MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Vol. 12, No. 1, Winter 2010, pp. 140–161
ISSN 1523-4614 | EISSN 1526-5498 | 10 | 1201 | 0140
DOI 10.1287/msom.1090.0285

Feasting on Leftovers: Strategic Use of Shortages in Price Competition Among Differentiated Products

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Two single-product firms with different quality levels and fixed limited capacities engage in sequential price competition in an essentially deterministic model where customers have heterogeneous valuations for both products. We develop conditions under which the leader (she) can take strategic advantage of her limited capacity by pricing relatively low, purposefully creating shortages and leaving some *leftovers* for the follower (him) to feast on, avoiding direct competition. The extent to which the leader benefits in this *Leftovers Equilibrium* depends on operational variables such as the capacity levels of the two firms and the sequence in which customers arrive at the market. We spell out the details for three different known arrival sequences within a specific subset of plausible fixed-capacity levels. The follower's *strategic shadow price* can be positive even when not all his capacity is used, and the leader's can be negative when all her capacity is used. We illustrate that Leftovers Equilibria can arise when some of our assumptions are relaxed.

Key words: operations strategy; quality management; service operations
History: Received: November 27, 2007; accepted: February 18, 2009. Published online in *Articles in Advance* December 29, 2009.

1. Introduction

Major hotels often are booked during busy seasons. For example, leading high-quality hotels frequently sell out their conference rate rooms during conferences. The shortage of these rooms forces left-out customers to purchase from lower-quality competitors who have rooms available—and possibly at higher prices than the original conference rate at the higher-quality hotels. We are accustomed to product/capacity shortages when there is uncertain demand, but can a shortage be predicted with near certainty in advance? If so, why does the higher-quality hotel not increase its price?

We are not the first to raise these questions. For example, Becker (1991, p. 1109), after observing a popular restaurant with many waiting customers and another comparable restaurant across the street with many empty seats, asked, "Why doesn't the popular restaurant raise prices, which would reduce the queue for seats but expand profit?" Similarly, we have observed popular chain restaurants with long lines at certain times of the day, while local, smaller competitors have half empty dining rooms—yet both restaurants are profitable enough to coexist. We offer a simple competitive setting in which one firm should purposefully leave its price so low that some customers who prefer its product are unable to obtain it and buy the less attractive competing product as a second choice.[1] The firm strategically uses shortages to increase its profits.

What drives the firm to create shortages? Before addressing this question, let us clarify the context of our model. We have a simple, static, effectively deterministic model of two competing firms, each with a single product, fixed limited and possibly different capacities, and negligible production/service costs. The products differ in their exogenously given quality levels but are substitutes in that a customer will buy at most one of them. Each firm decides only the price of its respective product. The firm with the higher-quality product is the price leader, setting her price first.[2] Because capacities are limited, some customers may find their preferred product unavailable

[1] We address Becker's single firm explanation of the phenomenon in the literature review.

[2] We designate the leader as female and the follower as male.

upon arrival and end up buying their second-choice product.

Given limited capacities, it is easy to imagine prices for the two products such that there is unmet demand for the higher-quality product and some customers end up buying the lower-quality product, even though they would prefer the higher-quality one: The higher-quality product "runs out" and these customers prefer to buy the lower-quality product than to not buy at all. It is not quite as easy to imagine that this can be an equilibrium, which we call a *Leftovers Equilibrium*,[3] as our usual intuition is that the leader should increase her price, because she should still sell all her capacity at a higher price. But this can be wrong, and we explain why.

Whether the leader chooses to create shortages and the extent of the resulting benefit depends on the firms' capacity levels and the customer arrival sequence, which specifies the order in which customer types arrive at the market to observe prices and availabilities of the two products and then decide what to buy, if anything.[4] The customer arrival sequence determines which customer types are left with only the option to buy the lower-quality product in a Leftovers Equilibrium. We examine three established, exogenously given arrival sequences: high-to-low (in which customers arrive in decreasing order of their willingness to pay (WTP)), independent (in which customers arrive independently of their WTP), and low-to-high (in which customers arrive in increasing order of WTP). We provide separate sufficient conditions for each of these three arrival sequences under which there is a unique Stackelberg equilibrium that is a Leftovers Equilibrium and that has an explicit characterization, which we call the *Baseline Leftovers Equilibrium*. These conditions represent a specific but plausible subset of the (exogenous) capacity region. We explore the impact of the different arrival sequences on this Baseline Leftovers Equilibrium.

[3] We formally define our italicized vocabulary later, in §§3.3, 4.1, and 4.2.

[4] See, e.g., §7 in Dana (1999). For convenience, we use the terminology "customer arrival sequence" throughout, whether the customers are consumers or businesses, and whether priorities, rationing, and/or allocations, which influence the effective customer arrival sequence, are used.

We show that, in the Baseline Leftovers Equilibrium, the strategic shadow price of capacity—the marginal value of additional exogenous capacity—can differ dramatically from what we would see in a single firm optimization model. For example, the follower's profits can strictly increase as his capacity increases, even though he had excess capacity in the first place and none of the added capacity will be used. He threatens the leader, who appeases him by lowering her price. This result depends on the customer arrival sequence: it holds for two of the three arrival sequences but not for all three. In addition, the leader's revenue can decrease as her capacity increases, even though her capacity constraint is binding throughout.

Section 2 reviews the related literature. Section 3 formulates the model and provides some preliminary analysis. Section 4 presents the Baseline Leftovers Equilibria under the different customer arrival sequences. Section 5 develops comparative statics and explores managerial incentives under Baseline Leftovers Equilibria and summarizes our explicit results. Section 6 discusses our model and extensions and illustrates that Leftovers Equilibria can arise in settings other than those specified in our basic model. Section 7 offers concluding remarks.

2. Literature Review

Following Angelus and Porteus (2008), asset management covers both traditional capacity management, where the assets are fixed and reusable for production and services, and inventory management, where the assets are liquid and represent inventory levels. Competitive asset management has recently drawn attention in the operations management literature, with a focus on stocking decisions under a fixed product price (e.g., Lippman and McCardle 1997, Mahajan and van Ryzin 2001, Netessine and Rudi 2003). In our model, asset levels are constant and two firms compete in prices, and we show how stockouts can be strategically induced.

Our paper is closely related to the literature that addresses price competition among firms with nonidentical substitutable products and unlimited capacities. Schmidt and Porteus (2000) examine price competition alone and find Nash equilibria. Other papers explore competition in quality choice followed by price competition (e.g., Moorthy 1988, Jones and Mendelson 1997, Chambers et al. 2006). Tyagi (2000) investigates duopoly competition in sequential product positioning under horizontal differentiation. We consider competition under limited capacities.

Studies in the revenue management literature explore dynamic pricing under limited capacity (e.g., Elmaghraby and Keskinocak 2003, Talluri and van Ryzin 2004, McAfee and te Velde 2006). Some recent papers in this stream take competition into consideration (e.g., Netessine and Shumsky 2005, Gallego and Hu 2008, and Levin et al. 2009). The single firm models in this literature typically assume low-to-high arrivals and conclude that firms should reserve some capacity for late arriving, high-value customers. Our pricing framework is static, so the price leader is not able to charge more to late arriving customers. We show that the price leader may want to purposefully sell out to low-end customers in our environment.

Our paper is also related to the branch of the economics literature that explores simultaneous price competition for an identical product under limited capacities. This branch tends to use the term *rationing*. For example, what we call high-to-low and independent arrivals correspond to *efficient* and *random* rationing, respectively. The usual starting point (e.g., Levitan and Shubik 1972) is efficient rationing. Kreps and Scheinkman (1983) study a two-stage duopoly game in which a capacity commitment stage is followed by a price competition stage. They show that quantity competition outcomes appear under efficient rationing. Other papers in this branch explore the equilibrium outcome under both efficient and random rationing (e.g., Davidson and Deneckere 1986, Maskin 1986, and Allen and Hellwig 1993). Sherman and Visscher (1982) add consideration of low-to-high rationing to the literature (see also Dana 1998, 1999).

Cui and Ho (2007), assuming both firms have the same capacity levels, argue that the equilibrium profits are smallest (for both firms) under efficient rationing and are largest under low-to-high rationing. Our model differs from this branch by examining differentiated (nonidentical) products, differing capacities, and sequential price competition. One feature of our approach is that when both firms' prices are equal, there is no ambiguity as to how many units are sold by each firm; in the identical product case, a splitting rule is needed. Another feature is that customer demand is continuous in prices under differentiated products but discontinuous under identical products. We reveal the strategic use of shortages and how it is impacted by the capacity levels and the customer arrival sequence. There are also papers that examine the strategic use of capacity (e.g., Gelman and Salop 1983). Among these, Rotemberg and Saloner (1989) show that in a repeated game setting, where competing firms tacitly collude, higher capacity levels can serve as credible threats to perpetuate collusion. We shall show that in a noncollusive setting the follower can have a strategic incentive to increase his capacity as a credible threat that he would initiate direct competition.

As mentioned in the introduction, Becker (1991) questioned why firms "do not raise prices even with persistent excess demand." He develops a single-firm *social influence* model in which the attractiveness of a product depends on how many others also want to buy it. He argues that by setting a low price, a monopolist firm can increase the demand for its product, which in turn can increase consumers' willingness to pay for it. Therefore, the monopolist can be better off setting a low price even if it induces shortages. Other papers in the literature study motivations such as building customer loyalty and signaling product quality (see Haddock and McChesney 1994 and Stock and Balachander 2005, respectively). Our model opens the competition dimension and points out a different incentive, namely the strategic use of shortages to keep a competitor in check.

3. Model Formulation and Preliminary Analysis

In this section, we formulate our model, describe its salient features, and provide some preliminary analysis, including useful notation and the equilibrium in the unlimited capacities case.

3.1. Model Description

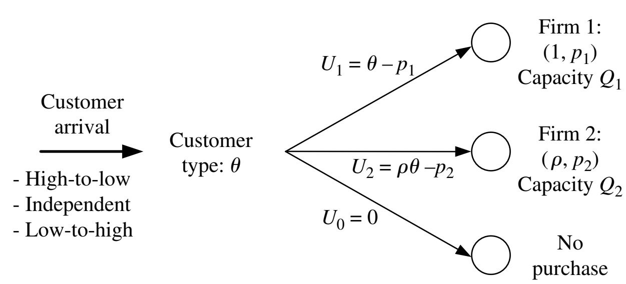
We consider the duopoly price competition model of product differentiation in quality, introduced by Shaked and Sutton (1983, 1987), under exogenously given qualities and limited capacities. Firm 1 has normalized quality level 1 and capacity Q_1, and Firm 2 has quality ρ (<1) and capacity Q_2. We assume Stackelberg pricing: Firm 1, with the higher-quality product, first sets her price, p_1, and then, after observing the leader's price, Firm 2 sets his price, p_2. We also assume that the unit production costs are negligible. We discuss our assumptions in detail in §6.

In this model, customers are heterogeneous in their WTP for quality, and all customers unambiguously prefer the higher-quality product to the lower-quality one at the same price. A customer will buy at most one product. On arrival at the market, each customer observes both firms' prices as well as availability and makes a purchase decision.

Fix the prices at (p_1, p_2) for the rest of this subsection. The market clears in the following way as depicted in Figure 1; each customer has her own type θ, WTP, which is assumed to be uniformly distributed (deterministically and continuously) between 0 and 1. A type θ customer gets (net) utility of $U_1 = \theta - p_1$ by buying from Firm 1, $U_2 = \theta\rho - p_2$ by buying from Firm 2, and $U_0 = 0$ by buying nothing. On arrival, each customer seeks to maximize her utility. When the product with her maximum utility is out of stock, she purchases her second-choice product if it is available and if doing so is better than not buying.

We assume without loss of generality that $p_1 \leq 1$ and $p_2 \leq \rho$, because neither firm can benefit from setting the price higher. Let θ_1 denote the type of customer who is indifferent between purchasing the leader's product and not buying, θ_2 the type of customer who is indifferent between buying the follower's product and not purchasing, and θ_0 the type of customer who is indifferent between the leader's and the follower's product. Note that $\theta_1 = p_1$, $\theta_2 = p_2/\rho$, and $\theta_0 = (p_1 - p_2)/(1 - \rho)$. Let $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ denote the *primary demands* of the leader

Figure 1 Customer Arrival and Purchase Process



and the follower, respectively.[5] Three different cases arise. First, consider the case in which p_1 is small in the sense that $p_1 \leq p_2/\rho$, as given in panel (a) of Figure 2. All customer types above θ_2 prefer the follower's product over nothing, but they also prefer the leader's over the follower's. The leader's primary demand is $D_1 = 1 - \theta_1$, and the follower has no primary demand. The follower can have leftover demand from the leader if the leader's capacity is less than her primary demand.

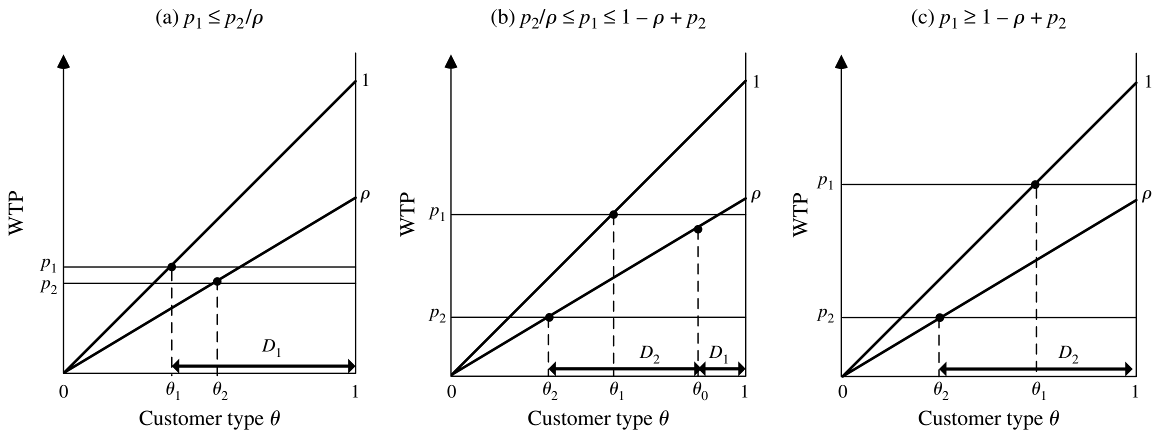
Next, consider the case in which the leader's price is moderate in the sense that $p_2/\rho \leq p_1 \leq 1 - \rho + p_2$, as illustrated in panel (b) of Figure 2. In this case, the follower's product is the first choice of customers whose types are between θ_2 and θ_0, and the leader's product is the first choice of customers whose types are greater than θ_0. That is, the leader's primary demand is $D_1 = 1 - \theta_0$, and the follower's primary demand is $D_2 = \theta_0 - \theta_2$. Depending on the capacity levels of both firms and the customer arrival sequence, either firm can have leftover demand from the other firm. Last, when the leader's price is sufficiently high, $p_1 \geq 1 - \rho + p_2$, as in panel (c) of Figure 2, which does not arise in equilibrium, $D_1 = 0$ and $D_2 = 1 - \theta_2$.

3.2. Customer Arrival Sequences

As already indicated, the firms' sales quantities can differ from their primary demands, and the customer arrival sequence s can impact the results. We focus on three different arrival sequences: high-to-low ($s = H$, higher-valuation customers arrive earlier than lower-valuation customers), independent ($s = I$, customers arrive randomly, independent of their valuations), and low-to-high ($s = L$, lower-valuation customers arrive earlier than higher-valuation customers). Although we take the arrival sequence as given, the high-to-low sequence can arise when both firms give priority to the higher-end customers. Independent arrivals can arise in retail environments where customers arrive randomly, independent of their type, and are served on a first-come, first-served basis. Low-to-high arrivals can arise, for example,

[5] We drop the arguments of D_1 and D_2 when they are clear. We use the terminology *primary demand* because a firm may have leftover demand from the other firm if the other firm's capacity is less than its primary demand.

