10%) of the average "play" probability; details are provided in Appendix II.

5. Empirical application

5.1. Data overview and summary statistics

The dataset in this paper is obtained from Appdata (www.appdata.com), a San Francisco, CA-based company that provides daily usage data and tracks ratings for Facebook, iOS, Android, and Windows apps. According to information provided on the company's website, their main clients are developers, publishers, investors, and industry analysts who use Appdata to assess where different apps rank in the Facebook and mobile app markets. In the case of daily usage data of Facebook games analyzed in this paper, Appdata gathers daily DAU and MAU data directly from Facebook through an application programming interface (API) and sells it to the general public through a monthly subscription plan.

From Appdata, I obtain daily usage data for a sample of 379 Facebook social games that are released between September, 2009 and September, 2011. As discussed earlier, the usage data for each game include DAU (the number of Daily Active User) and MAU (the number of Monthly Active User) on each day from the release date of each game up to October, 2011, the end of my data collection period. Several key summary statistics of the usage data are shown in

Table 2. As can be seen, the length of the data collection period for each title is around 386 days, or a little more than a year. The average DAU (across the entire history of each social game) is around 97,000, with a wide range from 20 to over 6 million. The peak DAU across games is 212,290 on average, with a range of 120 to over 11 million. In terms of monthly unique users, the average MAU across games ranges from 240 to over 37 million, with an average of 571,630. The peak MAU ranges from around one thousand users to around 66.5 million users, with an average of around 1.2 million users. The large MAU for some games suggests that, as discussed in Section 3.2, a full data augmentation approach ($R = M_i$) is infeasible because of large memory storage requirement and heavy computation burden. Finally, the median and mean DAU/MAU ratio across the set of 379 social games are 0.10 and 0.12 (respectively); these figures are well in line with industry standards, suggesting that the dataset collected here is fairly representative of social games in general.

[Insert Table 2 about here]

In addition to the usage data shown in Table 2, I supplement the dataset with key game characteristics. For each game, I record the genre of the game (e.g., Action & Sports, Arcade, Board & Card, Simulation, Virtual World, etc.). Specifically, each game is categorized into one of nine different genres listed in Table 3, along with several examples. As can be seen, the "Arcade" genre is the most popular genre in my dataset, accounting for more than 20% of all games. In contrast, the "Trivia" and "Strategy" genre are the least popular genres in the dataset, each accounting for around 2% of all games.

[Insert Table 3 about here]

Next, a research assistant conducts a content analysis for each game and specifies the mechanics by which the game encourages player retention. Specifically, I look for the following game mechanics that are suggested by Barnes (2010) as useful ways to reduce churn: (i) daily bonus, (ii) overdue punishment, and (iii) limited energy, described in the subsections below.

Daily bonus

Similar to a loyalty program (Bolton et al. 2000), gamers are given direct incentives to return and play the game each day. For example, in the game *Poker Texas Hold'em Cardstown*, players are given free bonus poker chips when they check-in on each day. Similarly, in the game *It Girl*, gamers receive a daily bonus in the form of "experience points" and (virtual) cash that they can spend on in-game items such as clothes and accessories every 24 hours, by clicking on the "newsstand" within the game. As shown in Table 3, around 20% of the social games provide some kind of reward to gamers when they "check-in" on each day.

Overdue punishment

A player is given a specific "punishment" if she does not play the game every day. For example, in *Ranch Town* (a farm simulation game), a player's crops will "rot" if she does not come back the next day to "harvest" them. A similar mechanic is used in the game *Smurf & Co*. In the game *Casino City*, visitors periodically leave "trash" on the casino floor that needs to be "cleaned" by the gamer (by clicking on the trash). Thus, like the "daily bonus" mechanic, this strategy also directly incentivizes gamers to return often to avoid being penalized. Overall, around 9% of the games in the dataset punish gamers in some ways for not checking in regularly.

Limited energy

Some games explicitly limit the amount of time or "actions" that players can perform on each day. Typically, the game assigns players with a certain amount of "energy" every day. Each in-game action requires players to spend a certain amount of energy; once a player is out of energy, she has to wait till the next day to play again. For instance, in the game *Mafia Wars*, players have limited energy for doing "jobs" (such as fighting with other gangs). Once out of energy, players have to wait for their energy to slowly recharge over time. Similarly, in the game *Criminal Case*, working on each crime scene investigation requires energy, thus limiting the number of crime cases that a gamer can solve on each day. More generally, the goal of the "limited energy" strategy is to "ration" how much players can play in order to "keep them hungry" (Barnes 2010). This strategy is closely related to behavioral research on satiation (Redden and Galak 2013), which shows that the rate of satiation is driven by consumption timing, yet consumers typically do not choose their consumption timing "optimally" (Galak et al. 2011). For instance, Galak et al. (2013) find that consumers are prone to consume a hedonic activity too rapidly when allowed to choose their own consumption timing, which leads to faster satiation and lower total consumption than they would with slower consumption. Thus, the "limited energy" strategy is similar in spirit to the recommendation by Nelson and Meyvis (2008) and Nelson et al. (2009), who propose that deliberate "breaks" be inserted within an enjoyable experience to help reduce satiation and raise overall consumption, as breaks disrupt the process of adaptation and partially restores consumer's enjoyment to its original level (Nelson and Meyvis 2008). Thus, inserting a short break in a positive experience (e.g., a commercial break within a TV show) makes the experience more pleasant. Around 26% of games in the dataset employ some form of "limited energy" mechanics.

Note that the above game mechanics are not mutually exclusive; a game can incorporate any combination of these mechanics. Later in Section 5.4, these game mechanics will be linked to the estimated retention rates across games to empirically assess their effectiveness in encouraging gamer retention.

5.2. An illustrative example: Estimating retention rate for the game *Bouncing Ball*

To illustrate the performance of the proposed model in fitting the observed DAU and MAU data, I look at the game title *Bouncing Ball* discussed in the introduction, an arcade game that was released on 11/1/2010, whose DAU and MAU data are shown in Figure 1. For this game, daily DAU and MAU data are recorded for a total of 347 days.

First, the fit of the model in terms of (log-) DAU and MAU are shown in Figure 3, where the solid line denotes the observed data and the blue line denotes the (scaled) estimates, computed using Equation [9]—[10], from the "augmented" play histories matrix $Y_{(i)}$. As can be seen, the scaled DAU and MAU estimates from the augmented data matrix track the observed DAU and MAU extremely well; the MSE for log(DAU) and log(MAU) are 0.0034 and 0.0006, respectively, indicating an excellent fit to the observed usage data.

[Insert Figure 3 about here]

Next, I turn to the parameters estimates for the title *Bouncing Ball*. Following the estimator proposed in Section 3.3, the "ballpark" estimate of market potential $\hat{M}$ for the game *Bouncing Ball* is 870,885. The posterior mean of the average daily adoption rate $E(\pi_{it}) = \frac{a_i^{(\pi)}}{a_i^{(\pi)} + b_i^{(\pi)}}$ is around 0.011, suggesting that about 9,600 new gamers join the game on each day. Next, I look at the posterior distribution of the churn parameters $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$, and hence the estimated 1-day retention rate $1 - E(\theta_i) = 1 - \frac{a_i^{(\theta)}}{a_i^{(\theta)} + b_i^{(\theta)}}$. The posterior mean of $a_i^{(\theta)}$ and $b_i^{(\theta)}$ are 7.50 and 11.67, respectively. Given the posterior draws of $a_i^{(\theta)}$ and $b_i^{(\theta)}$, a histogram displaying the posterior distribution of the 1-day retention rate is shown in Figure 4, where the posterior mean is depicted by a solid vertical line. As can be seen, the posterior mean of the 1-day retention rate is 60.9%, with a 95% posterior interval of (58.8%, 62.2%). The narrow posterior interval suggests that even given only aggregate DAU and MAU data, retention rates can be estimated fairly accurately.

[Insert Figure 4 about here]

Further, using Equation [8], the posterior mean of the 7-day retention rate is around 5.3%, with a 95% posterior interval of (4.4%, 6.1%). The posterior distribution of the 7-day retention rate is shown in Figure 5. Based on the above results, I find that the game *Bouncing Ball* has a huge week-by-week turnaround of players. The game loses almost all of its newly acquired players on a week-by-week basis and replaces them with another set of new players each week. Note that this result is fairly consistent with recent industry reports (e.g., Playnomics 2012) that suggest that, on average, around 95% of all gamers churn within the first 7 days of joining a social game.

