Competition among Proprietary and Open-Source Software Firms: The Role of Licensing on Strategic Contribution*

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Abstract

In enterprise software markets, firms are increasingly using *services*-based business models built on open-source software (OSS) to compete with established, proprietary software firms. Because third-party firms can also strategically contribute to OSS and compete in the services market, the nature of competition between OSS constituents and proprietary software firms can be complex. Moreover, their incentives are likely influenced by the licensing schemes that govern OSS. We study a three player game and examine how open-source licensing affects competition among an open-source originator, open-source contributor, and a proprietor competing in an enterprise software market. In this regard, we examine: (i) how quality investments and prices are endogenously determined in equilibrium, (ii) how license restrictiveness impacts equilibrium investments and the quality of offerings, and (iii) how license restrictiveness affects consumer surplus and social welfare. Although some in the open-source community often advocate restrictive licenses such as GPL, because it is not always in the best interest of the originator for the contributor to invest greater development effort, such licensing can actually be detrimental to both consumer surplus and social welfare when it exacerbates this incentive conflict. We find such an outcome in markets characterized by software providers with similar development capabilities yet cast in favor of the proprietor. On the other hand, when either these capabilities become more dispersed or remain similar but tilt in favor of open-source, a more restrictive license instead encourages greater effort from the OSS contributor, leads to higher OSS quality, and provides a larger societal benefit.

Keywords: Open-source software, software competition, licensing, collaborative development, co-creation, product quality, software services market, strategic contributions to open-source software

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1 Introduction

The co-founders of Apache Kafka, an open-source platform for handling real-time data streams, are also the co-founders of Confluent, a firm that has built a business based on "selling management tools and services that make it easier to run Kafka" to enterprise customers (Konrad 2015). This professional open-source business model, where profit-motivated firms are driven by revenues stemming from a market for professional services, is not a new one. In fact, the co-founders of Confluent are following the path of JBoss which, in 1999, successfully built a company on top of open-source software (OSS), driven purely by the provision of services for its enterprise java application server product. Watson et al. (2008, p. 43) refers to this type of OSS business model as second generation open-source or OSSg2 characterized by firms that "typically generate the bulk of their revenues by providing complementary services around their products" and "own or tightly control the software code and can exploit their intimate knowledge of the code to provide higher-quality service than could potential competing service providers."

In this paper, we focus on enterprise software markets where OSSg2 business models are viable and often observed in practice. Importantly, the economic incentives underlying these business models are commonly related to service revenue generation, and hence we study the complex interactions that arise among service-oriented firms competing in a services market.[1] In particular, we study three-way competition among a proprietary firm (*proprietor*), an OSS originating firm (*originator*), and an OSS contributing firm (*contributor*). The nature of the interaction between an originator and contributor is both collaborative and competitive. They collaborate on the development and management of enterprise OSS, but then compete against one another in the services market. For all practical purposes, OSS-based business models typically preclude charging for the software alone. However, services are generally required to achieve high-quality integrated software solutions. The originator and contributor compete for these services based on expertise generated from their activity on the OSS project.[2]

[1]Because of our focus on revenue-generating business models, we necessarily do not aim to comment generally on the vast OSS landscape spanning broadly defined contexts and governed by a wide range of intrinsic and extrinsic motivations for OSS contributions. In general, firm and developer motivations vary drastically across software classes (operating systems, platforms, client applications, server applications, virtual machines, middleware, and more) and individual capabilities (Roberts et al. 2006).

[2]Some examples of strategic originators and contributors include Shadow-Soft (JBoss), Synolia (SugarCRM), Bista Solutions (OpenERP), and Hortonworks (Hadoop).

These conflicting, collaborative and competitive interests in the OSS domain become even more salient in the presence of competition from a proprietary provider. How competition from a proprietary incumbent affects the economic incentives of firms who strategically contribute to OSS as part of OSSg2 business models has yet to be studied in the literature but has important and wide-ranging implications on the quality and competitiveness of the products that emerge in these enterprise software markets. In this paper, we study how the existence of a strategic contributor affects competition between enterprise open-source and proprietary software offerings. This is a unique research contribution because the nature of the firms involved differs fundamentally. In particular, the proprietor solely incurs production cost but then retains significant pricing power as it can charge for the product itself.[3] On the other hand, the OSS originator and contributor share production costs, cannot charge for an open-source product, and must compete for service revenues.[4]

In contrast to the belief that open source means free of restrictions and liberal in copyright, intellectual property (IP) concerns do not disappear in the open-source world. In fact, contributions from the open-source community may be substantially affected by IP regimes (Wen et al. 2013). In the open-source ecosystem, OSS projects are distributed under various license schemes as a means to govern intellectual property. Distinct licenses each have different restrictions on the use and modification of the software as well as its derivatives. For example, the GNU General Public License (GPL) is a widely-employed open-source license that is considered to be quite restrictive with regard to what a contributor can do with the software. On the other end of the spectrum lies the Berkeley Software Distribution (BSD) license which imposes minimal restrictions on anyone that uses or extends BSD licensed software (Laurent 2004).

It is then critical to examine the impact of licensing on the economic incentives of firms to contribute to OSS in contexts where contributors are commercial firms extrinsically motivated by service markets. Because the strategic development efforts made by these firms largely co-determine

[3]If there exist third-party agents who offer integration services for the proprietary software product, the proprietor would utilizes two pricing levers (product and services) and set the product price to exert control. The simplified view we explore here is the proprietor is not competing on the servicing of its product unlike the originator who provides services for the same underlying product as the contributor and therefore has amplified interactions.

[4]Unlike a typical duopolistic model (Lee and Mendelson 2008, Zhu and Zhou 2012, August et al. 2013) or unilaterally strategic model (Casadesus-Masanell and Ghemawat 2006, Jaisingh et al. 2008, Athey and Ellison 2014) of competition between OSS and proprietary software, our model necessarily requires all three players (the proprietor, originator and contributor) to be strategic.

the qualities of the fully integrated software solutions that emerge, different licenses can significantly impact enterprise software markets where OSSg2 business models are being employed. Moreover, because OSS competes with proprietary counterparts in these markets, licenses can greatly influence the quality of proprietary offerings brought to market as well. A second research contribution of our paper is we explore how the degree of OSS license restrictiveness affects the collaborative efforts of the OSS providers and thereby affects the incentives of the proprietor.

When one reflects on the open-source movement and the ideology behind it, an open question is what types of licensing genuinely lead to better outcomes in these enterprise, services-driven software markets. For example, do certain licenses lead to higher quality integrated software solutions in the market? To that end, our third research contribution is to characterize market conditions under which permissive and restrictive licenses each respectively help improve consumer surplus associated with deployed software solutions.

Finally, there is an intimate connection between how licensing and other OSS project primitives such as the maturity of development processes (e.g., whether frameworks such as the Capability Maturity Model are utilized), code architecture (e.g., modularity of design), and governance controls (e.g., contributor accreditation and code acceptance policy) affect the strategic interactions that unfold in this competitive setting. We discuss the relationships between licensing, these other factors, and even the extent to which the flexibility associated with OSS is valued by the specific enterprise software market in question, with regard to the manner in which they influence our findings.

2 Literature Review

Our study is related to the body of literature that examines how firms compete with OSS (Lerner and Tirole 2005a). As a prerequisite to analyzing competition, researchers have thoroughly studied the motivations of open-source developers which led to a stream of literature that connects contributions to various extrinsic and intrinsic motivations.[5] Several papers consider these diverse motivations to examine how a commercial firm competes with an open-source product. This literature can be classified into two groups dependent on whether the open-source investments are

[5] See, e.g., Hars and Ou (2002), von Krogh and von Hippel (2006), Roberts et al. (2006), Iansiti and Richards (2006), August et al. (2013).

driven by commercial or non-commercial interests.

First, we discuss the non-commercial case where the OSS product is generally available at zero price. Casadesus-Masanell and Ghemawat (2006) formulate a model motivated by competition between Windows and Linux, where Linux benefits from superior demand-side learning and lower (zero) prices that boost market share. Even under these conditions, they show that Windows can persist in the market. Lee and Mendelson (2008) examine a market characterized by network effects and compatibility issues where open-source developers maximize a weighted sum of consumer surplus and intrinsic benefit. They find that in some cases, a commercial firm has incentives to make its product *incompatible* with OSS and of higher quality. Casadesus-Masanell and Llanes (2011) also study compatibility issues when the commercial firm can open part of its codebase and benefit from additional effort stemming from the OSS community. Athey and Ellison (2014) model the evolution of OSS when developers contribute due to reciprocal altruism and study how a proprietary firm dynamically prices its product to compete. Prior work has also examined the impact of product heterogeneity (Bitzer 2004), network effects (Cheng et al. 2011), and lock-in strategies (Zhu and Zhou 2012) on competition between OSS and proprietary software.

Our work benefits from these studies but also differs from them by focusing on profit-maximizing, commercial open-source firms, and thus is closer to the second group of papers that examine commercial interests. In particular, we model open-source firms who invest in OSS and compete in the market for value-added services. In this portion of the literature, several papers study complementary products and services. Haruvy et al. (2008) examine a monopolist's decision on whether to open the source code when it can profit from a complementary product. Mustonen (2005) and Asundi et al. (2012) study a proprietary firm's incentives to support OSS when the firm can benefit from either network effects or through users' preferences on customization. In the context of platform competition between OSS and proprietary software with two-sided network effects, Economides and Katsamakas (2006) show that a proprietary system dominates an OSS platform when the demand potential for the latter is relatively limited. Kim et al. (2006) examine competition between proprietary software and OSS, where OSS is offered under either a dual licensing scheme or a support model. When the OSS firm uses a support model, they find that a proprietary firm squeezes the OSS firm out of the market by pricing at the marginal cost of support. Sen (2007a) studies competition among three vertically differentiated offerings, where the qualities of the proprietary and base OSS product without services is fixed, and the focus lies on how the OSS provider of support services invests in usability.

We complement these papers in several ways. First, in our model, we have three providers investing strategically such that the software qualities are endogenously determined in equilibrium. Second, we include a strategic contributor who helps to improve the OSS product (which also can benefit the OSS originator) but acts as a competitor in the market for services. Different from August et al. (2013), our work includes a proprietary firm in order to study the 3-way competition which significantly impacts the OSS originator's investment incentives. Further, while they study the decision whether to pursue a proprietary or OSS path, we focus on how *licensing* (especially the restrictiveness of licensing) affects competition due to its impact on strategic contributions from the third party.

Because proprietary and OSS can be seen as two systems of providers competing on quality under different cost structures, our work is also related to the literature on multi-firm quality competition in a broader context (Abbott 1953, Motta 1993, Chambers et al. 2006). While competitors in this literature are homogeneous in their pricing power according to quality, our context differs in that the proprietor has significantly more control and pricing power than its open-source counterparts. The closed nature of the proprietary software allows the proprietor to charge both for the product and the service (see footnote 3). But the OSS providers can only charge for their services because the product itself is free, which makes them more exposed to intensified competition for services. Another unique feature of our study is that the OSS originator and contributor both collaborate (on the project) and compete (for services). Even though the multi-firm competition literature has studied firm collaboration in terms of joint ventures (Choi 1993), strategic alliance (Amaldoss et al. 2000), and collaborative product development (Bhaskaran and Krishnan 2009), the collaboration are typically governed explicitly with contracts. In our study, the collaboration and competition are strongly influenced by the licensing and IP schemes, and the interactions are non-contractual. Our study focuses on how this licensing affects the incentives of both these OSS participants and a strategic proprietor who possesses greater control and competes in the same market.

In this respect, our work is also related to the emerging literature that explores licensing. Lerner and Tirole (2005b) explore the choice of open-source licenses as it relates to an OSS originator's ability to induce contributions from the community and generate returns from commercial clients that may prefer more permissive licenses. Singh and Phelps (2013) examine the relationship between OSS license choice and social influence. It is noteworthy that oftentimes the license type can already be pre-determined to some extent by industry. For example, Polanski (2007) shows that restrictive licenses such as GPL may be a rational choice for the first innovator in a sequential innovation setting. Firms often have no choice but to adhere to certain license restrictions in order to make their products compatible with other software they intend to leverage. August et al. (2017) study the policy implications of OSS licensing as it swings a software originator's decision to go either proprietary or open source.

3 Model

Three strategic players, a proprietor (denoted with subscript p), an OSS originator (denoted with subscript o), and an OSS contributor (denoted with subscript c) compete in an enterprise software market. To make the setting more concrete, one can think of Salesforce as the proprietor, SugarCRM as the OSS originator, and Synolia as the OSS contributor. In this market, in order to derive value from the enterprise software, a consumer[6] must install it, integrate it with existing business systems and processes, and acquire support going forward; i.e., the consumer needs to obtain *services* from a service provider of the software whose expertise determines to what extent this value is accessed.[7] It is clear that the total quality of solution increases when a provider has greater involvement in the OSS project and possesses greater expertise on how the software should be integrated and utilized.

For simplicity, we assume the proprietor is the only service provider for its proprietary, closed-source enterprise software. However, for the open-source enterprise software alternative, both the originator and contributor are capable of providing services, which ultimately leads to differentiated solution qualities. We denote the total quality of software solution for the proprietor, originator,

[6]In most cases, "consumer" refers to a firm that leverages the enterprise software as part of business operations.

[7]*Integration* refers to getting software installed on a company's servers and integrated with business processes and pre-existing information systems as effectively as possible to increase the overall quality of the solution provided. *Support* refers to ongoing maintenance of these integrated systems, from troubleshooting bugs which arise to efficiently providing security vulnerability patches and more. Information technology *consulting* often refers to the provision of human resources to advise, implement, and deploy IT solutions which may include custom-built applications that run on top of open source software. Altogether, the integration, support, and consulting offerings are generally referred to as *services* (August et al. 2013).

and contributor with Q_p, Q_o, and Q_c respectively.

There is a continuum of consumers who have heterogeneous sensitivities to the total quality of the software solution which we model as a uniformly distributed type characteristic $\theta \in \Theta = [0,1]$. Thus a consumer with type θ derives value θQ_p, θQ_o, or θQ_c, depending on whether she contracts with the proprietor, originator, or contributor, respectively. The total qualities are determined by effort investments which we next describe. For the proprietary case, the proprietor incurs all development costs alone. In particular, the proprietor chooses a development effort $e_p \in \mathbb{R}_+$ and correspondingly incurs a quadratic, convex cost of effort $\beta_p e_p^2/2$, where $\beta_p > 0$ is a measure of cost efficiency. Based on this effort investment, the proprietor's total quality is given by $Q_p = s_p e_p$, where $s_p > 0$ is a measure of how effectively it translates effort into quality.

Having observed the proprietor's effort investment and quality, the originator can invest in the design, development and management of a competing, OSS offering. Analogously, the originator chooses a development effort $e_o \in \mathbb{R}_+$ and incurs a cost of effort $\beta_o e_o^2/2$, where $\beta_o > 0$. Having observed both the proprietor and originator's investments, the contributor may commit effort to be involved with and improve the OSS while generating expertise. The contributor chooses a development effort $e_c \in \mathbb{R}_+$ and incurs a cost of effort $\beta_c e_c^2/2$, where $\beta_c > 0$. We also include an outside option for the contributor of value V_c, which accounts for the contributor's opportunity cost. The parameters β_p, β_o, and β_c are generally different due to firms' heterogeneity in cost efficiency (see, e.g., Oi 1983).[8]

The active involvement of the originator and contributor in the development, management and direction of the OSS leads to their having garnered extensive expertise that can be leveraged in the provision of services to render higher total quality software solutions to their consumers.[9] We model these providers as having a quality component due to this expertise, similarly proportional

[8]Effort in both proprietary and OSS development generally involves labor and resources, and, in the software industry, firms exhibit significant variation in size and worker abilities. Further, for any given project, there can be significant differences in resource availability and their shadow prices. In light of our central research questions, we employ a model of complete information. Also, an important feature of OSS projects is that contributions are typically public. For example, one can readily see the actual code contributions and identities of committers for most of the OSS projects hosted on Sourceforge and GitHub. Thus, in the OSS domain, efforts are typically observable and reflected that way in our model. For the case of the proprietor, the quality of its offering is typically observable and sufficient for our analysis.

[9]Firm efforts into OSS projects lead to both software being developed and service expertise being generated. Because these two are intertwined and ultimately lead to a total quality of solution, we focus on that simpler construct which is more relevant to consumers.

to their efforts.[10] In addition, the originator and contributor's total qualities can be affected by each other's investments which leads to cross effects and captures the collaborative nature of OSS development, which is an important feature of our model. In particular, the contributor's total solution quality benefits from the efforts exerted by the originator toward development of the OSS; similarly, the contributor's subsequent developments can be open source and available to the public, hence the originator also benefits from the contributor's efforts.

Capturing these collaborative synergies of effort on the total quality of software solution provided, we model the originator's total quality as:

$$Q_o = s_o e_o + s_{oc} e_c \, , \tag{1}$$

where $s_{oc} > 0$ indicates the cross effect of the contributor's effort on the originator's total quality, while $s_o > 0$ is the direct effect of the originator's own effort on its own total quality. Similarly, the contributor's total quality also depends on the originator's effort through an analogous parameter, s_{co}, and is given by

$$Q_c = s_c e_c + s_{co} e_o \, , \tag{2}$$

where $s_{co} > 0$ represents the cross effect of the originator's effort on the contributor's total quality and $s_c > 0$ represents the direct effect.[11]

The magnitudes of the coefficients s_p, s_o, s_c, s_{oc}, and s_{co} depend critically on characteristics of the particular enterprise software market being studied as well as both the nature of and relationship between the service-providing firms. In our competitive framework, these coefficients carry substantial economic and strategic significance. The OSS-related parameters (s_o, s_c, s_{oc}, and

[10]Firms who invest heavily and take leadership roles can tightly control project direction. They have the ability to ensure the project meets the needs of their customers.

[11]It is worthwhile to note that a strategic service provider can also exist in the proprietary software market, but it is not modeled here for several reasons. First, unlike with OSS, a proprietor can always charge for its product and will thus leverage its pricing power to behave like a monopolist, providing a strategic provider with weak incentives to invest. In many cases, prices would be set to preclude the third-party from entering the proprietary market. Observationally, we may see third-party providers in existence, but proprietors also charge for certifications, attempting to extract back surplus via tariffs. Second, in the case of OSS, a strategic contributor can exert a lot of effort in developing the product and gaining expertise and then leverage that intimate knowledge of the code to integrate and customize solutions for customers; strategic contributors in the OSS domain can really stand out in the quality dimension. However, a strategic contributor for proprietary software is inherently limited in that the code base is closed, they are confined to working within the rules of the proprietary product and limited API support, and cannot fully customize solutions for customers. In this sense, the concept of strategic contribution is quite different under these two paradigms.

s_{co}) capture the collaborative and competitive nature of OSS contributions in a parsimonious way. An OSS provider's total quality of solution is ultimately determined by both the magnitude of these parameters and the equilibrium efforts of the OSS participants, and therefore the parameters necessarily capture a host of quality-impacting primitive factors. These factors include licensing, flexibility associated with openness, development processes and architecture, and governance controls. Please see Section 5.1 for a comprehensive discussion of these factors and their relationships with the OSS parameters which is provided just prior to our main results. The parameters strongly influence a firm's incentive to invest because having a competing service provider who also derives a large benefit can amplify or reduce the firm's willingness to invest in quality.

In (1) and (2), it is important to note that efforts are additive but not separable. We do not include any multiplicative terms (i.e., $e_o e_c$) of these efforts on the providers' qualities, preferring to demonstrate that even without such strong assumed interactions, in equilibrium, e_o^* and e_c^* can be either *strategic* complements or substitutes as the providers strategically adapt their equilibrium efforts to varying license scenarios (Bulow et al. 1985). Due to the sequential nature of the effort investments (proprietor, originator, and then contributor), these efforts will be intimately related.[12]

After effort investments take place, all three service providers simultaneously set the prices of their offerings: p_p, p_o, and p_c, respectively. These prices represent the total price a consumer must pay for fully integrated software solution offered by each provider. For simplicity, we assume that the unit cost of providing this integration and services work is the same for each provider and denoted $c \geq 0$.[13]

Consumers' usage decisions are made in the last stage. Denoting the net utility to consumer θ with $V(\theta)$, she obtains the following payoffs depending on her provider choice:

$$V(\theta) = \begin{cases} \theta Q_p - p_p & \text{if contracted with the proprietor;} \\ \theta Q_o - p_o & \text{if contracted with the originator;} \\ \theta Q_c - p_c & \text{if contracted with the contributor;} \\ 0 & \text{if not contracted.} \end{cases} \tag{3}$$

[12]In our analysis, we show how multiplicative terms $e_o e_c$ arise in the maximization problem faced by the firms and characterize the nature of the related best response function, $e_o(e_c^*)$.

[13]One can think of this cost representing a fixed number of hours of service/integration work. What varies here is the total quality of solution that is achieved after that fixed number of hours.

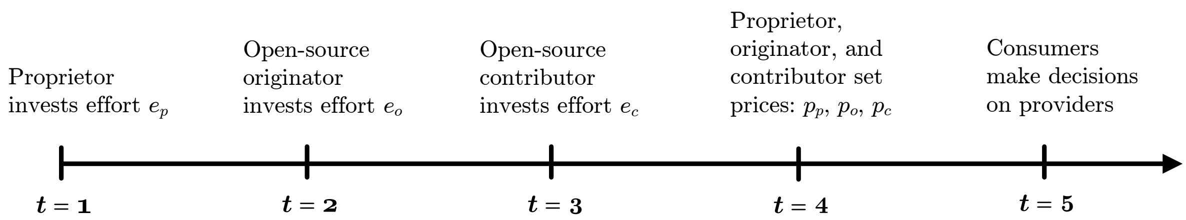

Figure 1: Sequence of events

In summary, the timeline for our model is depicted in Figure 1.

4 Consumer Market Equilibrium and Pricing

4.1 Consumer Market Equilibrium

We begin by examining the final stage of the game at which point each consumer either selects a service provider or chooses to remain out of the market. At this point, all providers' effort levels (e_p, e_o, and e_c) and prices (p_p, p_o, and p_c) have already been set. Hence, the consumers observe total qualities (Q_p, Q_o, and Q_c) and prices and make consumption decisions. Each consumer $\theta \in \Theta$ can select a strategy from one of four options: P (contract with the proprietor), O (contract with the originator), C (contract with the contributor), or N (not contract with any provider). The resulting consumer market equilibrium critically depends on the ordering of total qualities in magnitude and is indifferent with regard to who provides it. Thus, without loss of generality, we characterize this equilibrium when $Q_i > Q_j > Q_k$ where $i, j, k \in \{p, o, c\}$ and $i \neq j \neq k$.[14]

We highlight the consumer market outcome where all three service providers are present in the marketplace and service positive masses of consumers. This outcome is of particular interest because it is commonly observed in the services-driven enterprise software markets that are the focus of our study. We then obtain the following relevant result:

Lemma 1 *For fixed prices p_i, p_j, and p_k and qualities $Q_i > Q_j > Q_k$, the consumer market has the following threshold characterization if $p_i < Q_i$, $p_i - Q_i + Q_j \leq p_j < \frac{p_i Q_j}{Q_i}$, and $p_j - \frac{(p_i - p_j)(Q_j - Q_k)}{Q_i - Q_j} \leq p_k < \frac{p_j Q_k}{Q_j}$ are satisfied:*

[14]Because qualities are endogenous to the model, inclusion of consumer market equilibria for the cases where either $Q_i = Q_j$ or $Q_j = Q_k$ is not essential. We formally establish that such outcomes do not arise in subsequent analysis of the effort game's equilibrium.

(a) consumers with $\theta \in (\theta_{ij},\ 1]$ *contract with firm* i*,*

(b) consumers with $\theta \in (\theta_{jk},\ \theta_{ij}]$ *contract with firm* j*,*

(c) consumers with $\theta \in (\theta_k,\ \theta_{jk}]$ *contract with firm* k*,*

(d) consumers with $\theta \in [0,\ \theta_k]$ *do not use the software,*

where $\theta_{ij} = \frac{p_i - p_j}{Q_i - Q_j}$, $\theta_{jk} = \frac{p_j - p_k}{Q_j - Q_k}$, *and* $\theta_k = \frac{p_k}{Q_k}$.[15]

Intuitively, this lemma formalizes that the firms service the consumer market in a tiered fashion according to their qualities. Consumers with the lowest types find it preferable to remain out of the market considering the prices of the software solutions and the relatively low utility these consumers derive. To benefit the mathematical exposition, it is useful to denote the equilibrium strategy profile in the consumer market as $\sigma^*(\theta \,|\, \mathbf{Q}, \mathbf{p})$ which takes vectors (boldface) of qualities and prices as given and maps each consumer type to her equilibrium strategy as prescribed by Lemma 1; specifically, $\mathbf{Q} = (Q_p, Q_o, Q_c)$ and $\mathbf{p} = (p_p, p_o, p_c)$. In the following, we characterize the parameter region upon which this consumer market outcome obtains in equilibrium and then explore diverse behaviors within this region with a goal of better understanding how efforts are leveraged in the open-source domain as providers compete on the quality of their software solutions.

4.2 Strategic Pricing of Software Solutions

Given total qualities and an understanding of how prices influence the consumer market equilibrium, in the second-to-last stage, the three providers compete in the software market on prices. In particular, at this stage, the service providers consider their initial investments as sunk. Thus, the relevant profit functions for this stage can be defined in a straightforward manner by

$$\tilde{\Pi}_p(p_p \,|\, p_o, p_c, \mathbf{Q}) = (p_p - c) \int_\Theta \mathbf{1}_{\{\sigma^*(\theta \,|\, \mathbf{Q}, \mathbf{p}) = P\}} d\theta\,, \tag{4}$$

$$\tilde{\Pi}_o(p_o \,|\, p_p, p_c, \mathbf{Q}) = (p_o - c) \int_\Theta \mathbf{1}_{\{\sigma^*(\theta \,|\, \mathbf{Q}, \mathbf{p}) = O\}} d\theta\,, \tag{5}$$

and

$$\tilde{\Pi}_c(p_c \,|\, p_p, p_o, \mathbf{Q}) = (p_c - c) \int_\Theta \mathbf{1}_{\{\sigma^*(\theta \,|\, \mathbf{Q}, \mathbf{p}) = C\}} d\theta\,, \tag{6}$$

[15] For all formal statements of lemmas and propositions, the technical proofs are provided in the online appendix.

respectively. Each provider maximizes its current-stage profit function taking the other two providers' prices as given. Intersecting these best response price functions gives rise to the simultaneously set Nash equilibrium prices which we denote by $p_p^*(\mathbf{Q})$, $p_o^*(\mathbf{Q})$, and $p_c^*(\mathbf{Q})$. When $\mathbf{Q} > c$, the Nash equilibrium prices fall into one of five regions depending on the level of service costs.[16] As service costs rise, each of the lower quality providers are progressively squeezed out of the market by the equilibrium prices of the remaining higher quality providers. Next, we turn our attention to the sequential effort selection investment problems that determine these qualities.

4.3 Investment and Quality Contribution

In enterprise software markets, it is commonly observed that the leading proprietary offering establishes itself in the market prior to the leading OSS alternative. Typically, a proprietor innovates, invests in development, and begins providing its enterprise software solution in the market. Subsequently, in the OSS domain, an originator introduces an open-source alternative after incurring its own investments. Third, a contributor to the open-source product may also invest and compete in the services market. Given e_p and e_o, we first describe the contributor's effort investment decision problem. If it chooses to invest in OSS, its profit function at this stage can be written as

$$\Pi_c(e_c \,|\, e_p, e_o) = (p_c^*(\mathbf{Q}(e_c)) - c) \left(\int_\Theta \mathbf{1}_{\{\sigma^*(\theta \,|\, \mathbf{Q}(e_c), \mathbf{p}^*(\mathbf{Q}(e_c))) = C\}} d\theta \right) - \beta_c e_c^2/2 \,. \tag{7}$$

By maximizing (7) over $e_c \in \mathbb{R}_+$ and comparing the maximal value to V_c, which is the value of the contributor's outside option, we denote the contributor's best response to each possible set of proprietor and originator efforts with $e_c^*(e_p, e_o)$. Rolling back to the originator's decision problem, its profit function can be written as

$$\Pi_o(e_o \,|\, e_p) = (p_o^*(\mathbf{Q}(e_c^*(e_o), e_o)) - c) \left(\int_\Theta \mathbf{1}_{\{\sigma^*(\theta \,|\, \mathbf{Q}(e_c^*(e_o), e_o), \mathbf{p}^*(\mathbf{Q}(e_c^*(e_o), e_o))) = O\}} d\theta \right) - \beta_o e_o^2/2 \,, \tag{8}$$

and similarly, by maximizing (8) over $e_o \in \mathbb{R}_+$, we denote the originator's best response to each possible proprietor's effort level with $e_o^*(e_p)$. In particular, $\mathbf{Q}(e_c^*(e_o), e_o)$ highlights the strategic interactions between the originator and contributor's effort choices. Finally, we examine the initial

[16] Please see Lemma A.2 in the online appendix for complete details on the pricing equilibrium.

effort investment decision faced by the proprietor, whose profit function can be written as

$$\Pi_p(e_p) = (p_p^*(\mathbf{Q}(e_p)) - c) \left(\int_\Theta \mathbf{1}_{\{\sigma^*(\theta \,|\, \mathbf{Q}(e_p), \mathbf{p}^*(\mathbf{Q}(e_p))) = P\}} d\theta \right) - \beta_p e_p^2/2 \,, \tag{9}$$

where we use the shorthand notation $\mathbf{Q}(e_p) = \mathbf{Q}(e_c^*(e_p, e_o^*(e_p)), e_o^*(e_p), e_p)$. Similar to the other providers, the proprietor maximizes (9) over $e_p \in \mathbb{R}_+$. We denote the equilibrium to this sequential effort-selection game with $\mathbf{e}^* = (e_p^*, e_o^*, e_c^*)$.

5 Oligopolistic Competition in Enterprise Software Markets

As we laid out in Section 3, firms are generally heterogeneous with regard to their respective capabilities and software development efficiencies. In our model, the cost efficiency parameters are given by β_p, β_o, and β_c for the proprietor, originator, and contributor, respectively. Differences in these parameters reflect differences in the firms that produce enterprise software; such differences are connected to a variety of underlying firm characteristics. Some factors that directly drive costs include software development productivity and the availability of and/or access to IT labor resource capacity and capital. Higher productivity in development is the ability to achieve the same outcome at a lower amount of effort (and hence, cost). Productivity can be predicted by (i) level attainment in process-driven software development models such as the capability maturity model integration (CMMI) (see, e.g., Krishnan et al. 2000, Harter et al. 2000, Harter and Slaughter 2003), and (ii) developer talent level and experience (Turley and Bieman 1995, Huckman et al. 2009). Thus, firms who recruit in stronger IT labor markets may source more productive labor (Tambe 2014), and firms leveraging modern HR analytics strategies may retain more productive resources. This is important because productivity among programmers and teams can vary by an order of magnitude (Curtis 1981, Scacchi and Hurley 1995, McConnell 2008).

In order to invest a certain amount of effort into a project, a firm must be able to either allocate or source a sufficient number of developers to achieve it. Frictions in being able to do so go hand-in-hand with increased costs. These frictions include a lack of internally qualified labor resources and having to bring externally-sourced labor into the firm and train them on firm-specific development processes. Larger, established firms may more easily attract and retain labor resources (Idson and

Oi 1999); for example, multinational corporations such as Salesforce with access to worldwide IT labor markets can leverage resources in more affordable markets for gains in cost efficiency. Finally, firms have to fund development, and they cannot do so without access to capital. Firms with higher free cash flow would face significantly less frictions than those needing to borrow. Generally, smaller firms tend to face more financial constraints (Beck et al. 2005), and even start-ups can vary substantially in terms of their financial resources such as venture capital funding.