Figure 2 **Formation of the Demands for the Two Products** (D_1, D_2)



when lower-end customers tend to arrive early in hopes of finding a good deal. As mentioned earlier, this sequence is usually assumed in the revenue management literature. The customer arrival sequence can be thought of as the customer type transaction sequence if we interpret the transaction as including a customer's decision to buy nothing.

3.3. Leftover Demand

We now specify the leftover demands and how they depend on the arrival sequence. For our purposes, the *leftovers* from a firm are the excess, if any, of that firm's primary demand over its capacity. For example, the leftovers from Firm 1 are $(D_1(p_1, p_2) - Q_1)^+$. Not all of those necessarily become leftover demand for the other firm, because some of the customers who are unable to buy their preferred product may choose to buy nothing, rather than buy their second-choice product. The *leftover demand* for Firm i, denoted by $L_i^s(p_1, p_2, Q_1, Q_2)$,[6] consists of the number of customers, out of the leftovers from the other firm, who prefer to buy from Firm i than to buy nothing. Let $\pi_i^s(p_1, p_2)$ denote the resulting profits to Firm i, namely, $\pi_i^s(p_1, p_2) = p_i \min(D_i(p_1, p_2) + L_i^s(p_1, p_2), Q_i)$.

For the case of panel (a) in Figure 2, every buying customer prefers the leader's product, so there can be no leftover demand for the leader. That is, $L_1^s(p_1, p_2) = 0$ for every s. If the leader runs out of product, then only customer types between θ_2 and 1 are willing to buy from the follower; customer types between θ_1 and θ_2 choose not to purchase. For high-to-low arrivals, customer types between $1 - Q_1$ and 1 are served by the leader, so the leftover demand for the follower comes from all customer types between θ_2 and $1 - Q_1$: $L_2^H(p_1, p_2) = (1 - Q_1 - \theta_2)^+$. For low-to-high arrivals, the types between θ_1 and $\theta_1 + Q_1$ are served by the leader, so the leftover demand for the follower comes from types between $\max(\theta_2, \theta_1 + Q_1)$ and 1: $L_2^L(p_1, p_2) = \min(1 - \theta_2, (1 - \theta_1 - Q_1)^+)$. For independent arrivals, a uniform proportion of the types between θ_1 and 1 are served by the leader, so the leftover demand for the follower comes from the remaining unserved portion of the types between θ_2 and 1. This interval of types is uniformly thinned out by independent arrivals: $L_2^I(p_1, p_2) = [(1-\theta_2)/(1-\theta_1)] \times (1 - \theta_1 - Q_1)^+$. The leftover demands for the cases of panels (b) and (c) in Figure 2 are obtained in a similar manner.

3.4. Solution Approach

Given arrival sequence s, a Stackelberg equilibrium of our game[7] is the leader's price, p_1^s, and the follower's response function, $p_2^s(\cdot)$, such that

$$p_2^s(p_1) \in \arg\max_{p_2} \pi_2^s(p_1, p_2) = \arg\max_{p_2} [p_2 \min(D_2(p_1, p_2) + L_2^s(p_1, p_2), Q_2)], \quad \text{for all } p_1, \quad (1)$$

[6] The dependence on Q_1 and Q_2 is suppressed henceforth.

[7] Note that the Stackelberg equilibrium is a subgame perfect equilibrium of our game.

and

$$p_1^s \in \arg\max_{p_1} \pi_1^s(p_1, p_2^s(p_1))$$
$$= \arg\max_{p_1} [p_1 \min(D_1(p_1, p_2^s(p_1)) + L_1^s(p_1, p_2^s(p_1)), Q_1)]. \quad (2)$$

The pair of prices $(p_1^s, p_2^s(p_1^s))$ constitutes an equilibrium outcome of our game. Note that the primary demands depend on both prices, and the leftover demands and profits depend on the capacity levels and arrival sequence as well.

3.5. The Case of Unlimited Capacities

Consider the important benchmark case in which firms face no effective capacity limits. This case can arise when firms have sufficiently high capacity levels or when so little time is necessary to construct capacities that firms set their capacities essentially at the same time that they set their prices. Some examples for this case are digital goods and software sold over the Internet. We denote this case with the superscript N. Given ρ, the equilibrium prices[8] are $p_1^N = (1-\rho)/(2-\rho)$, and $p_2^N = \rho(1-\rho)/[2(2-\rho)]$. Consequently, the equilibrium sales quantities are $S_1^N = 1/2$ and $S_2^N = 1/[2(2-\rho)]$, which are equal to the equilibrium demand levels. We formally define this equilibrium as the *Unlimited Capacities Equilibrium* and call the vector of equilibrium sales quantities, (S_1^N, S_2^N), the *Benchmark Capacity Profile*.

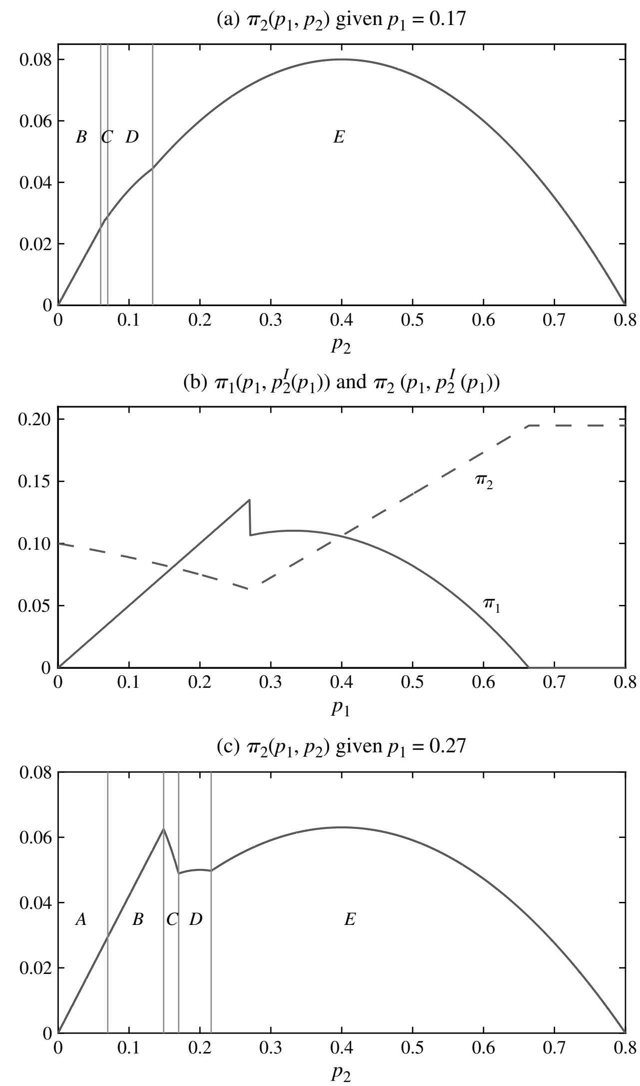
4. Leftovers Equilibria

We begin this section with an example that illustrates how and why the equilibrium of our model can change dramatically when capacities are limited, follow with our main existence result, and end with an examination of why the arrival sequence can make a substantial difference.

4.1. Consequences of Firms' Pricing Decisions

Suppose $\rho = 0.8$, so that in the Unlimited Capacities Equilibrium the leader would sell $S_1^N = 0.5$ at price $p_1^N = 0.17$, and the follower would sell $S_2^N = 0.42$ at price $p_2^N = 0.067$. Now suppose that those sales quantities, comprising the Benchmark Capacity Profile, are the respective capacities of the two firms: $Q_1 = 0.5$ and $Q_2 = 0.42$. To illustrate how and why the equilibrium changes so much, assume that we have independent arrivals and that this assumption holds for the rest of this subsection. Panel (a) of Figure 3 plots the follower's profits in this case as a function of his (own) price, given that the leader has set her price at $p_1^N = 0.17$. If the follower prices at $p_2^N = 0.067$ (the upper boundary of interval C), then the firms set

Figure 3 **Analysis under the Benchmark Capacity Profile ($Q_1 = 0.5$, $Q_2 = 0.42$) with $\rho = 0.8$ and Independent Arrivals**



Notes. Panel (a) plots the follower's profits given the leader prices at $p_1 = 0.17$. Panel (b) plots the firms' profits as functions of p_1 assuming the follower's best response ($p_2^I(p_1)$). Panel (c) plots the follower's profits given the leader prices at $p_1 = 0.27$.

[8] The derivation is given in the online supplement.

the same prices they would in the Unlimited Capacities Equilibrium, and they would get the same profits, too. However, the follower can do much better by increasing his price. That causes the leader's primary demand to increase above her capacity, which creates leftover customers for the follower. Many of these leftover customers prefer the follower's product over nothing at all, so, although his primary demand decreases, his leftover demand increases, leading to a net increase in his profits. In particular, the Unlimited Capacities Equilibrium is *not* an equilibrium in the limited capacities setting.

Panel (b) of Figure 3 helps explain how the equilibrium changes. It examines the leader's pricing problem assuming that the follower prices optimally in response. In particular, the leader can increase her profits by increasing her price above 0.17: the follower is reaping such high profits by pricing at 0.40 that his optimal response will not change until the leader increases her price to (about) 0.27. If the leader increases her price slightly above that point, then her profits take a discontinuous drop, and, although she can increase her profits a little from that (lower) point by increasing her price further, she cannot attain the profits she would get at 0.27.

Thus, the Stackelberg equilibrium price for the leader is 0.27. It is illuminating to examine the details of panel (c) in Figure 3, which plots the follower's profits as a function of his (own) price, assuming that $p_1 = 0.27$. There are five distinctly different intervals into which p_2 can fall. The follower's profits are concave within each interval, so the follower's optimal price response can be found by optimizing within each interval and taking the best result. It is interesting that in the neighborhood of an equilibrium, this objective function usually has at least two local maxima, and they play a key role in determining the equilibrium that arises. If p_2 is large enough to be in interval E, the follower obtains only leftover demand from the leader without any primary demand. If p_2 decreases to interval D, the follower has some primary demand as well as some leftover demand from the leader. Interval C represents what we call the *direct competition regime*,[9] which is when both firms have only their primary demands without any leftovers. Intervals B and A are analogous to intervals D and E, respectively, with firms' roles reversed: the leader gets leftover demand from the follower. However, in intervals A and B, the follower's profits increase as his price increases, because he sells all his capacity. Hence, his optimal price cannot be in interval A or the interior of interval B. It is straightforward to show that this holds in general in our setting, so there will be no equilibria with leftover demand from the follower to the leader. Thus, we unambiguously call interval E the *Pure Leftovers Regime*, interval D the *Partial Leftovers Regime*, and the union of intervals D and E the *Leftovers Regime*.[10]

The follower is indifferent between pricing low (at $p_2 = 0.15$) at the lower boundary of interval C, the direct competition regime, and pricing high (at $p_2 = 0.40$) in the interior of interval E, the Pure Leftovers Regime. However, the leader's profits are maximized only at $p_2 = 0.40$. The Stackelberg equilibrium outcome is therefore $(p_1, p_2) = (0.27, 0.40)$.[11]

We call the equilibrium in this example the *Benchmark Limited Capacities Equilibrium* because the firms have limited capacities equal to the Benchmark Capacity Profile. We formally define a *Leftovers Equilibrium* as a Stackelberg equilibrium of our game in which one firm runs out of capacity and the second firm sells at least some products to customers who prefer to buy from the first firm but are unable to do so. We also define a *Pure Leftovers Equilibrium* as a Leftovers Equilibrium in which all customers who buy from the second firm are those who prefer to buy from the first firm; i.e., there is no primary demand for the second firm. Thus, we can state the *three key properties* of the Benchmark Limited Capacities Equilibrium in this example: (a) It is a Pure Leftovers Equilibrium, (b) the follower does not use all his capacity, and (c) the follower is indifferent between his best price in the interior of the Pure Leftovers

[9] To clarify our terminology in the context of other competition models, we call this the *direct competition with primary demand-only regime*. We shorten it when the context is clear.

[10] Technically, for example, the *Pure Leftovers Regime* consists of prices such that p_2 lies in interval E.

[11] This is the unique (Stackelberg) equilibrium of the game because if the leader prices at 0.27 and the follower prices at 0.15, at the intersection of intervals B and C, we do not have a Stackelberg equilibrium. The leader could price a tiny bit below 0.27 to assure that the follower would strictly prefer the higher price in the Pure Leftovers Regime, giving much more to the leader.

Regime and the lowest price in the direct competition regime (namely, the price at which his primary demand equals his capacity). We find that the equilibrium that satisfies those three key properties arises not only under the Benchmark Capacity Profile but also under a wide intermediate range of capacities. We therefore define a *Baseline Leftovers Equilibrium* as a Stackelberg equilibrium that shares the cited three key properties, and most of this paper focuses on this equilibrium and conditions under which it arises.

4.2. Existence of Baseline Leftovers Equilibria

We now present our main result, which gives sufficient conditions, for each of the three arrival sequences, to ensure that there exists a unique Stackelberg equilibrium to our game and that it is a Baseline Leftovers Equilibrium.[12]

Proposition 1. *If* $\rho \in (2/3, 1)$, *then, for each customer arrival sequence* $s \in \{H, I, L\}$, *there exists a neighborhood* $R^s(\rho)(\subset \mathbb{R}^2)$ *of the Benchmark Capacity Profile such that if* $(Q_1, Q_2) \in R^s(\rho)$, *then there is a unique Stackelberg equilibrium under arrival sequence s and it is a Baseline Leftovers Equilibrium.*

Our proof defines $R^s(\rho)$ explicitly for each ρ and s and shows that $R^s(\rho)$ includes the Benchmark Capacity Profile.[13] Figure 4 illustrates Proposition 1 for the case of $\rho = 0.8$ by showing the (Q_1, Q_2) regions, for each arrival sequence in which a Baseline Leftovers Equilibrium arises. The intersection of these three regions, the darker shaded region, contains the Benchmark Capacity Profile (shown with the "+") and many cases where industry total capacity exceeds potential industry market size (e.g., $Q_1 = 0.45$, $Q_2 = 0.65$). In summary, shortages can be strategically induced for plausible levels of fixed capacities, including cases when there is plenty of capacity in the industry.[14]

[12] The price levels, sales quantities, and profits for all three arrival sequences are given explicitly in Table 1.

[13] The appendix gives the proof for independent arrivals and the online supplement gives the proof for the two other arrival sequences and all other proofs. In addition, our proof for the case of low-to-high arrivals gives an explicit region for all $\rho \in (0, 1)$.

[14] Subsection 6.1 reveals the other equilibria that arise in this example under independent arrivals.

4.3. Understanding the Baseline Leftovers Equilibrium

We first establish that an increase in the leader's price can have a counterintuitive negative effect on the follower's demand within the Pure Leftovers Regime. This phenomenon helps us understand how the Baseline Leftovers Equilibrium arises and how it depends on the arrival sequence. It also proves useful in understanding some of the comparative statics we obtain later.