[Insert Figure 5 about here]

Finally, given the posterior estimates of $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$ and the assumed values of $\left(a_i^{(\phi)}, b_i^{(\phi)}\right)$, I compute the "break-even" per-gamer acquisition cost of new gamers for *Bouncing Ball* by calculating the expected customer lifetime value (CLV) of a newly acquired gamer selected at random. Let X_i be the expected number of days that a newly acquired gamer (selected at random) of game *i* will return and play the game in the next 365 days.[8] Following Equation [8] in Fader et al. (2010), X_i can be computed as follows:

$$X_i = \left(\frac{a_i^{\phi}}{a_i^{\phi} + b_i^{\phi}}\right)\left(\frac{b_i^{\theta}}{a_i^{\theta} + b_i^{\theta}}\right)\left\{1 - \frac{\Gamma(a_i^{\theta} + b_i^{\theta})}{\Gamma(a_i^{\theta} + b_i^{\theta} + 365)} \frac{\Gamma(1 + b_i^{\theta} + 365)}{\Gamma(1 + b_i^{\theta})}\right\}. \qquad [15]$$

Social games generate revenue mainly from in-game advertising and the sales of in-game merchandise (e.g., a tractor in *Farmville*). Based on industry averages, the average revenue is

[8] Note that the "cutoff" of 365 here is immaterial, as virtually all gamers have churned within a year. Replacing 365 with a much larger number result in almost no change in expected CLV.

around 5 cents per DAU (De Vere 2011). Thus, given Equation [15] above, the break-even acquisition cost for a new gamer is $C = \$0.05 * (1 + X_i)$, where 1 is added to the second term to account for the fact that the gamer will definitely play on the first day she joins. For the game *Bouncing Ball*, X_i = 1.55 and hence C = \$0.128. In other words, the above analysis suggests that the maker of *Bouncing Ball* may spend up to 12.8 cents to acquire each new gamer.

5.3. Distribution of retention rates and per-gamer break-even acquisition costs across games

Next, I apply the proposal method separately to each of the 379 social games in the dataset. For each game, I sample from the posterior distribution of model parameters $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$ using the MCMC approach discussed in Section 3.3, and use the posterior distribution of $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$ to compute the estimated 1-day and 7-day retention rates. The distributions of the posterior mean estimates for 1-day and 7-day retention rates across games are shown in Figure 6 and Figure 7, respectively.

[Insert Figure 6 and Figure 7 about here]

As can be seen, the average 1-day retention rate across 379 games is 59.0%, with a range of 18.2% (for the title *The River Test*) to 97.1% (for the title *Empire Avenue*). The average 7-day retention rate (computed using Equation [8] as discussed earlier) across all games is 10.5%, with a range of 0.01% (for *Empire Avenue*) to 84.4% (for the title *Doc Martin*). In other words, social games on average lose almost 90% of their newly acquired users within a week, indicating an extremely high turnover rate. As discussed earlier, this finding is consistent with the intuition of many practitioners involved in social gaming (Playnomics 2012).

Next, using Equation [15], I compute the per-gamer break-even acquisition cost for each game in the dataset. The mean and median per-gamer break-even acquisition cost is 19.7 cents and 13.1 cents, respectively. In addition, I find that per-gamer break-even acquisition cost varies widely across games; across the dataset of 379 games, the per-gamer break-even acquisition costs range from 6.0 cents (from the game *A River Test*) to 6.2 dollars (for the game *Empire Avenue*). In the next section, I explore the relationship between retention rates and game characteristics such as game genre and game mechanics as shown in Table 3.

5.4. Relationship between retention rates, game genre, and game mechanics

Given the estimated 1-day and 7-day retention rates for each game, I analyze the relationship between the estimated retention rates and several game characteristics, i.e., genre and in-game mechanics (listed in Table 3). Four sets of multiple regressions, with untransformed and logit-transformed 1-day/7-day retention rates as dependent variables (respectively), are estimated and shown in Table 4. Given that the results across the four specifications of Table 4 are fairly consistent, I focus on discussing the results of the regression with untransformed 1-day retention rate as dependent variable (the first column of Table 4).

[Insert Table 4 about here]

As can be seen, games that belong to the "Strategy" genre are generally associated with lower 1-day retention rates ($\beta_{\text{strategy}} = -0.187; p < .01$). One possible reason is that strategy games typically have a "war/battle" theme which tends to attract younger male players, a segment that is known to be more "fickle" and generally less committed to games compared to female or older players (Laughlin 2013). In terms of game mechanics, providing a "daily bonus" such as in-game tokens and virtual currency to gamers when they check-in to play on each day is associated with an increase of 1-day retention rate by around 6.3% ($\beta_{\text{daily bonus}} = +0.063; p < .05$); this is consistent with the findings in the marketing literature that loyalty programs tend to increase relationship duration and usage level among customers (e.g., Bolton et al. 2000), by providing them a direct incentive to return. In contrast, "punishing" players for not returning does not appear to achieve the intended effect – controlling for other features, the 1-day retention rate for games that employ the "overdue punishment" mechanic is not significantly different from those that does not ($\beta_{\text{overdue punishment}} = 0.030; p = .40$). This suggests that, at least in the social gaming setting, a "carrot" (reward) seems to work better than a "stick" (punishment) for encouraging gamer retention.

Further, consistent with recent behavioral research on satiation (e.g., Galak et al. 2013), explicitly limiting the amount of time or actions (through the scarcity of in-game "energy" as discussed in Section 5.1) that players can spend on a game each day is associated with an increase of 1-day retention rate by about 7% ($\beta_{\text{limited energy}} = +0.069; p < .01$ on the first column of Table 4) and 7-day retention rate by almost 4% ($\beta_{\text{limited energy}} = +0.037; p < .1$ on the second column of Table 4). This result suggests that, similar to the TV setting (Nelson and Meyvis 2008; Nelson et al. 2009) where deliberately inserted commercial breaks improve the overall enjoyment of TV shows, slowing the rate by which gamers engage in social games seems to reduce the extent of satiation and increases retention rate. This also provides some confirming evidences for the belief among practitioners that rationing the amount that players can play would "keep them hungry for more" (Barnes 2010).

6. Discussion and conclusion

In this paper, I develop a Bayesian model to estimate retention rates using only aggregate DAU and MAU data. Building upon on the BG/BB model (Fader et al. 2010) at the individual level, the proposed method uses a data augmentation approach to simulate (latent) play histories and state information matrices for a representative sample of R players. The posterior distributions of model parameters are then sampled using MCMC (described in Appendix I), and used to compute 1-day/7-day retention rates and per-gamer break-even acquisition costs. After validating the proposed methodology using simulation studies, I apply the proposed method to analyze a sample of 379 social games. Results from the model reveal several key substantive insights. First, the average 1-day and 7-day retention rate for newly acquired players are 59.0% and 10.5% across games, respectively. Second, using the posterior estimates of churn parameters together with industry-average revenue per DAU, I estimate that the median break-even per-gamer acquisition cost is around 13.1 cents. To the best of my knowledge, this paper is the first to use a model-based approach to estimate retention rates and per-gamer break-even acquisition costs. Further, I analyze the relationship between the estimated retention rates and several game characteristics including genre and other in-game mechanics (daily bonus, overdue punishment, and limited energy). My results suggest that providing a daily incentive to players is associated with an increase of 1-day retention rate by around 6.3%, while limiting the amount of time and actions that a gamer can play each day is associated with an increase of 1-day retention rate by about 6.9%. Thus, game makers may consider incorporating both of these mechanics in their games to increase retention and hence boost the value of their customer base.

While the current paper provides some initial evidences for the effectiveness of the “daily bonus” and “limited energy” strategies, future research are needed to more concretely specific how to best utilize these in-game mechanics to maximize gamer retention. For instance, what is the optimal amount (or form) of daily bonus that should be given to gamers? What is the optimal time limit per day that would maximize retention? Is it better to directly limit the amount of playing time, or indirectly through the limiting the number of actions (through limited energy)? These and other specific issues can be addressed through a carefully designed A/B testing framework where game mechanics (e.g., the amount of daily bonus) can be systematically varied across different cohorts.