We focus on two parameter regions characterized by different cost structures. The first region, referred to as Region H, parameterizes a region where the cost efficiencies of the three providers are fairly dispersed (i.e., *high* cost dispersion). This region captures a setting in which the proprietor is more cost efficient than the originator who is in turn more cost efficient than the contributor. Per the above discussion, in this case it is more likely that the proprietary firm has more mature development processes, better access to IT talent, and faces less binding developer capacity constraints. Our running example for customer relationship management (CRM) software fits this region well.[17] In terms of modeling, the parameter space where $\beta_p \ll \beta_o \ll \beta_c$ characterizes Region H. Technically, we prove the existence of bounds such that if $\frac{\beta_o}{\beta_p}$ and $\frac{\beta_c}{\beta_o}$ are greater than these bounds, all of our results in this region hold. In layman's terms, the cost efficiencies simply need to be sufficiently separated.

The second region of focus, referred to as Region L, captures a contrasting scenario in which the cost efficiencies of the three providers are much less dispersed in the parameter space (i.e., *low* cost dispersion). Region L aims to capture settings where the proprietary and open-source firms have similar capabilities such that the gap in cost efficiencies is not as substantial as seen in Region H. As an example, Splunk (proprietor) and Elastic Inc. (originator) compete in the log management and analytics market.[18] In terms of modeling, the parameter space where $\beta_p \sim \beta_o \sim \beta_c < \bar{\beta}$

[17] Salesforce is a multinational company with approximately 25,000 employees, with locations in China, India, Japan, Korea, Australia, New Zealand and Singapore in addition to being headquartered in San Francisco. It has tremendous access to cost-effective worldwide IT labor resources as well as some of the top talent in the bay area and it also generates $8 billion in revenue and therefore likely has the financial ability to fund expansive new development. In contrast, SugarCRM is a much smaller firm with approximately 500 employees, revenues on the order of $100 million (approximately 1% of Salesforce's revenues), and a software engineering team located in the U.S. In short, SugarCRM would encounter significant frictions and difficulty if attempting to match the scale of development seen in Salesforce; this is the essence of Region H. Intuitively, there are many examples where the proprietor is a large, mature organization with extensive resources enabling it to develop products in a cost-efficient manner, whereas an open-source originator competing in the same market is a smaller, relatively less mature organization facing more constrained resources. A similar structure was observed with the large proprietor Oracle in the database management systems market, MySQL being a leading open-source originator prior to its acquisition, and a contributor such as Percona who leverages its own development expertise to contract with consumers in the same market.

characterizes Region L, where we intend to convey the meaning that the cost parameters are of the same order in the asymptotic analysis employed in the paper and that the firms are sufficiently cost efficient. Technically, we prove the existence of an upper and lower bound such that if $\frac{\beta_o}{\beta_p}$ and $\frac{\beta_c}{\beta_o}$ are within these bounds and $\beta_c < \bar{\beta}$, then all of our results in this region hold. In layman's terms, the cost efficiencies simply need to be sufficiently close which can be expected when relevantly connected firm characteristics are more similar.

In addition to the characterization of cost efficiencies that define Regions L and H, our results necessarily require conditions reflecting that the cross effect parameters s_{oc} and s_{co} should not be too large. For the benefit of the exposition, we assume the following sufficient conditions hold throughout the paper.

Assumption 1 *We assume that* $s_{oc} < \bar{s}_{oc}$ *and* $s_{co} < \bar{s}_{co}$.

The condition on s_{co} ensures that an OSS originator's effort comparatively boosts its own expertise and corresponding offerings to a sufficiently greater degree than it affects its collaborating OSS contributor. This is reasonable to assume as an OSS originator determines the governance of the project. The condition on s_{oc} similarly limits the extent to which an originator can leverage the contributor's effort above and beyond the contributor's ability to leverage its own efforts. This sufficient condition ensures the originator cannot simply free ride on the contributor's effort, which is important to keeping the contributor in the market.[19]

For the main analysis in the paper, we examine a region where the value of the contributor's outside option is initially limited such that the contributor finds it preferable to invest in the OSS project. In Section 5.4, we relax this assumption and contrast our findings with the economic

[18]Elastic Inc. is much more similar to Splunk which reflects comparatively less cost dispersion, as opposed to a comparison between SugerCRM and Salesforce. Splunk was founded in 2003, has grown a base of over 11,000 enterprise customers, generates approximately $1 billion in revenues, and has just under 3,000 employees (Beckerman 2016). Elastic, Inc. competes in this space with an OSSg2 business model, and widespread adoption of Elasticsearch's ELK stack is fueling revenue growth and making it an increasingly viable competitor to Splunk (Levy 2015). There are other examples of enterprise software markets that may also benefit from the insights generated from Region L. For example, the software development framework market is critical to enterprises and yet typically does not involve large scale development which suggests that the differences between firms would be less impactful.

[19]Without a practical upper bound on s_{oc}, implausible cases would arise. For example, an equilibrium possibility might have an originator who does nothing (i.e., exerts minimal effort), free rides completely on the contributor's significant efforts, and yet remains the quality leader in the marketplace. Such outcomes are not readily observed in practice. Sufficient bounds ($\bar{s}_{oc}$ and $\bar{s}_{co}$) are formally provided in the online appendix for each respective region (see Lemmas A.3 and A.4 in the online appendix). These bounds are fairly non-restrictive in the sense that all results being presented hold on broad regions of s_{oc} and s_{co}. We do however achieve a better focus on the regions of greater, practical interest.

outcomes that result under a higher opportunity cost for the contributor.

Assumption 2 *We assume that* $V_c < \bar{V}$.

Finally, due to the technical complexity of the equilibrium characterization, we employ asymptotic analysis throughout the paper's proofs. Similar to other papers using asymptotic analysis techniques, our goal is to identify regions under which each of our results and their corresponding insights are valid - these results are robust and satisfied for wide parameter regions.[20] For Region L, we provide some additional supporting technical conditions in Lemma A.4 in the online appendix for tractability.

5.1 Effort-to-Quality Parameters

In this section, we expound on primitive factors that influence the OSS effort-to-quality parameters (s_o, s_c, s_{oc}, and s_{co}) while particularly highlighting the connection between licensing and s_{oc}. We begin with factors that have the broadest impact, influencing all four of the OSS effort-to-quality parameters. First, the flexibility associated with openness in the OSS paradigm (i) enables service providers to fully customize their solutions to be closely aligned with the business processes they are designed to support, and (ii) permits OSS developers to collaboratively contribute code on top of existing code to add new functions, fix bugs, improve design and more. While a proprietor aims to compete similarly with APIs, the flexibility provided is much more limited than that under the OSS paradigm. In markets where flexibility is highly valued, we can expect s_o, s_c, s_{oc}, and s_{co} all to be positively affected as they each have a component that reflects how the originator and contributor's efforts on the codebase impact the shared good that is created.

Second, in addition to impacting productivity, development processes such as CMMI can also positively impact quality. As organizations rise up in their level of maturity, they can be expected to produce software that performs better on quality metrics such as defects per thousand lines of code.[21] To reflect this in our model, if an OSS participant such as the originator attains higher maturity, then both a higher s_o and s_{co} should be employed; similarly, if the contributor attains

[20] See, for example, the following papers and references therein: Li et al. (1987), Laffont and Tirole (1988), MacLeod and Malcomson (1993), Pesendorfer and Swinkels (2000), Muller (2000), Vereshchagina and Hopenhayn (2009), and Beil and Wan (2009).

[21] A positive relationship between process maturity and quality has been established in the academic literature (Harter and Slaughter 2003, Harter et al. 2000). Specifically, these studies have shown that for the same level of effort, a firm with a higher CMM rating creates higher quality code.

higher maturity, then s_c and s_{oc} are also higher. That is, there is a positive relationship between a firm's (whether originator or contributor) development maturity and its own effort-to-quality parameters.

Third, the manner in which an OSS project's code is architected can have wide-ranging impacts. A modular architecture is associated with a high option value in the sense that a developer always has the option to adopt new modules or new designs for existing modules but need not. In contrast, a monolithic architecture is more constraining in that contributions necessitate sweeping changes to the codebase (Baldwin and Clark 2006). Because of the inherently reusable nature of components in a modular architecture, OSS developers have the ability to innovate more quickly and add functionality with less effort, leveraging code reuse (MacCormack et al. 2006). For these reasons, more modular (monolithic) designs can be expected to positively (negatively) affect s_o, s_c, s_{oc}, and s_{co}.

Next, we discuss factors that have an impact that is narrower in scope (tending to influence a subset of the OSS effort-to-quality parameters) including governance controls and licensing (as well as learning curves which is not as specific to the OSS paradigm[22]). First, OSS project governance is an important aspect typically determined by the originator. Governance specifies the rules surrounding participation in the project. These rules impact the incentives for participation and the extent to which the originator can benefit from participation efforts. Consider the rules that govern who can contribute code to the OSS project (e.g., what requirements must be met for a developer to be permitted to commit code). Imposing stricter selection criteria may drive higher quality contributions but have fewer individuals participating. Also, the committed code itself is often governed by a set of processes that determine whether a given contribution is accepted into the OSS project or rejected. Both of these governance controls, developer accreditation and code acceptance, have been shown to increase developer intentions for continued participation in

[22]The qualities of the total integrated solutions offered by the OSS service providers reflect the accumulation of knowledge, experience and expertise. Conceptually, this is closely related to *learning curves*. Levy (1965) and Dutton and Thomas (1984) categorize learning into two types: autonomous and induced. Autonomous learning is the improvement that automatically results from sustained production over a long period of time (Dutton and Thomas 1984), hence cumulative production is often used as a proxy of experience (Spence 1981, Fine 1986). On the other hand, induced learning stems from investments made by the firm toward improvement, and therefore these cumulative investments are instead used as a proxy (Arrow 1962, Dorroh et al. 1994). Our work is consistent with others in the OSS literature where efficiencies of the OSS providers are a type of induced learning that stems from efforts invested into the OSS projects (Sen 2007b, August et al. 2013). Specifically, in our model, the impact of induced learning on the total quality of the strategic OSS providers can also be captured as components of the parameters s_o and s_c.

OSS projects in addition to improving quality directly (Ho and Rai 2017). Therefore, stricter (weaker) controls on the contributor should positively (negatively) affect the quality associated with the contributor's efforts; this can be represented by a higher (lower) s_c and s_{oc} in our model, respectively.

Another control with a different impact is documentation. Historically, OSS was associated with relatively poor documentation in comparison to its proprietary counterparts (Lerner and Tirole 2002), but the status quo has changed particularly in enterprise software markets where OSS offerings are increasingly competitive. On one hand, the act of documenting does not immediately benefit the OSS project and substitutes effort away from functionality contributions. On the other hand, documentation enables other developers to more readily understand code and thus makes it easier to build upon existing code. The net impact depends on these trade-offs. However, good documentation practices are more readily associated with higher cross effort-to-quality parameters in that the provider who benefits is less exposed to the downside of the trade-offs. Therefore, OSS project prioritizing of documentation can be expected to positively affect s_{oc} and s_{co}.

Second, license restrictiveness is an important factor that is focal to the paper. In the open-source community, there are many different licenses that are used, some more commonly than others.[23] One significant dimension along which open-source licenses vary is the degree of restrictiveness with regards to the rights granted to the community (including the contributor) who works with or uses the OSS in question. The degree of restrictiveness primarily involves two elements: (i) the "copyleft" element, which requires source code to be made available if the modified OSS version is distributed; and (ii) the "viral" element, which dictates that source code must be shared even if the OSS is used only as a component (Singh and Phelps 2013). Restrictive GPL-style licenses usually demand both elements, and permissive BSD-style licenses often require neither. Some less restrictive licenses, such as the GNU Lesser General Public License (LGPL), contain only the copyleft element, and thus are often adopted by software libraries. As the copyright holder of the OSS, the originator can enforce its copyright in court should any entity violate a clause of its license.[24]

[23]For example, the Open Source Initiative who maintains the definition of what it means to be "open source" lists around 70 different open-source licenses that satisfy their definition (OSI 2014) and provides detailed information on each license.

[24]For example, in *Artifex v. Hancom*, Artifex, the owner of Ghostscript (distributed under the GPL) won a judgment against Hancom who was asserted to have leveraged the OSS without distributing the source code of its own modifications.

Restrictiveness of a license determines the extent to which contributors must share source code back to a given OSS community. When contributors share more, the benefits to the originator increase as it leverages the contributions and associated collaborative exchanges to drive up the total quality of its own integrated solution to customers. This customized solution reflects both the enhanced OSS functionality and integration expertise. In our model, the parameter s_{oc} essentially captures this feature. The more restrictive the license, the more an originator can benefit from subsequent contributor efforts (i.e., s_{oc} increases), which are more forcibly leveraged.

On the other hand, as the copyright holder, the originator's exposure to license requirements is quite different. It is important to recognize that the originator's copyright enables it to essentially do whatever it wants. If it wants to share a lot of its efforts to the community under a restrictive license, it may, but if it does not want to, it needs not. The same is true under a permissive license. In this sense, the originator is much less constrained by licensing and can easily make license modifications to accommodate its own preference for sharing. For example, it might utilize a dual license model when it wants to impose restrictiveness on others but not itself (Watson et al. 2008).

At the end of the day, the originator of an OSS project already possesses the *intent* to both share and provide visibility into the source code of its product - this is why it has chosen an OSS business model in the first place. Therefore, it is typically the case that the originator makes significant shared contributions to the community (independent of licensing) in hopes to engender excitement and follow on communal contributions. To better reflect the freedoms associated with its copyright as well as its open-source intentions, we treat s_{co} as a free parameter unaffected by licensing for our main results. In this sense, we focus on the more significant relationship between license restrictiveness and contributor sharing. Throughout the paper, language indicating an increase in license restrictiveness refers to an increase in s_{oc}, and all formal results are likewise on movements in s_{oc}. For robustness, we also examine the case where licensing impacts s_{co}. While capturing that the degree of restrictiveness has a lesser influence on s_{co} in comparison to s_{oc}, we formally demonstrate that our main insights carry over to this relaxed scenario.[25]

[25]In Section A.3 of the Appendix, we demonstrate that as long as the primary impact of license restrictiveness is on s_{oc}, then our results are fairly robust to s_{co} also being partially impacted.

5.2 High Cost Dispersion

5.2.1 Efforts and Qualities

In Region H where the providers are characterized by larger differences in their abilities to invest efficiently in the development of their software offerings, we have fully characterized the equilibrium effort investment levels ($\mathbf{e}^*$) that arise in the sequential investment game that is typically found in these markets (see Lemma A.3 in the online appendix for the equilibrium effort levels). We establish that in equilibrium, the proprietor is a distanced, quality leader in the market while, because of cross effects, the originator and contributor compete slightly more aggressively although the originator's offering bears a higher quality in equilibrium. Because with OSS there exist cross-effects of effort investments on the qualities of the open-source offerings, we aim to explore how the strength of these effort interactions affects quality competition among the three providers.

Proposition 1 *Under the conditions of Region H, if the cross effect of the contributor's effort on the originator's total quality s_{oc} increases, then:*

(i) The proprietor's effort e_p^* decreases, whereas the contributor's effort e_c^* increases;

(ii) The originator's effort e_o^* decreases if the contributor's cost parameter is high. However, if the contributor's cost parameter is low, then e_o^* increases in s_{oc} up to a threshold value, $\tilde{s}_{oc}$, and then decreases in s_{oc} beyond $\tilde{s}_{oc}$. Technically, there exists a $\bar{\beta}_c > 0$ such that $\frac{de_o^*}{ds_{oc}} < 0$ if $\beta_c \geq \bar{\beta}_c$. However, if $\beta_c < \bar{\beta}_c$ then there exists $\tilde{s}_{oc} \in (0, \bar{s}_{oc})$ such that $\frac{de_o^*}{ds_{oc}} > 0$ for $s_{oc} \in (0, \tilde{s}_{oc})$ and $\frac{de_o^*}{ds_{oc}} < 0$ for $s_{oc} \in (\tilde{s}_{oc}, \bar{s}_{oc})$;[26]

(iii) The proprietor's quality $Q_p(\mathbf{e}^*)$ decreases, whereas both the originator and contributor's qualities, $Q_o(\mathbf{e}^*)$ and $Q_c(\mathbf{e}^*)$, respectively, increase.

Proposition 1 establishes that increases in s_{oc} resulting from changes in primitive factors such as license restrictiveness, contributor accreditation, code acceptance and maturity, will intensify competition between proprietary and open-source offerings in markets exhibiting properties consistent with Region H.[27] Specifically, part (iii) of Proposition 1 demonstrates that the proprietor's quality

[26] In the online appendix, $\bar{s}_{oc}$ is formally defined and an analytical expressions for $\bar{\beta}_c$ is provided.

[27] Our results focus on changes in the cross effort-to-quality parameter s_{oc}. In cases where factors that can affect multiple parameters such as developer accreditation, code acceptance and maturity are primarily impacting s_{oc}, the insights from our results will also be applicable.

decreases whereas both the originator and contributor's qualities increase in equilibrium. When s_{oc} increases, the contributor faces a trade-off. Because a higher s_{oc} serves to help the originator better leverage the contributor's efforts toward its own solution quality, the originator may partially substitute contributor efforts for its own efforts. This can reduce the contributor's incentives to incur these investments. On the other hand, the contributor may also have an incentive to help the originator become a stronger competitor against the proprietor. Specifically, if the total quality of the originator's offering becomes higher, then the originator also becomes closer in the quality space to the proprietor, creating space at the low end of the consumer market. This increased differentiation between originator and contributor quality relaxes price competition between them in the market for services which could be beneficial to the contributor; it can offer a higher quality offering at the low end where it can also charge a higher price.

It turns out that the latter effect dominates. More restrictive licenses that serve to increase s_{oc} actually increase the economic incentive for contributors to invest effort into OSS, as is stated in part (i) of Proposition 1. This result can be better understood by examining how the originator reacts to increases in s_{oc} which is summarized in part (ii) of Proposition 1. When the ability of a contributor to increase the quality of its own offering is limited because its investment cost function is highly convex, i.e., $\beta_c \geq \bar{\beta}_c$, then the originator has an incentive to scale back its own effort to induce the contributor to invest more. A similar incentive is found when the contributor has the ability to cost-efficiently increase its own quality, i.e., $\beta_c < \bar{\beta}_c$, but the originator can strongly leverage that effort toward its own quality offering, i.e., s_{oc} is at the high end of the range. Again, in this case, the originator scales back effort as s_{oc} increases within this range to induce the contributor to invest more to boost its own quality as well as the originator's quality. This behavior is illustrated in the right-hand portion of panels (b), (c), (e) and (f) in Figure 2. Examining the slope of the curves in panels (e) and (f), it becomes clear that greater differentiation of OSS solutions is achieved by increasing originator quality relatively more than contributor quality.[28]

In contrast, when the contributor is cost efficient ($\beta_c < \bar{\beta}_c$) and the originator does not benefit as much from the contributor's effort, i.e., s_{oc} is at the low end of its range, the originator's investment incentive is significantly altered. In particular, the originator is wary of the contributor

[28]In Section A.2 in the Appendix, we illustrate the contributor's outside option values under which our results presented in Figure 2 hold.

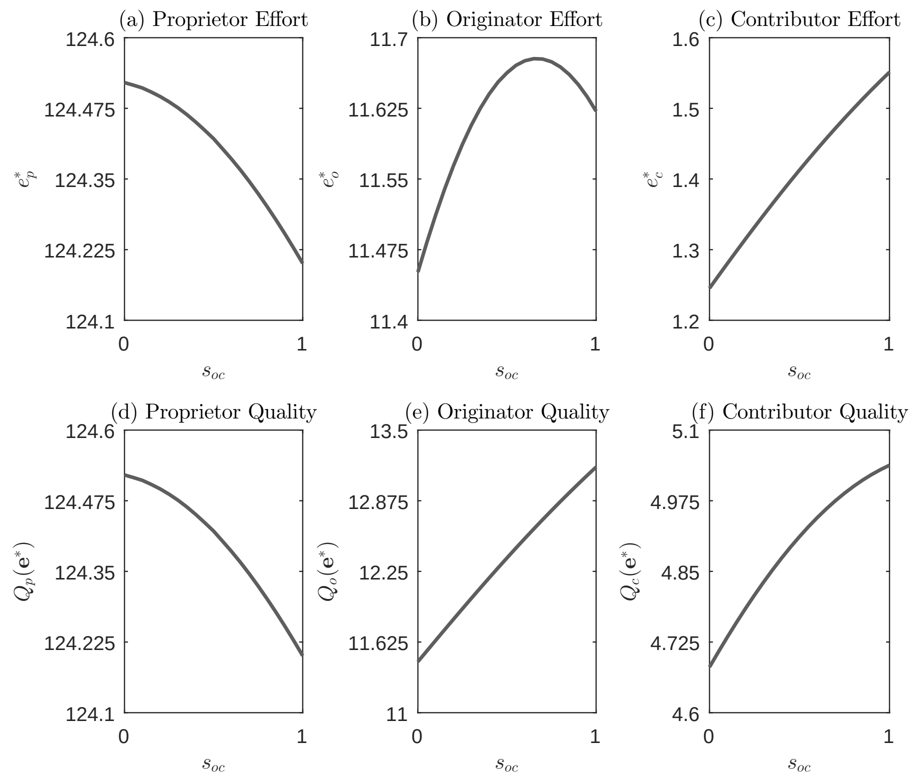

Figure 2: Equilibrium effort choices and resulting quality levels under Region H as affected by changes in s_{oc}. The parameter values are: $s_p = 1$, $s_o = 1$, $s_c = 1$, $s_{co} = 0.3$, $V_c = 0.01$, $c = 0.01$, $\beta_p = 0.002$, $\beta_o = 0.004$, and $\beta_c = 0.005$.

whose investment benefits its own quality significantly more than it benefits the originator's quality. Thus, as s_{oc} increases, the contributor has an incentive to significantly increase its own quality and compete more strongly against the originator whose quality is marginally higher as a result of this higher investment. Because of the contributor's strategic behavior, the originator finds it preferable to also make a larger effort investment and increase its own quality to maintain a sufficient quality gap and avoid excessive price competition. The left-hand side of panel (b) illustrates how the originator sharply increases effort as s_{oc} increases, which leads to a sharp increase in the originator's quality as can be seen in the same portion of panel (e).

When the cost functions of the three providers exhibit high dispersion (Region H), the proprietor is significantly more cost efficient in the development of its software offering. However, the open-source mode of production enables the originator and contributor to still achieve an increased quality in their offerings by leveraging the joint investment from both organizations which has an positive effect on quality. In fact, the inherent nature of open source helps to distribute development costs and enable both contributing providers to operate on the less convex part of their cost functions while still achieving higher quality levels through their separate, but collaborative contributions. Recognizing this open-source production advantage, the proprietor leverages its own cost efficiency and first mover advantage to invest heavily and bring a high quality offering to the market. This ensures that even though open-source offerings benefit from potentially strong cross-effort effects, there will still be sufficient distance between the qualities of software solutions brought to market in equilibrium. However, when s_{oc} further increases, the proprietor does not generate as significant of a return on its effort investments in the face of more competitive OSS offerings. Therefore, on the margin, it scales back its investment which is illustrated in panels (a) and (d) of Figure 2 and formalized in Proposition 1.

5.2.2 Consumer Surplus and Social Welfare

Because increases in s_{oc} greatly impact the incentives of all providers to incur investments and compete in an enterprise services market, we next turn attention to examining the aggregate effect on the market. In particular, we are concerned with how increases in s_{oc} stemming from increased license restrictiveness affect the market overall due to the qualities of the software solutions and the respective price competition that results. For our model, consumer surplus is defined as

$$CS = \sum_{k \in \{p,o,c\}} \int_{\Theta} (\mathbf{Q}_k(\mathbf{e}^*)\theta - p_k^*) \cdot \mathbf{1}_{\{\sigma^*(\theta \,|\, \mathbf{Q}(\mathbf{e}^*), \mathbf{p}^*) = k\}} d\theta \,, \tag{10}$$

and, similarly, social welfare can be measured as

$$SW = \sum_{k \in \{p,o,c\}} \Pi(e_k^*) + CS \,. \tag{11}$$

The following proposition establishes its impact on these measures.

Proposition 2 *Under the conditions of Region H, both consumer surplus and social welfare increase as the cross effect of the contributor's effort on the originator's total quality s_{oc} increases.*

Above, we saw how the equilibrium quality level of the proprietor decreases whereas the equilibrium quality levels of the originator and contributor both increase in s_{oc}. Because qualities are moving in mixed directions and price competition is stiffening between the proprietor and originator yet weakening between the originator and contributor, the overall impact on consumer surplus is unclear. In the analysis of Proposition 2, we establish that the increased price competition between the proprietor and originator more than offsets the impact of having a lower quality proprietary offering: Consumer surplus associated with both of their offerings increases in s_{oc}. Consumer surplus associated with the contributor's offering also tends to increase in s_{oc} because of its higher quality solution for most levels of license restrictiveness. However, in the upper range of s_{oc}, because the increase in contributor quality is only marginal whereas the contributor possesses additional pricing power at the low end of the market, consumer surplus associated with the contributor's offering decreases. Aggregating the impact across all providers, Proposition 2 formally establishes that in regions of high cost dispersion, consumer surplus increases with s_{oc} and therefore with increasingly restrictive licenses that govern OSS. Moreover, these effects extend to social welfare for the same rationale discussed above.

5.2.3 Transition from Region H to Region L

Part (ii) of Proposition 1 also examines a region where the contributor becomes more cost efficient ($\beta_c < \bar{\beta}_c$) and the cross effect of its effort on the originator is limited (i.e., low s_{oc}). In this case, the contributor is intuitively a stronger competitor. As β_c decreases it effectively becomes closer to β_o, moving from a highly dispersed ($\beta_o \ll \beta_c$) scenario to a less dispersed one in a relative sense ($\beta_o < \beta_c$). Between Region H and Region L which we study in the next section, there is another parameter space that is practically relevant: one in which the proprietor is much more efficient and the natural quality leader whereas the OSS participants are of similar caliber.

Combining the mathematical characteristics of Regions H and L, this space could be described as having cost parameters satisfying $\beta_p \ll \beta_o \sim \beta_c$. Because Proposition 1 analyzes a sub-case where β_c becomes smaller, it effectively extends its reach beyond only Region H; the analysis and insights produced in this sub-case are likely to be applicable even for this natural region of curiosity that lies in between. Intuitively, even if the contributor becomes a much stronger competitor, the originator continues to have a first-mover advantage and thus the opportunity to protect its quality leadership. The strategic behavior we characterize above and the associated economic insights can carry over even as far as the cost parameters being equal ($\beta_o = \beta_c$) because the originator utilizes his first-mover advantage to effectively manage the incentives of this more efficient contributor. In fact, even if we instead use $\beta_o = \beta_c = 0.004$ in Figure 2, aside from slight changes in the levels of equilibrium efforts and qualities, the qualitative results are essentially the same. However, if the proprietor's cost efficiency also becomes close to that of OSS participants, the economics differ significantly, which we now turn attention toward.

5.3 Low Cost Dispersion

In Region L where the providers have similar cost efficiencies, we characterize the equilibrium effort investment levels that arise in the sequential investment game; these are provided in Lemma A.4 in the online appendix. Because all three providers have similar cost efficiencies, our results depend critically on the relative magnitudes of the effort-to-quality parameters. We explore two scenarios in our analysis. First, we examine the case where the proprietor's effort-to-quality parameter s_p is relatively large such that it remains the quality leader in equilibrium. Contrasting with Region H, we demonstrate how the strategic interactions differ in Region L as a result of the reduced cost dispersion. Second, we examine the case where the relative strength of the OSS providers' effort-to-quality parameters favors the originator becoming the quality leader in equilibrium. We discuss such outcomes as they relate to OSS factors like customizability, flexibility and code modularity which can be highly valued in enterprise software markets.

5.3.1 Proprietor Quality Leadership

We begin by examining the case where s_p is relatively large in comparison to the effort-to-quality parameters that characterize the OSS providers. In this case, we demonstrate that even if the providers' cost efficiencies become more homogeneous as in Region L, their effort choices lead to equilibrium qualities that match the same ordering as established under Region H, albeit with less separation. However, the homogeneity in cost efficiencies that characterize Region L lead to significantly different strategic interactions. Because of their changing nature, the impact of s_{oc} on the equilibrium effort investments and resulting qualities also varies systematically from Region H, which can provide an enhanced understanding of how cross-effort quality effects interact with cost characteristics in the services market to determine market outcomes.

Proposition 3 *Under the conditions of Region L, if the proprietor has a relatively high effort-to-quality effect* (s_p)*, then as* s_{oc} *increases:*

(i) The proprietor's effort e_p^**, originator's effort* e_o^**, and contributor's effort* e_c^* *all decrease in equilibrium;*

(ii) The proprietor's quality $Q_p(\mathbf{e}^*)$ *and contributor's quality* $Q_c(\mathbf{e}^*)$ *both decrease. However, the originator's quality* $Q_o(\mathbf{e}^*)$ *increases and then decreases in* s_{oc}*. Technically, there exists* $\hat{s}_{oc} \in (0,\, \bar{s}_{oc})$ *such that* $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$ *for* $s_{oc} \in (0,\, \hat{s}_{oc})$ *and* $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} < 0$ *for* $s_{oc} \in (\hat{s}_{oc},\, \bar{s}_{oc})$*.*[29]

(iii) Similarly, consumer surplus and social welfare also increase and then decrease in s_{oc}*.*

Previously in Section 5.2, we learned that when firm capabilities are characterized by a high degree of cost dispersion (Region H), more restrictive licenses can actually increase the economic incentives for originators and contributors to invest effort into OSS by raising s_{oc}. In contrast, part (i) of Proposition 3 establishes that as firms become more similar in development capability, their incentives to invest in development can be substantially and negatively altered. As before, the originator has an incentive to leverage contributor's efforts, and the contributor also has an incentive to drive the originator's quality up for greater differentiation. However, because of the greater similarity in development capabilities in Region L, the proprietor can strategically limit the ability of the OSS firms from leveraging the complementarities that stem from restrictive licenses.

In particular, because cost efficiencies are similar, the proprietor is more wary of investing heavily in quality. If it does so, the originator can also invest efficiently and leverage contributor efforts to compete intensely and limit the proprietor's return on its own investment. Foreseeing this problem, the proprietor invests to a lesser degree and offers a lower quality total solution. Given the proprietor's quality, the originator can cost efficiently increase the quality of its offering in the market but, in this case, it becomes essentially constrained by the prior quality choice commitment of the proprietor. Thus, under a stronger cross effect that comes with more restrictive licenses, the originator simply has heightened incentives to scale back its own investment and substitute it with the contributor's investment to a greater degree. In a sense, the originator does not want its quality offering to increase too much because it only leads to more severe price competition with

[29] $\hat{s}_{oc}$ and $\bar{s}_{oc}$ are formally defined in the online appendix.