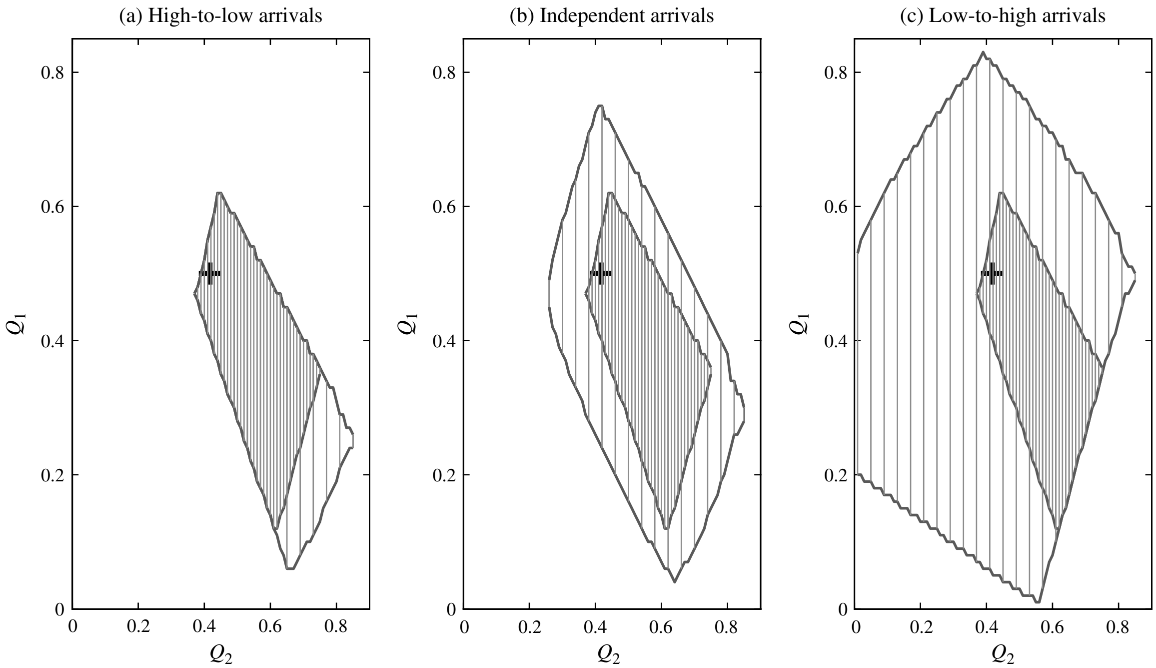
Proposition 2. *In the Pure Leftovers Regime, if the leader's price increases, then the follower's demand strictly decreases under independent arrivals, weakly decreases under low-to-high arrivals, and is unchanged under high-to-low arrivals.*

In the Pure Leftovers Regime (e.g., within interval E in panel (a) of Figure 3), where $p_2 \geq \rho p_1$ and $p_1 \leq 1 - Q_1$, the follower acts as a monopolist, exploiting the captive leftover demand he faces. The follower's captive demand curve comes by removing the customer types who buy from the leader.

Under independent arrivals, sales for the leader's product come uniformly from all customer types who want to buy it, so the leftovers for the follower have a uniform representation of the same customer types. The follower's captive demand curve can be viewed as a scaled-down version of his original monopoly demand curve (absent the leader) with a smaller market size. The scale factor $(1 - p_1 - Q_1)/(1 - p_1)$ depends only on Q_1 and p_1, so the follower applies his monopoly price, $p_2^I = \rho/2$, within this regime, regardless of the leftover demand he faces or of the leader's price. If the leader increases her price, then she removes some lower-valuation customers from the potential purchaser population. The scale factor decreases, so the follower's demand decreases. The leader still sells Q_1 units, but they are spread over a smaller interval of potential customers, so a higher proportion of the customers who prefer the leader's product will be able to buy it, which leaves fewer leftovers for the follower. That is, in contrast to the usual intuition, a firm gets hurt (because its demand is reduced) when its competitor raises the price of its competing product.

Under high-to-low arrivals, the follower's captive demand curve comes from slicing off the highest end

Figure 4 **Capacity Regions under which Baseline Leftovers Equilibria Emerge**



Notes. Lighter shaded regions are regions of Q_1 and Q_2, in which a unique Stackelberg equilibrium exists and is a Baseline Leftovers Equilibrium, under each arrival sequence ($\rho = 0.8$). The darker shaded regions are the intersection of the three lighter shaded regions. "+" depicts the Benchmark Capacity Profile.

customers from his monopoly demand curve. All the customer types below that cutoff point $(1 - Q_1)$ are his potential customers. Thus, his monopoly price for this captive market is $\rho(1 - Q_1)/2$. In this case the leftover demand for the follower does not depend on the leader's price. What happens here is that the leader's price increase simply reduces the number of customers who prefer her product, but the top Q_1 of them still buy it, so, given the follower's price, his sales quantities remain at $L_2^H = 1 - Q_1 - p_2/\rho$.

Under low-to-high arrivals, in the Pure Leftovers Regime, the leader sells Q_1 units to a middle slice of the customer type spectrum, and the follower gets to sell to the top slice. Regardless of whether he sells to all or part of that slice, his demand weakly decreases in the leader's price.

5. Comparative Statics and Managerial Incentives

We now explore the impact that changes in the operational environment have on the Baseline Leftovers Equilibrium outcomes. The changes we consider include (a) the customer arrival sequence, (b) the capacity levels, and (c) the quality ratio (ρ). We then reveal how those changes effect the Baseline Leftovers Equilibrium. We want to emphasize that all implications discussed in this section apply only under the assumptions of our simple model (such as zero production costs and Stackelberg pricing) and only to the Baseline Leftovers Equilibrium ($(Q_1, Q_2) \in R^s(\rho)$), so that as a parameter changes, for the conclusion to be valid we must not leave this region.[15]

5.1. The Impact of the Arrival Sequence on Prices and Profits

Proposition 3. *In the Baseline Leftovers Equilibrium,*

(a) $p_1^H \le p_1^I \le p_1^L$, $p_2^H \le p_2^I \le p_2^L$ *and*
(b) $\pi_1^H \le \pi_1^I \le \pi_1^L$ *and* $\pi_2^H \le \pi_2^I \le \pi_2^L$.

Both firms' prices and profits are highest under low-to-high arrivals, next highest under independent arrivals, and lowest under high-to-low arrivals. The intuition is that it is easiest for the leader to

[15] Subsection 6.1 illustrates the kinds of equilibria that can arise when we leave that region.

appease the follower with leftovers under low-to-high arrivals, less easy under independent arrivals, and hardest under high-to-low arrivals: Under low-to-high arrivals, the highest end customers are all unable to buy the leader's product and end up buying the follower's product. The follower can treat these customers like captive demand and set high prices. The leader knows this and does not need to provide much in the way of leftovers to ensure that the follower will not prefer direct competition. She therefore can price relatively high, leading to high profits.

Under independent arrivals, the mix of customer types within the leftovers is less attractive, with some low-end customers as well as high-end ones. The mix is worst under high-to-low arrivals. As the mix worsens, the follower must price lower (to attract the necessary customers), receiving lower profits and, hence, being more tempted by direct competition. The leader must appease more by providing more leftovers, which is done by lowering her price, which lowers her profits. This result reveals that both firms have the incentive to influence the arrival sequence so it is low-to-high. Further, in the Baseline Leftovers Equilibrium, the follower chooses to sell to all the leftover (high-valuation) customers only in the low-to-high sequence. This is noteworthy because this is the sequence under which both firms profit most.

5.2. The Impact of Firms' Capacities

We present how the strategic implications of capacity changes can depend dramatically on the arrival sequence in the Baseline Leftovers Equilibrium.

Proposition 4. *In the Baseline Leftovers Equilibrium, an increase in the leader's capacity* (Q_1) *decreases the leader's price and the follower's profits, and the leader's profits are quasiconcave in* Q_1. *Further, an increase in* Q_1 *has the following consequences*:

(a) *Under high-to-low arrivals, the follower's price decreases*.

(b) *Under independent arrivals, the follower's price is unchanged*.

(c) *Under low-to-high arrivals, the follower's price increases*.

As the leader's capacity increases in a Baseline Leftovers Equilibrium, then, regardless of the arrival sequence, all that capacity is fully utilized, which, without any price changes, would significantly decrease the leftovers available to the follower and thereby also decrease his profits. This increases the follower's incentive to switch to direct competition. To avoid inducing that switch, the leader must increase her own demand by decreasing her price, so that the reduction in the leftovers available to the follower is moderated and the follower still prefers to live on leftovers. The follower ends up with lower leftover demand and profits, but with profits at least as high as what he would get through direct competition.

It is interesting that the follower's price response differs dramatically depending on the arrival sequence. Under high-to-low arrivals, as the leader's capacity increases, she serves a larger number of high-end customers. Thus, the follower's customers are lower-end than before and his price must be reduced to compensate. Under low-to-high arrivals, the opposite holds: as the leader takes more of the lower-end customers, leaving a larger number of high-end customers in the leftovers, the follower raises his price. Under independent arrivals, the leftovers consist of a smaller uniform mix of the same customer types, so the follower does not change his price.

We define the *strategic shadow price* of additional capacity as the marginal value of increasing capacity in the model, analogous to the shadow price found in single firm optimization models. Here, the leader's profits are $p_1 Q_1$ in the equilibrium, with the price decreasing and the capacity increasing. When Q_1 is relatively small, the increase in the sales quantity dominates and her profits increase. However, for higher values of Q_1, the leader's price decrease can dominate and her profits decrease. Thus, as is usual when a firm's profits are quasiconcave in its capacity (e.g., Gelman and Salop 1983), that firm's strategic shadow price of additional capacity can be negative; hence, the firm can have a strategic incentive to limit capacity. Interestingly, this decrease in profits, which happens when capacity is added for free, arises in our model for the leader, even though she uses all her capacity and has many customers who are begging her to sell to them.

Proposition 5. *In the Baseline Leftovers Equilibrium, an increase in the follower's capacity* (Q_2) *decreases* *the leader's price and profits and has the following consequences*:

(a) *Under high-to-low arrivals*, *the follower's price and profits are unchanged*.

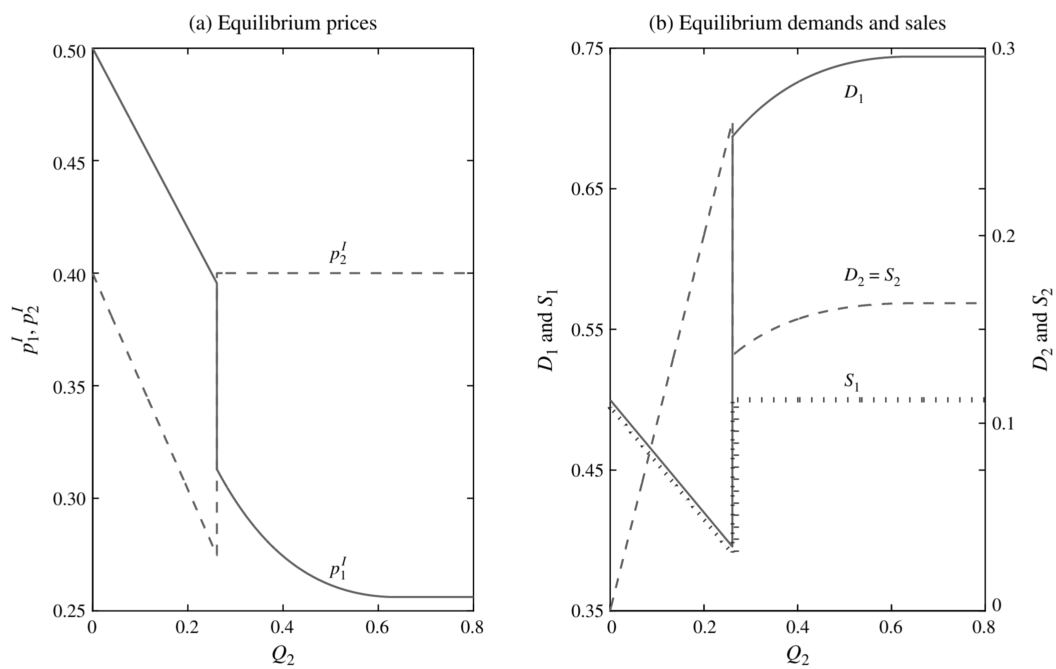
(b) *Under independent arrivals*, *the follower's price is unchanged*, *but his profits increase*.

(c) *Under low-to-high arrivals*, *the follower's price decreases and his profits increase*.

In a Baseline Leftovers Equilibrium, as the follower's capacity (Q_2) increases, the follower's threat to switch to direct competition increases, because his profits under the direct competition regime would increase. Thus, the leader must appease the follower to deter him from instigating direct competition. This is done under all three arrival sequences by the leader reducing her price and, consequently, decreasing her profits. One effect of the leader reducing her price is that the profits that the follower would get if he switched to direct competition are reduced. Under high-to-low arrivals, this is the only effect, because the leader cannot increase the follower's leftover demand by changing her price in this case: A small change in the leader's price only affects the number of lower-end customers in the leftovers, and the follower chooses not to sell to them. The follower leaves his price unchanged, and his profits are also unchanged.

However, under the other two arrival sequences, the leader lowering her price also leads to the follower increasing his profits. For example, Figure 5 illustrates what happens as the follower's capacity (Q_2) increases under independent arrivals. When Q_2 is small ($Q_2 < 0.26$), there are no leftovers, and both firms' capacities are binding ($D_1 = S_1$ and $D_2 = S_2$). However, once we get beyond about $Q_2 = 0.26$, our Baseline Leftovers Equilibrium appears. The leader lowers her price significantly (from $p_1 = 0.4$ to $p_1 = 0.32$), and the follower dramatically increases his price in response (from $p_2 = 0.27$ to $p_2 = 0.4$). As Q_2 continues to increase, the follower's mix of captive customer types does not change, so his optimal price will not change. By lowering her price, the leader increases her primary demand (D_1), which increases the follower's captive leftover demand (and profits). That adds a second disincentive for the follower to switch to direct competition.

Figure 5 Equilibrium Outcome as a Function of the Follower's Capacity, Q_2, under Independent Arrivals, $Q_1 = 0.5$ and $\rho = 0.8$



Notes. Panel (a) plots the firms' equilibrium prices (p_1^I and p_2^I). Panel (b) plots the equilibrium demand and sales quantities for the two firms (D_1, S_1 and D_2, S_2 respectively).

The intuition is similar for the case of low-to-high arrivals, except, in this case, the follower's price decreases in the process. As the leader's price decreases, more low-end customers are willing to buy her product, and, because of the arrival sequence, they do so. The leader still only sells Q_1, so she adds additional customers to the pool of leftovers. These are higher end than any she currently sells to but lower-end than any the follower previously sold to. The follower profits by selling to all these new customers; to do so, he lowers his price but does not even use all the capacity he had prior to the change. Thus, none of the additional follower capacity is used, but the follower is able to increase his profits.

The strategic shadow price of additional follower's capacity can therefore differ dramatically from the shadow price of capacity that arises in single-firm, limited capacity, deterministic demand optimization models. Here, the follower can have a positive strategic shadow price when his capacity constraint is not binding. Indeed, he will not utilize any of the additional capacity. He benefits from the threat value of that additional capacity.

Furthermore, one might imagine that changes in the arrival sequence would represent at most minor operational changes in the system and have no strategic consequences. However, we have just seen that the strategic shadow price (of additional follower capacity) can be strictly positive under two of the three arrival sequences but will be zero under high-to-low arrivals. This suggests that other results in the literature on oligopolistic competition under limited capacities may also depend on the arrival sequence (rationing rule). Sherman and Visscher (1982) and Dana (1999) also argue that arrival sequences other than high-to-low (efficient rationing) are more appropriate representations of reality in some settings. In other words, understanding the arrival sequence may be critical to operating effectively in a limited capacity competitive environment.

5.3. The Impact of the Quality Ratio

Proposition 6. (a) *In the Unlimited Capacities Equilibrium, as ρ increases, the leader's profits and industry profits decrease, whereas the follower's price and profits are quasiconcave.*

(b) *In the Baseline Leftovers Equilibrium, as ρ increases, the following results hold. The leader's profits decrease and the follower's price and profits increase. Industry profits increase under low-to-high arrivals but are indeterminate for the other two sequences. Furthermore,*

$$\frac{\partial \pi_1^N}{\partial \rho} \le \frac{\partial \pi_1^H}{\partial \rho} \le \frac{\partial \pi_1^I}{\partial \rho} \quad \text{and} \quad \frac{\partial \pi_2^N}{\partial \rho} \le \frac{\partial \pi_2^H}{\partial \rho} \le \frac{\partial \pi_2^I}{\partial \rho}. \tag{3}$$

There are two effects of an increase in the quality ratio (ρ): first, the relative attractiveness of the follower's product increases; second, competition between the two firms increases, because the products become more alike. In the Unlimited Capacities Equilibrium, the follower's price and profits both increase at first and then decrease as the quality ratio increases. That is, the first effect dominates the follower's perspective for low-quality ratios, whereas the second effect dominates for high-quality ratios. A follower's quality increase can end up hurting his profits if his quality gets too close to that of the leader. However, in the Baseline Leftovers Equilibrium (with capacity limits), the first effect dominates and the follower's price and profits both increase as the quality ratio increases. In this case, the strategic value of increased follower quality is always positive (provided the Baseline Leftovers Equilibrium still arises). Furthermore, this strategic marginal value to the follower is higher under independent arrivals than under high-to-low arrivals and in turn than in the Unlimited Capacities Equilibrium. An empirical implication of this result is that the follower will tend to have higher quality in environments with limited capacities and arrival sequences that resemble independent arrivals, than in environments with either unlimited capacity or high-to-low arrivals. For example, suppose we argue that in a high-technology new product market with innovations, the announcement of a new product, such as the iPhone, encourages those who value the new product the most to rush to buy, which resembles high-to-low arrivals. If we also argue that arrival sequences resemble independent arrivals in a generic customer retail environment, then we would expect to see a lower quality ratio in the first environment than in the second, but higher in the first than in the unlimited capacity environment. An empirical implication of part (a) is that when capacities are essentially unlimited, such as when the capacity lead time is minimal or nonexistent, as in the software and other digital goods industries, we would expect to see a distinct quality difference between the two products. In addition, when capacities are limited, such as in the hotel lodging industry, we would expect follower hotels to be less wary of building an almost identically high-quality hotel.

