

*Credit Default Swaps: Pricing and Finding the Sensitivity (of the Deviation of Theoretical Price from the Market Price) with Respect to Different Factors*

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## 1. Introduction: What we are going to test & the economic significance of the research

One of the risks of making a bank loan or investing in a debt security, say corporate bonds, is credit risk, the risk that borrower defaults. In response to this potential problem, new financial instruments called as credit derivatives have been developed in the past few years. Credit derivatives can help investors against adverse movements in the credit quality of the borrower. If the borrower defaults, the investor will suffer losses on the investment, but the losses can be offset by gains from credit derivatives. In particular, among these credit derivatives, credit default swaps (CDS) are very widely used instruments.

The simplest example of a single-name credit-default swap contract having an underlying of corporate bonds can be illustrated as follows. Suppose today, a protection buyer wishes to buy five years of protection against the default of the IBM bonds maturing December 12, 2008. Suppose the buyer holds 10,000 of these bonds each having a face value of \$1,000. Thus, the notional value of the buyer's position is \$10,000,000. The buyer contracts to buy full protection for the face value of the debt via single-name credit-default swap with a 123 basis points (say) premium. Thus the buyer shall pay  $A/360 * \text{Notional amount} (10,000,000) * 0.0123$  amount every quarter, where A is the actual number of days in a quarter. If there is a default, then the buyer delivers the 10,000 IBM bonds and receives from the seller a payment of \$10,000,000.

A wide range of institutions participate in the credit-derivatives market. Banks, security houses, and hedge funds dominate the protection-buyers market, with banks, representing about 50% of the demand. On the protection-sellers side, banks and insurance companies dominate (British Bankers' Association). However, the growth in the credit derivatives market has been accompanied by controversies. There have been concerns raised recently about whether the credit protection is priced fairly. A wrong pricing of the credit derivatives can push companies that are barely investment grade into speculative grade category. Similarly, many credit-protection sellers can also experience losses if the credit protection is priced too cheaply. In the recent years, there have been already such *claims* related to the credit derivatives market. Hence, the focus of our project is in the pricing the CDS' and finding the possible cause of the deviation of the price [obtained from theoretical models] from the market price. We are only going to consider the pricing of CDS', which have *corporate bonds* used as underlying (similar to the example mentioned in the first paragraph).

The next section describes the methodology in detail that we shall use for the pricing of the CDS'. After finding the theoretical price of the CDS', we shall try to find the causes of the error and run a regression to find the sensitivity of the error with respect to different factors.

## 2. Method that we shall use to test the main hypotheses

We shall be using the closed-form model for valuing the credit-default swaps within the well-known reduced-form framework of Duffie (1998), and Lando (1998), Duffie & Singleton (1999), and others. Following standard notations let  $r_T$  denote the riskless rate and  $\lambda_\tau$  the intensity of the Poisson process governing default. In our model, we assume stochastic behavior for both  $r_T$  and  $\lambda_\tau$ , we also assume that they follow independent processes. As in Lando (1998), we also assume

that the bondholder recovers a fraction of  $1 - w$  of the par value of the bond in the event of default. We shall assume  $w$  to be 0.5 (50% recovery rate). We are also assuming a mean-reversion model for the intensity process  $\lambda_t$ , as in the paper of Longstaff (2002), Mithal (2002), and Neis (2002), i.e.

$$d\lambda = (\alpha - \beta\lambda) dt + \sigma \lambda^{1/2} dZ \quad \text{----- (1)}$$

where  $\alpha, \beta$  and  $\sigma$  are positive constants and  $Z_T$  is a standard Brownian motion. These dynamics also allow for mean reversion & heteroskedasticity, in corporate spreads and guarantee that the intensity process is always nonnegative. Without going into the details of the gory mathematics, which can be found in the paper [1], the credit spread is given by:

$$s = \frac{E\left[w \int_0^T \lambda_t \exp\left(-\int_0^t (\alpha + \lambda_s) ds\right) dZ_t\right]}{E\left[w \int_0^T \lambda_t \exp\left(-\int_0^t (\alpha + \lambda_s) ds\right) dt\right]} \quad \text{-----(2)}$$

We are planning to use Monte Carlo simulation technique to evaluate both the numerator and the denominator of equation –(2) and then find the value of the spread. At this stage, we are considering the initial value of the default intensity to be based on the credit rating of the corporate bonds and hence it would be different for each kind of firm. The factors  $\alpha, \beta$  and  $\sigma$  for equation (1), shall be evaluated empirically as given in the next section. We can alternatively use the closed form solution to find the CS as mentioned in the paper of Longstaff, Mithal, and Neis [1].