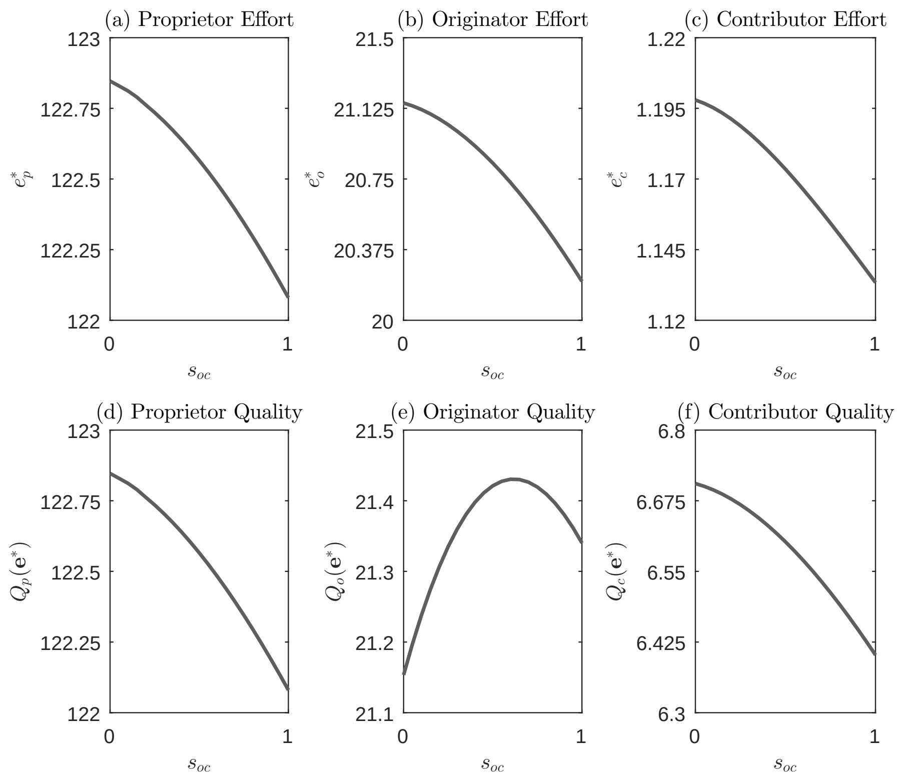

Figure 3: Equilibrium effort choices and resulting quality levels under Region L as affected by changes in s_{oc}. The parameter values are: $s_p = 1$, $s_o = 1$, $s_c = 0.3$, $s_{co} = 0.3$, $V_c = 0.01$, $c = 0.01$, $\beta_p = 0.002$, $\beta_o = 0.002$, and $\beta_c = 0.002$.

the proprietor who has strategically selected a lower position in the quality space. In this situation, the contributor also no longer has incentives to contribute heavily in order to push the originator further up in the quality space; indeed, the contributor realizes in equilibrium the originator will strategically not allow it.

In Figure 3, we show how the equilibrium efforts are impacted by s_{oc} and the consequence on the total quality brought to the market. Consistent with part (i) of Proposition 3, all three providers reduce their equilibrium effort investments as s_{oc} increases which is illustrated in panels (a)-(c). The originator takes a balanced approach toward managing the total quality of its joint offering. As s_{oc} initially increases, it leverages to a greater degree the contributor's effort while scaling back its own effort which is shown in panel (b) of Figure 3. Even though the contributor reduces its effort in response, the total contribution to the originator's quality from the contributor (i.e., $s_{oc}e_c^*$) still leads to a net increase in the quality of the originator's offering which is illustrated in the left-hand side of panel (e).[30] However, as s_{oc} increases to the more restrictive end of the spectrum, the proprietor is more concerned with the ability of the originator to leverage the contributor. As a result, the proprietor moves down sharply in the quality space, which is reflected in panels (a) and (d). Given the proprietor's behavior, the originator also reduces its effort more sharply to lower the quality of its offering and avoid head-to-head competition with the proprietor by creating distance in the quality space from above. In turn, the contributor's quality is also more sharply affected negatively as seen in panel (f) of Figure 3 which also serves to alleviate price competition in the market for services.

Part (iii) of Proposition 3 highlights an important insight into the role open-source licensing should play in software markets. In particular, although some in the open-source community often advocate restrictive licenses such as GPL, such licensing can sometimes be detrimental to both consumer surplus and social welfare. Specifically, in markets characterized by software providers with similar development capabilities, a restrictive license can amplify the incentive conflicts. Such a license makes OSS a greater threat to the proprietor due to how the collaborative nature of OSS development can potentially benefit from such licensing. Because of these concerns, the proprietor strategically brings to market a purposeful lower quality offering to force the open-source originator into a position where it prefers to limit the synergies stemming from the open-source cross effort effects. In these circumstances, it is important for less restrictive licenses (such as BSD) to be encouraged and supported by open-source communities. This insight will extend in a natural way to other OSS governance controls including developer accreditation, code acceptance controls, and documentation which can positively affect cross effort-to-quality parameters and ultimately exacerbate the same incentive conflict.

Contrasting this implication with Proposition 2, we see that more restrictive licenses seem to benefit society the most in markets where the characteristics among the providers exhibit significant dispersion. For example, this might occur when the proprietary firm is a clear and dominant leader such as in the case of Salesforce, SugarCRM and Synolia where the open-source firms may face greater resource constraints and lower cost efficiencies.

[30] In Section A.1 in the Appendix, we numerically illustrate the boundary between Region H and L as it relates to the differential impact of s_{oc} on the originator's quality. This section demonstrates the breadth of applicability of our results from Propositions 1 and 3 in terms of firm cost efficiencies.

5.3.2 Open Source Quality Leadership

Particular to Region L, it can even be the case that an OSS provider offers the highest total quality solution in the market despite the proprietor possessing a first-mover advantage. As is discussed in Section 5.1, there are some factors on which OSS maintains a relative advantage such as on flexibility and customizability. This advantage gets reflected in the effort-to-quality parameters, and if it is sufficiently strong, OSS quality leadership can emerge. The ability of OSS providers to integrate systems tailored to meet customers' idiosyncratic preferences can be highly valued in enterprise software markets. Parametrically, we capture greater flexibility with higher values for s_o, s_c, s_{co} and s_{oc}. For the last parameter, because our propositions are comparative statics on s_{oc} anyway, an increase in flexibility can be interpreted as an increased lower bound on the base level of s_{oc} above which the comparative statics apply. More generally, our formal results provide the more complete comparative statics results on s_{oc} and thus inform both cases where the base level of s_{oc} is marginally and heavily impacted by increased flexibility.

We study two cases that lead to OSS quality leadership. First, we examine a case where the impact of flexibility is limited but sufficient for the originator to become the quality leader in equilibrium. Second, we examine a case where the impact of flexibility is extensive and leads to both the originator and contributor ultimately offering higher total quality solutions than the proprietor. We begin with the first case. It is important to note that because the proprietor possesses a first mover advantage, it can assume quality leadership with a significant investment that effectively deters OSS providers. However, we establish in part (ii) of Lemma A.4 in the online appendix that it is not in the proprietor's best interest to do so, instead ceding that role to the originator. With the originator as the quality leader in equilibrium, the impact of additional gains in s_{oc} due to increased license restrictiveness is significantly different from what we characterized in Proposition 3. This is formalized in the following proposition.

Proposition 4 *Under the conditions of Region L, if the originator has a relatively high effort-to-quality effect (s_o) but the contributor's own effort-to-quality effect (s_c) and cross effect from the originator (s_{co}) remain limited, then as s_{oc} increases:*

(i) The proprietor's effort e_p^* and contributor's effort e_c^* both increase in equilibrium. However, the originator's effort e_o^* increases and then decreases in s_{oc}. Technically, there exists $\hat{s}_{oc} \in$

$(0, \bar{s}_{oc})$, such that $\frac{de_o^*}{ds_{oc}} > 0$ for $s_{oc} \in (0, \hat{s}_{oc})$ and $\frac{de_o^*}{ds_{oc}} < 0$ for $s_{oc} \in (\hat{s}_{oc}, \bar{s}_{oc})$;

(ii) The proprietor's quality $Q_p(\mathbf{e}^*)$ and originator's quality $Q_o(\mathbf{e}^*)$ both increase. However, the contributor's quality $Q_c(\mathbf{e}^*)$ increases and then decreases in s_{oc}. Technically, there exists $\hat{s}_{oc} \in (0, \bar{s}_{oc})$, such that $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} > 0$ for $s_{oc} \in (0, \hat{s}_{oc})$ and $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} < 0$ for $s_{oc} \in (\hat{s}_{oc}, \bar{s}_{oc})$.[31]

(iii) Both consumer surplus and social welfare increase.

Previously we showed that when the proprietor is the quality leader, an increase in s_{oc} decreases the investment incentives for all three providers. In contrast, once the originator's effort-to-quality effect becomes relatively stronger (in comparison to the proprietor), in equilibrium the originator makes greater investments and becomes the quality leader in the market. Once an OSS originator governs the top slot, the economic effects of licensing necessarily change. Specifically, the originator is no longer concerned about increased license restrictiveness causing its quality to move upward and too close to a proprietary offering that is the quality leader. By strategically investing to become the quality leader, these shackles are removed and the originator can once again benefit significantly from the positive impact of increased license restrictiveness on s_{oc}. In Proposition 4, we establish that the originator's total quality of solution once again increases in s_{oc}. Moreover, the proprietor also increases its effort and hence quality due to the relaxed pressure from above. The contributor also invests more as s_{oc} increases but the total quality of its offering hinges on the extent to which the originator substitutes the contributor's efforts for its own. In the upper range of s_{oc}, the originator's incentive to substitute efforts is quite high and ultimately leads to a reduction in the quality of the contributor's offering; this offering suffers from the originator's reduced investments via the cross-quality effect, $s_{co}e_o^*$.[32] Because increased license restrictiveness largely leads to increased investments and better quality offerings, the net impact of consumer surplus and social welfare is positive, which is reminiscent of our findings in Region H.

For the second case, we examine a scenario with a more extensive impact on the effort-to-quality parameters of the OSS participants. In particular, we look at a case where flexibility is highly valued in the market, or when other factors such as modular architecture or development maturity greatly enhance both OSS providers' effort-to-quality parameters. This leads to a much

[31] $\bar{s}_{oc}$ is formally defined in the online appendix.

[32] Notably, as we discuss next, if the impact of higher flexibility on s_{co} were sufficiently strong, the net impact on the contributor's total quality would instead be positive.

stronger contributor, and we formally establish in part (iii) of Lemma A.4 that the equilibrium is characterized as follows: the originator is the quality leader, the contributor now emerges to serve the middle tier of the market, and the proprietor serves the lower tier of the market. This outcome is consistent with a conceptualization of OSS flexibility being highly valued by the market and reflected by effective integration and the total quality provided by OSS participants. The following proposition formalizes the impact of additional gains in s_{oc} due to increased license restrictiveness in this case.

Proposition 5 *Under the conditions of Region L, if the originator has a relatively high effort-to-quality effect (s_o) and the contributor's own effort-to-quality effect (s_c) and cross effect from the originator (s_{co}) become moderate, then as s_{oc} increases:*

- *(i) The proprietor's effort e_p^* and contributor's effort e_c^* both increase in equilibrium.*
- *(ii) There exists $\bar{k}_c > 0$ such that if $\hat{k}_c \geq \bar{k}_c$, then the originator's effort e_o^* decreases in s_{oc}; If $\hat{k}_c < \bar{k}_c$, then the originator's effort e_o^* increases in s_{oc}.*[33]
- *(iii) The proprietor's quality $Q_p(\mathbf{e}^*)$, originator's quality $Q_o(\mathbf{e}^*)$, and contributor's quality $Q_c(\mathbf{e}^*)$ all increase.*
- *(iv) Both consumer surplus and social welfare increase.*

By contrasting Propositions 4 and 5, we see how the strength of an underlying primitive construct favoring OSS such as flexibility, modularity, and maturity can lead to different economic effects. In particular, the strategic interactions between the OSS participants and their interplay with the proprietor's incentives have been altered to some extent. In both cases, the originator largely benefits from an increase in the effort-to-quality cross effect from the contributor (s_{oc}). However, in the latter case, the originator must be more cognizant of the fact that the contributor has become inherently a stronger competitor in such a market. While the originator continues to leverage the contributor toward increasing its own quality (via substituted effort), part (ii) of Proposition 5 demonstrates how the originator strategically must increase its own effort as s_{oc} rises if the contributor is more efficient.

[33]Region L is defined in Lemma A.4 where $\beta_c = \hat{k}_c b$. $\hat{k}_c$ therefore captures the cost efficiency of the contributor. $\bar{k}_c$ is also formally defined in the online appendix.

This behavior highlights the difference between having OSS participants adjacent to one another in the quality space versus having their interactions intermediated when the proprietor serves the middle tier as in Proposition 4. When adjacent, the competitive side of co-opetition becomes much more dominant. In particular, a more efficient contributor (i.e., smaller $\hat{k}_c$) has a strong incentive to exert effort to drive up its own quality and be more competitive in the services market. With adjacency, an increase in s_{oc} further improves the return on effort because not only is its own quality improved but it also helps to push the originator further up in the quality space, essentially making it easier for it to extract surplus from the middle tier. A stronger contributor who exerts greater effort is an even bigger threat to the originator (notwithstanding the associated cross-effort benefits), and therefore the originator must also increase its own efforts to retain quality leadership and preserve a sufficient quality gap to maximize profitability. On the other hand, when the proprietor intermediates competition by serving the middle tier, the collaborative effects of OSS get muted. In this case, the benefit from any boost to the originator's quality stemming from increased contributor efforts greatly depends on the proprietor's reaction; its quality sits between the two competitors and essentially binds the contributor's behavior. Because the proprietor's quality is unaffected by contributor efforts, the contributor cannot create the same quality spacing. Therefore, it increases effort to a more limited extent which again enables substitution by the originator (as in Proposition 5).

5.4 Impact of the Value of the Contributor's Outside Option

In Section 3, we modeled the contributor as having an outside option value of V_c. For our main results, we focused on a region where V_c is not too high such that the contributor elects to invest in the OSS project in equilibrium; this region is perhaps most pertinent to the research questions being posed as we aim to better understand the strategic interactions underlying equilibria where all providers are in the market, which is an outcome frequently observed in practice. In this section, we also examine the impact of having a larger outside option, which can then become the contributor's preferred choice in equilibrium. We compare and contrast outcomes in this case with those characterized earlier under a smaller outside option value.

First, for Region H, we establish in Lemma A.3 that for a constant outside option value V_c, as long as the providers are sufficiently cost efficient, the resulting dispersion in the equilibrium quality levels enables the contributor's profits to be higher when involved in the OSS project; in this case, it can leverage both its expertise and better quality separation to drive larger revenues in the OSS domain. On the other hand, for Region L, the increased competitiveness that stems from greater similarity in providers' cost characteristics makes it more difficult for the contributor to generate revenues. As is shown in Lemma A.4, when the outside option becomes sufficiently large (i.e., satisfying $V_c > \bar{V}$), the contributor prefers that option to becoming involved with the OSS project. Thus, we use Region L where the proprietor's effort-to-quality effect (s_p) is relatively high to explore the impact of varying the value of the outside option. In the following proposition, we compare equilibrium outcomes for two cases: under a small outside option V_c^{in}, and under a large outside option V_c^{out} such that the contributor is in and out of the OSS market, respectively.

Proposition 6 *Under the conditions of Region L, if the proprietor's effort-to-quality effect (s_p) is relatively high, there exists $\underline{V}$ and $\bar{V}$ such that if $V_c^{in} < \underline{V}$ and $V_c^{out} > \bar{V}$, then relative to the case of the contributor being in the market (i.e., under V_c^{in}), when the contributor is out of the market (i.e., under V_c^{out}),*

(i) The originator exerts higher effort in equilibrium (i.e., $e_o^{out} > e_o^{in}$);

(ii) If the contributor is relatively cost efficient and the restrictiveness of the license is within an intermediate range, then both the originator's equilibrium quality and total consumer surplus are lower (i.e., $Q_o^{out} < Q_o^{in}$ and $CS^{out} < CS^{in}$); otherwise both the originator's quality and total consumer surplus are higher.

Proposition 6 provides insights into how the attractiveness of outside options for the contributor affects the originator's incentives to invest in OSS projects as well as the quality of software offerings that emerge in these enterprise software markets (and thus market competitiveness). Part (i) establishes that the originator essentially has to make greater effort investments in equilibrium when the contributor is no longer involved in the OSS project. This behavior can be expected as the originator now needs to at least partially offset the lost investments that would have been made by a strategic contributor.

What is more interesting is the extent to which the originator increases its investment. This is made clear in part (ii) of Proposition 6 where we establish that the quality offered by the originator

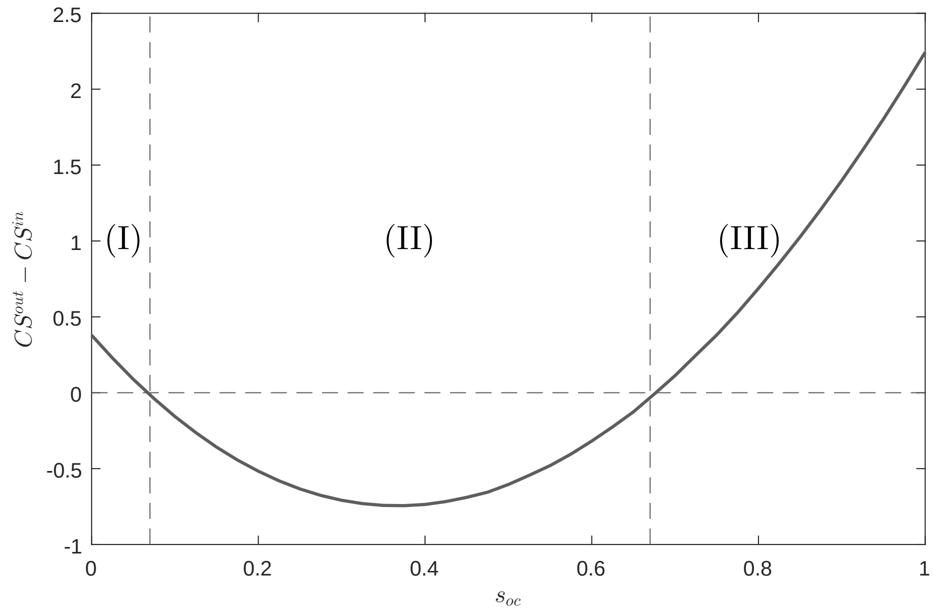

Figure 4: Comparing the difference in consumer surplus when the contributor is involved in the OSS project and when the contributor instead elects for its outside alternative. The parameter values are: $s_p = 1,\ s_o = 1,\ s_c = 0.3,\ s_{co} = 0.23,\ V_c^{in} = 0.01,\ V_c^{out} = 10,\ \beta_p = 0.00005,\ \beta_o = 0.00005,$ $\beta_c = 0.00005$, and $c = 0.001$.

can actually *increase*. It is not the case that the quality of its software solution necessarily goes down in equilibrium, as there exist distinct trade-offs. On one hand, the absence of a strategic contributor makes it more costly for the originator to improve the quality of its offering. On the other hand, the absence of a strategic contributor also relaxes competition in the services market for the premium offerings (i.e., ones offered by firms who have developed significant expertise resulting from their activities with the OSS project).

We demonstrate that the total quality of the originator's offering in the absence of a strategic contributor is only lower if both the contributor is relatively cost efficient and the licensing parameter is moderately restrictive. This set of conditions creates an incentive for the contributor to make a moderate contribution to the OSS project that is beneficial to the originator while not being so large that the proprietor aggressively reduces its own quality, thereby causing increased competition. As we discussed earlier in describing the effects seen in panel (e) of Figure 3, the relationship between the originator's total quality and license restrictiveness is non-monotonic. As the license moves from permissive to moderately restrictive, the originator's quality *increases*. However, as the license moves from moderately restrictive to very restrictive, the originator's quality *decreases*. Therefore, whether the license is permissive or very restrictive (i.e., at either end), the originator is incentivized to adjust its own effort to maintain a relatively lower quality offering. In this sense, the absence of a strategic contributor and the associated relaxation of price competition in the market for services can lead the originator to offer a higher quality offering than would otherwise be offered. Moreover, when the improvement in quality is substantial, the total consumer surplus in the market is also higher. The impact on consumer surplus is depicted by the areas labeled (I) and (III) in Figure 4. However, for a moderate level of license restrictiveness, the equilibrium contributions from the originator and contributor lead to a higher total quality offering by the contributor, one that it would choose not to match if forced to provide such a substantial investment independently. In a similar vein, consumer surplus can be relatively lower when the market loses the contributor's investments. The area labeled (II) of Figure 4 illustrates this fact.

6 Discussion and Conclusions

In this paper, we study competition among open-source and proprietary software firms while considering the strategic interplay of open-source contributors who also invest in the development of OSS and compete in the services market. Although prior work has mostly focused on a mixed-model of duopolistic competition, the existence of strategic contributors who vie for service revenues can significantly alter competitive forces. We add to the literature by analyzing a model that includes all three strategic players competing in the market and explores the important role that OSS governance controls such as licensing plays in moderating strategic contributions. In particular, because the open-source originator and contributor both collaborate toward developing the OSS product and compete against each other and the proprietary firm, the economic incentives associated with investment in OSS vary substantially depending on the license governing it as well as the market conditions. For example, the originator may prefer to leverage the contributor to compete more fiercely with the proprietor, while the contributor faces its own dilemma whether to exert more or less investment in contributing to the OSS project (i.e., more effort will contribute to quality yet allow the originator to reduce its own effort). The degree of license restrictiveness is a critical attribute that can either amplify or diminish such incentives.

Our model demonstrates that increased license restrictiveness often leads to preferable market outcomes. As long as the proprietor's behavior does not inordinately and adversely impact OSS development incentives, then restrictiveness tends to be helpful. Under high cost dispersion and a quality-leading proprietor as well as low cost dispersion and a quality-leading OSS originator, the proprietor does not possess incentives to strategically impede OSS development. Whether being a distal leader or a resigned low-to-mid tier provider, the proprietor essentially permits increases in license restrictiveness (such as with GPL) to positively affect OSS participants by increasing quality differentiation and relaxing price competition. This generally leads to higher contributor efforts in equilibrium which results in higher consumer surplus and social welfare. An originator may begin to increase free-riding behavior as license restrictiveness becomes severe. However, in a more competitive environment characterized by low cost dispersion, a quality-leading proprietor has incentives to scale back efforts and effectively impede OSS development under the threat of increased license restrictiveness. The proprietor's strategic behavior yields lower equilibrium efforts, consumer surplus and social welfare, and therefore less restrictive licenses (such as BSD) would be more beneficial in these more competitive environments.

6.1 Managerial and Policy Implications

Our results have important implications for OSS firms' licensing strategies. When adopting an open-source business model, an OSS firm should carefully consider its licensing strategy because the role that license restrictiveness plays changes significantly depending on market conditions. First, when the firms competing in a particular market (i.e., proprietor, OSS originator and OSS contributor) have disparate capabilities that lie in favor of the proprietor, more restrictive licenses are quite beneficial. In this case, when facing a more restrictive license (such as GPL), the contributor is motivated to invest greater effort which pushes the originator's offering higher in the quality space (and closer to the proprietor) enabling the contributor to be more profitable while servicing the lower tier of the market. On the other hand, when the firms have more homogeneous development capabilities but still in favor of the proprietary firm, more permissive licenses (such as BSD) are beneficial. In this case, the proprietor views OSS offerings as a threat and strategically limits the originator's ability to leverage the stronger complementarities that stem from restrictive licenses by occupying a position in the quality space that serves as a deterrent. Lastly, in markets that highly value the flexibility associated with OSS, open-source providers can actually become quality leaders. In this case, more restrictive licenses are beneficial once again as the proprietor loses the ability to strategically limit OSS quality. Instead, the OSS firm can bring to market a higher quality service, charge a higher price, and benefit consumers.

Connected to our main findings on the role of licensing restrictiveness, OSS firms can also influence the cross effect of the contributor's effort on the originator's total quality (s_{oc}) using a variety of instruments. First, as part of the governance of its OSS project, the originator can impose stricter selection criteria for developer accreditation as well as higher quality standards for code acceptance. Both of these actions would positively affect the quality associated with the contributor's effort, and as we discussed above, would benefit the originator in markets characterized by (i) high quality dispersion in favor of the proprietor, and (ii) low cost dispersion in favor of the OSS originator. In these two cases, the positive impact to the OSS originator's quality would ultimately be more profitable. A second instrument with a similar net impact is for the OSS originator to require higher development process maturity from its partners (e.g., resellers and systems integrators) who are often contributors to the OSS project.

The OSS originator can also influence the cross effect of its own effort on the contributor's total quality (s_{co}) via the underlying primitive factors. For example, a more modular software architecture would enable the contributor to better understand the structure of the OSS project's source code and more easily make changes. As we formally discuss in Section A.3 of the Appendix, a modular design enhances OSS service qualities as long as, overall, the originator can sufficiently benefit from the contributor's effort to maintain a leading, quality differential. This outcome can often be the case as the originator is the designer of the software. Also, improved documentation has a similar impact in that it enhances the contributor's comprehension of the OSS project and code. In markets that highly value the flexibility of OSS, the originator may want to make documentation a priority provided its own efforts can assure its position as quality leader. In this case, better documentation can bring forth a more impactful contributor who significantly drives up quality and emerges to serve the middle tier of the market (rather than the lower tier). Specifically, the contributor has incentives to boost the originator's quality to create more space for itself to serve the middle tier; the net result being higher qualities of both OSS originator and contributor offerings.

Our results provide important insights and implications to policy makers. We show that the impact of OSS license restrictiveness (s_{oc}) on consumer surplus and social welfare depends heavily on market conditions. When firms have homogeneous development capabilities that lie in slight favor of the proprietary firm, more permissive licenses (such as BSD) can lead to higher consumer surplus and social welfare. Thus, in software markets that are more competitive in nature, policy makers may want to encourage the adoption of permissive licenses. In fact, the government can exercise leadership through its own OSS adoption decisions by preferring the use of products with BSD, MIT, and Apache style licenses when such options exist and are on par with other alternatives in terms of quality. This recommendation highlights an important insight into the role of OSS licensing in software markets. Although some in the OSS community might universally advocate restrictive licenses such as GPL which have copyleft characteristics, the *right* license to employ necessarily needs to be more discerning. As we have seen, GPL-type license restrictiveness tends to amplify the incentive conflicts in competitive markets where OSS firms are strategically vying for service revenue, which ultimately becomes detrimental to both consumer surplus and social welfare. However, policy makers should be more supportive of GPL licenses in less competitive software markets as they can be quite helpful.

Policy makers may also want to positively influence OSS firms' positions on primitive factors such as a more modular architecture and higher maturity of software development processes. For example, in software markets where customization and flexibility will be highly valued, there are immense social benefits to having OSS providers become the quality leaders who bring very high quality and open offerings to the market. For example, container orchestration platforms like Kubernetes, which cloud computing service providers use to manage containers such as Docker, may fit the bill quite well. Because enterprises have heterogeneous demands, it is important to produce modules that are optional and pluggable. In software markets like this, it may be helpful for policy makers to encourage standards that favor modular and inter-operable designs as well as development practices that are particularly rigorous in that the bugs and security vulnerabilities that arise are at the infrastructure level. Moreover, in markets where OSS has an opportunity to become the leading quality solution, the government may even consider subsidizing documentation efforts in order to springboard these outcomes. Because these markets value customizability, better documentation by OSS originators will feed forward to provide incentives for OSS contributors to also exert higher efforts, which in aggregate will yield the desirable, high quality offerings that benefit consumers. Our model cautions that if the cross effect from the originator to the contributor (s_{co}) is too high, the originator may dramatically scale back its effort and shift to freeriding. Therefore, the incentives described above will help but need to be properly calibrated.

6.2 Future Directions

While the abstractions we made are fitting to answer our primary research questions and reasonable, they are not without limitation. We employ a model of a single-shot game in which a proprietor invests first in bringing its offering to market, followed by an OSS originator, followed by an OSS contributor. Within our framework, there are three notable simplifications and chosen areas of focus worth discussing: (i) dynamics, (ii) sequencing, and (iii) parameter regimes. First, the focus of our work lies on explaining how a proprietor adjusts its effort investments in the face of entry of an OSS originator and possibly OSS contributor and how license restrictiveness impacts the incentives of all players. These initial investments to create original software, integration, and support solutions are quite substantial, sticky and determine the state of the market for an extended amount of time. However, once these offerings have been produced, all firms naturally then continue to innovate and release improved versions which reflects that these markets certainly also have a dynamic nature. Although subsequent dynamics are beyond the scope of our study, it would be interesting to explore the impact of upgrade cycle lengths, switching costs, pricing models (SaaS vs. on-premises) and other factors relevant to market dynamics on the early market strategic interactions that we examine in this work.

Second, as motivated and discussed in Section 4.3, in many enterprise software markets, it is typical for a proprietor to emerge first, followed by an OSS originator, and then potentially a strategic contributor who becomes involved with the OSS project. Our assumption on effort investment sequencing matches these industry observations. However, there are certainly other software markets that exhibit different sequencing. For example, it can be the case that a leading OSS provider arises first in certain markets. This is not commonly the case for enterprise software markets driven by services (OSSg2), but it may unfold in commercial business models centered on other revenue sources such as advertisements and even non-profit business models focused on altruistic and other intrinsic motivations underlying OSS participants. It would be a worthwhile endeavor to study OSS and proprietary competition in these markets which are governed by incentives different than those explored in our paper. For example, one interesting direction for future research would be to study the role of uncertainty in these markets in that the incentives for strategic contributors to become involved are greatly affected by the risk of OSS project failure. Also, with the pervasive use of cloud computing in recent years, OSS has become even more relevant to proprietary firms operating as cloud service providers. Amazon built its successful Amazon Web Services with the help of many OSS products. Other proprietary firms have also increased their participation in OSS, as evidenced by Google's Kubernetes, Microsoft's acquisition of Github, and IBM's acquisition of RedHat. Another interesting direction for future research would be to study the open-source strategies of proprietary firms.

Third, within our modeling framework, we focus on particular parameter regions of interest that give rise to equilibria commonly observed in enterprise software markets. Because of the generality of the model, it is certainly possible to examine other parameter regions that lead to different orderings in the equilibrium qualities that emerge. Our model can be leveraged to study contexts such as those with cross effort-to-quality parameters that are relatively high, thus falling outside our focal area specified in Assumption 1. It is also feasible to formally study comparative statics in the originator to contributor cross effect (i.e., s_{co}) as well as contexts where s_{oc} and s_{co} are more tightly coupled. For the latter, we have shown that as long as the primary impact of licensing changes lies on s_{oc}, then our results are fairly robust to s_{co} also being partially impacted. This analysis can be found in Section A.3 of the Appendix. However if the impact on s_{co} becomes comparable or even greater, then new analysis is required. We believe a complete study into the role of s_{co} (as influenced by a particular primitive construct whose primary impact lies on s_{co}) would be an interesting direction for future research. In that the sequencing in our game gives the originator a first-mover advantage, movements in s_{co} may lead to different strategic behavior than characterized in our work.

Finally, it would also be interesting to explore endogenizing the cross quality effect, s_{oc}. Frictions exist that can sometimes limit an originator's ability to choose s_{oc} (for example, the viral nature of GPL for an OSS product attempting to leverage other GPL-licensed libraries). However, originators ultimately have some degree of freedom in choosing licenses. While s_{oc} is a parameter in our model, throughout our paper we provide some insights into what an originator might prefer in terms of s_{oc}, based on computed equilibrium measures, to provide the reader with some intuition into license choice. However, fully endogenizing s_{oc} would be quite interesting and lead to new interesting insights stemming from the originator's selection between effort and restrictiveness. Given the complexity of the current study, we believe this would be an ambitious task although significant progress could be made numerically leveraging the model we have put forth.

6.3 Concluding Remarks

Open-source software has become a mainstay for businesses as they compete in dynamic environments that reward flexibility and agility. Provision of value-added services is critical in this context and is an essential aspect of the commercial OSS business model. By developing a better understanding of competition among open-source contributors and proprietary firms, we aim to provide both organizations and policy makers with insights into how licensing can affect market outcomes. These insights can help guide software firms as they determine the appropriate strategy to participate in OSS and influence policy makers as they examine regulations that govern the intellectual property rights associated with OSS. We hope the work reported in this paper will help stimulate more research efforts in this growing area.