Industry profits respond differently to an increase in the quality ratio. In the Unlimited Capacities Equilibrium, the second effect dominates and industry profits decrease. In contrast, the dominant effect in the Baseline Leftovers Equilibrium depends on the arrival sequence. Under low-to-high arrivals, the first effect dominates such that industry profits increase. Thus, the effect on the industry of increasing the quality ratio depends in fundamental ways on the arrival sequence.

In addition, although the leader's profits decrease in the quality ratio (ρ) for all arrival sequences within the Baseline Leftovers Equilibrium, Equation (3) says that the rate of decrease is less for independent arrivals than for high-to-low arrivals, which, in turn, is less than for the Unlimited Capacities Equilibrium.

5.4. Equilibrium Price Ratios

We now turn our attention to the ratio of the follower's price to the leader's in the Baseline Leftovers Equilibrium.

Proposition 7. *Let $r^s \triangleq p_2^s/p_1^s$ denote the price ratio under customer arrival sequence s. Under the Baseline Leftovers Equilibrium,*

(a) $\rho/2 = r^N \leq \rho \leq r^H \leq r^I \leq r^L$, *and*

(b) *All (four) price ratios are increasing in ρ, Q_1, and Q_2.*

For all customer types, ρ is the ratio of their utility for the follower's product to their utility for the leader's product. In the Unlimited Capacities Equilibrium, the price ratio is $\rho/2$, half of the utility ratio. Thus, the follower's product is a much better value in this case in the sense that customers get more bang per buck (utility per dollar spent). However, in a Baseline Leftovers Equilibrium, the follower's product is relatively high priced and the leader's product is a better value. Returning to the hotels example, this result says that we should expect the lower-quality hotel that satisfies the customers who cannot get into the higher-quality hotel to have a higher quality-adjusted price than the higher-quality hotel. Furthermore, if one argues that the arrival sequence in the lodging industry resembles low-to-high, in the sense that low-end customers book earlier, then we would expect the lower-quality hotel to be a particularly poor value, something that may resonate with those who have booked lodging late for professional conferences.

Similarly, if we argue that generic retail environments emulate independent arrivals and arrival processes following new product introductions emulate high-to-low arrivals, our model would predict higher price ratios for the former environment controlling for the quality levels. We also expect the price ratio to be lowest in duopolies where capacity plays a minimal or no role.

Part (b) of Proposition 7 says that in the Baseline Leftovers Equilibrium, as the follower's product quality or the capacity of either firm increases, the price ratio increases. The empirical implication of this is that in industries where the follower's relative product quality is higher or where industry capacity is higher, customers get less bang per buck from the follower's product.

5.5. Summary and Interpretation of the Results

Table 1 provides a compact summary of our results on the Baseline Leftovers Equilibrium.[16] To interpret the results, note that by the third key property of a Baseline Leftovers Equilibrium, the leader's equilibrium price is the largest price she can charge such that the follower prefers to sell solely to leftovers, instead of engaging in direct competition and selling all his capacity. (Under our conditions, all best follower responses in other regimes lead to lower profits for the follower.) In the latter scenario, the follower's profits do not depend on the arrival sequence and are given by $\pi_2 = \rho(p_1 - \beta)Q_2$, where $\beta = (1-\rho)Q_2$. By our analysis in §3.1, to have any primary demand, the follower must set $p_2 \leq \rho p_1$, and his resulting primary demand consists of consumers with types $\theta \in [p_2/\rho, (p_1 - p_2)/(1-\rho)]$. Thus,

$$Q_2 = \frac{p_1 - p_2}{1-\rho} - \frac{p_2}{\rho}. \tag{4}$$

[16] Table 1 also includes some comparative statics that are not specifically mentioned in our propositions. Proofs of those results are in the online supplement.

Table 1 **Equilibrium Outcome and Comparative Statics with Respect to ρ, Q_1, and Q_2 for the Unlimited Capacities Equilibrium and the Baseline Leftovers Equilibrium**

	Unlimited capacities		Customer arrival sequence: High-to-low				Independent				Low-to-high			
		ρ		ρ	Q_1	Q_2		ρ	Q_1	Q_2		ρ	Q_1	Q_2
p_1	$\frac{1-\rho}{2-\rho}$	$\searrow$	$\frac{\alpha^2}{4Q_2}+\beta$	$\searrow$	$\searrow$	$\searrow$	$\frac{1}{2}\Big(\frac{1}{4Q_2}+1+\beta - \sqrt{(1/(4Q_2)+1+\beta)^2-4\gamma}\Big)$	$\searrow$	$\searrow$	$\searrow$	$\frac{1}{2}(\alpha-(Q_1+Q_2) + \sqrt{(\alpha-(Q_1+Q_2))^2+4\delta})$	$\searrow$	$\searrow$	$\searrow$
p_2	$\frac{\rho(1-\rho)}{2(2-\rho)}$	$\curvearrowright$	$\frac{\rho\alpha}{2}$	$\nearrow$	$\searrow$	$\rightarrow$	$\frac{\rho}{2}$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rho(p_1^L+Q_1)$	$\nearrow$	$\nearrow$	$\searrow$
S_2	$\frac{1}{2(2-\rho)}$	$\nearrow$	$\frac{\alpha}{2}$	$\rightarrow$	$\searrow$	$\rightarrow$	$\frac{1}{2}\Big(1-\frac{Q_1}{1-p_1^I}\Big)$	$\nearrow$	$\searrow$	$\nearrow$	$1-p_1^L-Q_1$	$\nearrow$	$\searrow$	$\nearrow$
π_1	$\frac{1-\rho}{2(2-\rho)}$	$\searrow$	$p_1^H Q_1$	$\searrow$	$\curvearrowright$	$\searrow$	$p_1^I Q_1$	$\searrow$	$\curvearrowright$	$\searrow$	$p_1^L Q_1$	$\searrow$	$\curvearrowright$	$\searrow$
π_2	$\frac{\rho(1-\rho)}{4(2-\rho)^2}$	$\curvearrowright$	$p_2^H S_2^H$	$\nearrow$	$\searrow$	$\rightarrow$	$p_2^I S_2^I$	$\nearrow$	$\searrow$	$\nearrow$	$p_2^L S_2^L$	$\nearrow$	$\searrow$	$\nearrow$
π_T		$\searrow$		$\uparrow\downarrow$	$\curvearrowright$	$\searrow$		$\uparrow\downarrow$	$\curvearrowright$	$\searrow$		$\nearrow$	$\curvearrowright$	$\uparrow\downarrow$

Notes. $\alpha = 1-Q_1$, $\beta = (1-\rho)Q_2$, $\gamma = (\alpha/4+\beta Q_2)/Q_2$, and $\delta = \alpha Q_1 + \beta Q_2$. In addition, $\pi_T = \pi_1 + \pi_2$. $\curvearrowright$ indicates that the corresponding value is quasi-concave, so it can increase up to a point and decrease afterwards. $\uparrow\downarrow$ indicates that the corresponding value can either increase or decrease within different intervals depending on values of ρ, Q_1, and Q_2.

Solving (4), we obtain $p_2 = \rho(p_1 - \beta)$; thus, π_2 is as given.

The follower's best profits in the interior of the Pure Leftovers Regime depend strongly on the arrival sequence, and this dependence leads to different expressions for the equilibria. In all cases of a Baseline Leftovers Equilibrium, the leader sells Q_1, all of her capacity. Let $\alpha = 1 - Q_1$, which is the size of the captive market (including leftover customers) for the follower to exploit. Under high-to-low arrivals, the leader sells her Q_1 units to those who value the product the most, so the customer valuations within the follower's resulting captive market range from zero to $\rho\alpha$. His monopoly price for this captive market is $\rho\alpha/2$, his monopoly sales quantity is $\alpha/2$, and his profits are the product of the two: $\rho\alpha^2/4$. Thus, the leader's equilibrium price p_1^H—the largest price she can charge and still have the follower prefer to sell solely to leftovers—can be expressed as $p_1^H = \max\{p_1 \in [0,1] \mid \rho\alpha^2/4 \ge \rho(p_1-\beta)Q_2\}$. It therefore follows that

$$p_1^H = \frac{1}{Q_2}\left(\frac{\alpha^2}{4}\right) + \beta. \tag{5}$$

Under independent arrivals (within the Pure Leftovers Regime), as discussed in §4.3, the follower applies his monopoly price, $\rho/2$, his sales quantity is $(1-Q_1-p_1)/2(1-p_1)$, and his profits are the product of the two. Thus, the leader's equilibrium price is $p_1^I = \max\{p_1 \in [0,1] \mid \rho(1-Q_1-p_1)/4(1-p_1) - \rho(p_1-\beta Q_2)Q_2 \ge 0\}$. The boundary of the inequality is a convex quadratic equation, and p_1^I equals the smaller root (the other root is strictly larger than 1), which is given in Table 1.

Last, under low-to-high arrivals, the leader sells Q_1 units to the customer types between p_1^L and $p_1^L + Q_1$. The leftover customers comprise a captive market with a size of $1 - p_1^L - Q_1$, with customer valuations ranging from $\rho(p_1^L + Q_1)$ to ρ. The best price for the follower in the Pure Leftovers Regime is $p_2 = \rho(p_1^L + Q_1)$, which corresponds to selling to all leftover customers from the leader in the high-valuation segment. The sales quantity is therefore $1 - p_1^L - Q_1$ and, hence, the resulting follower's profits are $\rho(p_1^L + Q_1)(1 - p_1^L - Q_1)$. Thus, the leader's equilibrium price is $p_1^L = \max\{p_1 \in [0,1] \mid \rho(p_1+Q_1)(1-p_1-Q_1) - \rho(p_1-(1-\rho)Q_2)Q_2 \ge 0\}$. As in the case of independent arrivals, the boundary of this constraint is a quadratic equation in p_1, with a unique solution in $[0,1]$ given by the entry in Table 1.

6. Discussion and Extensions

6.1. Equilibria in Other Capacity Regions

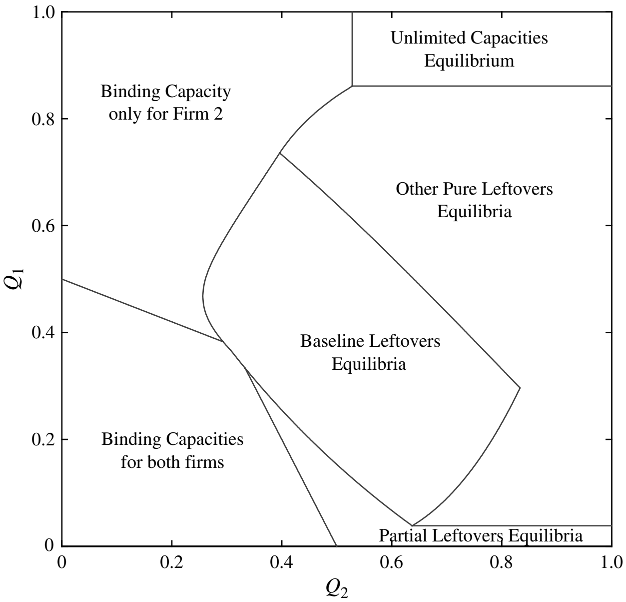
We have shown that the Baseline Leftovers Equilibrium arises in an intermediate capacity region that includes the Benchmark Capacity Profile. Figure 6 illustrates the unique Stackelberg equilibria that arise outside of this region under independent arrivals and when $\rho = 0.8$.[17] In the Baseline Leftovers Equilibrium, the leader prices so that the follower is indifferent between the Pure Leftovers Regime and the boundary of the direct competition regime, where his capacity is binding. In contrast, in the Other Pure Leftovers Equilibria, the leader prices so that the follower is indifferent between the Pure Leftovers Regime and the direct competition regime in which his capacity is *not* binding.[18] Because of that, the follower's strategic shadow price of his capacity in the other Pure Leftovers Equilibria is zero: the follower does not threaten the leader if his capacity is increased in this case because his profits under direct competition and nonbinding capacity will not change. The *Partial Leftovers Equilibrium* is a Leftovers Equilibrium in which some customers buy from the follower as their first choice. When the follower's capacity is smaller, as in the regions of Binding Capacity only for Firm 2 or Binding Capacities for both firms, the follower sells all his capacity, and there are no leftovers in equilibrium.

Some interesting observations can be made from this figure. First, the Unlimited Capacities Equilibrium arises only when industry capacity exceeds the potential market size (maximum customer demand) by at least 37%. Second, Pure Leftovers Equilibria can arise when industry capacity exceeds the potential market size by as much as 80%. Third, the capacity levels have a substantial impact on the kinds of equilibria that arise. When the follower's capacity is relatively low, all his capacity is utilized in equilibrium and there will be no leftovers. However, once the follower's capacity gets sufficiently large, the leader finds it in her interest to strategically use shortages. This is true even when the leader has very little capacity. It is also true even when the follower has

[17] There are ties on the boundaries.

[18] These are all Pure Leftovers Equilibria, and the follower has excess capacity in each. The main difference is that the explicit equilibrium expressions differ.

Figure 6 **Different Types of Equilibria That Can Arise for Different Capacity Levels under Independent Customer Arrivals** ($\rho = 0.8$)



unlimited capacity, provided that the leader can meet less than 80% of the potential market demand. Fourth, the large-capacity region in which Leftovers Equilibria arise suggests that they may have more practical relevance in industries with excess, but not excessive, capacity. It would be worth specifying the conditions under which all different forms of equilibria arise in our model, the explicit forms of those equilibria, and the nature of the comparative statics for them. Such a task is well beyond the scope of this paper, as the analysis just for the Baseline Leftovers Equilibrium is relatively lengthy.

It is possible that the leader's price in the Baseline Leftovers Equilibrium can be lower than it is in the Unlimited Capacities Equilibrium. For example, under independent arrivals, if $Q_1 = 0.73$, $Q_2 = 0.4$, and $\rho = 0.8$, then a Baseline Leftovers Equilibrium arises with $p_1^I = 0.161$, which is lower than $p_1^N = 0.166$. Thus, prices in a Leftovers Equilibrium are not necessarily both "high," and prices in an equilibrium within the direct competition regime, such as the Unlimited Capacities Equilibrium, are not necessarily both "low." The key is whether the follower can price relatively high and thereby induce leftovers from the leader, sell to some of them, and find it best to do so.

6.2. Discussion of Assumptions

We assume that (a) prices are set sequentially, (b) firms set their price only once, (c) the higher-quality firm sets prices first, and (d) firms are given their capacity levels exogenously.

In some settings, such as a (high-quality) conference hotel posting the price it will charge during a future conference, our basic assumptions seem reasonable:[19] A conference hotel essentially commits to its conference rate early and would be limited in responding to price adjustments of any competing hotel. For each particular conference, capacities cannot be changed much on the margin. For example, we recently observed at a conference that rooms still available in three-star hotels sold for about twice the conference rate that was no longer available at the four-star main conference hotel. This situation is consistent with Leftovers Equilibria in the sense that the conference hotel is the price leader and prices are such that one could easily predict that there would be significant leftover demand.