Beyond the "daily bonus" and "limited energy" strategies, other game mechanics that are outside of the scope of this paper can be employed by game makers to further reduce churn. For instance, a game mechanic that can potentially increase retention is the introduction of a "leadership board" by which a gamer can compare her performance with that of her peer group, hence creating "social pressure" to play more. Another related issue is how "levels" within a game can be designed to be appropriately challenging, so that gamers are incentivized to play more. For instance, discussions with game makers reveal that they often appeal to the "x+1 rule", a concept that originates from education and organizational management (e.g., Sapolsky and Reynolds 2006). The key idea is that when gamers achieve a certain level (x), the difficult of the next level should be designed such that it is "just a bit", but not a whole lot, harder than the previous one (x+1). How this intuitive rule should be translated into optimal level design for different games, however, is often unclear. Future research may look into the effectiveness of these and other mechanics in driving retention rates using the framework and methodology developed here.

Despite efforts in reducing churn and maximizing retention, social games, which by nature belong to the "causal gaming" genre, are characterized by low emotional investment on the part of the consumer and thus high turnover rates. One way that game makers can capture more value from their existing customer base is through cross-selling (Gupta and Zeithaml 2006). For instance, at some point after a game is released, game makers may start cross-promoting their other offerings to their current gamer base. The goal is that when gamers invariably become satiated and churn, they will more likely move to another game offered by the current game maker, rather than "deflecting" to a competitor's offering. While the current research provides some initial suggestions regarding the timing of such cross-promotion efforts (within a few days to a week, depending on the game), further research needs to be conducted to identify the best cross-selling strategy. For example, which new game(s) should be suggested to the current gamers? Should games from the same genre by recommended, or should a game from a completely different genre be suggested to maximize perceived variety and hence reduce satiation (Redden 2008)?

Finally, from a methodological standpoint, this research also speaks to the data storage and processing issues that researchers from the area of "big data analytics" are facing today. Specifically, the current research shows that it is not always necessary to retain all individual-level data if the goal is to estimate certain model parameters of interest (see, e.g., Fader et al. 2007). For instance, in the context of social games analyzed in this paper, full individual-level play histories are too large to be stored at a daily level and thus only aggregate DAU and MAU data are retained; however, despite the lack of individual-level data, retention rates can still be estimated fairly accurately. Future research can explore how data-augmentation approach such as that developed in this paper or in the previous literature can be used in conjunction with data summaries to generate managerial insights, which would greatly reduce the amount of storage and computational overhead in the analysis of big data in consumer analytics applications.

Reference

Barnes, David (2010), "DAU/MAU Crash Course – Your Measure of Game Design Quality," available at http://fbindie.posterous.com/daumau-crash-course-the-main-measure-of-game.

Bass, Frank (1969), "A New Production Growth Model for Consumer Durables," *Management Science*, 15(5), 215-227.

Bolton, Ruth, P. K. Kannan, Matthew Bramlett (2000), "Implications of Loyalty Program Membership and Service Experiences for Customer Retention," *Journal of the Academy of Marketing Science*, 28(1), 95-108.

Casella, George, and Edward I. George (1992), "Explaining the Gibbs Sampler," *The American Statistician*, 46(3), 167-174.

Chen, Andrew (2009), "How to Create a Profitable Freemium Startup," available at http://andrewchen.co/2009/01/19/how-to-create-a-profitable-freemium-startup-spreadsheet-model-included/.

Chen, Yuxin, and Sha Yang (2007), "Estimating Disaggregate Models Using Aggregate Data Through Augmentation of Individual Choice," *Journal of Marketing Research*, 44 (Nov), 613-621.

De Vere, Kathleen (2011), "46 Cents in Revenue Per Daily Active User?" available at http://www.insidemobileapps.com/2011/11/16/a-thinking-ape-interview-kenshi-arasaki/.

Doshi, Suhail (2010), "How to Analyze Traffic Funnels and Retention in Facebook Applications," available at http://www.insidefacebook.com/2010/01/28/how-to-analyze-traffic-funnels-and-retention-in-facebook-applications/.

Duryee, Tricia (2012), "Why Zynga Should Have Seen Draw Something's Fall Coming," available at http://allthingsd.com/20121022/why-zynga-should-have-seen-draw-somethings-fall-coming/.

Fader, Peter, and Bruce G. Hardie (2009), "Probability Models for Customer-Base Analysis," *Journal of Interactive Marketing*, 23, 61-69.

Fader, Peter S., Bruce G. Hardie, and Kinshuk Jerath (2007), "Estimating CLV Using Aggregated Data: The Tuscan Lifestyles Case Revisited," *Journal of Interactive Marketing*, 21(3), 55-71.

Fader, Peter S., Bruce G. Hardie, and Ka Lok Lee (2005), "'Counting Your Customers' the Easy Way: An Alternative to the Pareto/NBD Model," *Marketing Science*, 24(2), 275-286.

Fader, Peter S., Bruce G. Hardie, and Jen Shang (2010), "Customer-Base Analysis in a Discrete-Time Noncontractual Setting," *Marketing Science*, 29(6), 1086-1108.

Farago, Peter (2012), "App Engagement: The Matrix Reloaded," available at http://blog.flurry.com/bid/90743/App-Engagement-The-Matrix-Reloaded.

Galak, Jeff, Justin Kruger, and George Loewenstein (2011), "Is Variety the Spice of Life? It All Depends on the Rate of Consumption," *Judgment and Decision Making*, 6(3), 230-238.

Galak, Jeff, Justin Kruger, and George Loewenstein (2013), "Slow Down! Insensitivity to Rate of Consumption Leads to Avoidable Satiation," *Journal of Consumer Research*, 39(5), 993-1009.

Gelman, Andrew, John B. Carlin, Hal S. Stern, and Donald B. Rubin (2003), *Bayesian Data Analysis, 2nd Edition*, Chapman & Hall.

Gupta, Sunil, Donald R. Lehmann, and Jennifer Ames Stuart (2004), "Valuing Customers," *Journal of Marketing Research*, 41 (Feb), 7-18.

Gupta, Sunil, and Valarie Zeithaml (2006), "Customer Metrics and Their Impact on Financial Performance," *Marketing Science*, 25(6), 718-739.

Hyatt, Nabeel (2009), "How to Measure the True Stickiness (and Success) of a Facebook App," available at http://techcrunch.com/2009/10/29/how-to-measure-the-true-stickiness-and-success-of-a-facebook-app/.

Jiang, Renna, Puneet Manchanda, and Peter E. Rossi (2009), "Bayesian Analysis of Random Coefficient Logit Models using Aggregate Data," *Journal of Econometrics*, 149, 136-148.

Laughlin, Dau (2013), "Love, Courtship and the Promiscuous Male Mobile Gamer," The Flurry Blog (3/29/2013), available at http://blog.flurry.com/bid/95605/Love-Courtship-and-the-Promiscuous-Male-Mobile-Gamer.

Lehmann, Donald R. (1971), "Television Show Preference: Application of a Choice Model," *Journal of Marketing Research*, 8(1), 47-55.

Lovell, Nicholas (2011), "DAU/MAU = Engagement," available at http://www.gamesbrief.com/2011/10/daumau-engagement/.

MacMillan, Douglas, and Brad Stone (2011), "That Listless Feeling Down on the Virtual Farm," *Bloomberg Businessweek* (Nov 28—Dec 4, 2011), 43-44.

Musalem, Andres, Eric T. Bradlow, and Jagmohan S. Raju (2008), "Who's Got the Coupon? Estimating Consumer Preferences and Coupon Usage from Aggregate Information," *Journal of Marketing Research*, 45 (Dec), 715-730.

Musalem, Andres, Eric T. Bradlow, and Jagmohan S. Raju (2009), "Bayesian Estimation of Random-Coefficients Choice Models Using Aggregate Data," *Journal of Applied Econometrics*, 24, 490-516.

Nelson, R. B. (1999), *An Introduction to Copulas*, Springer, New York.

Nelson, Leif D., and Tom Meyvis (2008), "Interrupted Consumption: Disrupting Adaptation to Hedonic Experiences," *Journal of Marketing Research*, 45 (December), 654-664.

Nelson, Leif D., Tom Meyvis, and Jeff Galak (2009), "Enhancing the Television Viewing Experience Through Commercial Interruptions," *Journal of Consumer Research*, 36 (August), 160-172.

Netzer, Oded, James M. Lattin, and V.Srinivasan (2008), "A Hidden Markov Model of Customer Relationship Dynamics," *Marketing Science*, 27(2), 185-204.