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Appendix A - Ranges of Applicability for Primary Results

A.1 Separation between Regions H and L

In Sections 5.2 and 5.3, we demonstrate the differential impact that open-source licensing has with changing market characteristics. In particular, we vary the extent to which providers' cost efficiencies are dispersed. The beginning of Section 5 provides a discussion of why firm cost efficiencies can vary and the underlying nature of Regions H and L (and the sub-regions). In this section, we illustrate how the equilibrium quality of the OSS originator responds to increasing license restrictiveness over the space of cost efficiencies. This illustration is numerical in nature, but we hope it can help give the reader a more concrete depiction of the focal regions examined in this study.

In panel (a) of Figure A.1, we use a region plot to delineate the portions of the parameter space upon which our results and insights for Regions H and L (for the sub-region where the proprietor has a larger effort-to-quality effect, i.e., higher s_p) remain valid as they relate to Propositions 1 and 3. A primary insight in our paper is that increased license restrictiveness will tend to reduce the quality of the OSS originator's software solution (Q_o) due to strategic interactions if there is low dispersion in cost efficiencies of the providers. The x-axis in Figure A.1 represents β_o as a proportion of β_c, whereas the y-axis represents β_p as a proportion of β_o. Thus, the plot illustrates the region where the proprietor is the most cost efficient, followed by the originator, followed by the contributor.

The gray shaded area, labeled (I), depicts the range of β_p and β_o relative to β_c where Q_o strictly increases in license restrictiveness which we formally established in Proposition 1 as being the equilibrium outcome in Region H. In particular, when the cost dispersion is relatively high (β_p relatively small compared to β_o, which in turn is relatively small compared to β_c), the equilibrium outcomes are consistent with those proven to unfold under Region H. On the other hand, when the differences between the cost efficiencies decrease (β_p becomes closer to β_o, which in turn is closer to β_c), the area labeled (II) in panel (a) of Figure A.1 demonstrates that Q_o initially increases and then decreases in license restrictiveness. We analytically showed this to be the nature of the equilibrium outcome in Proposition 3 in Region L, pointing to the negative impact such license restrictiveness can have on consumer surplus and social welfare.

In panel (b) of Figure A.1, we further illustrate how the parameter space relates to our regions

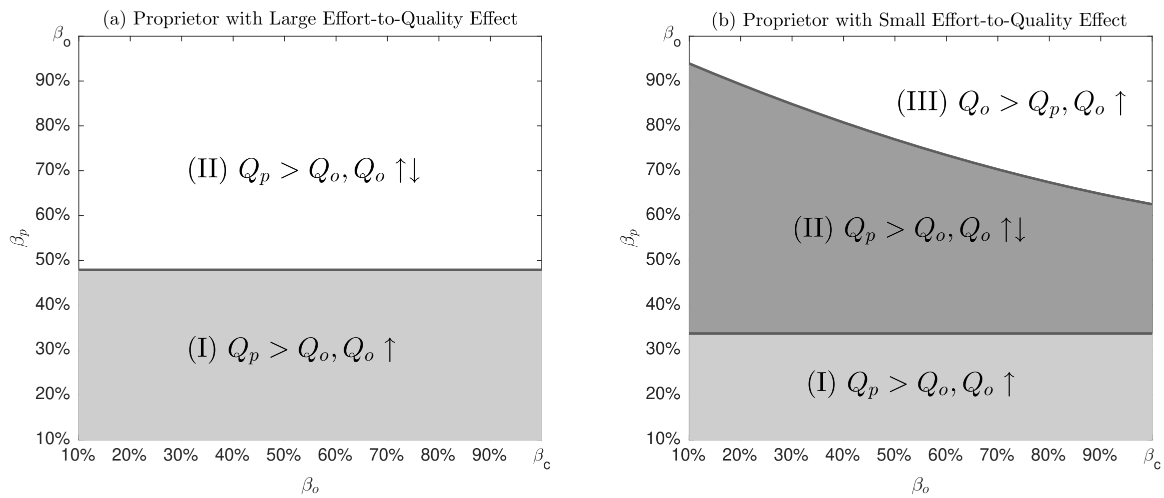

Figure A.1: The impact of increasing license restrictiveness on the OSS originator's equilibrium quality as determined by cost dispersion among providers. The originator's cost parameter β_o ranges from 10% to 100% of $\beta_c = 0.001$, the proprietor's cost parameter β_p ranges from 10% to 100% of β_o, and license restrictiveness s_{oc} is evaluated from 0 to 1. Panels (a) and (b) depict scenarios where the proprietor has relatively larger ($s_p = 1$) and smaller ($s_p = 0.8$) effort-to-quality effects, respectively. The other parameter values are: $s_o = 1$, $s_c = 0.1$, $s_{co} = 0.2$, $V_c = 0.001$, and $c = 0.001$.

when the originator has a relatively high effort-to-quality effect (s_o). In this plot, a third area labeled (III) emerges. Sub-region (III) depicts the range of cost parameters where the originator becomes the quality leader ($Q_o > Q_p$) and Q_o strictly increases in license restrictiveness, which we formally established in Proposition 4 as being the equilibrium outcome in Region L when the originator has a relatively high effort-to-quality effect (s_o) but the contributor's own effort-to-quality effect (s_c) and cross effect from the originator (s_{co}) remain limited. Another noteworthy feature from panel (b) is that sub-region (III) becomes larger as the cost efficiency parameters compress (technically, β_p and β_o become closer to β_c), which is illustrated by the top right-hand side of panel (b). As compression occurs (and hence less cost dispersion), the proprietor feels greater pressure to relinquish quality leadership due to the increased competition as we establish in Proposition 4.

Overall, although the characterization of Regions H and L (and its sub-regions) are constructed to support the asymptotic analysis used in the paper, Figure A.1 demonstrates that nature of the insights we generate extends to wide regions in the parameter space.

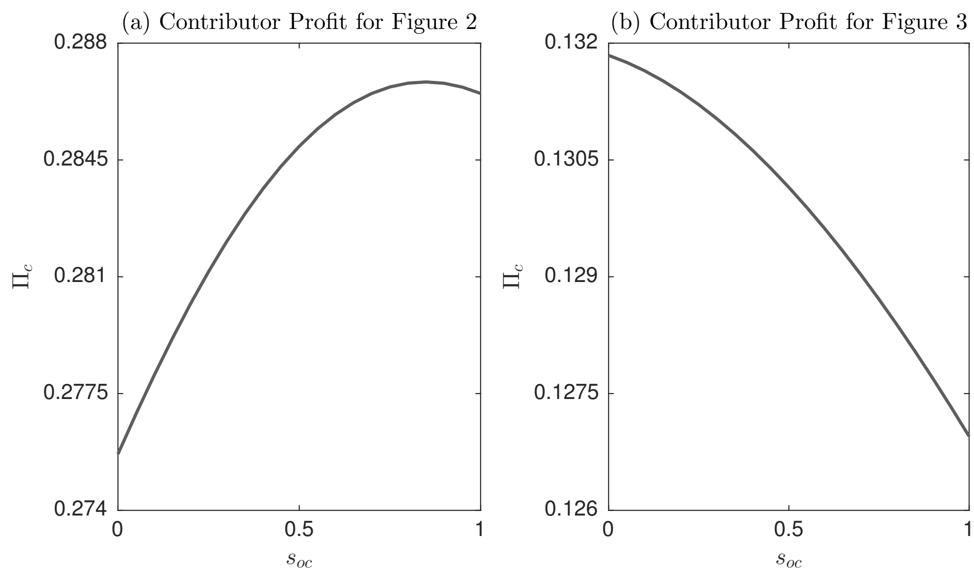

Figure A.2: The impact of license restrictiveness on OSS originator's equilibrium profits.

A.2 Sensitivity of Contributor's Outside Option Values

In support of the numerical illustrations of our primary results, it is useful to examine sensitivity to the value of the contributor's outside option. In particular, how the contributor's equilibrium profits change with respect to s_{oc} is quite relevant. In Figure A.2, we plot the curve of equilibrium profits for the contributor which is essentially an upper bound on the outside option value for the contributor to remain a participant. Panels (a) and (b) depict the contributor's profits in Regions H and L, corresponding to Figures 2 and 3, respectively. As Figure A.2 shows, even through equilibrium profits shift with license restrictiveness, the magnitude of change is relatively small. As long as the value of the outside option is smaller than the lower bounds of the contributor's profits in these regions, the contributor will remain a participant of the OSS project.

A.3 Licensing Impacting Both Cross Effects

In the main text, we focus on moving only a single reduced-form parameter (s_{oc}) with license restrictiveness. However, it is not the case that our results only obtain when moving a single parameter. The major insights presented in the paper tend to hinge at a higher level on the extent of the relative difference between the strength of these parameters holding a cost context fixed. In this section, we show that relaxations can permit a broader impact of a change in license restrictiveness on more system parameters and still provide the same qualitative insights. Specifically, we show the breadth of applicability of our results even if both s_{oc} and s_{co} are affected by license restrictiveness.

Proposition A.1 *Under the conditions of Region H, if both cross effects (s_{oc} and s_{co}) increase, and the increase in the cross effect from the contributor to the originator (s_{oc}) is relatively stronger than the increase in the cross effect from the originator to the contributor (s_{co}), then:*

(i) The proprietor's quality $Q_p(\mathbf{e}^*)$ decreases, whereas both the originator and contributor's qualities, $Q_o(\mathbf{e}^*)$ and $Q_c(\mathbf{e}^*)$, respectively, increase.

(ii) Both consumer surplus (CS) and social welfare (SW) increase.

Proposition A.1 indicates that our main insights on the aggregate equilibrium measures of quality levels, consumer surplus, and social welfare retain the nature of their characterization as in Propositions 1 and 2. As long as the aggregate impact on the parameters as influenced by license restrictiveness maintains the necessary relative quality differentials, our primary insights continue to apply.

To show to what extent key insights continue to hold as we take a broader interpretation of the impact of license restrictiveness, we visualize the region where the claims in Proposition A.1 hold using a specification $s_{co} = J + K \cdot s_{oc}$. Having K be zero is the orthogonal representation used to demonstrate the formal comparative statics on s_{oc} that reside in the paper. Relaxing K to be a positive constant is akin to license restrictiveness impacting both parameters with K representing the strength of the impact of license restrictiveness on s_{co} relative to s_{oc}. In Figures A.3 and A.4, we depict the regions in (J, K) space on which these comparative statics continue to apply under this relaxation of the assumption on orthogonality. As can be seen, the results are quite robust and still obtain under very broad regions on which licensing restrictiveness affects both parameters. Similarly, for Region L, the key insights in Proposition 3 are also robust under broad regions where licensing affects both parameters.[34]

Finally, the main model and equilibrium are sufficiently rich to study many other scenarios that go beyond our study which primarily focuses on license restrictiveness. For example, our numerical results show that when we get outside the robustness regions in Figures A.3 and A.4,

[34] The related figures are omitted for brevity.

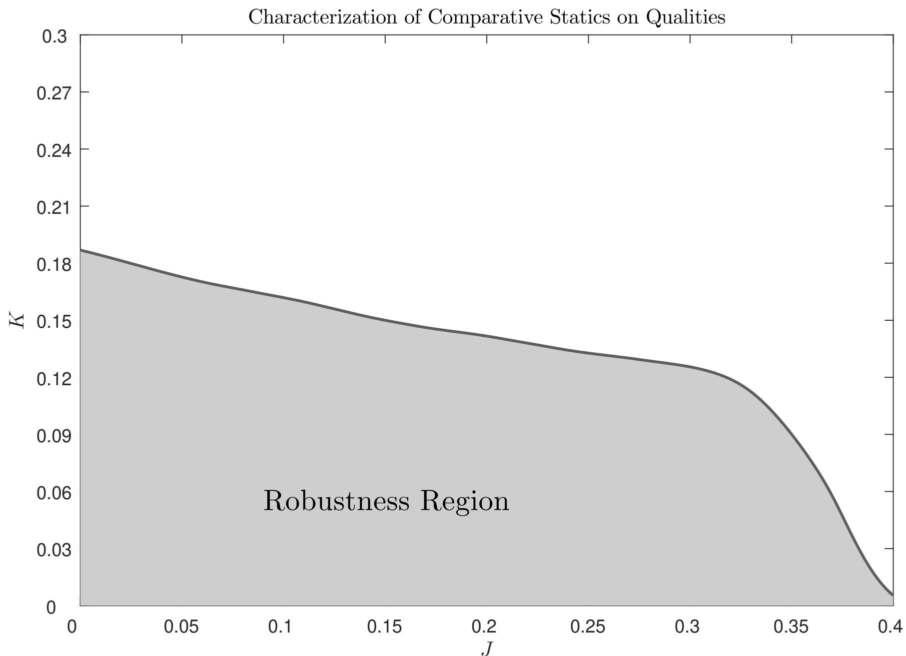

Figure A.3: Robustness of comparative statics results on equilibrium quality levels (i.e., $Q_p(\mathbf{e}^*)$, $Q_o(\mathbf{e}^*)$ and $Q_c(\mathbf{e}^*)$) when licensing can impact both s_{oc} and s_{co}. We specify $s_{co} = J + K \cdot s_{oc}$ and vary over (J, K) space while testing whether each outcome matches that under $J = 0.30$ and $K = 0$ as in Figure 2 and Proposition 1. The other parameter values are: $s_p = 1$, $s_o = 1$, $s_c = 1$, $V_c = 0.01$, $c = 0.01$, $\beta_p = 0.002$, $\beta_o = 0.004$, and $\beta_c = 0.005$, also consistent with Figure 2.

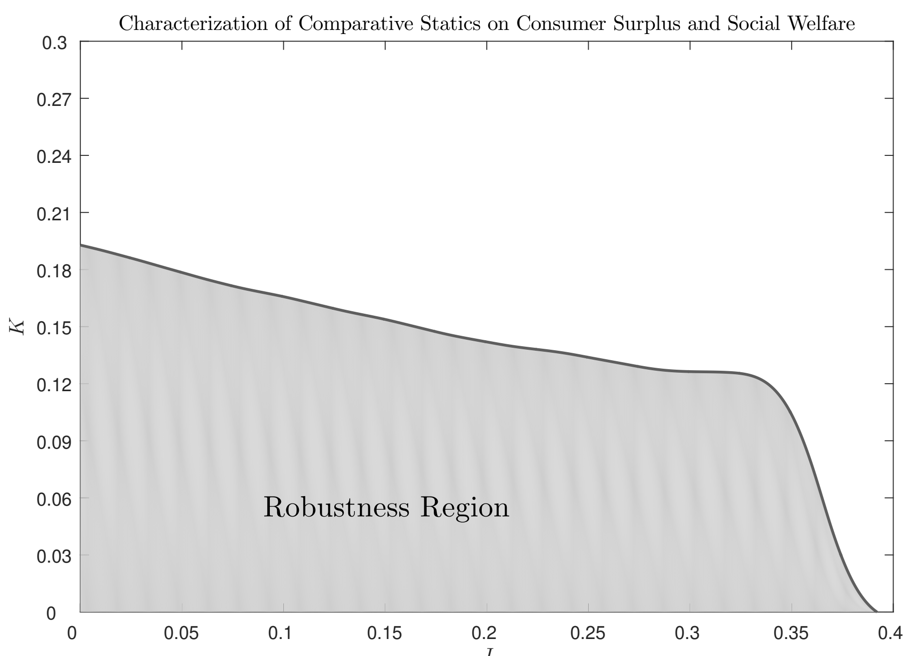

Figure A.4: Robustness of comparative statics results on equilibrium consumer surplus and social welfare levels (i.e., $CS(\mathbf{e}^*)$ and $SW(\mathbf{e}^*)$) when licensing can impact both s_{oc} and s_{co}, using the same specification as in Figure A.3

the equilibrium quality from the originator ($Q_o(\mathbf{e}^*)$) begins to fall because now the contributor can free-ride on the originator's effort to a large extent. Due to the high value of s_{co}, the originator scales back its effort dramatically. Consumer surplus and social welfare both decrease as a result of the originator's change in strategy. Part of the intrinsic value of our model and equilibrium characterization is that it is sufficiently general and flexible enough to examine other scenarios. While the analysis itself can be complex (as can be seen in the proofs), equilibria can be solved numerically for any interesting case one desires to explore; in this sense, one contribution is our model itself that facilitates further study.

Online Supplement for

Competition among Proprietary and Open-Source Software Firms: The Role of Licensing on Strategic Contribution

Terrence August, Wei Chen, Kevin Xiaoguo Zhu

Proofs of Propositions

Lemma A.1 *For fixed prices p_i, p_j, p_k, and qualities $Q_i > Q_j > Q_k$, the consumer market has the following characterization of regions:*

Region I: If $p_i \geq Q_i$, $p_j \geq Q_j$, and $p_k \geq Q_k$, then no consumer uses the software;

Region II: If $p_i < Q_i$, $p_j \geq \frac{p_i Q_j}{Q_i}$, and $p_k \geq \frac{p_i Q_k}{Q_i}$, then only firm i is active in the market, and

(a) consumers with $\theta \in [0,\, \theta_i)$ do not use the software, and

(b) consumers with $\theta \in [\theta_i,\, 1]$ purchase from firm i;

Region III: If $p_i < Q_i$, $p_i - Q_i + Q_j < p_j < \frac{p_i Q_j}{Q_i}$, and $p_k \geq \frac{p_j Q_k}{Q_j}$, then only firms i and j are active in the market, and

(a) consumers with $\theta \in [0,\, \theta_j)$ do not use the software,

(b) consumers with $\theta \in [\theta_j,\, \theta_{ij})$ purchase from firm j, and

(c) consumers with $\theta \in [\theta_{ij},\, 1]$ purchase from firm i;

Region IV: If either (i) $p_i < Q_i$, $p_j \geq \frac{p_i Q_j}{Q_i}$, and $p_i - Q_i + Q_k < p_k < \frac{p_i Q_k}{Q_i}$, or (ii) $p_i < Q_i$, $p_i - Q_i + Q_j < p_j < \frac{p_i Q_j}{Q_i}$, and $p_i - Q_i + Q_k < p_k \leq p_j - \frac{(p_i - p_j)(Q_j - Q_k)}{Q_i - Q_j}$, then only firms i and k are active in the market, and

(a) consumers with $\theta \in [0,\, \theta_k)$ do not use the software,

(b) consumers with $\theta \in [\theta_k,\, \theta_{ik})$ purchase from firm k, and

(c) consumers with $\theta \in [\theta_{ik},\, 1]$ purchase from firm i;

Region V: If $p_i < Q_i$, $p_i - Q_i + Q_j < p_j < \frac{p_i Q_j}{Q_i}$, and $p_j - \frac{(p_i - p_j)(Q_j - Q_k)}{Q_i - Q_j} < p_k < \frac{p_j Q_k}{Q_j}$, then all three firms are active in the market, and

(a) consumers with $\theta \in [0,\, \theta_k)$ do not use the software,

(b) consumers with $\theta \in [\theta_k,\, \theta_{jk})$ purchase from firm k,

(c) consumers with $\theta \in [\theta_{jk},\, \theta_{ij})$ purchase from firm j, and

(d) consumers with $\theta \in [\theta_{ij},\, 1]$ purchase from firm i;

Region VI: If either (i) $p_i \geq Q_i$, $p_j < Q_j$, and $p_k \geq \frac{p_j Q_k}{Q_j}$, or (ii) $p_i < Q_i$, $p_j \leq p_i - Q_i + Q_j$, and $p_k \geq \frac{p_j Q_k}{Q_j}$, then only firm j is active in the market, and

(a) consumers with $\theta \in [0,\, \theta_j)$ do not use the software, and

(b) consumers with $\theta \in [\theta_j,\, 1]$ purchase from firm j;

Region VII: If one of the following holds: (i) $p_i \geq Q_i$, $p_j \geq Q_j$, *and* $p_k < Q_k$*; (ii)* $p_i \geq Q_i$, $p_j < Q_j$, *and* $p_k \leq p_j - Q_j + Q_k$*; (iii)* $p_i < Q_i$, $p_j \geq p_i - Q_i + Q_j$, *and* $p_k < p_i - Q_i + Q_k$*; (iv)* $p_i < Q_i$, $p_j < p_i - Q_i + Q_j$, *and* $p_k < p_j - Q_j + Q_k$, *then only firm* k *is active in the market, and*

(a) consumers with $\theta \in [0,\ \theta_k)$ *do not use the software, and*

(b) consumers with $\theta \in [\theta_k,\ 1]$ *purchase from firm* k*;*

Region VIII: If either (i) $p_i \geq Q_i$, $p_j < Q_j$, *and* $p_j - Q_j + Q_i < p_k < \frac{p_j Q_k}{Q_j}$, *or (ii)* $p_i < Q_i$, $p_j < p_i - Q_i + Q_j$, *and* $p_j - Q_j + Q_k < p_k < \frac{p_j Q_k}{Q_j}$, *then only firms* j *and* k *are active in the market, and*

(a) consumers with $\theta \in [0,\ \theta_k)$ *do not use the software,*

(b) consumers with $\theta \in [\theta_k,\ \theta_{jk})$ *purchase from firm* k*, and*

(c) consumers with $\theta \in [\theta_{jk},\ 1]$ *purchase from firm* j*;*

where $\theta_i = p_i/Q_i$, $\theta_j = p_j/Q_j$, $\theta_k = p_k/Q_k$, $\theta_{ij} = (p_i - p_j)/(Q_i - Q_j)$, $\theta_{ik} = (p_i - p_k)/(Q_i - Q_k)$, *and* $\theta_{jk} = (p_j - p_k)/(Q_j - Q_k)$.

Proof. A consumer with type θ: (a) prefers to purchase from firm i rather than to not use the software if and only if $\theta Q_i - p_i \geq 0$, i.e., $\theta \geq \theta_i$; (b) prefers to purchase from firm j rather than to not use the software if and only if $\theta Q_j - p_j \geq 0$, i.e., $\theta \geq \theta_j$; (c) prefers to purchase from firm k rather than to not use the software if and only if $\theta Q_k - p_k \geq 0$, i.e., $\theta \geq \theta_k$; (d) prefers to purchase from firm i rather than firm j if and only if $\theta Q_i - p_i \geq \theta Q_j - p_j$, i.e., $\theta \geq \theta_{ij}$; (e) prefers to purchase from firm i rather than firm k if and only if $\theta Q_i - p_i \geq \theta Q_k - p_k$, i.e., $\theta \geq \theta_{ik}$; (f) prefers to purchase from firm j rather than firm k if and only if $\theta Q_j - p_j \geq \theta Q_k - p_k$, i.e., $\theta \geq \theta_{jk}$.

By (a), (d), and (e) above and the definition of Θ, $\sigma(\theta) = i$ if and only if

$$\theta \geq t_A \triangleq \min\left(\max\left(\theta_i,\ \theta_{ij},\ \theta_{ik}\right),\ 1\right). \tag{A.1}$$

Similarly, $\sigma(\theta) = j$ if and only if

$$t_B \triangleq \max(\theta_j,\ \theta_{jk}) \leq \theta < t_C \triangleq \min\left(\theta_{ij},\ 1\right), \tag{A.2}$$

and $\sigma(\theta) = k$ if and only if

$$\theta_k \leq \theta < t_D \triangleq \min\left(\theta_{ik},\ \theta_{jk},\ 1\right). \tag{A.3}$$

Finally, $\sigma(\theta) = \emptyset$ if and only if

$$0 \leq \theta < t_E \triangleq \min\left(\theta_i,\ \theta_j,\ \theta_k,\ 1\right). \tag{A.4}$$

To see Region V, first define

$$t_V \triangleq p_j - \frac{(p_i - p_j)(Q_j - Q_k)}{Q_i - Q_j}. \tag{A.5}$$

By (A.1), $t_A = \theta_{ij} < 1$ because $t_V < p_k$ implies $\theta_{ik} < \theta_{ij}$, $p_j < p_i Q_j/Q_i$ implies $\theta_i < \theta_{ij}$, and $p_i - Q_i + Q_j < p_j$ implies $\theta_{ij} < 1$. Hence, $\sigma = i$ for $\theta \geq \theta_{ij}$. Further, because $p_k < p_j Q_k/Q_j$ implies $\theta_{jk} > \theta_j$, we obtain that $t_B = \theta_{jk}$. Because $t_V < p_k$ implies $t_B < t_C = t_A = \theta_{ij}$, $\sigma = j$ for $\theta \in [\theta_{jk},\ \theta_{ij})$. Because $t_V < p_k$ implies $\theta_{ik} > \theta_{jk}$ and $p_k < p_j Q_k/Q_j$ implies $\theta_k < \theta_{jk}$, we obtain that $t_D = t_B = \theta_{jk}$, and $\sigma = k$ for $\theta \in [\theta_k,\ \theta_{jk})$. Finally, $t_E = \theta_k$ because $p_i < Q_i$, $p_j < p_i Q_j/Q_i$, and $p_k < p_j Q_k/Q_j$. Therefore, $\sigma = \emptyset$ for $t \in [0,\ \theta_k)$, which finishes the characterization presented in Region V. The proofs of the remaining regions follow closely with that of Region V. ■

Lemma A.2 (Generalized Statement) *(i) If all three service providers are in the market, and $Q_i > Q_j > Q_k > c$ where $i, j, k \in \{p, o, c\}$ and $i \neq j \neq k$, there exist threshold values $0 < \tau_A < \tau_B < \tau_C < \tau_D$ such that*

Region (i): If $c < \tau_A \triangleq \frac{Q_k(Q_i - Q_j)}{4Q_i - Q_j - 3Q_k}$, then

$$p_i^* = \frac{(Q_i - Q_j)\left(Q_i\left(4Q_j - Q_k\right) - 3Q_jQ_k\right) + c\left(Q_i\left(7Q_j - Q_k\right) - Q_j\left(Q_j + 5Q_k\right)\right)}{2\left(Q_i\left(4Q_j - Q_k\right) - Q_j\left(Q_j + 2Q_k\right)\right)}, \tag{A.6}$$

$$p_j^* = \frac{(Q_i - Q_j)\, Q_j\left(Q_j - Q_k\right) + 3cQ_j\left(Q_i - Q_k\right)}{Q_i\left(4Q_j - Q_k\right) - Q_j\left(Q_j + 2Q_k\right)}, \tag{A.7}$$

and

$$p_k^* = \frac{(Q_i - Q_j)\left(Q_j - Q_k\right) Q_k + c\left(4Q_iQ_j - Q_j^2 + 2Q_iQ_k - 2Q_jQ_k - 3Q_k^2\right)}{2\left(Q_i\left(4Q_j - Q_k\right) - Q_j\left(Q_j + 2Q_k\right)\right)}. \tag{A.8}$$

Region (ii): If $\tau_A \leq c < \tau_B \triangleq \frac{Q_kQ_jQ_i - Q_kQ_j^2}{4Q_jQ_i - (Q_kQ_j + Q_j^2 + 2Q_kQ_i)}$, then

$$p_i^* = \frac{cQ_k + cQ_j - Q_kQ_j + Q_kQ_i}{2Q_k},\ p_j^* = \frac{Q_jc}{Q_k},\ p_k^* = c. \tag{A.9}$$

Region (iii): If $\tau_B \leq c < \tau_C \triangleq \frac{Q_j}{2}$, then

$$p_i^* = \frac{(2Q_i + 3c - 2Q_j)Q_i}{4Q_i - Q_j},\ p_j^* = \frac{Q_j(Q_i - Q_j) + c(Q_j + 2Q_i)}{4Q_i - Q_j},\ p_k^* = c. \tag{A.10}$$

Region (iv): If $\tau_C \leq c < \tau_D \triangleq \frac{Q_jQ_i}{2Q_i - Q_j}$, then

$$p_i^* = \frac{Q_i}{Q_j}c,\ p_j^* = c,\ p_k^* = c. \tag{A.11}$$

Region (v): If $c \geq \tau_D$*, then*

$$p_i^* = \frac{Q_i + c}{2},\ p_j^* = c,\ p_k^* = c. \tag{A.12}$$

(ii) If the contributor is out of the market, and $Q_i > Q_j > c$ *where* $i, j \in \{p, o\}$ *and* $i \neq j$*, there exist threshold values* $0 < \tau_E < \tau_F$ *such that*

Region (vi): If $c < \tau_E \triangleq \frac{Q_j}{2}$*, then*

$$p_i^* = \frac{Q_i(3c + 2Q_i - 2Q_j)}{4Q_i - Q_j}\ ,\ p_j^* = \frac{c(2Q_i + Q_j) + Q_j(Q_i - Q_j)}{4Q_i - Q_j}\ ; \tag{A.13}$$

Region (vii): If $\tau_E \leq c < \tau_F \triangleq \frac{Q_i Q_j}{2Q_i - Q_j}$*, then*

$$p_i^* = \frac{Q_i}{Q_j} c\ ,\ p_j^* = c\ ; \tag{A.14}$$

Region (viii): If $c \geq \tau_F$*, then*

$$p_i^* = \frac{c + Q_i}{2}\ ,\ p_j^* = c\ . \tag{A.15}$$

Proof. For part (i), given $Q_i > Q_j > Q_k > c$, it is easy to verify that τ_A, τ_B, τ_C, and τ_D satisfy $0 < \tau_A < \tau_B < \tau_C < \tau_D$. We first examine Region (i) where $c < \tau_A$ is satisfied. Suppose the price equilibrium satisfies the conditions for any of Region I, VI, VII, or VIII of Lemma A.1. Then, $\tilde{\Pi}_i = 0$. In each case, since $Q_i > c$, there exists $p > c$ such that $\tilde{\Pi}_i(p \,|\, p_j, p_k) > 0$. Hence, firm i can deviate and none of these regions can occur in equilibrium. For Region IV of Lemma A.1, fixing any set of parameters which satisfy the conditions, we have $\tilde{\Pi}_j = 0$. However, since $Q_j > Q_k > c$, there exists $c < p < p_k Q_j / Q_k$ such that $\tilde{\Pi}_j(p \,|\, p_i, p_k) > 0$. Thus, firm j would deviate, and Region IV cannot occur in equilibrium. Suppose the price equilibrium satisfies the conditions for Region II, then only firm i is in the market and $\tilde{\Pi}_j = 0$. Given that only firm i is in the market, the equilibrium price $p_i^{II} = \frac{c + Q_i}{2}$. Because $Q_i > Q_j > Q_k > c$ and $c < \tau_A$ imply $c < p_i^{II} Q_j / Q_i$, then firm j can deviate to a price $p \in (c,\, p_i^{II} Q_j / Q_i)$ such that $\tilde{\Pi}_j(p \,|\, p_i, p_k) > 0$. Hence, Region II cannot occur in equilibrium. We can rule out Region III of Lemma A.1 in a similar way to Region II. Therefore, we can focus on Region V for candidate equilibria.