More generally, if a firm incurs a cost to change its price and a second firm incurs a sufficiently low cost to do so, it is plausible that the first firm should price first and once, followed by the second firm. If a firm (she) knows that her single competitor (he) will always respond with his best price to any price she sets and it will be much more costly for her to change her price in response, then she may be better off choosing her Stackelberg price in the first place and sticking to it. There is evidence in the literature that firms incur costs to change their prices. Such costs can arise because of effort and delays in executing decisions in organizational hierarchies and tend to increase with the firm's size (Tirole 1988, Dixit 1991, Levy et al. 1997). A relevant and interesting research problem would therefore study pricing behavior under sequential pricing, possibly multiple times, with heterogeneous (transactions) costs incurred by firms each time they change their prices.

It is worth discussing the phenomenon of heterogeneous price-changing costs in the specific context of competing restaurants. It may be easier for a local independent shop to change prices than the chain restaurant, as the latter belongs to a bigger, hierarchical structure, where many menu and pricing decisions are controlled at a higher level.[20] For example, if a chain restaurant has a national or regional promotion at advertised prices for some menu items, it may be committed to those prices for the duration of the promotion. Thus, it may be appropriate to consider the local independent restaurants as price followers. This situation is consistent with Leftovers Equilibria in the sense that the chain restaurant is the price leader and sets prices in a way that at least in some locations and times of day, one could easily predict that significant excess demand would arise. Note that with prices staying steady, predictable demand fluctuations over days of the week and times of the day would naturally result in variations in the emergence of shortages. An extension of this sort would be merited.

In addition, suppose one of the firms can respond to price changes much more quickly than the other firm. For example, Miller Oil, a small retailer of gasoline in Virginia whose main competitors are large national chains, uses a system that automatically tracks competitor prices and enables Miller to change its prices within 12 minutes in response to competitor price changes (KSS 2008). A competing national chain may be unable to respond so quickly. If both firms respond as fast as they can to their competitor, then most of the time the prices in effect represent Miller's best response to the chain's price. The chain may conclude that choosing its Stackelberg price in the first place and sticking to it is the best option. Having a higher cost of changing her price would add further support to the argument. Leftovers are a recognized phenomenon in the context of competing gasoline stations: In an empirical study, Deacon and

[19] Our model does not capture the role of the conference organizers, who negotiate with conference hotel(s) on rates, the conference rate expiration date, and the specific rooms to which the conference rate applies. Furthermore, there are often several conference hotels and several competing (nonconference) hotels. Conference hotels achieve substantial revenue from other related activities when hosting a conference, and there can be significant uncertainty about how many will attend the conference. So our simple model naturally does not capture all features of this setting.

[20] We would make this argument regardless of whether the local shop is of higher or lower quality. We illustrate in §6.3 that Leftovers Equilibria can also arise when the lower-quality firm prices first.

Sonstelie (1985) observed a situation in which some gasoline stations were required to price substantially lower (averaging \$0.185 per gallon lower) than others. The higher-priced stations still served a substantial percentage of the customers because the lower-priced stations had long lines and customers with high waiting costs were willing to pay the higher price to avoid waiting. Even though the low prices were not selected by the gas stations themselves, we speculate that a form of Leftovers Equilibria can arise in a model with queues and waiting costs, where one firm creates leftover demand for its competitor by pricing relatively low to channel some customers, who are less willing to wait when the lower-priced station is congested, to higher-priced competitors.[21]

Our assumption that capacity levels are exogenous can be plausible in some environments. For example, suppose that a hotel is built with certain quality and capacity levels, neither of which can be easily changed, and that the hotel will be used at many different times of the year, each of which has different demand characteristics. We can envision separate seasons such that the prices set in one season have negligible effect on demands in the other seasons. Assuming there is a competitor hotel, say a price follower, then the two hotels will compete in price in a number of independent periods, each time with the same quality and capacity levels. It seems plausible that there can be one or more off-peak seasons in which the two firms have excess, but not excessive, capacity such that a Leftovers Equilibrium arises.

6.3. Robustness of Leftovers Equilibria

We now discuss the extent to which Leftovers Equilibria may emerge in settings not covered (or not obviously covered) by our assumptions. As indicated earlier, in the kinds of models we consider, leftovers only make sense when there is limited capacity.

We assume zero marginal (unit) production costs for simplicity. This assumption is technically equivalent to assuming positive unit production costs proportional to quality.[22] The prices in our model are the unit profit margins in the model with positive production costs.

Leftovers Equilibria can emerge in other production cost settings, such as when unit production costs are not proportional to quality and when production costs are not linear in quantity.[23] In addition, Leftovers Equilibria can also arise when capacity is endogenous.[24] This result is not surprising because we already know that the strategic shadow price of the leader's capacity can be negative under the Baseline Leftovers Equilibrium, which suggests that even if capacity were free to her, she could have the incentive to select a limited amount of it.

We assume that the higher-quality firm prices first. However, Leftovers Equilibria can still arise when the lower-quality firm prices first.[25] Furthermore, the comparative statics results from §5.2 on capacities are also valid in this case. Even when both firms' qualities are almost identical (i.e., as ρ gets arbitrarily close to 1), Leftovers Equilibria can still arise in a broad range of capacity levels and the main comparative statics are maintained.

We hypothesize that leftovers can arise when price setting is done simultaneously, in the context of a mixed-strategy Nash equilibrium. For example, if a mixed strategy Nash equilibrium entails exercise of the efficient rationing rule under at least one outcome, leftovers may well appear.

[21] We are grateful to our colleague Seungjin Whang for drawing our attention to this reference, to the role queues can serve in channeling customers to different providers, and to the possibility that a situation like this, with heterogeneous prices and an almost identical product, might be part of an equilibrium. The context of competing restaurants may be better represented by such a model as well.

[22] Suppose that each firm has linear production costs (in quantity) and each firm's unit production cost is proportional to its exogenously given quality level. By Cremer and Thisse (1991), if we subtract each firm's unit production cost from the customer's utility function for that product, we obtain an equivalent representation of the problem with zero unit production costs. Furthermore, the newly converted utility functions both intersect the X-axis at the same point. Customer types below this point would not buy either product even if it were sold at cost, so they can be ignored. The remaining customer types would comprise the addressable market.

[23] Specific examples are given in the online appendix.

[24] Suppose that (a) the firms select their capacity levels, sequentially, before they set their prices, (b) each firm incurs a unit capacity cost for capacity that it builds (Firm 1's unit capacity cost is 0.3 and Firm 2's is 0.1), (c) $\rho = 0.8$, and (d) we have low-to-high arrivals. Then a Baseline Leftovers Equilibrium arises, with the leader leaving 0.23 in leftovers for the follower ($Q_1^* = 0.32$, $Q_2^* = 0.49$, $p_1^* = 0.45$, and $p_2^* = 0.62$).

[25] A numerical example is provided in the online appendix.

We assume static pricing, in the sense that each firm prices just once. In certain cases (such as in airline flight pricing), we see dynamic pricing, where prices are adjusted over time as units of capacity are sold. We have already mentioned extensions to dynamic pricing in §6.2, but these incorporated either transactions costs for changing a price or differential response times. In the case of airline flight pricing on the Internet, one can argue that the price-changing transaction costs and response times are negligible. It would be interesting to explore whether Leftovers Equilibria can/will arise in models of dynamic pricing with negligible price-changing transactions costs and response times. For example, we have found some cases of our model in which both firms prefer a specific firm to price first (over the reverse sequence or simultaneous pricing), and we speculate that Leftovers Equilibria may arise among these cases when firms endogenously decide on the sequence in which they set their prices.

7. Concluding Remarks

In this paper, we identify the strategic use of shortages in a model of two firms competing in prices with differentiated, substitutable products and limited exogenous capacities. The leader prices relatively low, purposefully letting her primary demand exceed her available capacity. Some of the customers who cannot get their first-choice product choose to buy the follower's product. The follower feasts on these leftovers. We focus on one form of this phenomenon, the Baseline Leftovers Equilibrium, in which the follower prices so high that he is not the first choice of any customers who buy from him (it is a Pure Leftovers Equilibrium) and the follower does not use all his capacity.

Our model is simple, being essentially deterministic. One advantage of using such a simple model is that it is clear that shortages arise intentionally, rather than as a result of random demand or capacity. This is interesting from an operations management perspective for several reasons. First, there is much research in operations management on capacity expansion that provides guidance about when and how much capacity a firm should add over time. The deterministic models in this literature, such as Manne (1967) and Luss (1982), tend to include economies of scale in the expansion cost function. Our model addresses only a single period and takes capacity as given. However, as discussed earlier, it reveals that the existence of competition can change the incentives for additional capacity dramatically. Indeed, the strategic shadow price of capacity can differ significantly from the shadow price of capacity that arises in single-firm, limited capacity, deterministic demand-optimization models. Of course, our model and analysis are far from comprehensive, and we can only speculate about what will happen in general in more complex settings. For example, will there still be such a distinctive strategic shadow price of capacity when there are economies of scale and expansions are made over time?

Second, our analysis reveals that the customer arrival sequence affects not only the extent of an influence, but also the kind of influence. For example, within the Baseline Leftovers Equilibrium, the strategic shadow price of additional follower's capacity is zero if the arrival sequence is high-to-low, but is strictly positive for the other two sequences considered. The strategic value of quality improvement also depends on the arrival sequence. The fact that both firms prefer low-to-high arrivals, as in Cui and Ho (2007), implies that there may be a rich line of inquiry into customer arrival sequence management. For example, what can one do to influence the arrival sequence of customer types?

Third, there is also a large and growing literature on revenue management (e.g., Talluri and van Ryzin 2004). This literature mainly focuses on a *monopolist* firm's optimal pricing and capacity-allocation problem under essentially low-to-high arrivals. A main consideration of the analysis in this literature is that the monopolist should reserve capacity for the late-arriving high-valuation customers. We show that this intuition can change dramatically when one considers the effect of competition. We demonstrate that under the threat of a strategic competitor, in a Leftovers Equilibrium, a firm may in fact prefer not to reserve capacity for high-valuation customers, but rather price relatively low to sell out all its capacity to lower-valuation customers to leave some high-valuation customers to the competitor and avoid direct competition. Our model is less general than most in this literature in the sense that our pricing is done only once, rather than dynamically. However, we reveal the possibly important concept of strategic shortages. It would be interesting to explore the role, if any, of strategic shortages in the competitive revenue-management context (under dynamic pricing).

Our model lays another stepping stone on the way to developing the theory of operations strategy. We hope that it will open and stimulate new avenues of research on the effects of pricing under competition with limited capacity, including the role of customer arrival sequences.

Electronic Companion

An electronic companion to this paper is available at http://msom.pubs.informs.org/ecompanion.

Acknowledgments

The authors thank Gérard Cachon, the anonymous Associate Editor, and the three anonymous referees for their detailed and extremely helpful comments and suggestions.

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Appendix: Proof of Proposition 1 for Independent Arrivals

We define $R^I(\rho)$ explicitly, show that it includes the Benchmark Capacity Profile, and then prove that the Baseline Leftovers Equilibrium is the unique Stackelberg equilibrium in $R^I(\rho)$ under independent arrivals.

Lemma A.1. *Let $R^I(\rho)$ denote the subset of (Q_1, Q_2) that satisfies all of the following conditions*:

(i) $Q_1 + (2-\rho)Q_2 \le 1$,

(ii) $Q_1 + 2Q_2 \ge 1$,

(iii) $(2-Q_1)(1-\rho/2) \le 1$,

(iv) $\rho Q_2(2Q_2-1)^2 \le Q_1$, *and*

(v) $Q_1^2(4+16Q_2\beta) - Q_1(1+4Q_2(1+\beta))(1-Q_2\rho)^2 + Q_2(1-Q_2\rho)^4 > 4Q_1^3$, *where $\beta = (1-\rho)Q_2$. If $2/3 < \rho < 1$, then $R^I(\rho)$ contains an open subset and $(S_1^N(\rho), S_2^N(\rho)) \in R^I(\rho)$, where $S_1^N(\rho) = 1/2$ and $S_2^N(\rho) = 1/[2(2-\rho)]$.*[26]

The proof uses simple algebra and is omitted.

The next lemma addresses how much each firm gets in profits under the three collectively exhaustive regimes illustrated in Figure 2 and now distinguished by p_2 instead of p_1: In regime (*a*), given by $p_2 \ge \rho p_1$, only the leader has primary demand. In regime (*b*), given by $p_1 - (1-\rho) \le p_2 \le \rho p_1$, both firms have primary demands. In regime (*c*), given by $p_2 \le p_1 - (1-\rho)$, only the follower has primary demand.

[26] We have explicitly recognized the dependence of the Benchmark Capacity Profile on the quality ratio ρ.

Lemma A.2. *Under independent arrivals, the profits to each firm are given as follows, as functions of their respective prices.*

$$\pi_1 = \begin{cases} p_1 \min(1-p_1,\, Q_1), & \text{in regime } (a), \\ p_1 \min\left[1 - \dfrac{p_1-p_2}{1-\rho} + \rho\left(\dfrac{\rho p_1 - p_2}{\rho(1-\rho)} - Q_2\right)^+,\, Q_1\right], & \text{in regime } (b), \\ p_1 \min\left[(1-p_1)\dfrac{(\rho(1-Q_2)-p_2)^+}{\rho - p_2},\, Q_1\right], & \text{in regime } (c), \end{cases} \tag{6}$$

and

$$\pi_2 = \begin{cases} p_2 \min\left[\left(\dfrac{\rho-p_2}{\rho}\right)\dfrac{(1-p_1-Q_1)^+}{1-p_1},\, Q_2\right], & \text{in regime } (a), \\ p_2 \min\left[\dfrac{\rho p_1 - p_2}{\rho(1-\rho)} + \left(1 - \dfrac{p_1-p_2}{1-\rho} - Q_1\right)^+,\, Q_2\right], & \text{in regime } (b), \\ p_2 \min\left(1 - \dfrac{p_2}{\rho},\, Q_2\right), & \text{in regime } (c). \end{cases} \tag{7}$$

The proof entails using definitions and simple algebra and is therefore omitted.