### 3. Estimation of the parameters & Data requirement

The parameters  $\alpha, \beta$  and  $\sigma$  are evaluated from the yields of bonds having the maturity in the range of 3 – 8 years but the yields/prices of the bonds are observed at dates when the CDS is issued. We will try to get as many data points as possible for these bonds. After that, we first make an assumption about the three parameters  $\alpha, \beta$  and  $\sigma$ , and then find a value of the bond yield using the closed form solution mentioned in the paper [1]. This is compared to the observed yield/price of the bond and then we shall try to minimize the error between the calculated yield and the observed yield by making another assumption of the three parameters. This is done until the error is minimized to an acceptance level. It's worth mentioning that the notional amount of these bonds should not be less than that of the notional amount of CDS and the bonds chosen should be registered with SEC, having no embedded options. For a firm to be considered for pricing in our project, we need at least two such bonds [data points]. *Some of this data is available in Bloomberg. We also contacted Credittrade for seeking this data.*

Once we obtain the values of  $\alpha, \beta$  and  $\sigma$ , we can easily find the default intensity  $\lambda_T$  as say:

$$d\lambda = (0.0054 - 0.0534\lambda) dt + 0.0422 \lambda^{1/2} dZ$$

Then, those values of  $\alpha, \beta$  and  $\sigma$  shall be used to price the credit spread from the equation no. (2.)

For finding the discount factor, we shall be taking the T-bill of different maturities, and bootstrap to estimate the discount factor of six months interval. *This information can be obtained from the Wall Street Journal.*

Last but not least, we need the credit spread, or the premium of the CDS' transaction, having a maturity of five years, having an underlying of corporate bonds. We have contacted and have obtained a trial version of *Credittrade [www.credittrade.com]* for seeking the data related to the credit spreads. Alternatively, we can also look for this data from the *Bloomberg terminal*.

Having found the credit spread for a number of observations, we compare it with the market price of each CDS transaction, (i.e., the premium of the transaction) and then find the error. Again, it's worth mentioning here that the market premium that we take into account is the average of the bid and ask quotations. And then we shall plot the two credit spreads to know if there is any trend, and find the t-statistics between the two means, so as to get an estimate as to how significant is the difference between the two credit spreads.

Next, we shall run regression to find the sensitivity of the error with respect to that of the other parameters. Given the time constraint, we shall try to mainly find the error as a function of different variables (as many of the following):

1. Volatility of the market (estimated in a way different for the pricing of the CDS'), say GARCH or Riskmetrics approach.
2. Rate of recovery in the event of default.
3. Risk free rate
4. S & P index change (to know how macro economic factors affect the premium)
5. Log of the volume of the equity shares traded
6. Default risk associated with the issuer of the CDS'.
7. Liquidity factor associated with the bonds that were used for pricing the CS.

#### **4. Possible Conclusions**

Our conclusions would include what factors affect the errors the most by estimating different statistical tests say  $R^2$ , etc and then report how much are the theoretical prices different from the market prices. Is the implied CS always higher than the market CS and if yes then why? From the paper of [1], we gather that the implied spread is always higher than the market spread. We shall try to see if we obtain the same results and then find the causes of the errors, using different volatility models, etc as mentioned previously. We shall report the regression equation and hence the sensitivity of the error with respect to the different parameters mentioned above. This will be valuable because we could consider alternative model that might take into account different factors for finding the theoretical credit spread.

## 5. References

1. Longstaff Francis, Mithal Sanjay, and Eric Neis, 2002. The Credit-Default Swap Market: Is Credit Protection Priced Correctly? *The Journal of Finance* (December 2002).
2. Hull, John, and Alan White, 2001, Valuing Credit Default Swaps II: Modeling Default Correlations, *The Journal of Derivatives* 8, No.3, Spring, 12-22
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