For Region V, by (4), (5), (6), and Lemma A.1, we obtain

$$\tilde{\Pi}_i(p_i \,|\, p_j, p_k) = (p_i - c)\left(1 - \frac{p_i - p_j}{Q_i - Q_j}\right), \tag{A.16}$$

$$\tilde{\Pi}_j(p_j \,|\, p_i, p_k) = (p_j - c)\left(\frac{p_i - p_j}{Q_i - Q_j} - \frac{p_j - p_k}{Q_j - Q_k}\right), \tag{A.17}$$

and

$$\tilde{\Pi}_k(p_k \,|\, p_i, p_j) = (p_k - c)\left(\frac{p_j - p_k}{Q_j - Q_k} - \frac{p_k}{Q_k}\right). \tag{A.18}$$

Because $Q_i > Q_j > Q_k$, by (A.16), (A.17), and (A.18), all three residual profit functions are strictly concave, with unconstrained maximizers characterized by

$$p_i = \frac{p_j + c + Q_i - Q_j}{2}, \tag{A.19}$$

$$p_j = \frac{(c + p_k)\, Q_i + (p_i - p_k)\, Q_j - (c + p_i)\, Q_k}{2\,(Q_i - Q_k)}, \tag{A.20}$$

and

$$p_k = \frac{cQ_j + p_j Q_k}{2Q_j}. \tag{A.21}$$

Simultaneously solving (A.19), (A.20), and (A.21), we obtain the equilibrium prices in (A.6), (A.7), and (A.8). By (A.6), (A.7), (A.8), and $c < \tau_A$, it is straightforward to verify the conditions of Region V in Lemma A.1 are satisfied. Therefore, p_i^*, p_j^*, and p_k^* given in (A.6), (A.7), and (A.8) are the unique candidate equilibrium prices in Region V of Lemma A.1.

To ensure that no firm would deviate to another region, we fix p_j^* and p_k^* and consider the pricing of firm i. Suppose it sets $p_i \le p_k^* Q_i/Q_k$, then Region II of Lemma A.1 applies. Suppose firm i sets $p_k^* Q_i/Q_k < p_i \le (p_k^*\,(-Q_i + Q_j) + p_j^*\,(Q_i - Q_k))/(Q_j - Q_k)$, then Region IV applies. Suppose it sets $(p_k^*\,(-Q_i + Q_j) + p_j^*\,(Q_i - Q_k))/(Q_j - Q_k) < p_i \le p_j^* - Q_j + Q_i$, then Region V applies. Finally, suppose $p_i > p_j^* - Q_j + Q_i$, then Region VIII applies. In summary, the profit function of firm i is given by

$$\tilde{\Pi}_i(p_i \,|\, p_j^*, p_k^*) = \begin{cases} (p_i - c)(1 - \frac{p_i}{Q_i}) & if\ p_i \le \frac{p_k^* Q_i}{Q_k}\ (Region\ II)\,; \\ (p_i - c)(1 - \frac{p_i - p_k^*}{Q_i - Q_k}) & if\ \frac{p_k^* Q_i}{Q_k} < p_i \le \frac{p_k^*(-Q_i+Q_j)+p_j^*(Q_i-Q_k)}{Q_j - Q_k}\ (Region\ IV)\,; \\ (p_i - c)(1 - \frac{p_i - p_j^*}{Q_i - Q_j}) & if\ \frac{p_k^*(-Q_i+Q_j)+p_j^*(Q_i-Q_k)}{Q_j - Q_k} < p_i \le p_j^* - Q_j + Q_i\ (Region\ V)\,; \\ 0 & if\ p_i > p_j^* - Q_j + Q_i\ (Region\ VIII)\,. \end{cases} \tag{A.22}$$

By (A.22), $\tilde{\Pi}_i(\cdot \,|\, p_j^*, p_k^*)$ is continuous. Further, because $(Q_i + c)/2 \ge p_k^* Q_i/Q_k$ under $c < \tau_A$, $\tilde{\Pi}_i(\cdot \,|\, p_j^*, p_k^*)$ increases in Region II. Also, $\tilde{\Pi}_i(\cdot \,|\, p_j^*, p_k^*)$ is increasing in Region IV if and only if $p_i \le (c + p_k^* + Q_i - Q_k)/2$, which is satisfied because $Q_i > Q_j > Q_k > c$ and $c < \tau_A$ imply $(p_k^*\,(-Q_i + Q_j) + p_j^*\,(Q_i - Q_k))/(Q_j - Q_k) \le (c + p_k^* + Q_i - Q_k)/2$. Therefore, p_i^* given in (A.6) is the unique price that maximizes (A.22).

Similarly, fixing prices of firms i and k, we examine the price setting problems of firm j. Its profit function is given by

$$\tilde{\Pi}_j(p_j \,|\, p_i^*, p_k^*) = \begin{cases} (p_j - c)(1 - \frac{p_j}{Q_j}) & if\ p_j \leq p_i^* - Q_i + Q_j\ (Region\ VI)\,; \\ (p_j - c)(\frac{p_i^* - p_j}{Q_i - Q_j} - \frac{p_j}{Q_j}) & if\ p_i^* - Q_i + Q_j < p_j \leq \frac{p_k^* Q_j}{Q_k}\ (Region\ III)\,; \\ (p_j - c)(\frac{p_i^* - p_j}{Q_i - Q_j} - \frac{p_j - p_k^*}{Q_j - Q_k}) & \frac{p_k^* Q_j}{Q_k} < p_j \leq \frac{p_k^*(Q_i - Q_j) + p_i^*(Q_j - Q_k)}{Q_i - Q_k}\ (Region\ V)\,; \\ 0 & if\ p_j > \frac{p_k^*(Q_i - Q_j) + p_i^*(Q_j - Q_k)}{Q_i - Q_k}\ (Region\ IV)\,. \end{cases} \tag{A.23}$$

By (A.23), $\tilde{\Pi}_j(\cdot|p_i^*, p_k^*)$ is continuous. Further, it is increasing on $[0,\ p_i^* - Q_i + Q_j]$ if and only if $p_j \leq (c + Q_j)/2$, which is satisfied because $Q_i > Q_j > Q_k > c$ implies $p_i^* - Q_i + Q_j < (c + Q_j)/2$. Also, $\tilde{\Pi}_j(\cdot|p_i^*, p_k^*)$ is increasing on $(p_i^* - Q_i + Q_j,\ p_k^* Q_j/Q_k]$ if and only if $p_j \leq (c + p_i^* Q_j/Q_i)/2$, which is satisfied since $p_k^* Q_j/Q_k \leq (c + p_i^* Q_j/Q_i)/2$. Therefore, p_j given in (A.7) maximizes (A.23).

Finally, fixing p_i^* and p_j^*, we examine the price setting problem of firm k, whose profit function is given by

$$\tilde{\Pi}_k(p_k \,|\, p_i^*, p_j^*) = \begin{cases} (p_k - c)(1 - \frac{p_k}{Q_k}) & if\ p_k \leq p_i^* - Q_i + Q_k\ (Region\ VII)\,; \\ (p_k - c)(\frac{p_i^* - p_k}{Q_i - Q_k} - \frac{p_k}{Q_k}) & if\ p_i^* - Q_i + Q_k < p_k \leq \frac{p_j^*(Q_i - Q_k) + p_i^*(-Q_j + Q_k)}{Q_i - Q_j}\ (Region\ IV)\,; \\ (p_k - c)(\frac{p_j^* - p_k}{Q_i - Q_k} - \frac{p_k}{Q_k}) & if\ \frac{p_j^*(Q_i - Q_k) + p_i^*(-Q_j + Q_k)}{Q_i - Q_j} < p_k \leq \frac{p_j^* Q_k}{Q_j}\ (Region\ V)\,; \\ 0 & if\ p_k > \frac{p_j^* Q_k}{Q_j}\ (Region\ III)\,. \end{cases} \tag{A.24}$$

By (A.24), $\tilde{\Pi}_k(\cdot|p_i^*, p_j^*)$ is continuous. Further, it is increasing on $[0,\ p_i^* - Q_i + Q_k]$ if and only if $p_k \leq (c + Q_k)/2$, which is satisfied because $Q_i > Q_j > Q_k > c$ implies $p_i^* - Q_i + Q_k < (c + Q_k)/2$. Also, $\tilde{\Pi}_k(\cdot|p_i^*, p_j^*)$ is increasing on $(p_i^* - Q_i + Q_k,\ (p_j^*\,(Q_i - Q_k) + p_i^*\,(-Q_j + Q_k))/(Q_i - Q_j)]$ if and only if $p_k \leq (c + p_i^* Q_k/Q_i)/2$, which is satisfied since $(p_j^*\,(Q_i - Q_k) + p_i^*\,(-Q_j + Q_k))/(Q_i - Q_j) \leq (c + p_i^* Q_k/Q_i)/2$. Therefore, p_k given in (A.8) maximizes (A.24). This completes the proof of Region (i) in part (i) of Lemma A.2. Equilibrium prices for the rest regions in part (i) and (ii) follow a similar train of logic and are omitted for brevity. ∎

Lemma A.3 *For a technical description of Region H, we define* $\beta_p = k_p b^2$, $\beta_o = k_o b$, $\beta_c = k_c$, $c = kb$, *and study the region where* $b < \bar{b}$ *for any constants* k_p, k_o, $k_c, k > 0$. *Further, suppose* $s_{oc} < \bar{s}_{oc} = \frac{s_c}{s_{co}} \cdot s_o$, *and* $s_{co} < \bar{s}_{co} = \lambda_H s_o$.[1] *Then, the equilibrium efforts of the proprietor, originator, and contributor satisfy*

$$e_p^* = \frac{s_p}{4k_p} \cdot \frac{1}{b^2} - \frac{4k_p s_o^5 \left(2s_{co}^2 + 13 s_{co} s_o + 3s_o^2\right) K_1^3}{k_o^2 s_p^3 K_2^6} + \frac{2k_p s_o^2 K_1 K_3}{k_c k_o^3 s_p^5 K_2^9} \cdot b + O(b^2)\,, \tag{A.25}$$

[1] $\lambda_H \leq 0.37$ is required for all conclusions in Region H to hold.

$$e_o^* = \frac{s_o^2 K_1}{k_o K_2^2} \cdot \frac{1}{b} - \frac{8k_p s_o^4 (s_{co} + 2s_o) K_1^3}{k_o^2 s_p^2 K_2^5} - \frac{K_4}{16 k_c^2 k_o^3 s_o^4 s_p^4 K_1^2 K_2^8} \cdot b + O(b^2)\,, \tag{A.26}$$

and

$$e_c^* = \frac{s_c s_o^2 (4s_o - 7s_{co}) + s_{co}^2 (s_{co} + 2s_o) s_{oc}}{4k_c K_2^3} - \frac{K_5}{8k_o k_c^2 s_o^2 s_p^2 K_1 K_2^6} \cdot b + O(b^2)\,, \tag{A.27}$$

respectively.[2]

Proof. For convenience, we define $z = 1/b$ and then examine e_p^*, e_o^*, and e_c^* as b becomes small. We can express $e_p^* \sim z^m$, $e_o^* \sim z^n$, and $e_c^* \sim z^q$ for some constants $m,\ n,\ q \in \mathbb{R}$. In other words, e_p^* is in the order of z^m, e_o^* is in the order of z^n, and e_c^* is in the order of z^q. Our first goal is to determine the value of m, n, and q in equilibrium. By $Q_p = s_p e_p$, (1), and (2), we obtain that $Q_p \sim z^m$, $Q_o \sim z^{\max(n,\,q)}$, and $Q_c \sim z^{\max(n,\,q)}$.

Suppose $m \leq \max(n, q)$ in equilibrium. First, suppose $\max(n,\, q) > 0$ and $n > q$. By Lemma A.2, p_o is $O(z^n)$. Because $|\Theta| = 1$, $\tilde{\Pi}_o$ is $O(z^n)$. The costs are $O(z^{2n-1})$ by (8). To generate non-negative profit for the originator, $z^n \geq z^{2n-1}$ should be satisfied, which gives $n \leq 1$. Second, suppose $\max(n, q) > 0$ and $n \leq q$. Similarly, $\tilde{\Pi}_c$ is $O(z^q)$ and costs are $O(z^{2q})$, which requires $q \geq 2q$, i.e., $q \leq 0$. Hence, $\max(n, q) \leq 1$. Because $m \leq \max(n, q)$, we obtain $m \leq 1$. Third, suppose $\max(n,\, q) \leq 0$. Together with $m \leq \max(n, q)$, it follows that $m \leq 0$. Therefore, from these three cases, Π_p is at most $O(z^1)$. However, the proprietor could instead set $m = \frac{3}{2}$ such that $\tilde{\Pi}_p \sim z^{\frac{3}{2}}$ and costs are $O(z^{2(\frac{3}{2})-2}) = O(z)$, and hence $\Pi_p \sim z^{\frac{3}{2}}$. This contradicts $m \leq \max(n, q)$. Therefore, $m > \max(n, q)$.

Second, suppose $\max(n, q) \leq 0$. Then, for the originator, Π_o is $O(1)$. However, suppose the originator instead selects $n = \frac{1}{2}$. Then $Q_o \sim z^{\frac{1}{2}}$ and $Q_c \sim z^{\frac{1}{2}}$. Because from the previous argument, $m \geq \frac{3}{2}$ and we obtain $\tau_A \sim z^{\frac{1}{2}}$, which implies Region (i) of Lemma A.2 is satisfied. Thus, $p_o \sim p_c \sim z^{\frac{1}{2}}$, which implies $\tilde{\Pi}_o \sim z^{\frac{1}{2}}$, whereas costs are $O(z^{2(\frac{1}{2})-1}) = O(1)$, and hence $\Pi_o \sim z^{\frac{1}{2}}$ such that the originator will deviate. Therefore $\max(n, q) > 0$.

Suppose $q \geq n$. Because there exists $\bar{b} > 0$ such that $m > \max(n, q) > 0$, $\tau_A > c$ as $b < \bar{b}$. Thus, by Region (i) of Lemma A.2, $\tilde{\Pi}_c \sim z^q$ and costs are $O(z^{2q})$, which requires $q \geq 2q$, i.e., $q \leq 0$. However, we have shown that $\max(n, q) > 0$, which is a contradiction. Therefore, $q < n$. Further, because $s_o > s_{co}$ in Region H, we have $Q_o > Q_c$. By $m > \max(n, q)$, $\max(n, q) > 0$, and $q < n$, we obtain $Q_p > Q_o > Q_c > c$. The pricing equilibrium falls into Region (i) of part (i) in Lemma A.2.

Substituting the equilibrium prices in Lemma A.2 into (7), we obtain the contributor's profit as

$$\Pi_c(e_c|e_p,\, e_o) = \frac{Q_o(Q_o - Q_c)(c(3Q_c + Q_o - 4Q_p) + Q_c(Q_p - Q_o))^2}{4Q_c(Q_o(Q_o - 4Q_p) + Q_c(2Q_o + Q_p))^2} - \frac{1}{2}\beta_c e_c^2\,. \tag{A.28}$$

Substituting $Q_p = s_p e_p$, (1), and (2) into (A.28) and differentiating twice with respect to e_c, and

[2] The constants, K_i for $i = 1,\, 2,\, \ldots, 5$, are fully characterized in the proof.

then plugging in $e_p \sim z^m$, $e_o \sim z^n$, and $e_c \sim z^q$, we find that the second order condition is satisfied, i.e., $d^2\Pi_c/de_c^2 < 0$ for all e_c. Analyzing the first order condition and collecting terms in powers of z, we find that it can be written as

$$E_1 z^q + E_2 + Y_1(z) = 0\,, \tag{A.29}$$

where $E_1,\ E_2 \in \mathbb{R}$ and $Y_1(z)$ is a polynomial in z with terms that will be dominated by E_2 in absolute value. As $z \to \infty$, equation (A.29) has to hold for all z values. This is not possible if $q > 0$ because in that case $E_1 z^q$ dominates all other terms in absolute value and explodes. Therefore, in equilibrium, $E_1 z^q$ and E_2 have to cancel each other, which implies $q = 0$.

Because $m > n > q = 0$, by (A.28), $\Pi_c \sim z^n$, which directly implies the existence of $\bar{b}$ such that $\Pi_c > V_c$ when $b < \bar{b}$. Effectuating $\Pi_c < V_c$ would require $n \leq 0$ due to cross effects, which is suboptimal from the point of view of the originator. Arguments as to why the proprietor lacks incentive to push the contributor out (whether directly or indirectly via pressure on the originator) are similar in spirit.

Next, for the originator's effort problem, we obtain the profit function of the originator by substituting the equilibrium prices in Region (i) of Lemma A.2 into (8):

$$\Pi_o(e_o|e_p) = \frac{(Q_p - Q_o)(Q_p - Q_c)(Q_o - Q_c)(Q_o - c)^2}{(Q_o(Q_o - 4Q_p) + Q_c(2Q_o + Q_p))^2} - \frac{1}{2}\beta_o e_o^2. \tag{A.30}$$

Taking a total derivative of (A.30) with respect to e_o (and another one to show concavity as before), and substituting $e_p \sim z^m$, $e_o \sim z^n$ and $e_c \sim z^0$ into the first order condition, we obtain

$$E_3 z^{n-1} + E_4 + Y_2(z) = 0, \tag{A.31}$$

where $E_3,\ E_4 \in \mathbb{R}$ and $Y_2(z)$ is a polynomial in z with terms that will be dominated by E_4 in absolute value. From a similar argument as in the contributor's problem, we can obtain $n = 1$.

For the proprietor's effort problem, the profit function of the proprietor is

$$\Pi_p(e_p) = \frac{(Q_p - Q_o)(c(Q_o - Q_c) - 4Q_oQ_p + Q_c(3Q_o + Q_p))^2}{4(Q_o(Q_o - 4Q_p) + Q_c(2Q_o + Q_p))^2} - \frac{1}{2}\beta_p e_p^2\,. \tag{A.32}$$

Similarly, we can obtain the first order condition for the proprietor:

$$E_5 z^{m-2} + E_6 + Y_3(z) = 0, \tag{A.33}$$

where $E_5,\ E_6 \in \mathbb{R}$ and $Y_3(z)$ is a polynomial in z with terms that will be dominated by E_6 in absolute value. Similarly, we establish concavity and obtain that, by (A.33), $m = 2$.

Based on the values $m=2$, $n=1$, and $q=0$, substituting $e_p=P_1z^2+O(z)$, $e_o=O_1z+O(1)$, and $e_c=C_1+O(1/z)$ into the three first order conditions and equating the lead coefficients of the highest order terms with respect to z to zero, we obtain P_1, O_1, and C_1 from the following equations:

$$4C_1k_c(s_{co}-4s_o)^3+s_cs_o^2(-7s_{co}+4s_o)+s_{co}^2(s_{co}+2s_o)s_{oc}=0\,, \tag{A.34}$$

$$k_oO_1\left(s_{co}-4s_o\right)^2+\left(s_{co}-s_o\right)s_o^2=0\,, \tag{A.35}$$

and

$$4k_pP_1-s_p=0\,. \tag{A.36}$$

Then we have

$$C_1=\frac{s_cs_o^2\left(4s_o-7s_{co}\right)+s_{co}^2\left(s_{co}+2s_o\right)s_{oc}}{4k_cK_2^3}, \tag{A.37}$$

$$O_1=\frac{s_o^2K_1}{k_oK_2^2},\ \textit{and}\ P_1=\frac{s_p}{4k_p}, \tag{A.38}$$

where $K_1=s_o-s_{co}$ and $K_2=4s_o-s_{co}$. Note that given $s_{co}/s_o<\lambda_H$, we obtain that $C_1>0$ and hence $e_c^*>0$ in Region H.

Repeating similar steps as above, we obtain the optimal effort investment levels to the order of b^2 as in Lemma A.3 by obtain that

$$e_p^*=\frac{s_p}{4k_p}\cdot\frac{1}{b^2}-\frac{4k_ps_o^5\left(2s_{co}^2+13s_{co}s_o+3s_o^2\right)K_1^3}{k_o^2s_p^3K_2^6}+\frac{2k_ps_o^2K_1K_3}{k_ck_o^3s_p^5K_2^9}\cdot b+O(b^2)\,, \tag{A.39}$$

where $K_3=-16K_1^3k_ck_ps_o^5\big(4s_{co}^4+55s_{co}^3s_o+58s_{co}^2s_o^2-66s_{co}s_o^3+30s_o^4\big)-K_2k_o^2\big(3s_c^2(4s_{co}-s_o)s_o^4\big(6s_{co}^2+41s_{co}s_o-20s_o^2\big)+s_cs_o^2\big(-19s_{co}^5-258s_{co}^4s_o-11s_{co}^3s_o^2+94s_{co}^2s_o^3-468s_{co}s_o^4+176s_o^5\big)s_{oc}+s_{co}\big(s_{co}^6+30s_{co}^5s_o+32s_{co}^4s_o^2-40s_{co}^3s_o^3+204s_{co}^2s_o^4-176s_{co}s_o^5+192s_o^6\big)s_{oc}^2\big)s_p^2$.

We can also obtain the equilibrium effort of the originator as

$$e_o^*=\frac{s_o^2K_1}{k_oK_2^2}\cdot\frac{1}{b}-\frac{8k_ps_o^4(s_{co}+2s_o)K_1^3}{k_o^2s_p^2K_2^5}-\frac{K_4}{16k_c^2k_o^3s_o^4s_p^4K_1^2K_2^8}\cdot b+O(b^2)\,, \tag{A.40}$$

where $K_4=256K_1^6k_c^2k_p^2(4s_{co}-s_o)s_o^{10}\big(s_{co}^2+7s_{co}s_o+s_o^2\big)+32K_1^3K_2k_ck_o^2k_ps_o^5\big(9s_c^2s_o^4\big(8s_{co}^2+5s_{co}s_o-4s_o^2\big)+s_cs_o^2\big(-19s_{co}^4-104s_{co}^3s_o+129s_{co}^2s_o^2-248s_{co}s_o^3+80s_o^4\big)s_{oc}+s_{co}\big(s_{co}^5+17s_{co}^4s_o-15s_{co}^3s_o^2+50s_{co}^2s_o^3-20s_{co}s_o^4+48s_o^5\big)s_{oc}^2\big)s_p^2+K_2^2k_o^4(s_cs_o-s_{co}s_{oc})^2\big(s_cs_o^2(-7s_{co}+4s_o)+s_{co}^2(s_{co}+2s_o)s_{oc}\big)\big(3s_cs_o^2\big(7s_{co}^2+25s_{co}s_o+4s_o^2\big)-\big(s_{co}^4+35s_{co}^3s_o+8s_{co}s_o^3+64s_o^4\big)s_{oc}\big)s_p^4$.

Lastly, the contributor's equilibrium effort is

$$e_c^*=\frac{s_cs_o^2\left(4s_o-7s_{co}\right)+s_{co}^2\left(s_{co}+2s_o\right)s_{oc}}{4k_cK_2^3}-\frac{K_5}{8k_ok_c^2s_o^2s_p^2K_1K_2^6}\cdot b+O(b^2)\,, \tag{A.41}$$

where $K_5 = 96K_1^3 k_c k_p s_o^5\big(s_c s_o^2(-5s_{co}+2s_o)+s_{co}\big(s_{co}^2+2s_o^2\big)s_{oc}\big)+K_2 k_o^2(7s_{co}+8s_o)(s_c s_o-s_{co}s_{oc})^2\big(s_c s_o^2(-7s_{co}+4s_o)+s_{co}^2(s_{co}+2s_o)s_{oc}\big)s_p^2$. ■

Proof of Proposition 1: Technically, we show that under the conditions of Region H

(i) $\frac{de_p^*}{ds_{oc}} < 0$, and $\frac{de_c^*}{ds_{oc}} > 0$;

(ii) There exists $\bar{k}_c > 0$ such that if $k_c \geq \bar{k}_c$, then $\frac{de_o^*}{ds_{oc}} < 0$ for all $s_{oc} \in (0,\ \bar{s}_{oc})$; if $k_c < \bar{k}_c$, then there exists $\tilde{s}_{oc} \in (0,\ \bar{s}_{oc})$ such that $\frac{de_o^*}{ds_{oc}} > 0$ for $s_{oc} \in (0,\ \bar{s}_{oc})$, and $\frac{de_o^*}{ds_{oc}} < 0$ for $s_{oc} \in (\tilde{s}_{oc},\ \bar{s}_{oc})$;

(iii) $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} < 0$, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$, and $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} > 0$,

where $\bar{s}_{oc} = \frac{s_c s_o}{s_{co}}$ and $\bar{k}_c = \frac{K_2 k_o^2 s_c^2 (s_{co}+2s_o)\left(-7s_{co}^4-149s_{co}^3 s_o+72s_{co}^2 s_o^2-56s_{co}s_o^3+32s_o^4\right)s_p^2}{8K_1^3 k_p s_o^3(-19s_{co}^4-104s_{co}^3 s_o+129s_{co}^2 s_o^2-248s_{co}s_o^3+80s_o^4)}$.

First, differentiating (A.27) with respect to s_{oc}, we obtain

$$\frac{de_c^*}{ds_{oc}} = \frac{s_{co}^2\left(s_{co}+2s_o\right)}{4{K_2}^3 k_c} + O(b), \tag{A.42}$$

which is positive because $s_o > s_{co}$ in Region H implies $K_2 = 4s_o - s_{co} > 0$.

By differentiating (A.25) with respect to s_{oc}, we obtain

$$\frac{de_p^*}{ds_{oc}} = -\frac{2K_1 s_o k_p\left(A_1 s_c s_o^2 + 2A_2 s_{co} s_{oc}\right)b}{K_2^8 s_p^3 k_o k_c} + O(b^2)\,, \tag{A.43}$$

where $-19s_{co}^5 - 258s_{co}^4 s_o - 11s_{co}^3 s_o^2 + 94s_{co}^2 s_o^3 - 468s_{co}s_o^4 + 176s_o^5$, and $A_2 = s_{co}^6 + 30s_{co}^5 s_o + 32s_{co}^4 s_o^2 - 40s_{co}^3 s_o^3 + 204s_{co}^2 s_o^4 - 176s_{co}s_o^5 + 192s_o^6$.

Because $s_o > s_{co}$ in Region H, it is easy to see that $K_1 = s_o - s_{co} > 0$, $K_2 = 4s_o - s_{co} > 0$, and $A_2 > 0$. Let $\lambda = s_{co}/s_o \in [0,\ 1)$. We can rewrite $A_1 = f_1(\lambda)s_o^5$, where $f_1(\lambda) = 176 - 468\lambda + 94\lambda^2 - 11\lambda^3 - 258\lambda^4 - 19\lambda^5$. By Sturm's Theorem, $f_1(\lambda) = 0$ has only one real root for $\lambda \in (0,\ 1]$. Because $f_1(0) = 176 > 0$, $f_1(1) = -486$, and $f_1(\lambda)$ is continuous, there exists a unique root $\bar{\lambda} \in (0,\ 1]$ such that $f_1(\lambda) > 0$ for $\lambda \in (0,\ \bar{\lambda})$. And we can find the value of $\bar{\lambda}$ numerically: $\bar{\lambda} \approx 0.392 > \lambda_H$. Hence, $A_1 > 0\ \forall\lambda \in [0,\ \lambda_H)$ in Region H. Therefore, $de_p^*/ds_{oc} < 0$ in Region H. This completes the proof of part (i). Because s_p is positive, this also implies that $dQ_p(\mathbf{e}^*)/ds_{oc} < 0$.

Differentiating (A.26) with respect to s_{oc}, we obtain

$$\frac{de_o^*}{ds_{oc}} = \frac{(A_3 s_{oc}^3 + A_4 s_{oc}^2 + A_5 s_{oc} + A_6)b}{8K_1^2 K_2^7 k_c^2 k_o s_o^4 s_p^2} + O(b^2), \tag{A.44}$$

where $A_3 = 2K_2 k_o^2 s_{co}^4 s_p^2(s_{co}+2s_o)\big(s_{co}^4 + 35s_{co}^3 s_o + 8s_{co}s_o^3 + 64s_o^4\big)$ and $A_4 = -3K_2 k_o^2 s_c s_{co}^2 s_o s_p^2\big(s_{co}^6 + 51s_{co}^5 s_o + 249s_{co}^4 s_o^2 + 19s_{co}^3 s_o^3 + 120s_{co}^2 s_o^4 + 336s_{co}s_o^5 - 128s_o^6\big)$.

Above, we also define

$A_5 = s_{co}s_o^2\big(-32K_1^3 k_c k_p s_o^3\big(s_{co}^5 + 17s_{co}^4 s_o - 15s_{co}^3 s_o^2 + 50s_{co}^2 s_o^3 - 20s_{co}s_o^4 + 48s_o^5\big) + K_2 k_o^2 s_c^2\big(s_{co}^6 + 93s_{co}^5 s_o + 933s_{co}^4 s_o^2 + 493s_{co}^3 s_o^3 + 24s_{co}^2 s_o^4 + 912s_{co}s_o^5 - 512s_o^6\big)s_p^2\big)$,

and

$A_6 = 2s_cs_o^4\big(8K_1^3k_ck_ps_o^3\big(19s_{co}^4 + 104s_{co}^3s_o - 129s_{co}^2s_o^2 + 248s_{co}s_o^3 - 80s_o^4\big) - K_2k_o^2s_c^2(s_{co}+2s_o)\big(7s_{co}^4 + 149s_{co}^3s_o - 72s_{co}^2s_o^2 + 56s_{co}s_o^3 - 32s_o^4\big)s_p^2\big)$.

Because the denominator of the first term is positive, we only need to examine the sign of the numerator. Let $f_2(s_{oc}) = A_3s_{oc}^3 + A_4s_{oc}^2 + A_5s_{oc} + A_6$. Then, $f_2(s_{oc})$ is a third degree polynomial of s_{oc} and has at most three real roots. It is easy to see that $A_3 > 0$. We can obtain that $f_2(s_cs_o/s_{co}) = -16K_1^4K_2k_ck_ps_cs_o^6\big(2s_{co}^3 + 25s_{co}^2s_o - 17s_{co}s_o^2 + 44s_o^3\big) < 0$. Next, we can obtain that

$$f_2(s_{oc} = -s_cs_o/s_{co}) = 2s_cs_o^3\left(8A_{10}K_1^3k_ck_ps_o^3 + A_{11}K_2k_o^2s_c^2s_p^2\right),$$

where $A_{10} = 2s_{co}^5 + 53s_{co}^4s_o + 74s_{co}^3s_o^2 - 29s_{co}^2s_o^3 + 208s_{co}s_o^4 + 16s_o^5$, and $A_{11} = -3s_{co}^6 - 167s_{co}^5s_o - 1073s_{co}^4s_o^2 - 509s_{co}^3s_o^3 - 184s_{co}^2s_o^4 - 1168s_{co}s_o^5 + 512s_o^6$.

Given $s_o > s_{oc}$, it is easy to show that $A_{10} > 0$. We can write $A_{11} = f_3(\lambda)s_o^6$, where $f_3(\lambda) = 512 - 1168\lambda - 184\lambda^2 - 509\lambda^3 - 1073\lambda^4 - 167\lambda^5 - 3\lambda^6$ and $\lambda = s_{co}/s_o$. Sturm's theorem shows that $f_3(\lambda) = 0$ has only one real root for $r \in [0, 1)$. Because $f_3(0) = 512$, $f_3(1) = -2592$, and $f_3(r)$ is continuous, we know that there exists $\tilde{\lambda} \in [0, 1)$ such that $f_3(\lambda) > 0\ \forall\lambda \in [0, \tilde{\lambda})$. We can then find $\tilde{\lambda}$ numerically that $\tilde{\lambda} \approx 0.374 > \lambda_H$. Hence, $A_{11} > 0\ \forall\lambda \in [0, \lambda_H)$. And we can obtain that $f_2(-s_cs_o/s_{co}) > 0$ in Region H.