Park, Sung-Ho, and Sachin Gupta (2009), "Simulated Maximum Likelihood Estimator for the Random Coefficient Logit Model Using Aggregate Data," *Journal of Marketing Research*, 46(4), 531-542.

Playnomics (2012), "Playnomics Quarterly U.S. Player Engagement Study," Q3 (July to September), 2012.

PopCap Research (2011), "2011 PopCap Games Social Gaming Research," available at http://www.infosolutionsgroup.com/pdfs/2011_PopCap_Social_Gaming_Research_Results.pdf.

Redden, Joseph P. (2008), "Reducing Satiation: The Role of Categorization Level," *Journal of Consumer Research*, 34 (Feb), 624-634.

Redden, Joseph P., and Jeff Galak (2013), "The Subjective Sense of Feeling Satiated," *Journal of Experimental Psychology: General*, 142(1), 209-217.

Robert, Christian, and George Casella (2004), *Monte Carlo Statistical Methods*, Springer.

Sapolsky, Robert, and Marcia Reynolds (2006), "Zebras and Lions in the Workplace: An Interview with Dr. Robert Sapolsky," *International Journal of Coaching in Organization*, 4(2), 7-15.

Schmittlein, David C., Donald G. Morrison, and Richard Colombo (1987), "Counting Your Customers: Who They Are and What Will They Do Next?" *Management Science*, 33 (Jan), 1-24.

Schweidel, David A., and George Knox (2013), "Incorporating Direct Marketing Activity into Latent Attrition Models," *Marketing Science*, 32(3), 471-487.

Stark, Heather (2010), "Facebook DAU and MAU: What They Tell You (and What They Don't)," available at http://insightanalysis.wordpress.com/2010/07/21/facebook-dau-and-mau-what-they-tell-you-and-what-they-dont/.

Tan, Gerald (2012), "Business Model of Social Games," *Working Paper*, presented at TIGA-Event (Causal Games Meetup), Jan 25, 2012.

Tanner, Martin A., and Wing Hung Wong (1987), "The Calculation of Posterior Distributions by Data Augmentation," *Journal of the American Statistical Association*, 82 (398), 528-540.

Von Coelln, Eric (2009), "The Sticky Factor: Creating a Benchmark for Social Gaming Success," available at http://www.insidesocialgames.com/2009/10/27/the-sticky-factor-creating-a-benchmark-for-social-gaming-success/.

Winkler, Rolfe (2011), "Testing the Durability of Zynga's Virtual Business," *The Wall Street Journal*, 9/28/2011.

Appendix

I. MCMC computational procedure

I describe the MCMC procedure used to calibrate the model. In each iteration, I draw from the full conditional distributions of model parameters in the following order: $(S_{(i)}, Y_{(i)}), \pi_{it}, \theta_{ij}, \phi_{ij}, (a_i^{(\pi)}, b_i^{(\pi)}), (a_i^{(\theta)}, b_i^{(\theta)})$. An independent Metropolis-Hasting algorithm is used to sample each row of $(S_{(i)}, Y_{(i)})$; given $(S_{(i)}, Y_{(i)})$, $\pi_{it}, \theta_{ij}, \phi_{ij}$ are drawn using a Gibbs sampler, and the hyperparameters $(a_i^{(\pi)}, b_i^{(\pi)})$ and $(a_i^{(\theta)}, b_i^{(\theta)})$ are drawn using a random-walk Metropolis-Hastings algorithm (Gelman et al. 2003). Each step is outlined as follows.

1) Drawing each row of $(S_{(i)}, Y_{(i)})$:

For each representative gamer *j* (i.e., the *j*-th row in $(S_{(i)}, Y_{(i)})$), I simulate a "proposed play history" (including both the time series of her play history and state transitions) using the hidden Markov model specified in Equation [1]—[6] with on the current draw of θ_{ij} and ϕ_{ij}. Then, I compute the likelihood of the proposed play history given Equation [11] and [12], and accept or reject the new draw based on the Metropolis-Hastings acceptance probability (Gelman et al. 2003).

2) Drawing π_{it}:

Denote the number of gamers (in the representative sample of size *R*) who are in the "Unaware" state at the start of the *t*-period by $n_{it}^{(U)}$, and denote the number of transitions from the "Unaware" state to the "Active" state during the *t*-th period by $n_{it}^{(U \to A)}$. Then, π_{it} can be sampled from a $Beta(a_i^{(\pi)} + n_{it}^{(U \to A)}, b_i^{(\pi)} + n_{it}^{(U)} - n_{it}^{(U \to A)})$ distribution.

3) Drawing θ_{ij}:

For gamer *j*, denote her total number of "Active"→"Active" transitions by $n_{ij}^{(A \to A)}$, and denote her total number of "Active"→"Dead" transitions by $n_{ij}^{(A \to D)}$ (by definition, $n_{ij}^{(A \to D)}$ takes either the value of 0 or 1 since "Dead" is an absorbing state). Then, θ_{ij} can be sampled from a $Beta(a_i^{(\theta)} + n_{ij}^{(A \to D)}, b_i^{(\theta)} + n_{ij}^{(A \to A)})$ distribution.

4) Drawing ϕ_{ij} :

Denote the number of days that gamer *j* stays in the "Active" state (excluding the first day when she first becomes "Active") by $n_{ij}^{(A)}$, and denote the number of days that gamer *j* plays the game (excluding the first day) by $n_{ij}^{(P)}$. Then, ϕ_{ij} can be sampled from a $Beta(a_i^{(\phi)} + n_{ij}^{(P)}, b_i^{(\phi)} + n_{ij}^{(A)} - n_{ij}^{(P)})$ distribution.

5) Drawing $\left(a_i^{(\pi)}, b_i^{(\pi)}\right)$ and $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$:

Because standard conjugate computations are not available to sample the hyperparameters $\left(a_i^{(\pi)}, b_i^{(\pi)}\right)$ and $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$, I use a random walk Metropolis-Hastings algorithm to sample from their posterior distributions. To sample $\left(a_i^{(\pi)}, b_i^{(\pi)}\right)$, I first log-transform both parameters, and use a bivariate Gaussian random walk proposal distribution with the mean centered on the value of the previous draw; the variance of the proposal distribution is adjusted to achieve an acceptance rate close to 50% (Gelman et al. 2003). Similarly, the hyperparameters $\left(a_i^{(\theta)}, b_i^{(\theta)}\right)$ can be sampled using an analogous procedure.

II. Robustness checks with respect to $\left(a_i^{(\phi)}, b_i^{(\phi)}\right)$

To explore the robustness of retention rate estimates with respect to the chosen values of $\left(a_i^{(\phi)} = 0.76, b_i^{(\phi)} = 0.12\right)$ (which corresponds to an average play probability of around 0.86), I conduct two sets of simulation studies that increase/decrease the average "play" probability by around 10%, respectively. More specifically, I repeat the simulation studies in Section 4, but instead specifies $\left(a_i^{(\phi)} = 0.95, b_i^{(\phi)} = 0.05\right)$ (which corresponds to an average play probability of 0.95), and $\left(a_i^{(\phi)} = 0.70, b_i^{(\phi)} = 0.20\right)$ (which corresponds to an average play probability of 0.78), to simulate the data. In other words, the assumption of $\left(a_i^{(\phi)} = 0.76, b_i^{(\phi)} = 0.12\right)$ is now inaccurate; thus, the analyses below study the extent to which estimated retention rates are biased in the presence of misspecification errors in $\left(a_i^{(\phi)}, b_i^{(\phi)}\right)$. Table A1 and Figure A1 summarize the results with data simulated using $\left(a_i^{(\phi)} = 0.95, b_i^{(\phi)} = 0.05\right)$ on the left panel and data simulated using $\left(a_i^{(\phi)} = 0.70, b_i^{(\phi)} = 0.20\right)$ on the right panel.

As can be seen in Table A1 and Figure A1, misspecification of $\left(a_i^{(\phi)}, b_i^{(\phi)}\right)$ introduces an additional estimation bias on estimated retention rates. Specifically, when the true play probability is higher (lower) than the assumed value, the estimated retention rates tend to over- (under-) estimate the true retention rates (respectively). Overall, however, the estimated retention rates still track the true retention rates reasonably well and are fairly close to the 45-degree line. The RMSE for the specifications with $\left(a_i^{(\phi)} = 0.95, b_i^{(\phi)} = 0.05\right)$ and $\left(a_i^{(\phi)} = 0.70, b_i^{(\phi)} = 0.20\right)$ are 0.021 and 0.031, respectively, which suggests that the accuracies of the estimated retention rates are reasonably insensitive to minor misspecification errors in the values of $\left(a_i^{(\phi)}, b_i^{(\phi)}\right)$.