It is useful to partition the regimes so that, within each subregime, the profit expressions for each firm are simplified. In particular, the profit expressions for the follower will be concave in his price. Subregime (*a*1) adds the requirement that $Q_1 \ge 1 - p_1$, so the leader meets all her demand. Subregime (*a*2) requires $Q_1 \le 1 - p_1$ and $Q_2 \ge (\rho - p_2)\cdot(\alpha - p_1)/[\rho(1-p_1)]$, where $\alpha = 1 - Q_1$, so the leader provides leftovers to the follower, who satisfies them all. Subregime (*a*3) requires $Q_1 \le 1 - p_1$ and $Q_2 \le (\rho - p_2)(\alpha - p_1)/[\rho(1-p_1)]$, so the leader provides leftovers to the follower, who does not satisfy them all. Hence the leader's profits are

$$\pi_1 = \begin{cases} p_1(1-p_1), & \text{in subregime } (a1), \\ p_1 Q_1, & \text{in subregimes } (a2) \text{ and } (a3), \end{cases} \tag{8}$$

and the follower's profits are

$$\pi_2 = \begin{cases} 0, & \text{in subregime } (a1), \\ p_2\left(\dfrac{\rho - p_2}{\rho}\right)\dfrac{\alpha - p_1}{1-p_1}, & \text{in subregime } (a2), \\ p_2 Q_2, & \text{in subregime } (a3). \end{cases} \tag{9}$$

For regime (*b*), subregime (*b*1) adds the requirements that $Q_1 \ge 1 - (p_1 - p_2)/(1-\rho)$ and $Q_2 \ge (\rho p_1 - p_2)/[\rho(1-\rho)]$, so both firms satisfy their primary demands. Subregime (*b*2) requires $Q_1 \le 1 - (p_1 - p_2)/(1-\rho)$ and $Q_2 \ge \alpha - p_2/\rho$, so the leader leaves leftovers for the follower, who satisfies all of his demand. Subregime (*b*3) requires $Q_1 \le 1 - (p_1 -$

$p_2)/(1-\rho)$ and $(\rho p_1 - p_2)/[\rho(1-\rho)] \le Q_2 \le \alpha - p_2/\rho$, so the leader leaves leftovers for the follower, who satisfies all of his primary demand but not all demand. Subregime ($b4$) requires $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$ and $Q_1 \ge 1 - p_1 - \rho Q_2$, so the follower leaves leftovers for the leader, who satisfies all of her demand. Subregime ($b5$) requires $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$ and $1-(p_1-p_2)/(1-\rho) \le Q_1 \le 1 - p_1 - \rho Q_2$, so the follower leaves leftovers for the leader, who satisfies all primary demand but not all demand. Subregime ($b6$) requires $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$ and $Q_1 \le 1-(p_1-p_2)/(1-\rho)$, so both firms use all of their capacities to service their primary demands. Therefore, the leader's profits in each subregime can be written as

$$\pi_1 = \begin{cases} p_1\left(1-\dfrac{p_1-p_2}{1-\rho}\right), & \text{in subregime } (b1), \\ p_1\left[1-\dfrac{p_1-p_2}{1-\rho}+\rho\left(\dfrac{\rho p_1-p_2}{\rho(1-\rho)}-Q_2\right)\right], & \text{in subregime } (b4), \\ p_1 Q_1, & \text{in subregimes } (b2), (b3), (b5), \text{ and } (b6), \end{cases} \tag{10}$$

and the follower's profits are

$$\pi_2 = \begin{cases} p_2\dfrac{\rho p_1-p_2}{\rho(1-\rho)}, & \text{in subregime } (b1), \\ p_2\left(\alpha-\dfrac{p_2}{\rho}\right), & \text{in subregime } (b2), \\ p_2 Q_2, & \text{in subregimes } (b3), (b4), (b5), \text{ and } (b6). \end{cases} \tag{11}$$

For regime (c), subregime ($c1$) adds the requirement that $Q_2 \ge 1 - p_2/\rho$, so the follower meets all his primary demand. Subregime ($c2$) requires $Q_2 \le 1 - p_2/\rho$ and $Q_1 \ge (1-p_1)[\rho(1-Q_2)-p_2]/(\rho-p_2)$, so the follower leaves leftovers for the leader, who satisfies them all. Subregime ($c3$) requires $Q_2 \le 1 - p_2/\rho$ and $Q_1 \le (1-p_1)[\rho(1-Q_2)-p_2]/(\rho-p_2)$, so the follower leaves leftovers for the leader, who does not satisfy them all. Then the leader's profits for each subregime are

$$\pi_1 = \begin{cases} 0, & \text{in subregime } (c1), \\ p_1(1-p_1)\dfrac{\rho(1-Q_2)-p_2}{\rho-p_2}, & \text{in subregime } (c2), \\ p_1 Q_1, & \text{in subregime } (c3), \end{cases} \tag{12}$$

while the follower's profits are

$$\pi_2 = \begin{cases} p_2\left(1-\dfrac{p_2}{\rho}\right), & \text{in subregime } (c1), \\ p_2 Q_2, & \text{in subregimes } (c2) \text{ and } (c3). \end{cases} \tag{13}$$

Lemma A.3 provides the conditional optimal response of the follower to the price p_1 of the leader, conditioned on each regime (or a set of subregimes) that the follower can feasibly select under $R^I(\rho)$.

Lemma A.3. *If $(Q_1, Q_2) \in R^I(\rho)$ under independent arrivals, then the conditional optimal profits of the follower are given as follows, within the regimes in which the follower can restrict himself*:

(i) *Within regime (a), if $p_1 \ge \alpha$, then $\pi_2^* = 0$.*

(ii) *Within regime (a), if $p_1 < \alpha$, then*[27]

$$\pi_2^* = \begin{cases} \dfrac{\rho(\alpha-p_1)}{4(1-p_1)} & \text{if } p_1 \le \frac{1}{2} \quad [a2], \\ \rho p_1(\alpha-p_1) & \text{if } \frac{1}{2} \le p_1 \quad [a2]. \end{cases} \tag{14}$$

(iii) *Within regime (b), if $p_1 \ge \alpha$, then*

$$\pi_2^* = \begin{cases} \dfrac{\rho p_1^2}{4(1-\rho)} & \text{if } p_1 \le 2\beta \quad [b1], \\ \rho(p_1-\beta)Q_2 & \text{if } 2\beta \le p_1 \le 1-\rho Q_2 \quad [b1], \\ (p_1-(1-\rho))\dfrac{1-p_1}{\rho} & \text{if } 1-\rho Q_2 \le p_1 \quad [b1]. \end{cases} \tag{15}$$

(iv) *Within regime (b), if $p_1 \le \alpha$ and $p_2 \ge p_1 - (1-\rho)\alpha$, then only sub-regimes ($b2$) and ($b3$) can arise and*

$$\pi_2^* = \begin{cases} \rho p_1 Q_2 & \text{if } p_1 \le \alpha - Q_2 \quad [b3], \\ \rho p_1(\alpha-p_1) & \text{if } \alpha - Q_2 \le p_1 \le \dfrac{\alpha}{2} \quad [b2], \\ \dfrac{\rho\alpha^2}{4} & \text{if } \dfrac{\alpha}{2} \le p_1 \le \left(1-\dfrac{\rho}{2}\right)\alpha \quad [b2], \\ (p_1-(1-\rho)\alpha)\dfrac{\alpha-p_1}{\rho} & \text{if } \left(1-\dfrac{\rho}{2}\right)\alpha \le p_1 \quad [b2]. \end{cases} \tag{16}$$

(v) *Within regime (b), if $p_1 \le \alpha$ and $p_2 \le p_1 - (1-\rho)\alpha$, then only subregimes ($b1$), ($b4$), ($b5$), and ($b6$) can arise, and*

$$\pi_2^* = \begin{cases} (p_1-(1-\rho)\alpha)Q_2 & \text{if } p_1 \le \alpha-\rho Q_2 \quad [b6], \\ \rho(p_1-\beta)Q_2 & \text{if } \alpha-\rho Q_2 \le p_1 \le 1-\rho Q_2 \quad [b1], \\ (p_1-(1-\rho))\dfrac{1-p_1}{\rho} & \text{if } 1-\rho Q_2 \le p_1 \quad [b1]. \end{cases} \tag{17}$$

(vi) *Within regime (c), if $Q_2 \le 1/2$, then*

$$\pi_2^* = \begin{cases} \rho(1-Q_2)Q_2 & \text{if } p_1 \ge 1-\rho Q_2 \quad [c1], \\ (p_1-(1-\rho))Q_2 & \text{if } p_1 \le 1-\rho Q_2 \quad [c2] \text{ or } [c3]. \end{cases} \tag{18}$$

(vii) *Within regime (c), if $Q_2 \ge 1/2$, then*

$$\pi_2^* = \begin{cases} \dfrac{\rho}{4} \quad \text{if } 1-\dfrac{\rho}{2} \le p_1 & [c1], \\ (p_1-(1-\rho))\dfrac{1-p_1}{\rho} \quad \text{if } 1-\rho Q_2 \le p_1 \le 1-\dfrac{\rho}{2} & [c1], \\ (p_1-(1-\rho))Q_2 \quad \text{if } p_1 \le 1-\rho Q_2 & [c2] \text{ or } [c3]. \end{cases} \tag{19}$$

[27] The chosen subregime is provided in square brackets.

For part (i), if $p_1 \ge \alpha$, then the follower has neither primary nor leftover demand and hence has zero profits. The outline of the rest of the proof is as follows. The best follower's price within a regime will fall within one of the subregimes. Each subregime is specified by an interval in which p_2 must lie. Each profit function is concave over its interval, so the (conditional) optimal price is the closest point in that interval to the unconstrained maximizer. For example, in subregime ($a2$), $\pi_2 = p_2(\rho - p_2)(\alpha - p_1)/[\rho(1-p_1)]$, and the unconstrained maximizer is $p_2 = \rho/2$, which is the follower's monopoly price. The feasible region for subregime ($a2$) consists of two lower bound constraints on p_2, namely ρp_1 and $[\rho(1-p_1)(1-Q_2) - \rho Q_1]/(\alpha - p_1)$. The optimal response is the closest feasible price to the unconstrained optimal price. By condition (ii) of $R^I(\rho)$, it follows that the unconstrained optimal price always satisfies the second constraint. Thus, the only possible binding lower bound on p_2 is ρp_1. Hence, the follower's best price response within ($a2$) is $\rho/2$ if $\rho/2 \ge \rho p_1$, and ρp_1 otherwise. The follower's optimal profits within subregime ($a2$) follow. Once we have found the conditional optimal profits within subregimes ($a1$), ($a2$), and ($a3$), we pick the best and part (ii) follows. The other parts follow similarly.[28]

Let $p_2^*(p_1)$ denote an optimal price response by the follower as a function of the leader's price.

Lemma A.4. *If $(Q_1, Q_2) \in R^I(\rho)$ under independent arrivals, then the optimal profits of the follower and the resulting profits of the leader firm are given as follows, as functions of the price that the leader sets.*

(i) *If $p_1 \ge \alpha$ and $Q_2 \le 1/2$, then*

$$\pi_2(p_1, p_2^*(p_1)) = \begin{cases} \dfrac{\rho p_1^2}{4(1-\rho)} & \text{if } p_1 \le 2\beta \quad [b1], \\ \rho(p_1-\beta)Q_2 & \text{if } 2\beta \le p_1 \le 1-\rho Q_2 \quad [b1], \\ \rho(1-Q_2)Q_2 & \text{if } 1-\rho Q_2 \le p_1 \quad [c1]. \end{cases} \tag{20}$$

(ii) *If $p_1 \ge \alpha$ and $Q_2 \ge 1/2$, then*

$$\pi_2(p_1, p_2^*(p_1)) = \begin{cases} \dfrac{\rho p_1^2}{4(1-\rho)} & \text{if } p_1 \le 2\beta \quad [b1], \\ \rho(p_1-\beta)Q_2 & \text{if } 2\beta \le p_1 \le 1-\rho Q_2 \quad [b1], \\ (p_1-(1-\rho))\dfrac{1-p_1}{\rho} & \text{if } 1-\rho Q_2 \le p_1 \le 1-\dfrac{\rho}{2} \quad [c1], \\ \dfrac{\rho}{4} & \text{if } 1-\dfrac{\rho}{2} \le p_1 \quad [c1]. \end{cases} \tag{21}$$

(iii) *If $p_1 \ge \alpha$, then,*

$$\pi_1(p_1, p_2^*(p_1)) = \begin{cases} p_1(1-\rho Q_2 - p_1) & \text{if } p_1 \le 1-\rho Q_2 \quad [b1], \\ 0 & \text{if } 1-\rho Q_2 \le p_1 \quad [c1]. \end{cases} \tag{22}$$

(iv) *If $p_1 \le \alpha$ and $Q_2 \le 1/2$, then*

$$\pi_2(p_1, p_2^*(p_1)) = \begin{cases} \dfrac{\rho(\alpha-p_1)}{4(1-\rho)} & \text{if } p_1 \le p_1^I \quad [a2], \\ \rho(p_1-\beta)Q_2 & \text{if } p_1^I \le p_1 \le 1-\rho Q_2 \quad [b1], \\ \rho(1-Q_2)Q_2 & \text{if } 1-\rho Q_2 \le p_1 \quad [c1], \end{cases} \tag{23}$$

where

$$p_1^I = \frac{1}{2}\left(\frac{1}{4Q_2} + 1 + \beta - \sqrt{\left(\frac{1}{4Q_2}+1+\beta\right)^2 - 4\gamma}\right), \tag{24}$$

and $\gamma = (\alpha/4 + \beta Q_2)/Q_2$.

(v) *If $p_1 \le \alpha$ and $Q_2 \ge 1/2$, then*

$$\pi_2(p_1, p_2^*(p_1)) = \begin{cases} \dfrac{\rho(1-p_1-Q_1)}{4(1-\rho)} & \text{if } p_1 \le p_1^I \quad [a2], \\ \rho(p_1-\beta)Q_2 & \text{if } p_1^I \le p_1 \le 1-\rho Q_2 \quad [b1], \\ (p_1-(1-\rho))\dfrac{1-p_1}{\rho} & \text{if } 1-\rho Q_2 \le p_1 \le 1-\dfrac{\rho}{2} \quad [c1], \\ \dfrac{\rho}{4} & \text{if } 1-\dfrac{\rho}{2} \le p_1 \quad [c1]. \end{cases} \tag{25}$$

(vi) *If $p_1 \le \alpha$, then,*

$$\pi_1(p_1, p_2^*(p_1)) = \begin{cases} p_1 Q_1 & \text{if } p_1 \le p_1^I \quad [a2], \\ p_1(1-\rho Q_2 - p_1) & \text{if } p_1^I \le p_1 \le 1-\rho Q_2 \quad [b1], \\ 0 & \text{if } 1-\rho Q_2 \le p_1 \quad [c1]. \end{cases} \tag{26}$$

Note that the leader's equilibrium price p_1^I is the largest price she can charge and still have the follower prefer to sell solely to leftover customers rather than to engage in direct competition while using all his capacity. We compare the follower's optimal profits, derived in Lemma A.3, across different regimes to prove this lemma. For example, consider the case of $p_1 \le \alpha$ and $Q_2 \le 1/2$. Using conditions (ii) and (iii) of $R^I(\rho)$, we have $\alpha - \rho Q_2 \le (1-\rho/2)\alpha \le 1/2 \le 1-\rho Q_2$. We first consider the region, $Q_1 + 2\rho Q_2 \le 1$, which is equivalent to $\alpha/2 \le \alpha - Q_2$. Comparing (14), (16), (17), and (18), we first notice that (16) is dominated by the maximum of the remaining others. Comparing the remaining ones, we

[28] We use conditions (ii) and (iii) in $R^I(\rho)$ to derive the optimal π_2 within regime (b)—parts (iii), (iv), and (v).

obtain (23). We also obtain (23) for the case of $Q_1+2\rho Q_2 \geq 1$. In obtaining the results for the other cases, we utilize the conditions (iii) and (iv) of $R^I(\rho)$ when considering the case of $p_1 \leq \alpha$ and $Q_2 \geq 1/2$.

Finally, consider the leader's problem of setting her price p_1 to maximize her profits given the best response of $p_2^*(p_1)$. Note that from condition (v) (of $R^I(\rho)$) we have $p_1^I Q_1 \geq (1-\rho Q_2)^2/4$, and from condition (i) we have $p_1^I \leq \alpha$. It follows that the optimal p_1^I is as given in (24) and the corresponding optimal follower's price is $p_2^I(p_1^I) = \rho/2$. Note that in Baseline Leftovers Equilibrium, the follower is indifferent between the best price within (*a*2), the Pure Leftovers Regime with excess capacity, and the best price within (*b*1), the direct competition regime with binding capacity. □

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Online Supplement for "Feasting on Leftovers: Strategic Use of Shortages in Price Competition Among Differentiated Products"

Proofs of Propositions

Proof of Proposition 1 under high-to-low arrivals. We follow analogous steps to the proof under independent arrivals, as given in the Appendix of the paper.

Lemma EC.1 *Denote $R^H(\rho)$ as the subset of (Q_1, Q_2) that satisfies all of the following conditions: (i) $Q_1 + (2-\rho)Q_2 \le 1$, (ii) $Q_1 + 2Q_2 \ge 1$, (iii) $(1-Q_2)Q_2 \ge \alpha^2/4$, and (iv) $Q_1\alpha^2 + 4\beta Q_1 Q_2 > (1-\rho Q_2)^2 Q_2$, where $\alpha = 1 - Q_1$ and $\beta = (1-\rho)Q_2$. If $2/3 < \rho < 1$, then $R^H(\rho)$ contains an open subset and $(S_1^N(\rho), S_2^N(\rho)) \in R^H(\rho)$.*

The proof uses simple algebra and is omitted.