Given $f_2(s_cs_o/s_{co}) < 0$, $f_2(-s_cs_o/s_{co}) > 0$, and $A_3 > 0$, we can see that the equation $f_2(s_{oc}) = 0$ has one root in $(\frac{s_cs_o}{s_{co}}, \infty)$, another in $(-\infty, -\frac{s_cs_o}{s_{co}})$, and the third in $(-\frac{s_cs_o}{s_{co}}, \frac{s_cs_o}{s_{co}})$. We are interested in the third one which falls into our definition of Region H, where we have defined $s_{oc} < \bar{s}_{oc} = \frac{s_cs_o}{s_{co}}$. Let $\tilde{s}_{oc}$ be the root in $(-\frac{s_cs_o}{s_{co}}, \frac{s_cs_o}{s_{co}})$. If $f_2(s_{oc} = 0) > 0$, then $\tilde{s}_{oc} \in (0, \frac{s_cs_o}{s_{co}})$. Otherwise, $\tilde{s}_{oc} \in (-\frac{s_cs_o}{s_{co}}, 0)$. Solving for $f_2(0) = 0$, we obtain

$$\bar{k}_c = \frac{K_2k_o^2s_c^2(s_{co}+2s_o)\left(-7s_{co}^4 - 149s_{co}^3s_o + 72s_{co}^2s_o^2 - 56s_{co}s_o^3 + 32s_o^4\right)s_p^2}{8K_1^3k_ps_o^3\left(-19s_{co}^4 - 104s_{co}^3s_o + 129s_{co}^2s_o^2 - 248s_{co}s_o^3 + 80s_o^4\right)}, \tag{A.45}$$

and $\bar{k}_c > 0$ in Region H. Therefore, if $k_c > \bar{k}_c$, then $f_2(0) < 0$. And $f_2(s_{oc}) < 0$ for $s_{oc} \in (0, \bar{s}_{oc})$, which implies $\frac{de_o^*}{ds_{oc}} < 0$. However, if $k_c \leq \bar{k}_c$, then $f_2(0) \geq 0$, which means for $s_{oc} \in (0, \tilde{s}_{oc})$, $f_2(s_{oc}) > 0$, and hence, $de_o^*/ds_{oc} > 0$; for $s_{oc} \in (\tilde{s}_{oc}, \bar{s}_{oc})$, $f_2(s_{oc}) < 0$, and thus, $de_o^*/ds_{oc} < 0$. This completes the proof of part (ii).

We can now turn to the impact of s_{oc} on qualities of the originator and the contributor:

$$\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} = \frac{s_cs_o^2(-7s_{co}+4s_o) + 2s_{co}^2(s_{co}+2s_o)s_{oc}}{4K_2^3k_c} + O(b) \tag{A.46}$$

and

$$\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} = \frac{s_cs_{co}^2(s_{co}+2s_o)}{4K_2^3k_c} + O(b). \tag{A.47}$$

Since $K_2 > 0$, $dQ_c(\mathbf{e}^*)/ds_{oc} > 0$. Because $\lambda_H < 4/7$, $dQ_o(\mathbf{e}^*)/ds_{oc} > 0$ in Region H. This completes the proof. ■

Proof of Proposition 2: Technically, we show that $\frac{dSW}{ds_{oc}} > 0$ and $\frac{dCS}{ds_{oc}} > 0$ under the conditions of Region H.

Substituting (A.25), (A.26), (A.27) into (10), and differentiating with respect to s_{oc}, we obtain

$$\frac{dCS}{ds_{oc}} = \frac{s_o\left(A_1 s_{oc} + A_2\right)}{32\left(4s_o - s_{co}\right)^6 k_c} + O(b)\,, \tag{A.48}$$

where $A_1 = 4s_{co}^2(s_{co} + 2s_o)\left(s_{co}^2 - 42s_{co}s_o + 56s_o^2\right)$ and $A_2 = s_c s_o\big(-s_{co}^4 + 36s_{co}^3 s_o + 700s_{co}^2 s_o^2 - 1120s_{co}s_o^3 + 448s_o^4\big)$. Because $s_{co}/s_o < \lambda_H$, $A_1 > 0$ and $A_2 > 0$, which implies $\frac{dCS}{ds_{oc}} > 0$.

Similarly, substituting (A.25), (A.26), (A.27) into (11), and differentiating with respect to s_{oc}, we obtain

$$\frac{dSW}{ds_{oc}} = \frac{A_3 s_{oc} + A_4}{32\left(4s_o - s_{co}\right)^6 k_c} + O(b)\,, \tag{A.49}$$

where $A_3 = 2s_{co}^2(s_{co} + 2s_o)\left(-s_{co}^3 + 20s_{co}^2 s_o - 60s_{co}s_o^2 + 80s_o^3\right)$ and $A_4 = s_c s_o^2\big(3s_{co}^4 - 160s_{co}^3 s_o + 484s_{co}^2 s_o^2 - 800s_{co}s_o^3 + 320s_o^4\big)$. Because $s_{co}/s_o < \lambda_H$ implies $A_3 > 0$ and $A_4 > 0$, we obtain $dSW/ds_{oc} > 0$. This completes the proof. ■

Lemma A.4 *For the analytical definition of Region L, we define $\beta_p = \hat{k}_p b$, $\beta_o = \hat{k}_o b$, $\beta_c = \hat{k}_c b$, $c = \hat{k} b$, and study the region where $b < \bar{b}$ for any constants $\hat{k}_p$, $\hat{k}_o$, $\hat{k}_c$, and $\hat{k} > 0$. Then, there exist $\underline{V}$ such that if $V_c \leq \underline{V}$, the contributor remains in the market.*

(i) (Proprietor Quality Leadership) Suppose s_p, s_o, and s_{oc} are constants, and $s_c = \hat{k}_{s_c} b$, $s_{co} = \hat{k}_{s_{co}} b$, and $s_{oc} < \bar{s_{oc}} = \frac{\lambda_L \hat{k}_{s_c}}{\hat{k}_{s_{co}}} s_o$ for constants $\hat{k}_{s_c}$, $\hat{k}_{s_{co}}$, and $\lambda_L > 0$. There exist $\bar{t} > 0$ such that if $t = \frac{s_p^2/\hat{k}_p}{s_o^2/\hat{k}_o} \geq \bar{t}$, the proprietor is the quality leader, and the equilibrium efforts of the proprietor, originator, and contributor satisfy

$$e_p^* = \frac{M_1 s_o^2}{r\hat{k}_o M_2^3 s_p} \cdot \frac{1}{b} - \frac{2r^2 M_6 s_p}{\hat{k}_c M_2^3\left(\hat{k}_p M_4^3 s_o^2 + 24\hat{k}_o M_5 r^3 s_p^2\right)} + O(b), \tag{A.50}$$

$$e_o^* = \frac{M_1 s_o}{k_o M_2^3} \cdot \frac{1}{b} + \frac{M_7}{2\hat{k}_c \hat{k}_o M_2^3 s_o^2\left(\hat{k}_p M_4^3 s_o^2 + 24\hat{k}_o M_5 r^3 s_p^2\right)} + O(b), \tag{A.51}$$

and

$$e_c^* = \frac{M_3(\hat{k}_{s_c} M_2 M_3 s_o - 6\hat{k}_{s_{co}} r s_{oc})}{4\hat{k}_c M_2^3 s_o} + O(b), \tag{A.52}$$

respectively[3], *where there exists a unique* $r \in (0,\ \frac{4}{7})$ *satisfying*

$$-64 + 176r - 196r^2 + 147r^3 + 4r\left(64 - 112r + 132r^2 - 139r^3 + 28r^4\right)t = 0\,. \tag{A.53}$$

(ii) (OSS Quality Leadership, Limited Contributor) Suppose s_p, s_o, s_{oc} *are constants, and* $s_c = \hat{k}_{s_c} b$, $s_{co} = \hat{k}_{s_{co}} b$, *and* $s_{oc} < \bar{s_{oc}} = \frac{\lambda_L \hat{k}_{s_c}}{\hat{k}_{s_{co}}} s_o$ *for constants* $\hat{k}_{s_c}$, $\hat{k}_{s_{co}}$, *and* $\lambda_L > 0$. *There exist* $\bar{t} > 0$ *such that if* $t = \frac{s_p^2/\hat{k}_p}{s_o^2/\hat{k}_o} < \bar{t}$, *the originator is the quality leader, and the equilibrium efforts of the proprietor, originator, and contributor satisfy*

$$e_p^* = \frac{4\tilde{M}_1 s_o^2}{\hat{k}_o \tilde{M}_2^3 s_p} \cdot \frac{1}{b} + \frac{\left(8\hat{k}_c \hat{k}_{s_{co}} \tilde{M}_6 s_o^2 + \hat{k}_o s_{oc}(\hat{k}_{s_c} \tilde{M}_7 s_o + 6\hat{k}_{s_{co}} \tilde{M}_8 s_{oc})\right) s_p}{4\hat{k}_c \tilde{M}_2^3 \tilde{r}^2 s_o \left(2\hat{k}_p \tilde{M}_4^3 s_o^2 + \hat{k}_o \tilde{M}_5 s_p^2\right)} + O(b), \tag{A.54}$$

$$e_o^* = \frac{4\tilde{M}_1 \tilde{r} s_o}{\hat{k}_o \tilde{M}_2^3} \cdot \frac{1}{b} + \frac{\tilde{M}_9}{2\hat{k}_c \hat{k}_o \tilde{M}_2^3 \tilde{r}^2 s_o^2 \left(\hat{k}_o \tilde{M}_5 s_p^2 + 2\hat{k}_p \tilde{M}_4^3 s_o^2\right)} + O(b), \tag{A.55}$$

and

$$e_c^* = \frac{\tilde{M}_3(\hat{k}_{s_c} \tilde{M}_2 \tilde{M}_3 s_o + 6\hat{k}_{s_{co}} \tilde{r} s_{oc})}{4\hat{k}_c \tilde{M}_2^3 s_o} + O(b), \tag{A.56}$$

respectively[4], *where there exists a unique* $\tilde{r} \in (\frac{7}{4},\ \infty)$ *satisfying*

$$\left(2 + 14\tilde{r} - 127\tilde{r}^2 + 196\tilde{r}^3 - 176\tilde{r}^4 + 64\tilde{r}^5\right)t - 4\left(42 - 95\tilde{r} + 164\tilde{r}^2 - 112\tilde{r}^3 + 64\tilde{r}^4\right) = 0\,. \tag{A.57}$$

(iii) (OSS Quality Leadership, Moderate Contributor) Suppose s_p, s_c, s_{oc} *and* s_{co} *are constants, and* $s_o = \frac{\hat{k}_o}{b}$ *for constants* $\hat{k}_{s_o} > 0$. *The equilibrium quality order follows the originator, the contributor, and the proprietor (i.e.,* $Q_o^* > Q_c^* > Q_p^*$*). The equilibrium efforts of the proprietor, originator, and contributor satisfy*

$$e_p^* = \frac{s_p}{64\hat{k}_p} \cdot \frac{1}{b} - \frac{\left(\hat{k}_o s_p^3 + 24\hat{k}_p s_{co}^2 s_p\right)}{1024\hat{k}_p^2 s_{co} \hat{k}_{s_o}} + \frac{\hat{M}_1}{\hat{M}_2} \cdot b + \frac{\hat{M}_3 + \hat{M}_4 s_{oc}}{\hat{M}_5} \cdot b^2 + O(b^3), \tag{A.58}$$

$$e_o^* = \frac{\hat{k}_{s_o}}{4\hat{k}_o} \cdot \frac{1}{b^2} - \frac{s_{co}}{8\hat{k}_o} \cdot \frac{1}{b} - \frac{5s_{co}^2}{64\hat{k}_o \hat{k}_{s_o}} + \frac{\hat{M}_6}{\hat{M}_7} \cdot b + \frac{\hat{M}_8 + \hat{M}_9 s_{oc}}{\hat{M}_{10}} \cdot b^2 + O(b^3), \tag{A.59}$$

and

$$e_c^* = \frac{s_c}{16\hat{k}_c} \cdot \frac{1}{b} - \frac{s_c s_{co}}{16\hat{k}_c \hat{k}_{s_o}} + \frac{\hat{M}_{11} + \hat{M}_{12} s_{oc}}{\hat{M}_{13}} \cdot b + O(b^2), \tag{A.60}$$

respectively.[5]

[3]The constants, M_i for $i = 1, 2, \ldots, 6$, are fully characterized in the proof.
[4]The constants, $\tilde{M}_i$ for $i = 1, 2, \ldots, 9$, are fully characterized in the proof.
[5]The constants, $\hat{M}_i$ for $i = 1, 2, \ldots, 7$, are fully characterized in the proof.

Proof. For parts (i) and (ii), we define $z = 1/b$ and examine e_p^*, e_o^*, and e_c^* as b becomes small. Further, we can express $e_p^* \sim z^m$, $e_o^* \sim z^n$, and $e_c^* \sim z^q$ for some $m,\ n,\ q \in \mathbb{R}$. By $Q_p = s_p e_p$, (1), (2), and the definition of Region L, $Q_p \sim z^m$, $Q_o \sim z^{\max(n,q)}$, and $Q_c \sim z^{\max(n,q)-1}$.

Suppose $m \le \max(n,q) - 1$ in equilibrium. First suppose that $\max(n,q) > 1$ and $n > q$. By Lemma A.2, p_o is $O(z^n)$. Because $|\Theta| = 1$, $\tilde{\Pi}_o$ is $O(z^n)$. The effort of the originator costs $O(z^{2n-1})$. To generate non-negative profit, $z^n \ge z^{2n-1}$ should be satisfied, which gives us $n \le 1$ and contradicts with $\max(n,q) > 1$ and $n > q$. Suppose $\max(n,q) > 1$ and $n \le q$. We can similarly obtain that $\tilde{\Pi}_c$ is $O(z^{q-1})$ and costs are $O(z^{2q-1})$, which gives us $q \le 0$ and contradicts with $\max(n,q) > 1$ and $n \le q$. Therefore, $\max(n,q) \le 1$. Because $m \le \max(n,q) - 1$, we obtain $m \le 0$. Thus, Π_p is $O(1)$. However, the proprietor could instead set $m = 1/2$ such that $\tilde{\Pi}_p \sim z^{\frac{1}{2}}$ and costs are $O(z^{2(1/2)-1}) = O(1)$, and hence $\Pi_p \sim z^{\frac{1}{2}}$. Therefore, the proprietor will deviate and we conclude that $m > \max(n,q) - 1$.

Suppose $\max(n,q) \le 0$. Then, for the originator, Π_o is $O(1)$. However, suppose the originator instead selects $n = \frac{1}{2}$. Then $Q_o \sim z^{\frac{1}{2}}$ and $Q_c \sim z^{-\frac{1}{2}}$. Because from the previous argument, $m \ge 1/2$ and we obtain $\tau_A \sim z^{-\frac{1}{2}} > c \sim z^{-1}$, which implies Region (i) of Lemma A.2 is satisfied. Thus, $p_o \sim z^{\frac{1}{2}}$, which implies $\tilde{\Pi}_o \sim z^{\frac{1}{2}}$, whereas costs are $O(z^{2(\frac{1}{2})-1}) = O(1)$, and hence $\Pi_o \sim z^{\frac{1}{2}}$ such that the originator will deviate. Therefore, $\max(n,q) > 0$.

Suppose $q \ge n$. As noted above, it follows that $q \le 0$. However, we have shown that $\max(n,q) > 0$, which is a contradiction. Hence, $q < n$.

Suppose $m > n$, we have $Q_p > Q_o > Q_c > c$. The contributor's profit is given by (A.28). Substituting $Q_p = s_p e_p$, (1), and (2) into (A.28), differentiating twice with respect to e_c, and then plugging in $e_p \sim z^m$, $e_o \sim z^n$ and $e_c \sim z^q$, we find that the second order condition is satisfied, i.e., $d^2\Pi_c/de_c^2 < 0$ for all e_c. Analyzing the first order condition and collecting terms in powers of z, we obtain

$$E_1 z^0 + E_2 z^q + Y_1(z) = 0\,, \tag{A.61}$$

where $E_1,\ E_2 \in \mathbb{R}$ and $Y_1(z)$ is a polynomial in z with terms that will be dominated by $E_1 z^0$ in absolute value. As $z \to \infty$, equation (A.61) has to hold for all z values. This is not possible if $q > 0$ because in that case $E_1 z^q$ dominates all other terms in absolute value. Therefore, in equilibrium $E_1 z^0$ and $E_2 z^q$ have to cancel each other, which implies $q = 0$.

Because $m > n > q = 0$, by (A.28), $\Pi_c \sim z^0$, which directly implies that there exists a constant $\bar{V} > 0$ such that $\Pi_c < \bar{V}$. Hence, the contributor would prefer the outside option if $V_c > \bar{V}$. However, if $V_c < \underline{V} = O(b) > 0$, then there exists $\bar{b}$ such that $\Pi_c > V_c$ when $b < \bar{b}$. Therefore, the contributor will not deviate to the outside option in equilibrium under the conditions of Region L. To effectuate $\Pi_c < V_c$ would also require $n \le 0$ due to cross effects, which is suboptimal from the point of view of the originator. Arguments as to why the proprietor lacks incentive to push the contributor out (whether directly or indirectly via pressure on the originator) are similar in spirit.

Taking a total derivative of (A.30) with respect to e_o (and another one to show concavity as before), and substituting $e_p \sim z^m$, $e_o \sim z^n$ and $e_c \sim z^0$ into the first order condition, we obtain

$$E_3 z^0 + E_4 z^{n-1} + Y_2(z) = 0\,, \tag{A.62}$$

where $E_3,\ E_4 \in \mathbb{R}$ and $Y_2(z)$ is a polynomial in z with terms that will be dominated by $E_3 z^0$ in absolute value. Similar to previous argument, we can obtain $n = 1$.

Substituting $e_p \sim z^m$, $e_o \sim z^1$ and $e_c \sim z^0$ into the first order condition of the proprietor, we obtain

$$E_5 z^0 + E_6 z^{m-1} + Y_3(z) = 0\,, \tag{A.63}$$

where $E_5,\ E_6 \in \mathbb{R}$ and $Y_3(z)$ is a polynomial in z with terms that will be dominated by $E_5 z^0$ in absolute value. Similarly, we establish concavity and obtain $m = 1$. However, because $m = 1$ and $n = 1$, it contradicts $m > n$. Hence, $n - 1 < m \leq n$. Following the same process as above, we can rule out $m < n$, and obtain that $m = n = 1$ and $q = 0$. Therefore, we can write the expression of the efforts as

$$e_p = P_1 z + O(1),\ e_o = O_1 z + O(1),\ and\ e_c = C_1 + O(1/z)\,. \tag{A.64}$$

We can still have $Q_o > Q_p$ if $s_p P_1 < s_o O_1$. We discuss below the two scenarios where either the proprietor or the originator is the quality leader, corresponding to the statements in part (i) and (ii) in Lemma A.4. Substituting (A.64) into (A.61) and solving $E_1 + E_2 = 0$, we obtain that C_1 has to satisfy the following:

$$C_1 = \begin{cases} \frac{(O_1 s_o - P_1 s_p)(-6\hat{k}_{sco} O_1 P_1 s_{oc} s_p + \hat{k}_{sc}(O_1 s_o - 4P_1 s_p)(O_1 s_o - P_1 s_p))}{4\hat{k}_c (O_1 s_o - 4P_1 s_p)^3} & if\ s_p P_1 > s_o O_1\,; \\ \frac{(O_1 s_o - P_1 s_p)(6\hat{k}_{sco} O_1 P_1 s_{oc} s_p + \hat{k}_{sc}(O_1 s_o - P_1 s_p)(4O_1 s_o - P_1 s_p))}{4\hat{k}_c (4O_1 s_o - P_1 s_p)^3} & if\ s_p P_1 \leq s_o O_1\,. \end{cases} \tag{A.65}$$

We show below that under the conditions of Region L there exists a $\bar{t} > 0$ such that when $t = \frac{s_p^2 \hat{k}_o}{s_o^2 \hat{k}_c} > \bar{t}$, the proprietor will be the quality leader; when $t < \bar{t}$, the originator will be the quality leader.

Suppose that the proprietor is the quality leader. We can substitute (A.65) into (A.62) and obtain the first order condition for the originator:

$$\hat{k}_o O_1 (O_1 s_o - 4P_1 s_p)^3 + P_1^2 s_o s_p^2 (-7O_1 s_o + 4P_1 s_p) = 0\,. \tag{A.66}$$

Given P_1, if the originator is forced to optimize as a quality follower, we have the optimal $\hat{O}_1$ solves

$$f_o(O_1) = P_1^2 s_o s_p^2 \left(-7O_1 s_o + 4P_1 s_p\right) + O_1 \left(O_1 s_o - 4P_1 s_p\right)^3 \hat{k}_o = 0\,. \tag{A.67}$$

Suppose such real root of (A.67) exists. We can obtain the profit by substituting $\hat{O}_1$ into (8):

$$\hat{\Pi}_o = \frac{P_1^3 s_o^2 s_p^3(-7\hat{O}_1 s_o + 4P_1 s_p)\left(2\hat{O}_1^2 s_o^2 - 3\hat{O}_1 P_1 s_o s_p + 4P_1^2 s_p^2\right)}{2\hat{k}_o(\hat{O}_1 s_o - 4P_1 s_p)^6} \cdot z + O(1)\,. \tag{A.68}$$

By the envelope theorem and $s_p P_1 > s_o \hat{O}_1$, we obtain

$$\frac{d\hat{\Pi}_o}{dP_1} = \frac{\partial \hat{\Pi}_o}{\partial P_1} = \frac{\hat{O}_1 P_1^2 s_o^3 s_p^3(\hat{O}_1 s_o + 2P_1 s_p)\left(21\hat{O}_1^2 s_o^2 - 16\hat{O}_1 P_1 s_o s_p + 16P_1^2 s_p^2\right)}{\hat{k}_o(4P_1 s_p - \hat{O}_1 s_o)^7} \cdot z + O(1) > 0. \tag{A.69}$$

Note that a positive profit requires that $\hat{O}_1 < 4P_1 s_p/7s_o$. Because $f_o(0) = 4P_1^3 s_o s_p^3 > 0$, $f_o(4P_1 s_p/7s_o) = -55296 k_o P_1^4 s_p^4/2401 s_o < 0$, and

$$\frac{df_o(O_1)}{dO_1} = -7P_1^2 s_o^2 s_p^2 + 4\left(O_1 s_o - 4P_1 s_p\right)^2\left(O_1 s_o - P_1 s_p\right)\hat{k}_o < 0 \tag{A.70}$$

for all $O_1 \leq P_1 s_p/s_o$, we obtain the unique root $\hat{O}_1 \in (0,\ \frac{4P_1 s_p}{7s_o})$ by the intermediate value theorem. Let $r = \frac{s_o \hat{O}_1}{s_p P_1}$, then there exists a unique $r \in (0,\ \frac{4}{7})$. Plugging r into (A.67), we can obtain

$$P_1 = \frac{(-4+7r)s_o^2}{(-4+r)^3 r s_p \hat{k}_o} \text{ and } O_1 = \frac{(-4+7r)s_o}{(-4+r)^3\hat{k}_o}. \tag{A.71}$$

Turning to the proprietor's decision as the quality leader, we can obtain the first order condition of the proprietor as

$$\begin{aligned} f_p(P_1) \quad &= 2P_1^2 s_o^2 s_p^2(7\hat{O}_1 s_o + 8P_1 s_p)\left(\hat{k}_p(\hat{O}_1 s_o - 4P_1 s_p)^2 + 4s_p^2(\hat{O}_1 s_o - P_1 s_p)\right) + \\ &\hat{k}_o(\hat{O}_1 s_o - 4P_1 s_p)^3\left(\hat{k}_p(\hat{O}_1 s_o - 4P_1 s_p)^3 + 4s_p^2\left(2\hat{O}_1^2 s_o^2 - 3\hat{O}_1 P_1 s_o s_p + 4P_1^2 s_p^2\right)\right) \\ &= 0\,. \end{aligned} \tag{A.72}$$

Substituting (A.71) into (A.72), we obtain

$$4r\left(64 - 112r + 132r^2 - 139r^3 + 28r^4\right)t + \left(-64 + 176r - 196r^2 + 147r^3\right) = 0\,, \tag{A.73}$$

where $t = \frac{\hat{k}_o s_p^2}{\hat{k}_p s_o^2}$. Given t, we can solve (A.73) for the unique solution r. Substituting (A.71) into (A.65), we can obtain the originator's profit $\hat{\Pi}_o$ and the proprietor's profit $\hat{\Pi}_p$ when the proprietor is the quality leader as

$$\hat{\Pi}_o = \frac{(4-7r)(4-3r+2r^2)s_o^2}{2\hat{k}_o(4-r)^6} \cdot z + O(1)\,, \tag{A.74}$$

and

$$\hat{\Pi}_p = \frac{(4-7r)\left(8rt(4-r)(1-r)-(4-7r)\right)s_o^2}{2r^2t(4-r)^6\hat{k}_o}\cdot z + O(1)\,. \tag{A.75}$$

Now suppose that the originator is the quality leader. Substituting (A.65) into (A.62), we can obtain the originator's first order condition:

$$4s_o^2\left(4O_1^2s_o^2 - 3O_1P_1s_os_p + 2P_1^2s_p^2\right) - \hat{k}_o(4O_1s_o - P_1s_p)^3 = 0\,. \tag{A.76}$$

Given P_1, if the originator is forced to optimize as the quality leader, we have the optimal $\tilde{O}_1$ solves

$$g_o(O_1) = 4s_o^2\left(4O_1^2s_o^2 - 3O_1P_1s_os_p + 2P_1^2s_p^2\right) - \left(4O_1s_o - P_1s_p\right)^3\hat{k}_o = 0\,. \tag{A.77}$$

Suppose such $\tilde{O}_1$ exists. We obtain the profit of the originator in this case:

$$\tilde{\Pi}_o = \frac{8\tilde{O}_1^3s_o^5(4\tilde{O}_1s_o - 7P_1s_p)\left(4\tilde{O}_1^2s_o^2 - 3\tilde{O}_1P_1s_os_p + 2P_1^2s_p^2\right)}{k_o(-4\tilde{O}_1s_o + P_1s_p)^6}\cdot z + O(1). \tag{A.78}$$

By envelope theorem and $P_1s_p \le s_o\tilde{O}_1$, we have

$$\frac{d\tilde{\Pi}_o}{dP_1} = \frac{\partial\tilde{\Pi}_o}{\partial P_1} = -\frac{16\tilde{O}_1^3s_o^5s_p(2\tilde{O}_1s_o + P_1s_p)\left(16\tilde{O}_1^2s_o^2 - 16\tilde{O}_1P_1s_os_p + 21P_1^2s_p^2\right)}{k_o(4\tilde{O}_1s_o - P_1s_p)^7}\cdot z + O(1) < 0. \tag{A.79}$$

We can see that g_o is a third order polynomial of O_1, and

$$\frac{dg_o}{dO_1} = -192O_1^2s_o^3\hat{k}_o + O_1\left(32s_o^4 + 96P_1s_o^2s_p\hat{k}_o\right) - 12P_1s_o^3s_p - 12P_1^2s_os_p^2\hat{k}_o\,, \tag{A.80}$$

which is a quadratic function of O_1. We use (A.77) and (A.80) to examine the existence of optimal $\tilde{O}_1$. First, note that a positive profit in (A.78) requires that $\tilde{O}_1 > 7P_1s_p/4s_o$. From (A.77), we can obtain that $g_o(\infty) < 0$, $g_o(\frac{7P_1s_p}{4s_o}) \le 0$ if $P_1 \ge \frac{s_o^2}{6\hat{k}_os_p}$, and $g_o(\frac{7P_1s_p}{4s_o}) > 0$ if $P_1 < \frac{s_o^2}{6\hat{k}_os_p}$. Second, if $P_1 > \frac{s_o^2}{3s_p\hat{k}_o}$, $\frac{dg_o}{dO_1} < 0$ according to (A.80), and (A.77) has only one real root. Third, if $P_1 \le \frac{s_o^2}{3s_p\hat{k}_o}$, the polynomial (A.80) has two roots:

$$r_1 = \frac{s_o^3 + 3P_1s_os_p\hat{k}_o - \sqrt{s_o^6 - 3P_1s_o^4s_p\hat{k}_o}}{12s_o^2\hat{k}_o} \text{ and } r_2 = \frac{s_o^3 + 3P_1s_os_p\hat{k}_o + \sqrt{s_o^6 - 3P_1s_o^4s_p\hat{k}_o}}{12s_o^2\hat{k}_o}. \tag{A.81}$$

We can obtain that $g_o(r_1) > 0$ and $g_o(r_2) > 0$. Further, $r_2 > \frac{7P_1s_p}{4s_o}$ if $P_1 < \frac{11s_o^2}{108\hat{k}_os_p}$. Combining the above three results, we obtain that (A.77) has one unique real root $\tilde{O}_1 \in (\frac{7P_1s_p}{4s_o}, \infty)$ if $P_1 < \bar{P}_1 = \frac{s_o^2}{6\hat{k}_os_p}$. However, if $P_1 \ge \bar{P}_1$, $g_o < 0$ for any $O_1 \in (\frac{7P_1s_p}{4s_o}, \infty)$ and (A.77) has no real root in the range. This means that the originator would prefer to set $\tilde{O}_1 = \frac{7P_1 s_p}{4s_o}$ and make zero profit if $P_1 \geq \bar{P}_1$ and the originator is forced to optimize as the quality leader. But the originator can obtain a positive profit by becoming a quality follower when $P_1 \geq \bar{P}_1$ as we have shown above.

Now suppose $P_1 < \bar{P}_1$, we can also obtain the first order condition for the proprietor as

$$\begin{aligned} g_p(P_1) = -8P_1^2 s_o^2 s_p^2 (5\tilde{O}_1 s_o + P_1 s_p) \left(\hat{k}_p P_1 (-4\tilde{O}_1 s_o + P_1 s_p)^2 + \tilde{O}_1 s_o s_p (-\tilde{O}_1 s_o + P_1 s_p)\right) - \\ \hat{k}_o (4\tilde{O}_1 s_o - P_1 s_p)^3 \left(\hat{k}_p P_1 (4\tilde{O}_1 s_o - P_1 s_p)^3 + \tilde{O}_1^2 s_o^2 s_p (-4\tilde{O}_1 s_o + 7P_1 s_p)\right) = 0. \end{aligned} \tag{A.82}$$

We have shown that (A.77) has only one unique root $\tilde{O}_1 \in (\frac{7P_1 s_p}{4s_o},\ \infty)$ if $P_1 < \bar{P}_1$. Let $\tilde{r} = \frac{s_o \tilde{O}_1}{s_p P_1}$, then there exists a unique $\tilde{r} \in (\frac{7}{4},\ \infty)$. Combining (A.82) and (A.77), we can obtain

$$P_1 = \frac{4\left(2 - 3\tilde{r} + 4\tilde{r}^2\right) s_o^2}{(-1 + 4\tilde{r})^3 s_p \hat{k}_o},\ O_1 = \frac{4\tilde{r}\left(2 - 3\tilde{r} + 4\tilde{r}^2\right) s_o}{(-1 + 4\tilde{r})^3 \hat{k}_o}, \tag{A.83}$$

where $\tilde{r}$ solves

$$\left(2 + 14\tilde{r} - 127\tilde{r}^2 + 196\tilde{r}^3 - 176\tilde{r}^4 + 64\tilde{r}^5\right) t - 4\left(42 - 95\tilde{r} + 164\tilde{r}^2 - 112\tilde{r}^3 + 64\tilde{r}^4\right) = 0\,. \tag{A.84}$$

Given t, we can solve for $\tilde{r}$ and obtain the originator's profit $\tilde{\Pi}_o$ and the proprietor's profit $\tilde{\Pi}_p$ in this case:

$$\tilde{\Pi}_o = \frac{8\tilde{r}^3 (4\tilde{r} - 7)\left(4\tilde{r}^2 - 3\tilde{r} + 2\right) s_o^2}{\hat{k}_o (1 - 4\tilde{r})^6} \cdot z + O(1)\,, \tag{A.85}$$

and

$$\tilde{\Pi}_p = \frac{4\left(2 - 3\tilde{r} + 4\tilde{r}^2\right)\left(-4 + (6 + t)\tilde{r} - (8 + 5t)\tilde{r}^2 + 4t\tilde{r}^3\right) s_o^2}{t\,(1 - 4\tilde{r})^6\, \hat{k}_o} \cdot z + O(1)\,. \tag{A.86}$$

We now compare the two scenarios to determine the equilibrium efforts of the proprietor and originator. We first show that when the proprietor is the quality leader, there exists $\bar{t} > 0$ such that $P_1(t) > \bar{P}_1$ if $t > \bar{t}$ and $P_1(t) \leq \bar{P}_1$ if $t \leq \bar{t}$.