	$\left(a_i^{(\phi)} = 0.95, b_i^{(\phi)} = 0.05\right)$		$\left(a_i^{(\phi)} = 0.70, b_i^{(\phi)} = 0.20\right)$	
True retention rate	Estimated 1-day retention rate	95% posterior interval	Estimated 1-day retention rate	95% posterior interval
0.05	0.100	(0.081, 0.126)	0.081	(0.066, 0.100)
0.10	0.122	(0.104, 0.148)	0.102	(0.082, 0.130)
0.15	0.182	(0.159, 0.220)	0.137	(0.106, 0.168)
0.20	0.229	(0.205, 0.253)	0.192	(0.165, 0.220)
0.25	0.268	(0.240, 0.295)	0.242	(0.223, 0.259)
0.30	0.313	(0.288, 0.352)	0.292	(0.261, 0.328)
0.35	0.366	(0.346, 0.386)	0.326	(0.287, 0.353)
0.40	0.415	(0.394, 0.431)	0.382	(0.362, 0.402)
0.45	0.456	(0.431, 0.479)	0.394	(0.370, 0.413)
0.50	0.528	(0.498, 0.558)	0.457	(0.416, 0.489)
0.55	0.557	(0.539, 0.585)	0.492	(0.470, 0.513)
0.60	0.625	(0.605, 0.646)	0.570	(0.553, 0.585)
0.65	0.656	(0.641, 0.671)	0.615	(0.592, 0.633)
0.70	0.726	(0.709, 0.740)	0.649	(0.627, 0.674)
0.75	0.764	(0.751, 0.776)	0.718	(0.697, 0.735)
0.80	0.811	(0.800, 0.823)	0.782	(0.762, 0.798)
0.85	0.867	(0.858, 0.877)	0.836	(0.823, 0.847)
0.90	0.907	(0.900, 0.914)	0.887	(0.879, 0.895)
0.95	0.951	(0.945, 0.956)	0.926	(0.918, 0.933)

Table A1. Simulation results with $\left(a_i^{(\phi)} = 0.95, b_i^{(\phi)} = 0.05\right)$ (left panel) and $\left(a_i^{(\phi)} = 0.70, b_i^{(\phi)} = 0.20\right)$ (right panel).

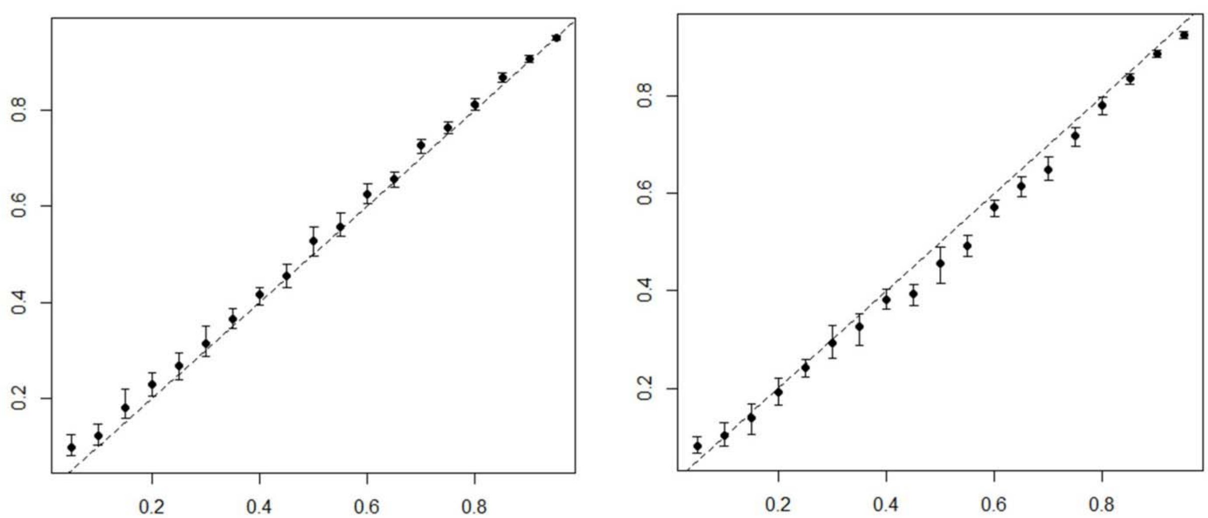

Figure A1. Results of simulation study with $\left(a_i^{(\phi)} = 0.95, b_i^{(\phi)} = 0.05\right)$ (left panel) and $\left(a_i^{(\phi)} = 0.70, b_i^{(\phi)} = 0.20\right)$ (right panel).

III. Robustness checks with respect to the assumption of independence between θ_{ij} and ϕ_{ij}

I now explore the robustness of retention rate estimates with respect to the a priori assumption that the individual-level parameters governing churn behavior (θ_{ij}) and play behavior (ϕ_{ij}) are independent, i.e., the same assumption that is made in the BG/BB (Fader et al. 2010) and Pareto/NBD models (Schmittlein et al. 1987). Towards that end, I use a bivariate Gaussian Copula approach (Nelson 1999) to simulate correlated samples of θ_{ij} and ϕ_{ij}, where the marginal distributions are $\theta_{ij} \sim Beta(a_i^{(\theta)}, b_i^{(\theta)})$ and $\phi_{ij} \sim Beta(a_i^{(\phi)}, b_i^{(\phi)})$ as before. Specifically, I use the following procedure to simulate correlated samples of θ_{ij} and ϕ_{ij}:

(i) First, I simulate two random variates $\begin{pmatrix} u_{ij} & v_{ij} \end{pmatrix}' \sim MVN\left(\vec{0}, \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}\right)$.

(ii) Then, I take $\theta_{ij} = F^{-1}_{Beta(a_i^{(\theta)}, b_i^{(\theta)})}(\Phi(u_{ij}))$ and $\phi_{ij} = F^{-1}_{Beta(a_i^{(\phi)}, b_i^{(\phi)})}(\Phi(v_{ij}))$,

where $\Phi(.)$ denotes the standard normal cumulative distribution function and $F^{-1}_{Beta(a_i^{(\theta)}, b_i^{(\theta)})}(.)$ denotes the inverse CDF of the $Beta(a_i^{(\theta)}, b_i^{(\theta)})$ distribution. After generating *R* correlated samples of θ_{ij} and ϕ_{ij} using the above procedure, the rest of the simulation procedure follows directly from Section 4. In the analysis below, I choose r = 0.2, 0.4, and 0.6 to explore the extent to which the proposed estimation procedure is robust to different strengths of correlation between θ_{ij} and ϕ_{ij}.

The results of this robustness check are summarized in Table A2 and Figure A2 below. As can be seen, the accuracies of the estimated retention rates are fairly robust with respect to moderate violations to the independence assumption between θ_{ij} and ϕ_{ij}. In all three cases, the estimated retention rates still track the true retention rates very well, despite violation to the independent assumption. Overall, the estimated retention rates are fairly close to the 45-degree line (see Figure A2). The RMSE for the specifications with $r = 0.2$, 0.4, and 0.6 are 0.024, 0.013, and 0.023, respectively, again suggesting that the accuracies of the estimated retention rates are reasonably insensitive to moderate correlations between θ_{ij} and ϕ_{ij}.