Recall that in regime (a), given by $p_2 \ge \rho p_1$, only the leader has primary demand, in regime (b), given by $p_1 - (1-\rho) \le p_2 \le \rho p_1$, both firms have primary demand, and, in regime (c), given by $p_2 \le p_1 - (1-\rho)$, only the follower has primary demand.

Lemma EC.2 *Under high-to-low arrivals, the profits to each firm are given as follows, as functions of their respective prices.*

$$\pi_1 = \begin{cases} p_1 \min(1-p_1, Q_1), & \text{in regime } (a), \\ p_1 \min\left[1 - \frac{p_1-p_2}{1-\rho} + \left(\frac{\rho p_1 - p_2}{1-\rho} - Q_2\right)^+, Q_1\right], & \text{in regime } (b), \\ p_1 \min\left[(1-p_1-Q_2)^+, Q_1\right], & \text{in regime } (c), \end{cases} \tag{EC.1}$$

and

$$\pi_2 = \begin{cases} p_2 \min\left[\left(1 - \frac{p_2}{\rho} - Q_1\right)^+, Q_2\right], & \text{in regime } (a), \\ p_2 \min\left[\frac{\rho p_1 - p_2}{\rho(1-\rho)} + \left(1 - \frac{p_1-p_2}{1-\rho} - Q_1\right)^+, Q_2\right], & \text{in regime } (b), \\ p_2 \min\left(1 - \frac{p_2}{\rho}, Q_2\right), & \text{in regime } (c). \end{cases} \tag{EC.2}$$

The proof entails using definitions and simple algebra and is therefore omitted.

Similar to the proof for independent arrivals, we partition the regimes so that, within each sub-regime, the profit expressions for each firm are simplified. Sub-regime $(a1)$ adds the requirement that $Q_1 \ge 1 - p_1$, so the leader meets all of her demand. Sub-regime $(a2)$ requires $1 - p_2/\rho \le Q_1 \le 1 - p_1$, so the leader has unmet demand but those customers are not willing to buy from the follower. Sub-regime $(a3)$ requires $Q_1 \le 1 - p_2/\rho \le Q_1 + Q_2$, so the leader provides leftovers to the follower who satisfies all of his demand. Sub-regime ($a4$) requires $Q_1+Q_2 \leq 1-p_2/\rho$, so the leader provides leftovers to the follower who does not satisfies them all. Hence the leader's profits are

$$\pi_1 = \begin{cases} p_1(1-p_1), & \text{in sub-regime (a1)}, \\ p_1 Q_1, & \text{in sub-regimes (a2), (a3) and (a4)}, \end{cases} \tag{EC.3}$$

and the follower's profits are

$$\pi_2 = \begin{cases} 0, & \text{in sub-regimes (a1) and (a2)}, \\ p_2\left(\alpha - \frac{p_2}{\rho}\right), & \text{in sub-regime (a3)}, \\ p_2 Q_2, & \text{in sub-regime (a4)}. \end{cases} \tag{EC.4}$$

Sub-regime ($b1$) adds the requirements that $Q_1 \geq 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \geq (\rho p_1-p_2)/[\rho(1-\rho)]$, so both firms satisfy their primary demands. Sub-regime ($b2$) requires $Q_1 \leq 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \geq \alpha - p_2/\rho$, so the leader provides leftovers to the follower who satisfies all of his demand. Sub-regime ($b3$) requires $Q_1 \leq 1-(p_1-p_2)/(1-\rho)$ and $(\rho p_1-p_2)/[\rho(1-\rho)] \leq Q_2 \leq 1-p_2/\rho-Q_1$, so the leader leaves leftovers to the follower, who satisfies all of his primary demand but not all demand. Sub-regime ($b4$) adds $Q_2 \leq (\rho p_1-p_2)/\rho(1-\rho)$ and $Q_1 \geq 1-p_1-Q_2$, so the follower leaves leftovers to the leader who satisfies all of her demand. Sub-regime ($b5$) requires $Q_2 \leq (\rho p_1-p_2)/\rho(1-\rho)$ and $1-(p_1-p_2)/(1-\rho) \leq Q_1 \leq 1-p_1-Q_2$, so the follower provides leftovers to the leader, who satisfies all primary demand but not all demand. Sub-regime ($b6$) requires $Q_1 \leq 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \leq (\rho p_1-p_2)/[\rho(1-\rho)]$, so neither firm satisfies all of its primary demand. Therefore, the leader's profits in each sub-regime can be written as

$$\pi_1 = \begin{cases} p_1\left(1-\frac{p_1-p_2}{1-\rho}\right), & \text{in sub-regime (b1)}, \\ p_1(1-p_1-Q_2), & \text{in sub-regime (b4)}, \\ p_1 Q_1, & \text{in sub-regimes (b2), (b3), (b5) and (b6)}, \end{cases} \tag{EC.5}$$

and the follower's profits are given at (A.6).

For regime (c), sub-regime ($c1$) adds the requirement that $Q_2 \geq 1-p_2/\rho$, so the follower meets all of his primary demand. Sub-regime ($c2$) requires $1-p_1 \leq Q_2 \leq 1-p_2/\rho$, so the follower has unmet demand but those customers are not willing to buy from the leader. Sub-regime ($c3$) requires $Q_2 \leq 1-p_1 \leq Q_1+Q_2$, so the follower provides leftovers to the leader who satisfies all of her demand. Sub-regime ($c4$) requires $Q_1+Q_2 \leq 1-p_1$, so the follower provides leftovers to the leader who does not satisfies them all. Therefore the leader's profits for each sub-regime are

$$\pi_1 = \begin{cases} 0, & \text{in sub-regimes (c1) and (c2)}, \\ p_1(1-p_1-Q_2), & \text{in sub-regime (c3)}, \\ p_1 Q_1, & \text{in sub-regime (c4)}, \end{cases} \tag{EC.6}$$

while the follower's profits are

$$\pi_2 = \begin{cases} p_2\left(1-\frac{p_2}{\rho}\right), & \text{in sub-regime (c1)}, \\ p_2 Q_2, & \text{in sub-regimes (c2), (c3) and (c4)}. \end{cases} \tag{EC.7}$$

Lemma EC.3 provides the conditional optimal response of the follower to the leader's price, conditioned on each regime (or a set of sub-regimes) that the follower can feasibly select under $R^H(\rho)$.

Lemma EC.3 *If $(Q_1, Q_2) \in R^H(\rho)$ under high-to-low arrivals, then the optimal profits of the follower are given as follows, within the regimes in which the follower can restrict himself.*

(a) Within regime (a), if $p_1 \geq \alpha$, then $\pi_2^* = 0$.

(b) Within regime (a), if $p_1 < \alpha$, then[29]

$$\pi_2^* = \begin{cases} \frac{\rho\alpha^2}{4} & \text{if} \quad p_1 \leq \frac{\alpha}{2} \quad [a3]\,, \\ \rho p_1(\alpha - p_1) & \text{if} \quad \frac{\alpha}{2} \leq p_1 \quad [a3]\,. \end{cases} \tag{EC.8}$$

(c) Within regime (b), if $p_1 \geq \alpha$, then the follower's optimal profits are given at (A.10).

(d) Within regime (b), if $p_1 \leq \alpha$ and $p_2 \geq p_1 - (1-\rho)\alpha$, the follower's optimal profits are given at (A.11).

(e) Within regime (b), if $p_1 \leq \alpha$ and $p_2 \leq p_1 - (1-\rho)\alpha$, the follower's optimal profits are given at (A.12).

(f) Within regime (c), if $Q_2 \leq 1/2$, then

$$\pi_2^* = \begin{cases} \rho(1-Q_2)Q_2 & \text{if } p_1 \geq 1 - \rho Q_2 \quad [c1]\,, \\ (p_1 - (1-\rho))\,Q_2 & \text{if } p_1 \leq 1 - \rho Q_2 \quad [c2],\ [c3] \ \text{or } [c4]\,. \end{cases} \tag{EC.9}$$

(g) Within regime (c), if $Q_2 \geq 1/2$, then

$$\pi_2^* = \begin{cases} \frac{\rho}{4} & \text{if} \quad 1 - \frac{\rho}{2} \leq p_1 & [c1]\,, \\ (p_1 - (1-\rho))\,\frac{1-p_1}{\rho} & \text{if} \quad 1 - \rho Q_2 \leq p_1 \leq 1 - \frac{\rho}{2} & [c1]\,, \\ (p_1 - (1-\rho))\,Q_2 & \text{if} \quad p_1 \leq 1 - \rho Q_2 & [c2],\ [c3] \ \text{or } [c3]\,. \end{cases} \tag{EC.10}$$

We follow the analogous proof for independent arrivals; for parts (a) and (b), we utilize the condition (ii) of $R^H(\rho)$; for parts (c), (d) and (e), we use the conditions (i) and (ii) in $R^H(\rho)$.

Let $p_2^*(p_1)$ denote the optimal price response by the follower as a function of the leader's price.

Lemma EC.4 *If $(Q_1, Q_2) \in R^H(\rho)$ under high-to-low arrivals, then the optimal profits of the follower and the resulting profits of the leader, are given as follows, as functions of the leader's price.*

(a) If $p_1 \geq \alpha$ and $Q_2 \leq 1/2$, the follower's optimal profits are given at (A.15).

[29] The chosen sub-regime is provided in square brackets

(b) If $p_1 \geq \alpha$ and $Q_2 \geq 1/2$, the follower's optimal profits are given at (A.16).

(c) If $p_1 \geq \alpha$, then, the leader's corresponding profits are given at (A.17).

(d) If $p_1 \leq \alpha$ and $Q_2 \leq 1/2$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \frac{\rho\alpha^2}{4} & \text{if} \quad p_1 \leq p_1^H \quad [a3]\,, \\ \rho(p_1-\beta)Q_2 & \text{if} \quad p_1^H \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ \rho(1-Q_2)Q_2 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,, \end{cases} \tag{EC.11}$$

where

$$p_1^H = \frac{\alpha^2}{4Q_2} + \beta. \tag{EC.12}$$

(e) If $p_1 \leq \alpha$ and $Q_2 \geq 1/2$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \frac{\rho\alpha^2}{4} & \text{if} \quad p_1 \leq p_1^H \quad [a3]\,, \\ \rho(p_1-\beta)Q_2 & \text{if} \quad p_1^H \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ (p_1-(1-\rho))\frac{1-p_1}{\rho} & \text{if} \quad 1-\rho Q_2 \leq p_1 \leq 1-\frac{\rho}{2} \quad [c1]\,, \\ \frac{\rho}{4} & \text{if} \quad 1-\frac{\rho}{2} \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.13}$$

(f) If $p_1 \leq \alpha$, then,

$$\pi_1(p_1, p_2^*(p_1)) = \begin{cases} p_1 Q_1 & \text{if} \quad p_1 \leq p_1^H \quad [a3]\,, \\ p_1(1-\rho Q_2 - p_1) & \text{if} \quad p_1^H \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ 0 & \text{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.14}$$

As noted in the paper, the leader's equilibrium price p_1^H is the largest price she can charge and still have the follower prefer to sell only to leftover customers rather than to engage in direct competition. We compare the follower's optimal profits, derived in lemma EC.3, across different regimes. We utilize the conditions (iii) to prove (EC.11), and (ii) to prove (EC.13).

Finally, consider the leader's decision on p_1 to maximize $\pi_1(p_1,\, p_2^*(p_1))$. From the conditions, (i) and (ii), we have $p_1^H \leq \alpha$. Further, from the condition (iv) in $R^H(\rho)$, we have $Q_1(\beta+\alpha^2/(4Q_2)) > (1-\rho Q_2)^2/4$. Hence $p_1^* = p_1^H$ and the corresponding equilibrium $p_2^H(p_1^H) = \rho\alpha/2$. Thus, the Baseline Leftovers Equilibrium arises and is unique. □

Proof of Proposition 1 under low-to-high arrivals. We follow analogous steps to the proof under independent arrivals, as given in the Appendix of the paper.

Lemma EC.5 *Denote $R^L(\rho)$ as the subset of (Q_1, Q_2) that satisfies all of the following conditions: (i) $Q_2 \leq 1/2$, (ii) $Q_2 \leq Q_1$, (iii) $Q_1 \geq \frac{(1-\rho)^2}{1+\rho^2}$, (iv) $Q_1 + 2Q_2 \geq 1$, (v) $Q_1 + 2\beta \leq 1$, (vi) $\beta(4Q_1 + (5-4\rho)Q_2 - 2) \leq \alpha Q_1$, and (vii) $4Q_1(Q_1 - \rho Q_2) + Q_2(2 - 5Q_2 + (1+\rho)^2 Q_2) < 1$, where $\alpha = 1 - Q_1$ and $\beta = (1-\rho)Q_2$. If $0 < \rho < 1$, $R^L(\rho)$ contains an open subset and $(S_1^N(\rho), S_2^N(\rho)) \in R^L(\rho)$.*

The proof uses simple algebra and is omitted.

Recall that in regime (a), only the leader has primary demand, in regime (b), both firms have primary demand, and, in regime (c), only the follower has primary demand.

Lemma EC.6 *Under low-to-high arrivals, the profits to each firm are given as follows, as functions of their respective prices.*

$$\pi_1 = \begin{cases} p_1 \min(1-p_1,\, Q_1), & \text{in regime (a)}, \\ p_1 \min\left[1-\frac{p_1-p_2}{1-\rho} + \min\left\{\frac{\rho p_1 - p_2}{1-\rho},\, \left(\frac{\rho p_1 - p_2}{\rho(1-\rho)} - Q_2\right)^+\right\}, Q_1\right], & \text{in regime (b)}, \\ p_1 \min\left[1-p_1,\, \left(1-\frac{p_2}{\rho} - Q_2\right)^+,\, Q_1\right], & \text{in regime (c)}, \end{cases} \quad \text{(EC.15)}$$

and

$$\pi_2 = \begin{cases} p_2 \min\left[1-\frac{p_2}{\rho},\, (1-p_1-Q_1)^+,\, Q_2\right], & \text{in regime (a)}, \\ p_2 \min\left[\frac{\rho p_1 - p_2}{\rho(1-\rho)} + \left(1-\frac{p_1-p_2}{1-\rho} - Q_1\right)^+,\, Q_2\right], & \text{in regime (b)}, \\ p_2 \min\left(1-\frac{p_2}{\rho},\, Q_2\right), & \text{in regime (c)}. \end{cases} \quad \text{(EC.16)}$$

The proof entails using definitions and simple algebra and is therefore omitted.