According to (A.73), we can obtain

$$t = \frac{64 - 176r + 196r^2 - 147r^3}{4r\left(64 - 112r + 132r^2 - 139r^3 + 28r^4\right)}. \tag{A.87}$$

By the inverse function theorem, $r'(t) = \frac{1}{t'(r)}$. We then can obtain the sign of $r'(t)$ by looking at $t'(r)$'s sign:

$$t'(r) = \frac{-4096 + 14336r - 32512r^2 + 63232r^3 - 91760r^4 + 74200r^5 - 36897r^6 + 8232r^7}{4r^2\left(64 - 112r + 132r^2 - 139r^3 + 28r^4\right)^2} < 0$$

for $r \in (0,\ \frac{4}{7})$. Similarly, according to (A.71), we can obtain that

$$\frac{dP_1}{dr} = -\frac{(16-16r+21r^2)s_o^2}{(4-r)^2\hat{k}_o s_p} < 0 \tag{A.88}$$

for $r \in (0, \frac{4}{7})$. Hence, we obtain that P_1 increases with t. Then we can solve $P_1 = \bar{P}_1$ and obtain that $\bar{t} \approx 0.695$. Therefore, when the proprietor is the quality leader, $P_1 > \bar{P}_1$ if $t > \bar{t}$ and $P_1 \leq \bar{P}_1$ otherwise.

Because we have shown that the originator would prefer to be the quality follower if $P_1 > \bar{P}_1$, we now show that the proprietor would prefer to be the quality leader if $t > \bar{t}$ and the originator would prefer to be the quality leader when $t < \bar{t}$. Technically, we show that $\tilde{\Pi}_o > \hat{\Pi}_o \, \forall t < \bar{t}$, and $\hat{\Pi}_p > \tilde{\Pi}_p \, \forall t > \bar{t}$.

We first examine the originator's profit as the quality follower ($\hat{\Pi}_o$) and leader ($\tilde{\Pi}_o$). According to (A.68) and (A.85), $\hat{\Pi}_o(P_1 = 0) < \tilde{\Pi}_o(P_1 = 0)$ and $\hat{\Pi}_o(P_1 = \bar{P}_1) < \tilde{\Pi}_o(P_1 = \bar{P}_1)$. And we have shown that $\frac{d\hat{\Pi}_o}{dP_1} > 0$ in (A.69) and $\frac{d\tilde{\Pi}_o}{dP_1} < 0$ in (A.79). Therefore, $\forall t < \bar{t}$, $\tilde{\Pi}_o > \hat{\Pi}_o$. This means that the originator would prefer to be the quality leader when $t < \bar{t}$.

Comparing the profits in (A.75) and (A.86), we can obtain that $\hat{\Pi}_p(t = \bar{t}) > \tilde{\Pi}_p(t = \bar{t})$. We next evaluate $\hat{\Pi}_p$ and $\tilde{\Pi}_p$ when $t > \bar{t}$. By the envelope theorem, we can obtain

$$\frac{d\hat{\Pi}_p}{dt} = \frac{(4-7r)^2 s_o^2 z}{2(-4+r)^6 r^2 t^2 \hat{k}_o} + O(1) \tag{A.89}$$

and

$$\frac{d\tilde{\Pi}_p}{dt} = \frac{8\left(2-3\tilde{r}+4\tilde{r}^2\right)^2 s_o^2 z}{t^2\left(1-4\tilde{r}\right)^6 \hat{k}_o} + O(1). \tag{A.90}$$

Because $r_t \approx 0.254$ and $\tilde{r}_t \approx 6.935$ when $t = \bar{t}$, $r \in (0,\ r_t)$ and $\tilde{r} \in (\frac{7}{4},\ \tilde{r}_t)$ for $t > \bar{t}$. By investigating (A.89) and (A.90) within these ranges, we can obtain that $\frac{d\hat{\Pi}_p}{dt} > \frac{d\tilde{\Pi}_p}{dt}$ for all $t \geq \bar{t}$. Therefore, the proprietor would prefer be the quality leader when $t > \bar{t}$.

According to the analysis above, we can conclude that if $t > \bar{t}$, the proprietor is the quality leader, and

$$P_1 = \frac{M_1 s_o^2}{r M_2^3 \hat{k}_o s_p},\ O_1 = \frac{M_1 s_o}{M_2^3 \hat{k}_o},\ and\ C_1 = \frac{M_3(\hat{k}_{s_c} M_2 M_3 s_o - 6\hat{k}_{s_{co}} r s_{oc})}{4\hat{k}_c M_2^3 s_o}, \tag{A.91}$$

where $M_1 = 4-7r$, $M_2 = 4-r$, $M_3 = 1-r$, and $r \in (0,\ r_t)$ is the unique root of (A.73). Note for $C_1 > 0$, we need $s_{oc} < \frac{M_2 M_3 \hat{k}_{s_c}}{6r\hat{k}_{s_{co}}} s_o$. Define $\lambda_L = M_2 M_3 / 6r$. We can obtain $\lambda_L \geq 1.83\ \forall r \in (0,\ r_t)$. Therefore, given $s_{oc} < \bar{s}_{oc} = \lambda_L s_o \hat{k}_{s_c} / \hat{k}_{s_{co}}$, we have $C_1 > 0$ and hence $e_c^* > 0$ in Region L.

Substituting P_1, O_1, and C_1 into the first order conditions, following similar process as above, we can find the second term in the proprietor's equilibrium effort level as in (A.50), where $M_4 = 16-$

$16r+21r^2$ and $M_5=-256-1920r+1888r^2-1160r^3-189r^4+98r^5$.

We also define $M_6=\hat{k}_c\hat{k}_{s_{co}}\big(17408-151808r+437760r^2-479680r^3+211804r^4+24621r^5-100516r^6+19411r^7+588r^8\big)s_o^2+\hat{k}_o s_{oc}\big(\hat{k}_{s_c}\big(11264-21248r-41472r^2+113216r^3-100076r^4+46239r^5-874r^6-2093r^7+147r^8\big)s_o+6\hat{k}_{s_{co}}\big(2048-8192r-576r^2+19120r^3-16204r^4+7455r^5+3234r^6\big)s_{oc}\big)$ in the above expression.

Similarly, we can obtain the second term in the originator's equilibrium effort level as

$$\frac{M_7}{2\hat{k}_c\hat{k}_o M_2^3 s_o^2\left(\hat{k}_p M_4^3 s_o^2+24\hat{k}_o M_5 r^3 s_p^2\right)}, \tag{A.92}$$

where

$$\begin{aligned}M_7= & -2\hat{k}_c\hat{k}_{s_{co}}s_o^2\big(\hat{k}_p M_4^2\big(32-136r+128r^2+85r^3-116r^4+7r^5\big)s_o^2+4\hat{k}_o r^3\big(-3072-4352r+39680r^2\\ & -12672r^3-15620r^4-4783r^5-17520r^6+3367r^7+392r^8\big)s_p^2\big)+\hat{k}_o r s_{oc}\big(\hat{k}_p M_4^2 s_o^2\big(\hat{k}_{s_c}\big(-80+188r\\ & -102r^2-13r^3+7r^4\big)s_o-3\hat{k}_{s_{co}}\big(16-68r+40r^2+21r^3\big)s_{oc}\big)+8\hat{k}_o r^3\big(\hat{k}_{s_c}\big(-1792+23680r-\\ & 35040r^2+16712r^3-6611r^4-951r^5+308r^6+49r^7\big)s_o-3\hat{k}_{s_{co}}\big(1280-7616r+11184r^2-3164r^3\\ & +3506r^4+2373r^5+294r^6\big)s_{oc}\big)s_p^2\big).\end{aligned}$$

Similarly, if $t\leq\bar{t}$, the originator is the quality leader, we can follow the same procedure to obtain that the contributor's equilibrium effort level follows

$$e_c^*=\frac{\tilde{M}_3(\hat{k}_{s_c}\tilde{M}_2\tilde{M}_3 s_o+6\hat{k}_{s_{co}}\tilde{r}s_{oc})}{4\hat{k}_c\tilde{M}_2^3 s_o}+O(b), \tag{A.93}$$

where $\tilde{M}_2=4\tilde{r}-1$ and $\tilde{M}_3=\tilde{r}-1$. Also, we can obtain the originator's equilibrium effort level is

$$e_o^*=\frac{4\tilde{M}_1\tilde{r}s_o}{\hat{k}_o\tilde{M}_2^3}\cdot\frac{1}{b}+\frac{\tilde{M}_9}{2\hat{k}_c\hat{k}_o\tilde{M}_2^3\tilde{r}^2 s_o^2\left(\hat{k}_o\tilde{M}_5 s_p^2+2\hat{k}_p\tilde{M}_4^3 s_o^2\right)}+O(b), \tag{A.94}$$

where $\tilde{M}_1=2-3\tilde{r}+4\tilde{r}^2$, $\tilde{M}_4=21-16\tilde{r}+16\tilde{r}^2$, and $\tilde{M}_5=-415+1779\tilde{r}-720\tilde{r}^2+608\tilde{r}^3+1920\tilde{r}^4-2304\tilde{r}^5+2048\tilde{r}^6$. Above, we also define:

$$\begin{aligned}\tilde{M}_9= & -8\hat{k}_c\hat{k}_{s_{co}}\tilde{r}^2 s_o^2\big(6\hat{k}_p\tilde{M}_4^2\big(-2+5\tilde{r}-11\tilde{r}^2+14\tilde{r}^3-14\tilde{r}^4+8\tilde{r}^5\big)s_o^2+\hat{k}_o(1-4\tilde{r})^2\big(15-350\tilde{r}+470\tilde{r}^2\\ & -399\tilde{r}^3-70\tilde{r}^4+364\tilde{r}^5-384\tilde{r}^6+192\tilde{r}^7\big)s_p^2\big)+\hat{k}_o s_{oc}\big(2\hat{k}_p\tilde{M}_4^2\tilde{r}s_o^2\big(\hat{k}_{s_c}\big(1+5\tilde{r}-60\tilde{r}^2+122\tilde{r}^3-116\tilde{r}^4\\ & +48\tilde{r}^5\big)s_o+3\hat{k}_{s_{co}}\tilde{r}\big(4+17\tilde{r}-32\tilde{r}^2+36\tilde{r}^3-16\tilde{r}^4\big)s_{oc}\big)+\hat{k}_o\big(\hat{k}_{s_c}\big(\tilde{M}_2+\tilde{r}^2\big(24+1118\tilde{r}-6548\tilde{r}^2\\ & +8121\tilde{r}^3-4720\tilde{r}^4-5216\tilde{r}^5+12672\tilde{r}^6-13056\tilde{r}^7+6144\tilde{r}^8\big)\big)s_o+3\hat{k}_{s_{co}}\tilde{r}^2\big(24+519\tilde{r}+2243\tilde{r}^2\\ & -2178\tilde{r}^3+3332\tilde{r}^4+1104\tilde{r}^5-3648\tilde{r}^6+4864\tilde{r}^7-2048\tilde{r}^8\big)s_{oc}\big)s_p^2\big).\end{aligned}$$

Lastly, we can obtain the proprietor's equilibrium effort level as

$$e_p^* = \frac{4\tilde{M}_1 s_o^2}{\hat{k}_o \tilde{M}_2^3 s_p} \cdot \frac{1}{b} + \frac{\left(8\hat{k}_c \hat{k}_{s_{co}} \tilde{M}_6 s_o^2 + \hat{k}_o s_{oc}(\hat{k}_{s_c} \tilde{M}_7 s_o + 6\hat{k}_{s_{co}} \tilde{M}_8 s_{oc})\right) s_p}{4\hat{k}_c \tilde{M}_2^3 \tilde{r}^2 s_o \left(2\hat{k}_p \tilde{M}_4^3 s_o^2 + \hat{k}_o \tilde{M}_5 s_p^2\right)} + O(b), \tag{A.95}$$

where $\tilde{M}_6 = \tilde{r}^2(2175 - 17664\tilde{r} + 51550\tilde{r}^2 - 79716\tilde{r}^3 + 76302\tilde{r}^4 - 32792\tilde{r}^5 - 12672\tilde{r}^6 + 33792\tilde{r}^7 - 22016\tilde{r}^8 + 6144\tilde{r}^9)$.

The rest parameters are defined as $\tilde{M}_7 = -21 + 620\tilde{r} - 2380\tilde{r}^2 + 1983\tilde{r}^3 - 4565\tilde{r}^4 + 2684\tilde{r}^5 - 3936\tilde{r}^6 - 2816\tilde{r}^7 + 10496\tilde{r}^8 - 15360\tilde{r}^9 + 8192\tilde{r}^{10}$ and $\tilde{M}_8 = \tilde{r}^2(1082 - 183\tilde{r} + 185\tilde{r}^2 + 3252\tilde{r}^3 - 1776\tilde{r}^4 + 1600\tilde{r}^5 + 1024\tilde{r}^6)$.

Part (iii): Similar to parts (i) and (ii), we define $z = 1/b$ and examine e_p^*, e_o^*, and e_c^* as b becomes small. Further, we can express $e_p^* \sim z^m$, $e_o^* \sim z^n$, and $e_c^* \sim z^q$ for some m, n, $q \in \mathbb{R}$. By $Q_p = s_p e_p$, (1), (2), and the definition of Region L, $Q_p \sim z^m$, $Q_o \sim z^{\max(n+1,\, q)}$, and $Q_c \sim z^{\max(n,\, q)}$.

Suppose $\max(n+1, q) \leq \max(n, q)$ in equilibrium, which implies that $n + 1 \leq q$. By Lemma A.2, p_o and p_c are $O(1)$. Because $|\Theta| = 1$, $\tilde{\Pi}_o$ and $\tilde{\Pi}_c$ are $O(1)$. The effort of the contributor costs $O(z^{2q-1})$. To generate non-negative profit, $z^0 \geq z^{2q-1}$ should be satisfied, which gives us $q \leq \frac{1}{2}$. Because $n + 1 \leq q$, we can derive that $n \leq -\frac{1}{2}$. However, suppose the originator instead selects $n = \frac{1}{2}$. Then we can derive $p_o \sim z^{\frac{1}{2}}$ and $\tilde{\Pi}_o \sim z^{\frac{1}{2}}$, whereas costs are $O(z^{2(\frac{1}{2})-1}) = O(1)$, and hence $\Pi_o \sim z^{\frac{1}{2}}$ such that the originator will deviate. Therefore, $\max(n+1, q) > \max(n, q)$ and $n + 1 > q$.

Suppose $m \geq \max(n, q)$ in equilibrium. First, suppose that $m \leq 0$. Then, $\tilde{\Pi}_p$ is $O(1)$. However, suppose the proprietor instead select $m = \frac{1}{2}$. Then we can derive that $\Pi_p \sim z^{\frac{1}{2}}$ such that the proprietor will deviate. Hence, $m > 0$. Second, given $m > 0$, for the proprietor to generate non-negative profit, $z^m \geq z^{2m-1}$ should be satisfied, which gives us $m \leq 1$. Because $m \geq \max(n, q)$, $\max(n, q) \leq 1$. Then, for the originator, we can show that $\tilde{\Pi}_o \sim z^{n+1} \leq z^2$. However, suppose the originator instead selects $n = \frac{3}{2}$. Then $\Pi_o \sim z^{\frac{5}{2}}$, which means that the originator will deviate. Therefore, $m < \max(n, q)$ in equilibrium.

Suppose $m < 0$. Then, we can derive that $\tilde{\Pi}_p$ is $O(1)$. As noted above, we can show that the proprietor would derive by setting $m = \frac{1}{2}$. Hence, $m \geq 0$. Given that $n + 1 > q$, $m < \max(n, q)$, and $m \geq 0$, we obtain that $Q_o > Q_c > Q_p > c$.

Given $Q_o > Q_c > Q_p > c$, the contributor's profit is given by (A.28). Substituting $Q_p = s_p e_p$, (1), and (2) into (A.28), differentiating twice with respect to e_c, and then plugging in $e_p \sim z^m$, $e_o \sim z^n$ and $e_c \sim z^q$, we find that the second order condition is satisfied, i.e., $d^2\Pi_c/de_c^2 < 0$ for all e_c. Analyzing the first order condition and collecting terms in powers of z, we obtain

$$E_1 z^0 + E_2 z^{q-1} + Y_1(z) = 0\,, \tag{A.96}$$

where $E_1,\ E_2 \in \mathbb{R}$ and $Y_1(z)$ is a polynomial in z with terms that will be dominated by $E_1 z^0$ in absolute value. As $z \to \infty$, equation (A.96) has to hold for all z values. This is not possible if $q > 1$ because in that case $E_1 z^{q-1}$ dominates all other terms in absolute value. Therefore, in equilibrium $E_1 z^0$ and $E_2 z^{q-1}$ have to cancel each other, which implies $q = 1$.

Because $n + 1 > q = 1,\ n > 1$. By (A.28), $\Pi_c \sim z^n$, which directly implies the existence of $\bar{b}$ such that $\Pi_c > V_c$ when $b < \bar{b}$. Effectuating $\Pi_c < V_c$ would require $n \leq 0$ due to cross effects, which is suboptimal from the point of view of the originator. Arguments as to why the proprietor lacks incentive to push the contributor out (whether directly or indirectly via pressure on the originator) are similar in spirit.

Taking a total derivative of (A.30) with respect to e_o (and another one to show concavity as before), and substituting $e_p \sim z^m$, $e_o \sim z^n$ and $e_c \sim z^1$ into the first order condition, we obtain

$$E_3 z^1 + E_4 z^{n-1} + Y_2(z) = 0\,, \tag{A.97}$$

where $E_3,\ E_4 \in \mathbb{R}$ and $Y_2(z)$ is a polynomial in z with terms that will be dominated by $E_3 z^1$ in absolute value. Similar to previous argument, we can obtain $n = 2$.

Substituting $e_p \sim z^m$, $e_o \sim z^2$ and $e_c \sim z^1$ into the first order condition of the proprietor, we obtain

$$E_5 z^1 + E_6 z^m + Y_3(z) = 0\,, \tag{A.98}$$

where $E_5,\ E_6 \in \mathbb{R}$ and $Y_3(z)$ is a polynomial in z with terms that will be dominated by $E_5 z^1$ in absolute value. Similarly, we establish concavity and obtain $m = 1$. Therefore, we can write the expression of the efforts as

$$e_p = P_1 z + O(1),\ e_o = O_1 z^2 + O(z),\ and\ e_c = C_1 z + O(1)\,. \tag{A.99}$$

Substituting (A.99) into the three first order conditions and equating the lead coefficients of the highest order terms with respect to z to zero, we obtain P_1, O_1, and C_1 as:

$$C_1 = \frac{s_c}{16\hat{k}_c},\ O_1 = \frac{\hat{k}_{s_o}}{4\hat{k}_o},\ and\ P_1 = \frac{s_p}{64\hat{k}_p}. \tag{A.100}$$

Repeating similar steps as above, we obtain the optimal effort investment level for the proprietor as

$$e_p^* = \frac{s_p}{64\hat{k}_p} \cdot \frac{1}{b} - \frac{\left(\hat{k}_o s_p^3 + 24\hat{k}_p s_{co}^2 s_p\right)}{1024\hat{k}_p^2 s_{co}\hat{k}_{s_o}} + \frac{\hat{M}_1}{\hat{M}_2} \cdot b + \frac{\hat{M}_3 + \hat{M}_4 s_{oc}}{\hat{M}_5} \cdot b^2 + O(b^3), \tag{A.101}$$

where $\hat{M}_1 = \hat{k}_c\hat{k}_o^2 s_p^5 + 768\hat{k}_p^2 s_{co}^2 s_p\left(\hat{k}_c s_{co}^2 - 2\hat{k}_o s_c^2\right) + 64\hat{k}_o\hat{k}_p s_p^3\left(19\hat{k}_c s_{co}^2 + \hat{k}_o s_c^2\right)$ and $\hat{M}_2 = 262144\hat{k}_c\hat{k}_p^3\hat{k}_{s_o}^2 s_{co}^2$.

In the above expression, we also define that $\hat{M}_3 = 2048\hat{k}_c\hat{k}_p^3 s_{co}^2 s_p\big(512\hat{k}\hat{k}_c\hat{k}_o\hat{k}_{s_o}^2 + 6\hat{k}_c s_{co}^4 + 9\hat{k}_o s_c^2 s_{co}^2\big) + 21\hat{k}_c^2\hat{k}_o^3 s_p^7 - 256\hat{k}_o\hat{k}_p^2 s_p^3\big(108\hat{k}_c^2 s_{co}^4 + \hat{k}_o^2 s_c^4\big) - 8\hat{k}_c\hat{k}_o^2\hat{k}_p s_p^5\big(178\hat{k}_c s_{co}^2 + \hat{k}_o s_c^2\big)$, $\hat{M}_4 = 24576\hat{k}_c\hat{k}_o\hat{k}_p^3 s_c s_{co}^4 s_p$, and $\hat{M}_5 = 4194304\hat{k}_c^2\hat{k}_p^4\hat{k}_{s_o}^3 s_{co}^3$.

Similarly, we obtain the originator's equilibrium effort level as

$$e_o^* = \frac{\hat{k}_{s_o}}{4\hat{k}_o}\cdot\frac{1}{b^2} - \frac{s_{co}}{8\hat{k}_o}\cdot\frac{1}{b} - \frac{5s_{co}^2}{64\hat{k}_o\hat{k}_{s_o}} + \frac{\hat{M}_6}{\hat{M}_7}\cdot b + \frac{\hat{M}_8 + \hat{M}_9 s_{oc}}{\hat{M}_{10}}\cdot b^2 + O(b^3), \tag{A.102}$$

where $\hat{M}_6 = 3\hat{k}_o^2 s_p^4 - 1024\hat{k}_p^2 s_{co}^4$, $\hat{M}_7 = 32768\hat{k}_o\hat{k}_p^2\hat{k}_{s_o}^2 s_{co}$, and $\hat{M}_8 = -12\hat{k}_o^2\hat{k}_p s_{co}^2\big(11\hat{k}_c^2 s_p^4 + 96\hat{k}_c\hat{k}_p s_c^2 s_p^2 + 256\hat{k}_p^2 s_c^4\big) - 11264\hat{k}_c^2\hat{k}_p^3 s_{co}^6 + \hat{k}_c\hat{k}_o^3 s_p^4\big(32\hat{k}_p s_c^2 - 9\hat{k}_c s_p^2\big)$.

in the above expression, we also define that $\hat{M}_9 = -32\hat{k}_o^2\hat{k}_p s_c\big(5\hat{k}_c\hat{k}_o s_p^4 + 48\hat{k}_c\hat{k}_p s_{co}^2 s_p^2 - 512\hat{k}_p^2 s_c^2 s_{co}^2\big)$ and $\hat{M}_{10} = 1048576\hat{k}_c^2\hat{k}_o\hat{k}_p^3\hat{k}_{s_o}^3 s_{co}^2$.

Finally, we obtain the contributor's equilibrium effort level:

$$e_c^* = \frac{s_c}{16\hat{k}_c}\cdot\frac{1}{b} - \frac{s_c s_{co}}{16\hat{k}_c\hat{k}_{s_o}} + \frac{\hat{M}_{11} + \hat{M}_{12} s_{oc}}{\hat{M}_{13}}\cdot b + O(b^2), \tag{A.103}$$

where $\hat{M}_{11} = 5\hat{k}_c\hat{k}_o^2 s_c s_p^4 + 96\hat{k}_c\hat{k}_o\hat{k}_p s_c s_{co}^2 s_p^2 - 3840\hat{k}_c\hat{k}_p^2 s_c s_{co}^4 - 1024\hat{k}_o\hat{k}_p^2 s_c^3 s_{co}^2$, $\hat{M}_{12} = 2048\hat{k}_c\hat{k}_p^2 s_{co}^4$, and $\hat{M}_{13} = 65536\hat{k}_c^2\hat{k}_p^2\hat{k}_{s_o}^2 s_{co}^2$. ■

Proof of Proposition 3: Technically, we show that when $t = \frac{s_p^2/\hat{k}_p}{s_o^2/\hat{k}_o} \geq \bar{t}$,

(i) $\frac{de_p^*}{ds_{oc}} < 0$, $\frac{de_o^*}{ds_{oc}} < 0$, and $\frac{de_c^*}{ds_{oc}} < 0$;

(ii) $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} < 0$, and $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} < 0$;

(iii) There exists $\hat{s}_{oc} \in (0,\ \bar{s}_{oc})$, such that $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$ for $s_{oc} \in (0,\ \hat{s}_{oc})$, and $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} < 0$ for $s_{oc} \in (\hat{s}_{oc},\ \bar{s}_{oc})$.

(iv) There exists $\check{s}_{oc} \in (0,\ \bar{s}_{oc})$, such that $\frac{dCS}{ds_{oc}} > 0$ for $s_{oc} < \check{s}_{oc}$, and $\frac{dCS}{ds_{oc}} < 0$ for $s_{oc} > \check{s}_{oc}$;

(v) There exists $\tilde{s}_{oc} \in (0,\ \bar{s}_{oc})$, such that $\frac{dSW}{ds_{oc}} > 0$ for $s_{oc} < \tilde{s}_{oc}$, and $\frac{dSW}{ds_{oc}} < 0$ for $s_{oc} > \tilde{s}_{oc}$.

First, differentiating (A.52) with respect to s_{oc}, we obtain

$$\frac{de_c^*}{ds_{oc}} = -\frac{3\hat{k}_{s_{co}} M_3 r}{2\hat{k}_c M_2^3 s_o} + O(b)\,. \tag{A.104}$$

By our proof in Lemma A.4, $r \in (0,\ r_t)$, where $r_t \approx 0.254$. Hence, $M_2 = 4 - r > 0$, and $M_3 = 1 - r > 0$. Therefore, $\frac{de_c^*}{ds_{oc}} < 0$. Next, differentiating (A.51) with respect to s_{oc} and simplify the derivative with (A.73), we obtain

$$\frac{de_o^*}{ds_{oc}} = -\frac{r(A_1\hat{k}_{s_c} s_o + 6A_2\hat{k}_{s_{co}} s_{oc})}{2A_3\hat{k}_c M_2^4 s_o^2} + O(b)\,, \tag{A.105}$$

where $A_1 = 81920 - 417792r + 1068032r^2 - 2010112r^3 + 2796224r^4 - 2511328r^5 + 1398500r^6 - 421336r^7 - 37681r^8 + 35133r^9 - 3430r^{10}$.

Above, we also define that $A_2 = 16384 - 114688r + 326656r^2 - 633856r^3 + 888768r^4 - 763264r^5 + 388020r^6 - 90440r^7 - 78057r^8 + 8232r^9$, and $A_3 = 4096 - 14336r + 32512r^2 - 63232r^3 + 91760r^4 - 74200r^5 + 36897r^6 - 8232r^7$.

Given $r \in (0,\ r_t)$, we obtain that $A_1 > 0$, $A_2 > 0$, and $A_3 > 0$. Therefore, $\frac{de_o^*}{ds_{oc}} < 0$. Further, because $\frac{de_o^*}{ds_{oc}} < 0$ and $\frac{de_c^*}{ds_{oc}} < 0$, we have $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} = s_c \frac{de_c^*}{ds_{oc}} + s_{co} \frac{de_o^*}{ds_{oc}} < 0$.

For the proprietor,

$$\frac{de_p^*}{ds_{oc}} = -\frac{M_1 r (A_4 \hat{k}_{s_c} s_o + 12 A_5 \hat{k}_{s_{co}} s_{oc})}{2 A_3 \hat{k}_c M_2^4 s_o s_p} + O(b)\,, \tag{A.106}$$

where $A_4 = 11264 - 21248r - 41472r^2 + 113216r^3 - 100076r^4 + 46239r^5 - 874r^6 - 2093r^7 + 147r^8$ and $A_5 = 2048 - 8192r - 576r^2 + 19120r^3 - 16204r^4 + 7455r^5 + 3234r^6$. We can obtain that $A_4 > 0$ and $A_5 > 0$ $\forall r \in (0,\ r_t)$. Therefore, we have $\frac{de_p^*}{ds_{oc}} < 0$. Then $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} = s_p \frac{de_p^*}{ds_{oc}} < 0$. This completes the proof of part (i) and (ii).

We now turn to part (iii) in the proposition:

$$\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} = \frac{M_1^2 (A_6 \hat{k}_{s_c} s_o - 12 A_7 \hat{k}_{s_{co}} r s_{oc})}{4 A_3 \hat{k}_c M_2^4 s_o} + O(b)\,, \tag{A.107}$$

where $A_6 = 4096 - 20480r + 44800r^2 - 90624r^3 + 122992r^4 - 96752r^5 + 51825r^6 - 11936r^7 + 967r^8 - 28r^9$, and $A_7 = 2048 - 4864r + 9984r^2 - 16640r^3 + 11464r^4 - 7797r^5$. We can obtain that $r \in (0,\ r_t)$ implies $A_6 > 0$ and $A_7 > 0$, which means there exists $\hat{s}_{oc} = \frac{A_6 \hat{k}_{s_c} s_o}{12 r A_7 \hat{k}_{s_{co}}} \in (0,\ \bar{s}_{oc})$, such that when $s_{oc} < \hat{s}_{oc}$, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$, and when $s_{oc} > \hat{s}_{oc}$, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} < 0$. This completes the proof of part (iii)

For part (iv) and (v), substituting (A.50), (A.51), (A.52) into (10), and differentiating with respect to s_{oc}, we obtain

$$\frac{dCS}{ds_{oc}} = \frac{M_1 (A_8 \hat{k}_{s_c} s_o - 36 A_9 r \hat{k}_{s_{co}} s_{oc})}{8 A_3 \hat{k}_c M_2^6 s_o} + O(b), \tag{A.108}$$

where $A_8 = 114688 - 815104r + 2153472r^2 - 3530496r^3 + 4895808r^4 - 4831248r^5 + 3019908r^6 - 1228905r^7 + 34260r^8 + 51569r^9 - 3920r^{10}$ and $A_9 = 24576 - 95232r + 146176r^2 - 185216r^3 + 195136r^4 - 123172r^5 + 70847r^6 + 21560r^7$. Given $r \in (0,\ r_t)$, we can obtain that $A_8 > 0$ and $A_9 > 0$. Because we have already shown that $A_3 > 0$, we obtain $\check{s}_{oc} = \frac{A_8 s_o \hat{k}_{s_c}}{36 r A_9 \hat{k}_{s_{co}}} \in (0,\ \bar{s}_{oc})$ such that if $s_{oc} < \check{s}_{oc}$, $dCS/ds_{oc} > 0$; if $s_{oc} > \check{s}_{oc}$, $dCS/ds_{oc} < 0$.

Substituting (A.50), (A.51), (A.52) into (11), and differentiating with respect to s_{oc}, we obtain

$$\frac{dSW}{ds_{oc}} = \frac{M_1(A_{10}\hat{k}_{s_c}s_o - 12A_{11}r\hat{k}_{s_{co}}s_{oc})}{8A_3M_2^7M_4\hat{k}_cs_o} + O(b), \tag{A.109}$$

where $A_{10} = 5242880 - 45613056r + 169017344r^2 - 401014784r^3 + 720248832r^4 - 1031356416r^5 + 1280988672r^6 - 1309675776r^7 + 997415568r^8 - 552345224r^9 + 196524663r^{10} - 44932208r^{11} + 5576669r^{12} - 168756r^{13} - 12348r^{14}$.