	$r = .2$ (low correlation)		$r = .4$ (mid correlation)		$r = .6$ (high correlation)	
True ret. rate	Est. ret. rate	95% post. int.	Est. ret. rate	95% post. int.	Est. ret. rate	95% post. int.
0.05	0.113	(0.095, 0.137)	0.065	(0.046, 0.081)	0.089	(0.050, 0.118)
0.10	0.100	(0.072, 0.130)	0.099	(0.075, 0.131)	0.107	(0.081, 0.139)
0.15	0.169	(0.148, 0.190)	0.154	(0.131, 0.171)	0.155	(0.140, 0.191)
0.20	0.186	(0.156, 0.207)	0.197	(0.167, 0.221)	0.182	(0.166, 0.201)
0.25	0.241	(0.215, 0.270)	0.254	(0.220, 0.281)	0.232	(0.206, 0.261)
0.30	0.306	(0.274, 0.336)	0.299	(0.282, 0.316)	0.299	(0.276, 0.314)
0.35	0.338	(0.317, 0.361)	0.338	(0.306, 0.373)	0.319	(0.299, 0.341)
0.40	0.379	(0.351, 0.402)	0.394	(0.375, 0.420)	0.378	(0.345, 0.399)
0.45	0.428	(0.404, 0.446)	0.433	(0.414, 0.450)	0.426	(0.411, 0.442)
0.50	0.455	(0.422, 0.480)	0.486	(0.460, 0.510)	0.462	(0.447, 0.473)
0.55	0.521	(0.497, 0.545)	0.542	(0.529, 0.559)	0.525	(0.507, 0.547)
0.60	0.584	(0.571, 0.598)	0.585	(0.566, 0.605)	0.571	(0.553, 0.587)
0.65	0.630	(0.613, 0.646)	0.627	(0.611, 0.645)	0.626	(0.601, 0.645)
0.70	0.665	(0.643, 0.685)	0.676	(0.660, 0.692)	0.668	(0.651, 0.683)
0.75	0.733	(0.715, 0.754)	0.726	(0.711, 0.740)	0.722	(0.708, 0.740)
0.80	0.788	(0.776, 0.801)	0.787	(0.771, 0.799)	0.780	(0.765, 0.793)
0.85	0.855	(0.847, 0.862)	0.840	(0.829, 0.850)	0.844	(0.835, 0.854)
0.90	0.900	(0.890, 0.909)	0.894	(0.884, 0.903)	0.885	(0.876, 0.892)
0.95	0.938	(0.931, 0.944)	0.940	(0.933, 0.96)	0.933	(0.927, 0.940)

Table A2. Simulation results with $r = 0.2$, 0.4, and 0.6.

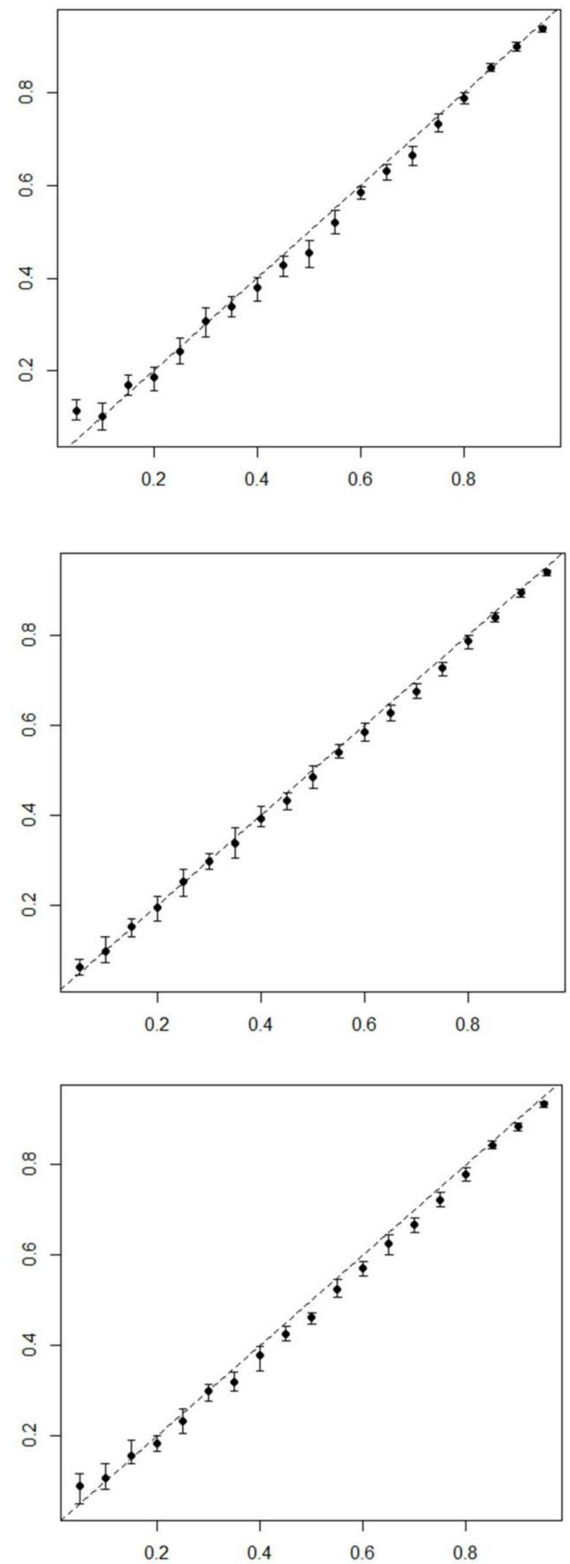

Figure A2. Results of simulation study with $r = 0.2$ (upper panel), $r = 0.4$ (middle panel), and $r = 0.6$ (lower panel).

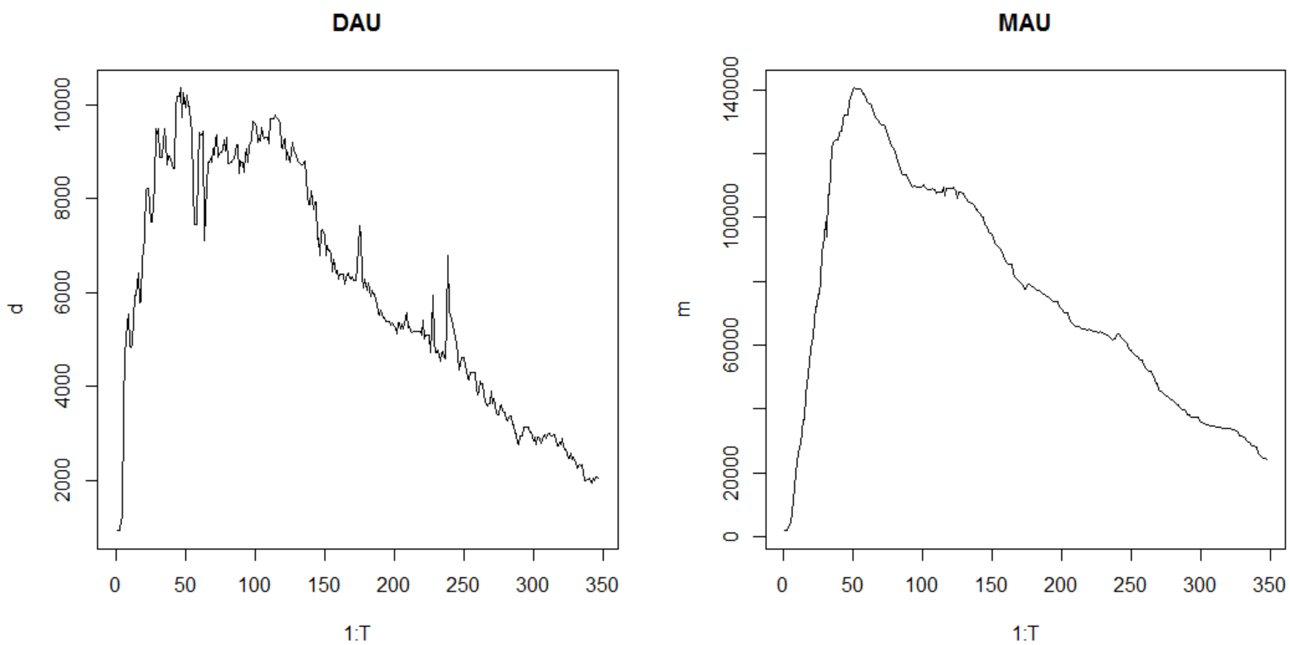

Figure 1. DAU (Daily Active User) and MAU (Monthly Active Users) data for the game *Bouncing Ball* from 11/1/2010 (the release date of the game) to 10/13/2011 (the end of my data collection period).

