Due: Before April 7, 11:59pm in Benn Eifert’s email box: benn@econ.berkeley.edu

All assignments must be completed using a software or programming language that allows the manipulation of matrices and vectors. No canned statistical packages are allowed.

To complete this assignment, you will need two data files of monthly and daily market stock returns. The two files are zipped into one file, called homework2_files.zip and can be found on my website. Once unzipped, they will be in an Excel spreadsheet. The first sheet contains monthly returns of the market portfolio. The second sheet contains daily market portfolio returns.

1. In this exercise, you will program your own variance ratio test. As mentioned in class, the VR tests one of the core hypotheses of modern finance—market efficiency. To conduct the variance ratio test, you will use as inputs the daily and monthly market portfolio returns. For the daily and monthly returns, you will run two versions of the VR test. All results must be presented in either a clear table or in graph form. Benn and I should be able to see the gist of your results in no more than a minute. (Note: For more details on this exercise, see Richardson and Stock (1989), Journal of Financial Economics).

(a) Your first VR function should take, as input, a vector of returns, and \( q \), the horizon at which we want to test the RW hypothesis. The procedure should then build the multiperiod returns \( r_{t,t+q} \) WITHOUT OVERLAP and the variation ratio statistic \( VR(q) = \frac{Var(r_{t,t+q})}{q^2 Var(r_{t+1})} = 1 \). Hence, we can compute the relevant statistic \( Z = \frac{VR(q) - 1}{\sqrt{2q - 1}} \) and report whether it falls within the 95% confidence interval. Your routine should have, as outputs, the following information: the numerical value of the VR test, the \( Z \) score from the statistic \( \frac{VR(q) - 1}{\sqrt{2q - 1}} \), and the corresponding \( p-value \). Compute the VR\((q)\) test for \( q = \{2, 3, \ldots, 24\} \).

(b) Build a second function that uses OVERLAPPING returns to conduct the VR test. Use the statistic \( Z = \frac{VR_{\text{overlap}}(q) - 1}{\sqrt{2q - (2q - 1) - 1}} \) to conduct your tests. The inputs and outputs would be identical to the first VR function. Again, compute the VR\((q)\) test for \( q = \{2, 3, \ldots, 24\} \). Do you see a pattern in the results, as the overlap increases?

(c) Do you obtain similar results from the overlapping and the non-overlapping procedures. Which results will you trust? Why?

2. In this exercise, you will test whether there is a positive relation between the conditional mean and the conditional variance of returns. Such a relation can be found in Merton’s (1979) work and is often referred to as the ICAPM (intertemporal CAPM). For a test of that model, please see French, Schwert, and Stambaugh’s (1987) paper in the Journal of Financial Economics.

(a) Using the daily returns, compute monthly variances \( \hat{\sigma}_t^2 \), where \( \hat{\sigma}_t^2 = \frac{1}{N} \sum_{j=1}^{N} \left( r_{t+\frac{j}{24}} - \bar{r}_t \right)^2 \), where \( r_{t+\frac{j}{24}} \) are daily returns in month \( t \) and \( \bar{r}_t \) is the average of daily return in month \( t \). First, use \( N = 22 \), i.e., use 22 daily returns to estimate the monthly variance.
(b) Once you have computed the monthly variances, fit an AR(1) model:

\[ \hat{\sigma}_t^2 = \nu + \phi \hat{\sigma}_{t-1}^2 + \psi_t \]

Is the variance process persistent?

(c) Now, please run the following regressions. The first regression links past monthly variance estimates to future monthly returns,

\[ r_{t+1} = \alpha + \gamma \sigma_t^2 + \varepsilon_{t+1} \]

The second regression links current returns and variances as

\[ r_{t+1} = \alpha + \gamma \sigma_{t+1}^2 + u_{t+1} \]

Discuss which one of the two regressions you think is a better test of the ICAPM and why?

(d) Now, change your estimate of the variance in part (a) by using \( N = 44 \) and \( N = 66 \) (i.e., using two and three months worth of daily data). Re-run regressions (1) and (2) using the new variance estimates. Do your results change? How can you explain the changes?