Instructions: You have 120 minutes to complete the exams. Please, write legibly. Succinct, clear, and accurate responses will get maximum points. The time and points allocated for each question are shown in parentheses. Don’t spend too much time on any given question. Good luck!

Problem 1 Answer by “True” or “False” and provide a short explanation for your answer.

1. The GARCH(1,1) is a simple and elegant model of volatility and its in-sample and out-of-sample performance are difficult to beat by other, more complicated models. (3 minutes/points)

2. Stock returns at 5-minute, daily, weekly, and monthly frequency are equally serially uncorrelated. (3 minutes/points)

3. A Variance Ratio test provides a less general way of testing for serial correlation in returns than if we were to fit an AR(1) model. (3 minutes/points)

4. The CAPM is a useful benchmark because it explains a great deal of the cross-sectional variation in returns. (3 minutes/points)

5. A 3-factor Fama-French model can be estimated using only time-series regressions. In fact, any 3-factor model can be estimated using only one time-series regression. (3 minutes/points)

6. In a linear model, the GMM and ML (maximum likelihood) estimators are always the same. (3 minutes/points)

7. Event studies are particularly prone to small sample biases. (3 minutes/points)

8. The difference between a bootstrap and a simulation is that in the latter we need to make distributional assumptions about the data while in the former we do not. (3 minutes/points)

9. A non-linear model with more parameters will always do a better job at forecasting out-of-sample than a simpler linear model with fewer parameters. (3 minutes/points)
10. We are interested in the following time varying CAPM model: \( R_{i,t} - r_f^t = \alpha_i + \beta_{i,t} (R_M^t - r_f^t) + \varepsilon_{i,t} \). In this setup, a rolling window estimator of \( \beta_{i,t} \) will be equivalent to a Kalman Filtering estimator because the system is linear. (3 minutes/points)

**Problem 2** Answer the following questions:

1. Suppose \( P_t \) is the price of the S&P 500 index. Is \( P_t \) stationary? Why? (3 minutes/points)

2. Suppose \( R_t \) is the return of the S&P 500 index. Is \( R_t \) stationary? Why? (3 minutes/points)

3. Suppose that you estimate the following process: \( Y_t = 0.04 + 0.94Y_{t-1} + \varepsilon_t \). Can the estimated process \( Y_t \) be the price of a stock? Can the estimated process \( Y_t \) be the return of a stock? Can the estimated process \( Y_t \) be the yield to maturity of a bond? Why? (3 minutes/points)

4. Provide a definition of a stationary process, \( X_t \). (3 minutes/points)

5. Why do we prefer to work with stationary time series in empirical work? Be as precise as possible. (4 minutes/points)

6. What would you do if you had to run a regression of \( Y_t \) on \( X_t \) and you found out that \( X_t \) is nonstationary? Be as precise as possible. (4 minutes/points)

**Problem 3** You are hired by a portfolio management company and your assignment is to extract information from option prices in order to make better portfolio allocation decisions. The company mainly invests in the largest 1000 companies. Suppose that the fraction of the portfolio invested in stock \( i \) (\( i = 1, \ldots, 1000 \)) at time \( t \), is \( w_{i,t} \). The portfolio return is defined as \( r_{p,t} = \sum_{i=1}^{1000} w_{i,t} r_{i,t} \). We will collect the portfolio weights in a vector \( w_t \) and the individual stock returns in a vector \( r_t \). The unconditional expected return of the stocks is collected in a vector \( \mu = E(r_t) \) and the unconditional variance is the matrix \( \Sigma = E(r_t r_t') \).

1. What are the dimension of vector \( \mu \)? (1 minutes/points)

2. What is the dimension of the matrix \( \Sigma \)? (1 minutes/points)

3. How would you estimate the unconditional mean \( \mu \)? (2 minutes/points)

4. Can you estimate the unconditional variance \( \Sigma \) in the same fashion? Why? How many distinct parameters are there in \( \Sigma \)? (4 minutes/points)

5. From the Markowitz mean variance results, express the portfolio weights as a function of \( \mu \) and \( \Sigma \). In this formulation of the problem, would the portfolio weights change from period to period? Be as specific as possible. (4 minutes/points)
6. Now, suppose we denote by \( \mu_{t|t-1} \) and \( \Sigma_{t|t-1} \) the conditional mean and the conditional covariance matrix of the returns at time \( t \) using all available information up to time \( t-1 \). When would the conditional first and second moment of the returns be different from the unconditional ones in parts 1 and 2. (4 minutes/points)

7. Would the portfolio weights in part 5 be time varying if you were to replace the unconditional moments by conditional moments? Explain. (4 minutes/points)

8. Now, suppose you want to estimate the conditional mean and the conditional variance. More specifically, you want to use information from the options market to help you get better estimates of the conditional mean and variance, which in turn will yield better estimates of the portfolio weights. What information from the options market do you think will be useful to do that? Explain how you would go about gathering the data, and how you would transform it so that it be useful for further empirical analysis. Clear, succinct and exhaustive answers will get maximum points. (10 minutes/points)

9. Suppose that the variables from the options market that you specified in part 8 are collected in a vector \( X_{i,t-1} \). Given these variables, how would you go about estimating the conditional mean \( \mu_{t|t-1} \). How would you compare the estimate of the conditional mean to the unconditional estimate, obtained in part 3. Discuss in-sample and out-of-sample comparisons. (10 minutes/points)

10. Explain why is it not a good idea to run the regressions

\[
R_{i,t} = \eta_i + \delta_i X_{i,t} + u_{i,t}
\]

(Hint: Would you be able to obtain a consistent estimate of \( \delta_i \)) (10 minutes/points)

11. How can you use the information in the options market (collected in \( X_{i,t} \)) to obtain a conditional estimate of the variance, \( \Sigma_{t|t-1} \)? Be as specific as possible. Please, keep in mind that \( \Sigma_{t|t-1} \) must be positive definite and symmetric. (10 minutes/points)

12. Can you jointly estimate the conditional mean \( \mu_{t|t-1} \) and conditional variance\( \Sigma_{t|t-1} \) into a GMM framework. Be as specific as possible. (10 minutes/points)