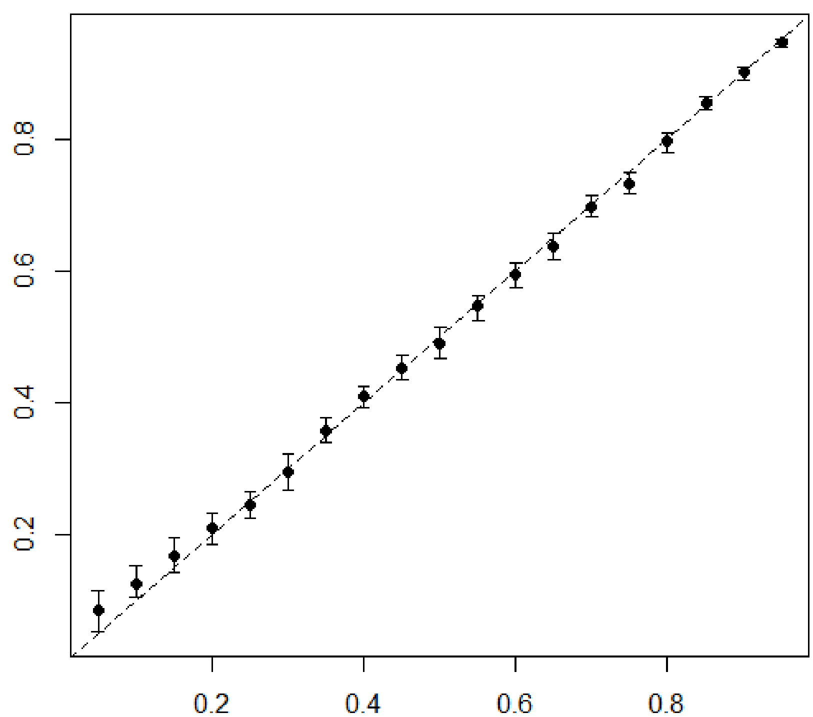

Figure 2. Results of the simulation study in Section 4. Each point plots the estimated 1-day retention rate (y-axis) versus the "true" 1-day retention rate (x-axis), and the error bars represent 95% posterior intervals. The broken line is the 45-degree line.

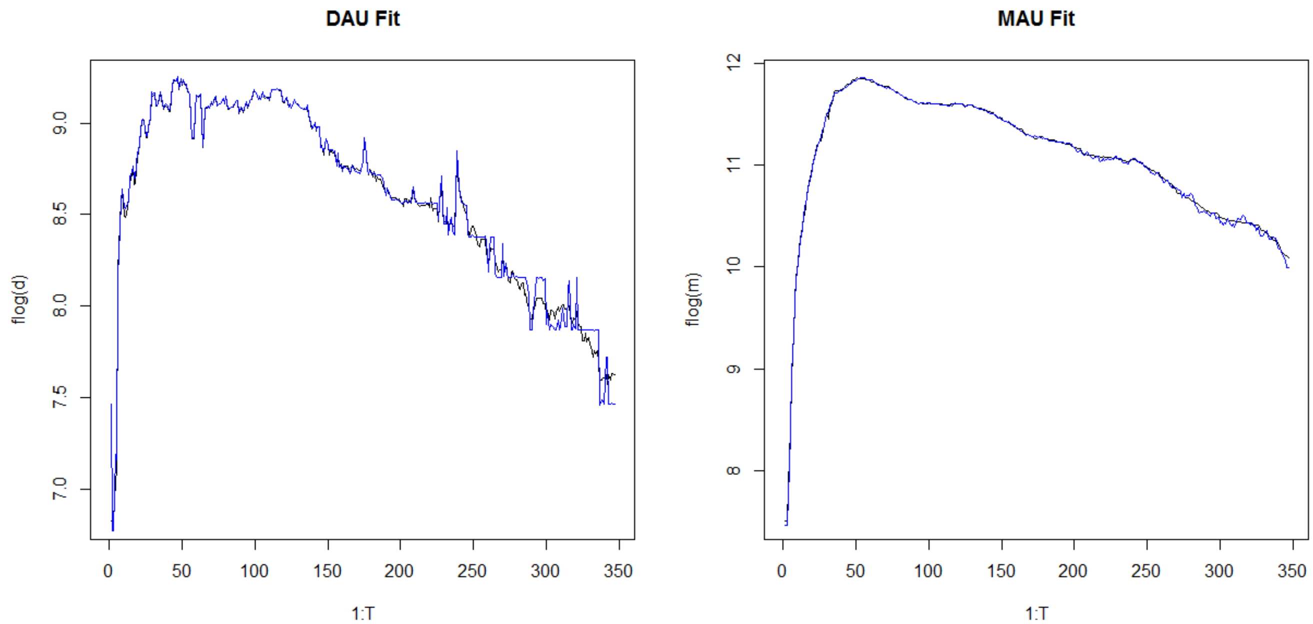

Figure 3. Fit of the proposed method in terms of observed (log-) DAU and MAU vs. the (scaled) estimates from the augmented "play histories" matrix. The solid black line denotes the observed data, while the blue line denotes the data summaries derived from the augmented data matrix *Y*.

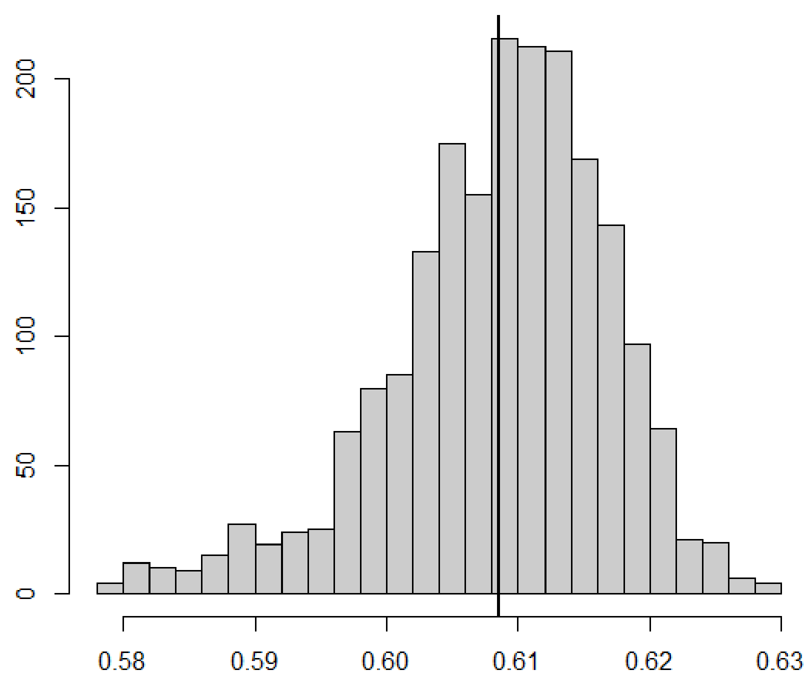

Figure 4. Posterior distribution of the 1-day retention rate for the title *Bouncing Ball*. The solid vertical line represents the posterior mean estimate.

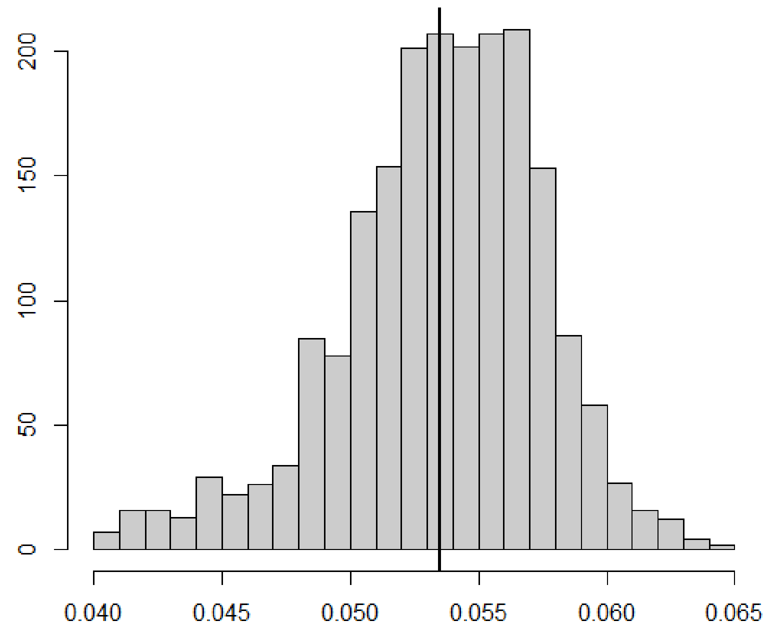

Figure 5. Posterior distribution of the 7-day retention rate for the title *Bouncing Ball*. The solid vertical line represents the posterior mean estimate.