And we also have $A_{11} = 3145728 - 17498112r + 44400640r^2 - 78249984r^3 + 103397376r^4 - 127801856r^5 + 151077504r^6 - 121846656r^7 + 71804536r^8 - 19019763r^9 + 3870510r^{10} - 74088r^{11}$.

Similarly, we obtain $A_{10} > 0$ and $A_{11} > 0$ for $r \in (0,\ r_t)$. Therefore, we obtain $\tilde{s}_{oc} = \frac{A_{10}s_o\hat{k}_{s_c}}{12rA_{11}\hat{k}_{s_{co}}} \in (0,\ \bar{s}_{oc})$, such that $dSW/ds_{oc} > 0$ for $s_{oc} < \tilde{s}_{oc}$, and $dSW/ds_{oc} < 0$ for $s_{oc} > \tilde{s}_{oc}$. ■

Proof of Proposition 4: Technically, we show that when $t = \frac{s_p^2/\hat{k}_p}{s_o^2/\hat{k}_o} < \bar{t}$,

(i) $\frac{de_p^*}{ds_{oc}} > 0$ and $\frac{de_c^*}{ds_{oc}} > 0$; There exists $\hat{s}_{oc} \in (0,\ \bar{s}_{oc})$, such that $\frac{de_o^*}{ds_{oc}} > 0$ for $s_{oc} \in (0,\ \hat{s}_{oc})$ and $\frac{de_o^*}{ds_{oc}} < 0$ for $s_{oc} \in (\hat{s}_{oc},\ \bar{s}_{oc})$;

(ii) $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} > 0$ and $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$; There exists $\hat{s}_{oc} \in (0,\ \bar{s}_{oc})$, such that $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} > 0$ for $s_{oc} \in (0,\ \hat{s}_{oc})$ and $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} < 0$ for $s_{oc} \in (\hat{s}_{oc},\ \bar{s}_{oc})$;

(iii) $\frac{dCS}{ds_{oc}} > 0$ and $\frac{dSW}{ds_{oc}} > 0$ for all $s_{oc} \in (0,\ \bar{s}_{oc})$.

First, differentiating (A.56) with respect to s_{oc}, we obtain

$$\frac{de_c^*}{ds_{oc}} = \frac{3\tilde{r}\tilde{M}_3\hat{k}_{s_{co}}}{2\hat{k}_c\tilde{M}_2^3s_o} + O(b). \tag{A.110}$$

By our proof in Lemma A.4, $\tilde{r} > \tilde{r}_t \approx 6.935$. Hence, $\tilde{M}_2 = 4\tilde{r} - 1 > 0$, and $\tilde{M}_3 = \tilde{r} - 1 > 0$. Therefore, $\frac{de_c^*}{ds_{oc}} > 0$. Next, differentiating (A.54) with respect to s_{oc} and simplify the derivative with (A.84), we obtain

$$\frac{de_p^*}{ds_{oc}} = \frac{\tilde{M}_1\left(A_1\hat{k}_{s_c}s_o + 12r^2A_2\hat{k}_{s_{co}}s_{oc}\right)}{2A_3r^2\hat{k}_c\tilde{M}_2^4s_os_p} + O(b)\,, \tag{A.111}$$

where $A_1 = 8192r^{10} - 15360r^9 + 10496r^8 - 2816r^7 - 3936r^6 + 2684r^5 - 4565r^4 + 1983r^3 - 2380r^2 + 620r - 21$.

We also have that $A_2 = 1024r^6 + 1600r^5 - 1776r^4 + 3252r^3 + 185r^2 - 183r + 1082$, and $A_3 = 4096r^8 - 14336r^7 + 38656r^6 - 65792r^5 + 78832r^4 - 64184r^3 + 35137r^2 - 11324r + 778$.

Given $\tilde{r} > \tilde{r}_t$, we can easily verify that A_1 to A_3 are positive. Therefore, $\frac{de_p^*}{ds_{oc}} > 0$. Because $Q_p = s_pe_p$, we also obtain that $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} > 0$.

For the originator, we can obtain

$$\frac{de_o^*}{ds_{oc}} = \frac{A_4 \hat{k}_{s_c} s_o - 6r^2 A_5 \hat{k}_{s_{co}} s_{oc}}{2r^2 A_3 \tilde{M}_2^4 \hat{k}_c s_o^2} + O(b)\,, \tag{A.112}$$

where $A_4 = 49152r^{13} - 253952r^{12} + 779264r^{11} - 1597440r^{10} + 2289728r^9 - 2369504r^8 + 1758508r^7 - 916508r^6 + 302344r^5 - 51227r^4 + 291r^3 + 536r^2 + 64r - 4$.

We also define constant $A_5 = 16384r^{11} - 81920r^{10} + 236544r^9 - 478208r^8 + 665536r^7 - 672896r^6 + 465524r^5 - 217456r^4 + 48005r^3 + 2276r^2 - 3694r - 264$. Given $\tilde{r} > \tilde{r}_t$, we can verify that A_4 and A_5 are positive. Solving $\frac{de_o^*}{ds_{oc}} = 0$, we can obtain $\hat{s}_{oc} = \frac{A_4 \hat{k}_{s_c} s_o}{6r^2 A_5 \hat{k}_{s_{co}}} \in (0,\, \bar{s}_{oc})$. Therefore, $\frac{de_o^*}{ds_{oc}} > 0$ for $s_{oc} \in (0,\, \hat{s}_{oc})$, and $\frac{de_o^*}{ds_{oc}} < 0$ for $s_{oc} \in (\hat{s}_{oc},\, \bar{s}_{oc})$. This completes the proof of part (i).

We next turn to the quality of the originator. We can obtain that

$$\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} = \frac{\tilde{M}_1^2 \left(A_6 \hat{k}_{s_c} s_o + 12r^2 A_7 \hat{k}_{s_{co}} s_{oc} \right)}{4r^2 A_3 \tilde{M}_2^4 \hat{k}_c s_o} + O(b)\,, \tag{A.113}$$

where $A_6 = 4096r^{10} - 12288r^9 + 26368r^8 - 35328r^7 + 28144r^6 - 15536r^5 + 5169r^4 - 3146r^3 + 553r^2 + 26r - 2$ and $A_7 = 256r^6 + 64r^4 + 1376r^3 - 837r^2 + 1316r + 66$.

Similarly, for the quality of the contributor, we can obtain

$$\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} = \frac{\hat{k}_{s_{co}} \left(A_8 \hat{k}_{s_c} s_o - 6r^2 A_9 \hat{k}_{s_{co}} s_{oc} \right) b}{2r^2 A_3 \tilde{M}_2^4 \hat{k}_c s_o^2} + O(b^2)\,, \tag{A.114}$$

where $A_8 = 98304r^{13} - 487424r^{12} + 1470464r^{11} - 3009792r^{10} + 4338560r^9 - 4519568r^8 + 3379408r^7 - 1772003r^6 + 586951r^5 - 96869r^4 + 2625r^3 + 536r^2 + 64r - 4$.

We also have that $A_9 = 16384r^{11} - 81920r^{10} + 236544r^9 - 478208r^8 + 665536r^7 - 672896r^6 + 465524r^5 - 217456r^4 + 48005r^3 + 2276r^2 - 3694r - 264$.

Given $\tilde{r} > \tilde{r}_t$, we can verify that A_6 to A_9 are positive. Therefore, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$. Solving $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} = 0$, we obtain $\tilde{s}_{oc} = \frac{A_8 \hat{k}_{s_c} s_o}{6r^2 A_9 \hat{k}_{s_{co}}} \in (0,\, \bar{s}_{oc})$. Therefore, $\frac{dQ_c^*}{ds_{oc}} > 0$ for $s_{oc} \in (0,\, \tilde{s}_{oc})$, and $\frac{dQ_c^*}{ds_{oc}} < 0$ for $s_{oc} \in (\tilde{s}_{oc},\, \bar{s}_{oc})$. This completes the proof of part (ii).

For the consumer surplus, we obtain

$$\frac{dCS}{ds_{oc}} = \frac{\tilde{M}_1 \left(A_{10} \hat{k}_{s_c} s_o + 12r^2 A_{11} \hat{k}_{s_{co}} s_{oc} \right)}{4r A_3 \tilde{M}_2^6 \hat{k}_c s_o} + O(b) \tag{A.115}$$

where $A_{10} = 32768r^{13} - 139264r^{12} + 395264r^{11} - 637440r^{10} + 616832r^9 - 387552r^8 + 105912r^7 + 8202r^6 - 48294r^5 + 21379r^4 - 22511r^3 + 4132r^2 + 291r - 20$.

We also have $A_{11} = 2048r^9 - 2560r^8 + 7936r^7 + 24896r^6 - 27832r^5 + 39022r^4 - 2206r^3 - 1855r^2 +$

$10192r + 660$; given $\tilde{r} > \tilde{r}_t$, it follows that $A_{10} > 0$ and $A_{11} > 0$.

For the social welfare, we obtain

$$\frac{dSW}{ds_{oc}} = \frac{\tilde{M}_1(A_{12}\hat{k}_{s_c}s_o + 24rA_{13}\hat{k}_{s_{co}}s_{oc})}{4rA_{14}\tilde{M}_2^7\hat{k}_c s_o} + O(b)\,. \tag{A.116}$$

Above, we also defined constants:

$$\begin{aligned}A_{12} = {}& 6291456r^{16} - 40894464r^{15} + 153747456r^{14} - 401473536r^{13} + 783728640r^{12} - 1188003840r^{11}\\ &+ 1419568128r^{10} - 1346869760r^9 + 1008807648r^8 - 586353168r^7 + 254197554r^6\\ &- 75587112r^5 + 12652590r^4 - 223767r^3 - 195927r^2 + 11409r - 358\\ A_{13} = {}& 524288r^{14} - 2949120r^{13} + 10387456r^{12} - 25362432r^{11} + 46766080r^{10} - 65768960r^9\\ &+ 71707520r^8 - 59769984r^7 + 37287552r^6 - 16314957r^5 + 4208664r^4 - 240277r^3 - 177349r^2\\ &+ 5561r - 778\end{aligned}$$

Finally, $A_{14} = 65536r^{10} - 294912r^9 + 933888r^8 - 1972224r^7 + 3125760r^6 - 3669888r^5 + 3244608r^4 - 2091240r^3 + 931509r^2 - 250252r + 16338$ and similar to A_{10}, given $\tilde{r} > \tilde{r}_t$, it follows that A_{12} to A_{14} are positive. Therefore, $\frac{dCS}{ds_{oc}} > 0$ and $\frac{dSW}{ds_{oc}} > 0$. This completes the proof of part (iii). ■

Proof of Proposition 5: Technically, we show that under the conditions of part (iii) in Lemma A.4,

(i) $\frac{de_p^*}{ds_{oc}} > 0$ and $\frac{de_c^*}{ds_{oc}} > 0$; $\frac{de_o^*}{ds_{oc}} > 0$ if $\hat{k}_c < \bar{k}_c$ and $\frac{de_o^*}{ds_{oc}} \le 0$ if $\hat{k}_c \ge \bar{k}_c$, where $\bar{k}_c = \frac{512\hat{k}_p^2 s_c^2 s_{co}^2}{s_p^2\left(5\hat{k}_o s_p^2 + 48\hat{k}_p s_{co}^2\right)}$;

(ii) $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} > 0$, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$ and $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} > 0$;

(iii) $\frac{dCS}{ds_{oc}} > 0$ and $\frac{dSW}{ds_{oc}} > 0$.

First, differentiating (A.58) and (A.60) with respect to s_{oc}, we obtain

$$\frac{de_p^*}{ds_{oc}} = \frac{s_{co}^2}{32\hat{k}_c\hat{k}_{s_o}^2} \cdot b + O(b^2), \tag{A.117}$$

and

$$\frac{de_c^*}{ds_{oc}} = \frac{3\hat{k}_o s_c s_{co} s_p}{512\hat{k}_c\hat{k}_p\hat{k}_{s_o}^3} \cdot b^2 + O(b^3). \tag{A.118}$$

Given the positive values of the parameters in (A.117) and (A.118), it is easy to verify that $\frac{de_p^*}{ds_{oc}} > 0$ and $\frac{de_c^*}{ds_{oc}} > 0$. Next, differentiating (A.59) with respect to s_{oc}, we obtain

$$\frac{de_o^*}{ds_{oc}} = \frac{\hat{k}_o s_c\left(512\hat{k}_p^2 s_c^2 s_{co}^2 - 5\hat{k}_c\hat{k}_o s_p^4 - 48\hat{k}_c\hat{k}_p s_{co}^2 s_p^2\right)}{32768\hat{k}_c^2\hat{k}_p^2\hat{k}_{s_o}^3 s_{co}^2} \cdot b^2 + O(b^3). \tag{A.119}$$

Solving $\frac{de_o^*}{ds_{oc}} = 0$, we obtain that $\hat{k}_c = \frac{512\hat{k}_p^2 s_c^2 s_{co}^2}{s_p^2(5\hat{k}_o s_p^2 + 48\hat{k}_p s_{co}^2)}$. Let $\bar{k}_c = \frac{512\hat{k}_p^2 s_c^2 s_{co}^2}{s_p^2(5\hat{k}_o s_p^2 + 48\hat{k}_p s_{co}^2)}$. Hence, $\frac{de_o^*}{ds_{oc}} > 0$ if $\hat{k}_c < \bar{k}_c$ and $\frac{de_o^*}{ds_{oc}} \le 0$ for $\hat{k}_c \ge \bar{k}_c$. This completes the proof of part (i).

We next consider the qualities. First, because $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} = \frac{s_p e_p^*}{ds_{oc}} = s_p \frac{e_p^*}{ds_{oc}}$ and $\frac{de_p^*}{ds_{oc}} > 0$, we obtain that $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} > 0$. For the originator and contributor, we can obtain

$$\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} = \frac{s_c}{16\hat{k}_c} \cdot \frac{1}{b} + O(1) \;\; and \;\; \frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} = \frac{s_c s_{co}^2}{32\hat{k}_c \hat{k}_{s_o}^2} \cdot b + O(b^2), \tag{A.120}$$

where we can easily verify that the dominant terms are positive. Therefore, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$ and $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} > 0$. This completes the proof of part (ii).

Regarding the consumer surplus and social welfare, we obtain

$$\frac{dCS}{ds_{oc}} = \frac{s_c}{128\hat{k}_c} \cdot \frac{1}{b} + O(1) \;\; and \;\; \frac{dSW}{ds_{oc}} = \frac{3s_c}{128\hat{k}_c} \cdot \frac{1}{b} + O(1)\,, \tag{A.121}$$

where we can easily verify that the dominant terms are positive. Therefore, $\frac{dCS}{ds_{oc}} > 0$ and $\frac{dSW}{ds_{oc}} > 0$. This completes the proof of part (iii). ■

Proof of Proposition 6: We denote the equilibrium efforts and qualities of the originator when the contributor is out of the market with superscript *out*, and when it is in the market with superscript *in*. For part (i), technically, we show that $e_o^{out} > e_o^{in}$. We have shown in Lemma A.4 that there exists $\bar{V} > 0$ such that if $V_c > \bar{V}$ the contributor will be out of the market. Under such conditions, similar to the proof of Lemma A.4, we can derive the optimal effort investment levels which satisfy

$$e_o^{out} = \frac{M_1 s_o}{\hat{k}_o M_2^3} \cdot \frac{1}{b} + O(b^2), \tag{A.122}$$

where M_1 and M_2 are the same as in Lemma A.4, and r satisfies condition (A.73) in Lemma A.4. Combining (A.51) and (A.122), we can obtain that

$$e_o^{out} - e_o^{in} = \frac{rA_1\hat{k}_o\hat{k}_{s_c}s_o s_{oc} + 3rA_2\hat{k}_o\hat{k}_{s_{co}}s_{oc}^2 + 4A_3\hat{k}_c\hat{k}_{s_{co}}s_o^2}{2A_4 M_2^4 \hat{k}_c \hat{k}_o s_o^2} + O(b)\,, \tag{A.123}$$

where $A_1 = 81920 - 417792r + 1068032r^2 - 2010112r^3 + 2796224r^4 - 2511328r^5 + 1398500r^6 - 421336r^7 - 37681r^8 + 35133r^9 - 3430r^{10}$ and $A_2 = 16384 - 114688r + 326656r^2 - 633856r^3 + 888768r^4 - 763264r^5 + 388020r^6 - 90440r^7 - 78057r^8 + 8232r^9$.

Above, we also define that $A_3 = 16384 - 114688r + 334848r^2 - 598528r^3 + 793280r^4 - 626368r^5 + 1412r^6 + 427274r^7 - 462925r^8 + 307384r^9 - 56889r^{10} + 686r^{11}$ and $A_4 = 4096 - 14336r + 32512r^2 - 63232r^3 + 91760r^4 - 74200r^5 + 36897r^6 - 8232r^7$. Given the range of r as shown in Lemma A.4, it is easy to verify that A_1 to A_4 are positive. Therefore, (A.123) is positive, which implies $e_o^{out} > e_o^{in}$.

For results regarding the originator's quality in part (ii), technically, we show that there exists $\tilde{k}_c > 0$ such that if $\hat{k}_c \geq \tilde{k}_c$, $Q_o^{out} \geq Q_o^{in}$; if $\hat{k}_c < \tilde{k}_c$, there exists $0 < s_1 < s_2 < \bar{s}_{oc}$ such that $Q_o^{out} < Q_o^{in}$ for all $s_{oc} \in (s_1,\, s_2)$, and $Q_o^{out} \geq Q_o^{in}$ otherwise.

Combining (A.51), (A.52), and (A.122), we can obtain that

$$Q_o^{out} - Q_o^{in} = \frac{M_1(8A_5\hat{k}_c\hat{k}_{s_{co}}s_o^2 - A_6M_1\hat{k}_o\hat{k}_{s_c}s_os_{oc} + 6rA_7M_1\hat{k}_o\hat{k}_{s_{co}}s_{oc}^2)}{4A_4M_2^4\hat{k}_c\hat{k}_os_o} + O(b), \tag{A.124}$$

where we define constants

$$\begin{aligned} A_5 &= 4096 - 21504r + 46080r^2 - 68992r^3 + 77584r^4 - 20820r^5 - 36082r^6 + 43675r^7 - 39300r^8 \\ &\quad + 8071r^9 - 98r^{10}, \\ A_6 &= 4096 - 20480r + 44800r^2 - 90624r^3 + 122992r^4 - 96752r^5 + 51825r^6 - 11936r^7 + 967r^8 - 28r^9, \\ A_7 &= 2048 - 4864r + 9984r^2 - 16640r^3 + 11464r^4 - 7797r^5\,. \end{aligned}$$

Given the range of r as shown in Lemma A.4, it is easy to verify that M_1 and M_2 as well as A_4 to A_7 are positive. Then the denominator of the first term in (A.124) is positive. We can write the numerator as $M_1 \cdot f$, where

$$f = 8A_5\hat{k}_c\hat{k}_{s_{co}}s_o^2 - A_6M_1\hat{k}_o\hat{k}_{s_c}s_os_{oc} + 6rA_7M_1\hat{k}_o\hat{k}_{s_{co}}s_{oc}^2\,, \tag{A.125}$$

which is a quadratic function of s_{oc}. We can obtain that if $\hat{k}_c \geq \tilde{k}_c = \frac{M_1A_6^2\hat{k}_o\hat{k}_{s_c}^2}{192rA_5A_7\hat{k}_{s_{co}}^2}$, $f \geq 0$, which means $Q_o^{out} \geq Q_o^{in}$. If $\hat{k}_c < \tilde{k}_c$, the equation $f(s_{oc}) = 0$ has two real roots. We denote them as

$$s_1 = \frac{A_6M_1\hat{k}_o\hat{k}_{s_c} - \sqrt{\hat{k}_oM_1\left(A_6^2M_1\hat{k}_o\hat{k}_{s_c}^2 - 192rA_5A_7\hat{k}_c\hat{k}_{s_{co}}^2\right)}}{12rA_7M_1\hat{k}_o\hat{k}_{s_{co}}} \cdot s_o \tag{A.126}$$

and

$$s_2 = \frac{A_6M_1\hat{k}_o\hat{k}_{s_c} + \sqrt{\hat{k}_oM_1\left(A_6^2M_1\hat{k}_o\hat{k}_{s_c}^2 - 192rA_5A_7\hat{k}_c\hat{k}_{s_{co}}^2\right)}}{12rA_7M_1\hat{k}_o\hat{k}_{s_{co}}} \cdot s_o\,. \tag{A.127}$$

The region where $Q_o^{out} < Q_o^{in}$ will depend on the relationship between s_1, s_2, and $\bar{s}_{oc}$. Since we can obtain that $s_1 > 0$ and $s_2 < \bar{s}_{oc}$, we have that $Q_o^{out} < Q_o^{in}$ for all $s_{oc} \in (s_1,\, s_2)$, and $Q_o^{out} \geq Q_o^{in}$ for $s_{oc} \in (0,\, s_1]$ and $s_{oc} \in [s_2,\, \bar{s}_{oc})$. In summary, if $\hat{k}_c < \tilde{k}_c$ and $s_{oc} \in (s_1,\, s_2)$, $Q_o^{out} < Q_o^{in}$; otherwise, $Q_o^{out} \geq Q_o^{in}$.

For results regarding consumer surplus in part (ii), we technically show that there exists $\check{k}_c > 0$ such that if $\hat{k}_c \geq \check{k}_c$, $CS^{out} \geq CS^{in}$; if $\hat{k}_c < \check{k}_c$, there exists $0 < s_3 < s_4 < \bar{s}_{oc}$ such that $CS^{out} < CS^{in}$ for all $s_{oc} \in (s_3,\, s_4)$, and $CS^{out} \geq CS^{in}$ otherwise.

With the equilibrium efforts when the contributor is in and out the market, we can obtain that

$$CS^{out} - CS^{in} = \frac{M_1(A_8\hat{k}_c\hat{k}_{s_{co}}s_o^2 - A_9\hat{k}_o\hat{k}_{s_c}s_o s_{oc} + 18rA_{10}\hat{k}_o\hat{k}_{s_{co}}s_{oc}^2)}{8A_4M_2^6\hat{k}_c\hat{k}_o s_o} + O(b)\,, \tag{A.128}$$

where $A_8 = 16384 + 208896r - 2612224r^2 + 8604416r^3 - 14949952r^4 + 18491600r^5 - 15244324r^6 + 8345813r^7 - 4073518r^8 + 645589r^9 + 7448r^{10}$.

We also define that $A_9 = 114688 - 815104r + 2153472r^2 - 3530496r^3 + 4895808r^4 - 4831248r^5 + 3019908r^6 - 1228905r^7 + 34260r^8 + 51569r^9 - 3920r^{10}$, and $A_{10} = 24576 - 95232r + 146176r^2 - 185216r^3 + 195136r^4 - 123172r^5 + 70847r^6 + 21560r^7$. Given the range of r as shown in Lemma A.4, it is easy to verify that M_1, M_2, A_4, A_8, A_9, and A_{10} are positive. Then the denominator of the first term in (A.128) is positive. We can write the numerator as $M_1 \cdot g$, where

$$g = A_8\hat{k}_c\hat{k}_{s_{co}}s_o^2 - A_9\hat{k}_o\hat{k}_{s_c}s_o s_{oc} + 18rA_{10}\hat{k}_o\hat{k}_{s_{co}}s_{oc}^2\,, \tag{A.129}$$

which is a quadratic function of s_{oc}. We can obtain that if $\hat{k}_c \geq \check{k}_c = \frac{A_9^2\hat{k}_o\hat{k}_{s_c}^2}{72rA_{10}A_8\hat{k}_{s_{co}}^2}$, $g \geq 0$, which means $CS^{out} \geq CS^{in}$. If $\hat{k}_c < \check{k}_c$, the equation $g(s_{oc}) = 0$ has two real roots. We denote them as

$$s_3 = \frac{A_9\hat{k}_o\hat{k}_{s_c} - \sqrt{A_9^2\hat{k}_o^2\hat{k}_{s_c}^2 - 72rA_{10}A_8\hat{k}_c\hat{k}_o\hat{k}_{s_{co}}^2}}{6rA_{10}\hat{k}_o\hat{k}_{s_{co}}}s_o \tag{A.130}$$

and

$$s_4 = \frac{A_9\hat{k}_o\hat{k}_{s_c} + \sqrt{A_9^2\hat{k}_o^2\hat{k}_{s_c}^2 - 72rA_{10}A_8\hat{k}_c\hat{k}_o\hat{k}_{s_{co}}^2}}{6rA_{10}\hat{k}_o\hat{k}_{s_{co}}}s_o\,. \tag{A.131}$$

The region where $CS^{out} < CS^{in}$ will depend on the relationship between s_3, s_4, and $\bar{s}_{oc}$. Since we can obtain that $s_3 > 0$ and $s_4 < \bar{s}_{oc}$, we have that $CS^{out} < CS^{in}$ for all $s_{oc} \in (s_3,\, s_4)$, and $CS^{out} \geq CS^{in}$ for $s_{oc} \in (0,\, s_3]$ and $s_{oc} \in [s_4,\, \bar{s}_{oc})$. In summary, if $\hat{k}_c < \check{k}_c$ and $s_{oc} \in (s_3,\, s_4)$, $CS^{out} < CS^{in}$; otherwise, $CS^{out} \geq CS^{in}$. This finishes the proof of part (ii). ■

Proof of Proposition A.1: Technically, we show that for Region H as defined in Lemma A.3, suppose $s_{co} = J + K \cdot s_{oc}$ and $K = k_n \cdot b^2$, then

(i) $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} < 0$, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$, and $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} > 0$;

(ii) $\frac{dCS}{ds_{oc}} > 0$ and $\frac{dSW}{ds_{oc}} > 0$.

Following the same logic as in Lemma A.3, the equilibrium effort of the proprietor satisfies

$$e_p^* = \frac{s_p}{4k_p} \cdot \frac{1}{b^2} - \frac{4k_p s_o^5\left(2J^2 + 13Js_o + 3s_o^2\right)K_1^3}{k_o^2 s_p^3 K_2^6} + \frac{2k_p s_o^2 K_1 K_3}{k_c k_o^3 s_p^5 K_2^9} \cdot b + O(b^2)\,, \tag{A.132}$$

where $K_1 = s_o - J$, $K_2 = 4s_o - J$, and $K_3 = -16K_1^3 k_c k_p s_o^5\big(4J^4 + 55J^3 s_o + 58J^2 s_o^2 - 66Js_o^3 + 30s_o^4\big) -$

$K_2 k_o^2 s_p^2\big(3s_c^2 s_o^4(s_o - 4J)\big(-6J^2 - 41Js_o + 20s_o^2\big) + s_c s_o^2 s_{oc}\big(-19J^5 - 258J^4 s_o - 11J^3 s_o^2 + 94J^2 s_o^3 - 468Js_o^4 + 176s_o^5\big) + Js_{oc}^2\big(J^6 + 30J^5 s_o + 32J^4 s_o^2 - 40J^3 s_o^3 + 204J^2 s_o^4 - 176Js_o^5 + 192s_o^6\big)\big)$.

The originator's equilibrium effort is

$$e_o^* = \frac{s_o^2 K_1}{k_o K_2^2}\cdot\frac{1}{b} - \frac{8k_p s_o^4(J+2s_o)K_1^3}{k_o^2 s_p^2 K_2^5} - \frac{K_4}{16k_c^2 k_o^3 s_o^4 s_p^4 K_1^2 K_8^8}\cdot b + O(b^2)\,, \tag{A.133}$$

where $K_4 = 256K_1^6 k_c^2 k_p^2 s_o^{10}(4J - s_o)\big(J^2 + 7Js_o + s_o^2\big) + 32K_1^3 K_2 k_c k_o^2 k_p s_o^5 s_p^2\big(9s_c^2 s_o^4\big(8J^2 + 5Js_o - 4s_o^2\big) + s_c s_o^2 s_{oc}\big(-19J^4 - 104J^3 s_o + 129J^2 s_o^2 - 248Js_o^3 + 80s_o^4\big) + Js_{oc}^2\big(J^5 + 17J^4 s_o - 15J^3 s_o^2 + 50J^2 s_o^3 - 20Js_o^4 + 48s_o^5\big)\big) + 16K_1^2 K_2^5 k_c^2 k_n k_o^2 s_o^6 s_{oc} s_p^4(J + 2s_o) + K_2^2 k_o^4 s_p^4\big(J^2 s_{oc}(J + 2s_o) + s_c s_o^2(4s_o - 7J)\big)(s_c s_o - Js_{oc})^2\big(3s_c s_o^2\big(7J^2 + 25Js_o + 4s_o^2\big) - s_{oc}\big(J^4 + 35J^3 s_o + 8Js_o^3 + 64s_o^4\big)\big)$.

And the contributor's effort level can be derived as

$$e_c^* = \frac{s_c s_o^2\,(4s_o - 7J) + s_{co}^2\,(J + 2s_o)\,s_{oc}}{4k_c K_2^3} - \frac{K_5}{8k_o k_c^2 s_o^2 s_p^2 K_1 K_2^6}\cdot b + O(b^2)\,, \tag{A.134}$$

where $K_5 = 96K_1^3 k_c k_p s_o^5\big(J^3 s_{oc} + Js_o^2(2s_{oc} - 5s_c) + 2s_c s_o^3\big) + K_2 k_o^2 s_p^2(7J + 8s_o)\big(J^3 s_{oc} + 2J^2 s_o s_{oc} - 7Js_c s_o^2 + 4s_c s_o^3\big)(s_c s_o - Js_{oc})^2$.

After obtaining the equilibrium efforts, we can derive that

$$\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} = \frac{s_c J^2(2s_o + J)}{4k_c K_2^3}\cdot b + O(b^2)\,. \tag{A.135}$$

Therefore, as long as $K_2 > 0$, i.e., $J < 4s_o$, we have $\frac{dQ_c(\mathbf{e}^*)}{ds_{oc}} > 0$. Similarly, we can show that if $7J < 4s_o$, $\frac{dQ_o(\mathbf{e}^*)}{ds_{oc}} > 0$, and $\frac{dQ_p(\mathbf{e}^*)}{ds_{oc}} < 0$ as long as $J < \lambda_H s_o$ where $\lambda_H = 0.392$. This finishes the proof of part (i). We next obtain that

$$\frac{dCS}{ds_{oc}} = \frac{s_o\,(A_1 s_{oc} + A_2)}{32k_c K_2^6} + O(b)\; and\; \frac{dW}{ds_{oc}} = \frac{A_3 s_{oc} + A_4}{32k_c K_2^6} + O(b)\,, \tag{A.136}$$

where $A_1 = 4J^2(J + 2s_o)\big(J^2 - 42Js_o + 56s_o^2\big)$ and $A_2 = s_c s_o\big(-J^4 + 36J^3 s_o + 700J^2 s_o^2 - 1120Js_o^3 + 448s_o^4\big)$.

We also define that $A_3 = -2J^6 + 36J^5 s_o - 40J^4 s_o^2 - 80J^3 s_o^3 + 320J^2 s_o^4$, and $A_4 = s_c s_o^2\big(3J^4 - 160J^3 s_o + 484J^2 s_o^2 + 800Js_o^3 + 320s_o^4\big)$. We can easily show that A_1 to A_4 are positive given $J < \lambda_H s_o$, which gives us that $\frac{dCS}{ds_{oc}} > 0$ and $\frac{dCS}{ds_{oc}} > 0$. This completes the proof of part (ii). ■