230E: Final Examination

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Instructions: You have 150 minutes to complete the exams. Please, write legibly. Succinct, clear, and accurate responses will get maximum points. The time and points allocated for each question are shown in parentheses. Don't spend too much time on any given question. Good luck!

Problem 1 *We want to estimate the linear model* $y_t = \beta x_t + \varepsilon_t$*, where all the processes are assumed to be stationary and we assume* $E(\varepsilon_t | x_t) = 0$.

1. *First, we want to construct an estimator without making any assumptions about the distributions of the processes. Show that the condition* $E(\varepsilon_t | x_t) = 0$ *implies that* $E(\varepsilon_t x_t) = 0$ *where the variables* ε_t *and* x_t *have well defined joint, conditional, and marginal distributions. (5 minutes/points)*

2. *Explain in detail how we can use the moment condition* $E(\varepsilon_t x_t) = 0$ *to estimate* β*. As a final answer, you should provide a formula that would help us estimate* β*. (10 minutes/points)*

3. *Now, suppose that we are willing to assume that* ε_t *are i.i.d. and* $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$*. We are going to estimate* β *using the maximum likelihood principle. Write down the log-likelihood that will help us estimate* β*. (10 minutes/points)*

4. *Using the log-likelihood function, write down the maximization problem that we have to solve in order to find* β*. (5 minutes/points)*

5. *Find the first order necessary and sufficient conditions for the above problem. What estimate of* β *do you obtain? As a final answer, you should provide a formula that would help us estimate* β*. (10 minutes/points)*

6. *Compare the estimates obtained in parts 2 and 5. Which one would be more efficient and under what conditions? (5 minutes/points)*

Problem 2 *Write down a three factor model of expected returns, where one of the factors is the market portfolio return and the other factors are two other portfolio returns (think Fama-French). Clearly label and explain your notation. Explain in details how would you test such a 3-factor model. (20 minutes/points)*

Problem 3 *Answer the following questions:*

1. *Suppose P_t is the price of the S&P 500 index. Is P_t stationary? Why? (3 minutes/points)*
2. *Suppose R_t is the return of the S&P 500 index. Is R_t stationary? Why? (3 minutes/points)*
3. *Suppose that you estimate the following process: $Y_t = 0.04 + .94Y_{t-1} + \hat{\varepsilon}_t$. Can the estimated process Y_t be the price of a stock? Can the estimated process Y_t be the return of a stock? Can the estimated process Y_t be the yield to maturity of a bond? Why? (3 minutes/points)*
4. *Provide a definition of a stationary process, X_t. (3 minutes/points)*
5. *Why do we prefer to work with stationary time series in empirical work? Be as precise as possible. (4 minutes/points)*
6. *What would you do if you had to run a regression of Y_t on X_t and you found out that X_t is nonstationary? Be as precise as possible. (4 minutes/points)*

Problem 4 *Suppose you have the following Kalman filtering (state-space) problem:*

$$\begin{aligned} Y_t &= \beta X_t + \varepsilon_t \\ X_t &= \phi X_{t-1} + u_t \end{aligned}$$

where ε_t and u_t are uncorrelated at all leads and lags, $E\left(\varepsilon_t^2\right) = \sigma_\varepsilon^2$, and $E\left(u_t^2\right) = \sigma_u^2$.

1. *What variables do we observe, and what variables are unobservable in the above system? (2 minutes/points)*
2. *Write down all the parameters of the Kalman filtering problem? (2 minutes/points)*
3. *Clearly write down all the steps that would help us produce the sequences $\left\{X_{t|t-1}\right\}_{t=1}^T, \{Y\}_{t=1}^T, \left\{X_{t|t}\right\}_{t=1}^T$, assuming that we know the parameters of the system. (6 minutes/points)*
4. *Discuss how would we go about estimating the parameters of the Kalman Filtering problem. Be as specific as possible. (5 minutes/points)*

Problem 5 *Show that in the AR(1) model $Y_t = c + \phi Y_{t-1} + \varepsilon_t$, with $var(\varepsilon_t) = \sigma^2$ and $|\phi| < 1$*

1. $E\left(Y_t\right) = \frac{c}{1-\phi}$. *(2 minutes/points)*
2. $Var\left(Y_t\right) = \frac{\sigma^2}{1-\phi^2}$. *(2 minutes/points)*

3. $E_t(Y_{t+1}) = c + \phi Y_t$. *(2 minutes/points)*

4. *Find* $Var_t(Y_{t+1})$? *(4 minutes/points)*

Problem 6 *Suppose that an estimator is NOT consistent, as is the case if* $E(\varepsilon_t x_t) \neq 0$. *In such a case, should we add more observations to improve on our estimate? Why? Then what should we do? Be as specific as possible. (10 minutes/points)*

Problem 7 *In finance, it is often believed that returns are conditionally heteroskedastic. We have seen a few models that capture such conditional heteroskedasticity.*

1. *Write down a GARCH (1,1) model, clearly labeling all parameters and placing the appropriate restrictions. (2 minutes/points)*

2. *Can you use GMM to estimate a GARCH(1,1) model? Explain. What are the virtues and deficiencies of such an approach? (4 minutes/points)*

3. *Can you use the maximum likelihood method to estimate a GARCH(1,1) model? Explain. What are the virtues and deficiencies of such an approach? (4 minutes/points)*

Problem 8 *Suppose we want to run the following regression:*

$$E_t r_{t+1} = \alpha + \beta\sigma_t + \varepsilon_t$$

where σ_t *is the conditional (at time t) volatility of returns.*

1. *What is the null hypothesis on* β? *What is the alternative? Why? (3 minutes/points)*

2. *We usually estimate the above system by running the regression* $r_{t+1} = \alpha + \beta\sigma_t + \varepsilon_t$. *What estimates of* β *do we observe in such equations? (3 minutes/points)*

3. *Suggest some possible explanations of the empirical findings. (6 minutes/points)*

4. *Suggest some possible remedies, either based on time domain or on frequency domain filtering. (8 minutes/points)*