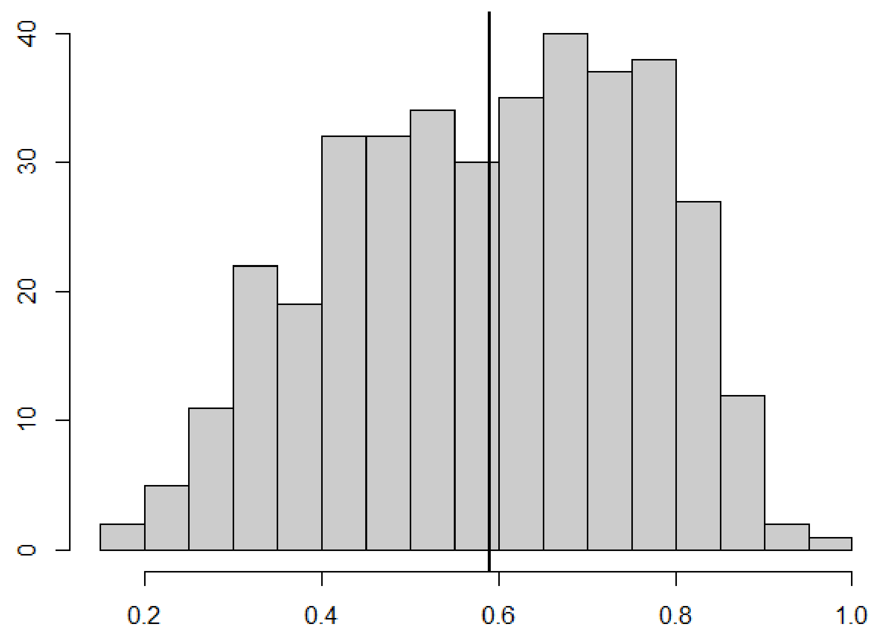

Figure 6. Distribution of the estimated 1-day retention rates across 379 social games. The solid vertical line represents the overall average across games.

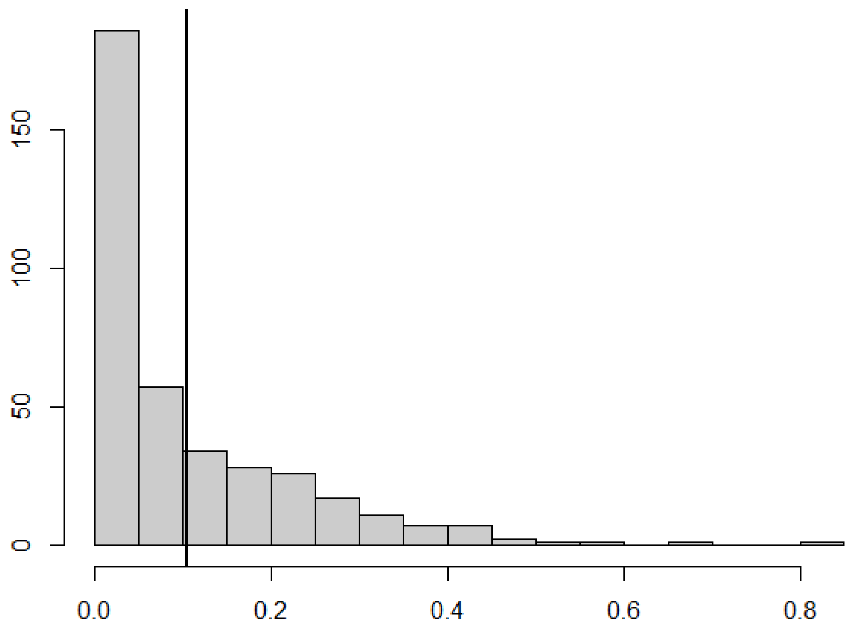

Figure 7. Distribution of the estimated 7-day retention rates across 379 social games. The solid vertical line represents the overall average across games.

True 1-day retention rate	$\hat{M}$	Estimated mean adoption rate	Estimated 1-day retention rate	95% posterior interval for 1-day retention rate	Average DAU/MAU ratio
0.05	99,968	0.023	0.085	(0.053, 0.115)	0.036
0.10	99,842	0.020	0.127	(0.105, 0.152)	0.037
0.15	100,061	0.022	0.168	(0.142, 0.195)	0.039
0.20	100,475	0.022	0.209	(0.185, 0.232)	0.041
0.25	100,269	0.022	0.246	(0.226, 0.266)	0.042
0.30	101,124	0.022	0.296	(0.267, 0.323)	0.047
0.35	101,138	0.021	0.358	(0.340, 0.377)	0.051
0.40	100,892	0.019	0.410	(0.393, 0.424)	0.053
0.45	101,699	0.019	0.453	(0.434, 0.471)	0.060
0.50	102,300	0.022	0.490	(0.468, 0.514)	0.062
0.55	102,431	0.020	0.546	(0.525, 0.562)	0.066
0.60	103,761	0.016	0.595	(0.574, 0.611)	0.077
0.65	104,516	0.019	0.636	(0.618, 0.656)	0.082
0.70	107,246	0.014	0.697	(0.681, 0.714)	0.100
0.75	109,662	0.011	0.731	(0.716, 0.748)	0.125
0.80	114,766	0.008	0.795	(0.780, 0.809)	0.156
0.85	122,982	0.006	0.854	(0.843, 0.864)	0.229
0.90	150,228	0.004	0.900	(0.889, 0.909)	0.407
0.95	256,529	0.002	0.945	(0.938, 0.952)	0.624

Table 1. Results of the simulation studies. RMSE(model-based estimate) = 0.013; RMSE (average DAU/MAU ratio) = 0.425.

	Mean	S.D.	Median	Min	Max
Length of data collection period (days)	385.73	150.63	381.00	31.00	748.00
Average DAU (in ,000)	97.24	525.24	3.36	0.02	6204.71
Average MAU (in ,000)	571.63	2707.74	42.11	0.24	37004.61
Peak DAU (in ,000)	212.29	991.94	17.36	0.12	11299.94
Peak MAU (in ,000)	1238.45	5296.46	122.30	1.09	66560.16
DAU/MAU ratio	0.12	0.08	0.10	0.04	0.73

Table 2. Summary statistics of the usage data (DAU and MAU).

Genre	Example games	% of games
Action & Sports	*Billiards Multiplayer Pool*, *Football Mania*	17.9%
Arcade	*Bubbles*, *Bouncing Balls*	20.3%
Board & Card	*Poker Texas Holdem Cardstown*, *Mahjong Saga*	12.4%
Simulation	*PyramidVille*, *CaveTown*	13.2%
Virtual world	*Stardrift Empires*, *Vinyl City*	12.1%
Trivia	*Scene It? Movies*, *Arithmetic Challenge*	1.6%
Strategy	*Overkings*, *Sword Quest*	1.8%
Role playing game (RPG)	*Digger*, *Legacy of a Thousand Suns*	5.6%
Other		12.4%
In-game mechanics	Example games	% of games
Daily bonus	*Poker Texas Holdem Cardstown*, *It Girl*	19.8%
Overdue punishment	*Ranch Town*, *Casino City*	9.2%
Limited energy	*Mafia Wars*, *Criminal Case*	25.9%

Table 3. Summary statistics of genre and in-game mechanics.

	1-day retention rate	logit(1-day retention rate)	7-day retention rate	logit(7-day retention rate)
Intercept	0.565**	0.310**	0.092**	-3.347**
Genre				
Action & Sports	-0.015	-0.080	-0.020	-0.206
Arcade	0.013	0.044	0.005	0.098
Board & Cards	0.026	0.077	0.001	0.288
Simulation	-0.015	-0.077	-0.005	-0.180
Virtual World	-0.030	-0.107	-0.001	-0.203
Trivia	-0.017	-0.045	0.014	-0.243
Strategy	-0.187**	-0.809*	-0.025	-0.957
Role play games	-0.032	-0.166	-0.040	-0.272
In-game mechanics				
Daily Bonus	0.063*	0.293**	0.032^	0.559*
Overdue punishment	0.030	0.189	0.034	0.302
Limited energy	0.069**	0.294*	0.037^	0.702*

(**: $p < .01$; *: $p < .05$; ^: $p < .10$)

Table 4. Regression of estimated retention rates versus game characteristics. (The "other" genre is the omitted category).