Similar to the proof of independent arrivals, we partition the regimes. Sub-regime $(a1)$ adds the requirement that $Q_1 \geq 1-p_1$, so the leader meets all of her demand. Sub-regime $(a2)$ requires $Q_1 \leq 1-p_1$, $Q_2 \geq 1-p_2/\rho$ and $1-p_2/\rho \leq \alpha - p_1$, so the follower's potential demand, if he were the monopolist, is less than his capacity and also his potential demand is less than the leftovers from the leader. Sub-regime $(a3)$ requires $Q_1 \leq 1-p_1$, $Q_2 \leq 1-p_2/\rho$ and $Q_2 \leq \alpha - p_1$, so the leader provides leftovers to the follower who does not satisfy them all. Sub-regime $(a4)$ adds $Q_1 \leq 1-p_1$, $\alpha - p_1 \leq 1-p_2/\rho$ and $Q_2 \geq \alpha - p_1$, so the leftovers from the leader are smaller than the follower's capacity as well as his potential demand. Hence the leader's profits are

$$\pi_1 = \begin{cases} p_1(1-p_1), & \text{in sub-regime (a1)}, \\ p_1 Q_1, & \text{in sub-regimes (a2), (a3) and (a4)}, \end{cases} \quad \text{(EC.17)}$$

and the follower's profits are

$$\pi_2 = \begin{cases} 0, & \text{in sub-regime (a1)}, \\ p_2\left(1-\frac{p_2}{\rho}\right), & \text{in sub-regime (a2)}, \\ p_2 Q_2, & \text{in sub-regime (a3)}, \\ p_2(\alpha - p_1), & \text{in sub-regime (a4)}. \end{cases} \quad \text{(EC.18)}$$

For regime (b), sub-regime $(b1)$ adds the requirements that $Q_1 \geq 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \geq (\rho p_1 - p_2)/[\rho(1-\rho)]$, so both firms satisfy their primary demands. Sub-regime $(b2)$ requires $Q_1 \leq 1-(p_1-p_2)/(1-\rho)$ and $Q_2 \geq \alpha - p_2/\rho$, so the leader leaves leftovers for the follower who satisfies all of his demand. Sub-regime $(b3)$ requires $Q_1 \leq 1-(p_1-p_2)/(1-\rho)$ and $(\rho p_1 - p_2)/[\rho(1-$

$\rho)] \le Q_2 \le \alpha - p_2/\rho$, so the leader leaves leftovers for the follower, who satisfies all of his primary demand but not all demand. Sub-regime $(b4)$ adds $Q_2 \le (\rho p_1 - p_2)/\rho$ and $Q_1 \ge 1 - p_1$, so the follower leaves enough leftovers for the leader so that she has all of her potential customers and she satisfies all of her demand. Sub-regime $(b5)$ requires $Q_2 \le (\rho p_1 - p_2)/\rho$ and $1 - (p_1 - p_2)/(1-\rho) \le Q_1 \le 1 - p_1$, so the follower leaves enough leftovers for the leader so that the leader has all of her potential customers, and she satisfies all of her primary demand but not all of her demand. Sub-regime $(b6)$ adds $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$ and $Q_1 \ge 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demand and she satisfies all of her demand. Sub-regime $(b7)$ adds $(\rho p_1 - p_2)/\rho \le Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$ and $1 - (p_1 - p_2)/(1-\rho) \le Q_1 \le 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demand, and she satisfies her primary demand but not all of her demand. Sub-regime $(b8)$ requires $Q_1 \le 1 - (p_1 - p_2)/(1-\rho)$ and $Q_2 \le (\rho p_1 - p_2)/[\rho(1-\rho)]$, so neither firm satisfies all of its primary demand. Therefore, the leader's profits in each sub-regime can be written as

$$\pi_1 = \begin{cases} p_1\left(1 - \frac{p_1 - p_2}{1-\rho}\right), & \text{in sub-regime (b1),} \\ p_1(1-p_1), & \text{in sub-regime (b4),} \\ p_1\left(1 - \frac{p_2}{\rho} - Q_2\right), & \text{in sub-regime (b6),} \\ p_1 Q_1, & \text{in sub-regimes (b2), (b3), (b5), (b7) and (b8),} \end{cases} \tag{EC.19}$$

and the follower's profits are

$$\pi_2 = \begin{cases} p_2 \, \frac{\rho p_1 - p_2}{\rho(1-\rho)}, & \text{in sub-regime (b1),} \\ p_2\left(\alpha - \frac{p_2}{\rho}\right), & \text{in sub-regime (b2),} \\ p_2 \, Q_2, & \text{in sub-regimes (b3), (b4), (b5), (b6), (b7) and (b8).} \end{cases} \tag{EC.20}$$

For regime (c), sub-regime $(c1)$ adds the requirement that $Q_2 \ge 1 - p_2/\rho$, so the follower meets all of his primary demand. Sub-regime $(c2)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \le 1 - p_2/\rho - Q_2$ and $Q_1 \ge 1 - p_1$, so the follower leaves enough leftovers for the leader, larger than the leader's potential demands, and the leader satisfies them all. Sub-regime $(c3)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \le 1 - p_2/\rho - Q_2$ and $Q_1 \le 1 - p_1$, so the follower leaves enough leftovers for the leader, larger than the leader's potential demands, but the leader does not satisfy them all. Sub-regime $(c4)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \ge 1 - p_2/\rho - Q_2$ and $Q_1 \ge 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demands and the leader satisfies all her demands. Sub-regime $(c5)$ requires $Q_2 \le 1 - p_2/\rho$, $1 - p_1 \ge 1 - p_2/\rho - Q_2$ and $Q_1 \le 1 - p_2/\rho - Q_2$, so the follower leaves leftovers for the leader but encroaches some of the leader's potential demands and the leader does not satisfy all her demands. Therefore the leader's profits for each sub-regime are

$$\pi_1 = \begin{cases} 0, & \text{in sub-regime (c1)}, \\ p_1(1-p_1), & \text{in sub-regime (c2)}, \\ p_1\left(1 - \frac{p_2}{\rho} - Q_2\right), & \text{in sub-regime (c4)}, \\ p_1 Q_1, & \text{in sub-regimes (c3) and (c5)}, \end{cases} \tag{EC.21}$$

while the follower's profits are

$$\pi_2 = \begin{cases} p_2\left(1 - \frac{p_2}{\rho}\right), & \text{in sub-regime (c1)}, \\ p_2 Q_2, & \text{in sub-regimes (c2), (c3), (c4) and (c5)}. \end{cases} \tag{EC.22}$$

Lemma EC.7 provides the conditional optimal response of the follower to the price of the leader, conditioned on each regime (or a set of sub-regimes) that the follower can feasibly select.

Lemma EC.7 *If $(Q_1, Q_2) \in R^L(\rho)$ under low-to-high arrivals, then the optimal profits of the follower are given as follows, within the regimes in which the follower can restrict himself.*

(a) Within regime (a),

$$\pi_2^* = \begin{cases} \rho(1-Q_2)Q_2 & \text{if} \quad p_1 \le \alpha - Q_2 \quad [a3], \\ \rho(p_1+Q_1)(\alpha - p_1) & \text{if} \quad \alpha - Q_2 \le p_1 \le \alpha \quad [a4], \\ 0 & \text{if} \quad \alpha \le p_1 \quad [a1]. \end{cases} \tag{EC.23}$$

(b) Within regime (b), if $p_1 \ge \alpha$, then

$$\pi_2^* = \begin{cases} \rho(p_1-\beta)Q_2 & \text{if} \quad p_1 \le 1-\rho Q_2 \quad [b1], \\ (p_1-(1-\rho))\frac{1-p_1}{\rho} & \text{if} \quad 1-\rho Q_2 \le p_1 \quad [b1]. \end{cases} \tag{EC.24}$$

(c) Within regime (b), if $p_1 \le \alpha$ and $p_2 \ge p_1 - (1-\rho)\alpha$, then

$$\pi_2^* = \begin{cases} \rho p_1 Q_2 & \text{if} \quad p_1 \le \alpha - Q_2 \quad [b3], \\ \rho p_1(\alpha - p_1) & \text{if} \quad \alpha - Q_2 \le p_1 \le \frac{\alpha}{2} \quad [b2], \\ \frac{\rho\alpha^2}{4} & \text{if} \quad \frac{\alpha}{2} \le p_1 \le \alpha\left(1-\frac{\rho}{2}\right) \quad [b2], \\ (p_1 - \alpha(1-\rho))\frac{\alpha - p_1}{\rho} & \text{if} \quad \alpha\left(1-\frac{\rho}{2}\right) \le p_1 \quad [b2]. \end{cases} \tag{EC.25}$$

(d) Within regime (b), if $p_1 \le \alpha$, $p_2 \le p_1 - (1-\rho)\alpha$ and $Q_1 + (2-\rho)Q_2 \ge 1$, then

$$\pi_2^* = \begin{cases} (p_1 - \alpha(1-\rho))\,Q_2 & \text{if} \quad p_1 \le \alpha - \rho Q_2 \quad [b8], \\ (p_1 - \alpha(1-\rho))\frac{\alpha - p_1}{\rho} & \text{if} \quad \alpha - \rho Q_2 \le p_1 \le \frac{2\alpha(1-\rho)}{2-\rho} \quad [b1], \\ \frac{\rho p_1^2}{4(1-\rho)} & \text{if} \quad \frac{2\alpha(1-\rho)}{2-\rho} \le p_1 \le 2\beta \quad [b1], \\ \rho(p_1-\beta)Q_2 & \text{if} \quad 2\beta \le p_1 \le 1-\rho Q_2 \quad [b1], \\ (p_1-(1-\rho))\frac{1-p_1}{\rho} & \text{if} \quad 1-\rho Q_2 \le p_1 \quad [b1]. \end{cases} \tag{EC.26}$$

(e) Within regime (b), if $p_1 \le \alpha$, $p_2 \le p_1 - (1-\rho)\alpha$ and $Q_1 + (2-\rho)Q_2 \le 1$, then

$$\pi_2^* = \begin{cases} (p_1 - \alpha(1-\rho))\,Q_2 & \text{if} \quad p_1 \le \alpha - \rho Q_2 \quad [b8], \\ \rho(p_1-\beta)Q_2 & \text{if} \quad \alpha - \rho Q_2 \le p_1 \le 1-\rho Q_2 \quad [b1], \\ (p_1-(1-\rho))\frac{1-p_1}{\rho} & \text{if} \quad 1-\rho Q_2 \le p_1 \quad [b1]. \end{cases} \tag{EC.27}$$

(f) Within regime (c),

$$\pi_2^* = \begin{cases} \rho(1-Q_2)Q_2 & \textit{if } p_1 \geq 1-\rho Q_2 \quad [c1]\,, \\ (p_1 - (1-\rho))\,Q_2 & \textit{if } p_1 \leq 1-\rho Q_2 \quad [c2], \quad [c3], \quad [c4] \textit{ or } [c5]\,. \end{cases} \tag{EC.28}$$

The proof is analogous to the case of independent arrivals. For part (a), we utilize the condition (i) of $R^L(\rho)$; for parts (b), (c), (d) and (e), we use the conditions (i), (iv) and (v); for part (f), we use the condition (i) of $R^L(\rho)$.

Lemma EC.8 *If $(Q_1, Q_2) \in R^L(\rho)$ under low-to-high arrivals, then the optimal follower's profits and the resulting leader's profits, are given as follows, as functions of the leader's price.*

(a) If $p_1 \geq \alpha$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \rho(p_1-\beta)Q_2 & \textit{if} \quad p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ \rho(1-Q_2)Q_2 & \textit{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,, \end{cases} \tag{EC.29}$$

$$\pi_1(p_1,\, p_2^*(p_1)) = \begin{cases} p_1(1-\rho Q_2 - p_1) & \textit{if} \quad p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ 0 & \textit{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.30}$$

(b) If $p_1 \leq \alpha$, then

$$\pi_2(p_1,\, p_2^*(p_1)) = \begin{cases} \rho(1-Q_2)Q_2 & \textit{if} \quad p_1 \leq \alpha - Q_2 \quad [a3]\,, \\ \rho(p_1+Q_1)(\alpha - p_1) & \textit{if} \quad \alpha - Q_2 \leq p_1 \leq p_1^L \quad [a4]\,, \\ \rho(p_1-\beta)Q_2 & \textit{if} \quad p_1^L \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ \rho(1-Q_2)Q_2 & \textit{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,, \end{cases} \tag{EC.31}$$

$$\pi_1(p_1,\, p_2^*(p_1)) = \begin{cases} p_1 Q_1 & \textit{if} \quad p_1 \leq p_1^L \quad [a3] \textit{ or } [a4]\,, \\ p_1(1-\rho Q_2 - p_1) & \textit{if} \quad p_1^L \leq p_1 \leq 1-\rho Q_2 \quad [b1]\,, \\ 0 & \textit{if} \quad 1-\rho Q_2 \leq p_1 \quad [c1]\,. \end{cases} \tag{EC.32}$$

where

$$p_1^L = \frac{1}{2}\left(\alpha - (Q_1+Q_2) + \sqrt{(\alpha-(Q_1+Q_2))^2 + 4\delta}\right)\,, \tag{EC.33}$$

and $\delta = \alpha Q_1 + \beta Q_2$.

As noted in the paper, the leader's equilibrium price p_1^L is the largest price she can charge and still have the follower prefer to sell solely to leftover customers rather than to engage in direct competition using all of his capacity. We use conditions (i) and (v) to prove (EC.29) and (EC.30); we use (iii), (iv), (v) and (vi) to prove (EC.31) and (EC.32).

Finally, for the optimal price for the leader, note that under the condition, (vii), $(1-\rho Q_2)/2 \leq p_1^L$. Hence π_1 is decreasing within the sub-regime $[b1]$. Therefore in equilibrium, $p_1^* = p_1^L$ as in (EC.33) and

$$p_2^*(p_1^L) = \rho(p_1^L + Q_1)\,, \tag{EC.34}$$

with the corresponding profits $\pi_1 = p_1^L Q_1$ and $\pi_2 = \rho(p_1^L + Q_1)(\alpha - p_1^L)$. □

Proof of Proposition 2: Under independent arrivals, given the leader's price p_1, the follower's optimal price p_2 within the Pure Leftovers Regime is $\rho/2$, which does not depend on p_1. Consequently, the follower's leftover demand from the leader is $(1 - Q_1/(1 - p_1))/2$, which decreases as p_1 increases. Under low-to-high arrivals, $L_2^L = \min(1 - p_2/\rho,\, \alpha - p_1)$, which is decreasing in p_1 when $p_2 \leq \rho(p_1 + Q_1)$. Otherwise L_2^L does not depend on p_1. If $\alpha - p_1 \leq Q_2$, the optimal p_2 is either $\rho/2$ or $\rho(1 - \alpha + p_1)$. In both cases, the follower's leftover demand weakly decreases in p_1. If $\alpha - p_1 \geq Q_2$, his leftover demand does not depend on p_1. Under high-to-low arrivals, given p_1, the optimal p_2 equals to $\rho\alpha/2$, which yields $L_2^H = \alpha/2$ in the Pure Leftovers Regime. Therefore the follower's demand is unchanged under high-to-low arrivals. □

Proof of Proposition 3: First, the leader sets p_1 at which the follower's profits under the Pure Leftovers Regime are the same as his profits under a direct competition in the Baseline Leftovers Equilibrium. Further follower's profits under direct competition are the same across all arrival sequences given p_1, $\pi_2 = \rho(p_1 - \beta)Q_2$. Consider the Pure Leftovers Regime. For independent arrivals, the follower's leftover demand from the leader is $L_2^I = 1 - p_2/\rho - Q_1(1 - p_2/\rho)/(1 - p_1)$. For high-to-low arrivals, it is $L_2^H = \alpha - p_2/\rho$, while for low-to-high arrivals, it is equal to $L_2^L = \min(1 - p_2/\rho,\, \alpha - p_1)$. First, since $p_1 \leq p_2/\rho$ in the Baseline Leftovers Equilibrium, $L_2^H \leq L_2^I$ for any given p_2 under the Pure Leftovers Regime. Second, since $1 - p_2/\rho \geq L_2^I$ and $\alpha - p_1 - L_2^I = (1 - Q_1/(1 - p_1))(p_2/\rho - p_1) \geq 0$, we obtain $L_2^I \leq L_2^L$ for any given p_2 under the Pure Leftovers Regime. Therefore at $p_1 = p_1^I$, the follower strictly prefers to be in the Pure Leftovers Regime under low-to-high arrivals, while he strictly prefers to be in direct competition under high-to-low arrivals. As a result, $p_1^H \leq p_1^I \leq p_1^L$ as in part (a), which in turn implies $\pi_1^H \leq \pi_1^I \leq \pi_1^L$ in part (b) since the leader's sales quantity in the Baseline Leftovers Equilibrium is Q_1 for all customer arrival sequences. In addition, the follower's profits under the Pure Leftovers Regime are the same as his profits under direct competition in the Baseline Leftovers Equilibrium for all customer arrival sequences. The follower's profits under a direct competition are the same for all arrivals and this follower's profits function are increasing in p_1 for any p_2. Hence $\pi_2^H \leq \pi_2^I \leq \pi_2^L$. Furthermore, from the follower's equilibrium price under the Baseline Leftovers Equilibrium, we have $p_2^H = \rho\alpha/2 \leq p_2^I = \rho/2$. Since $Q_2 \leq \sqrt{(\alpha - (Q_1 + Q_2))^2 + 4\delta}$ in $R^L(\rho)$, we also obtain $p_2^I \leq p_2^L$. □

Proof of Proposition 4: First, given that $Q_1 \in [0, 1]$, p_1^H is decreasing in Q_1. Using the fact that $(1 + 4Q_2(1 + \beta))^2 - 64Q_2^2\gamma = 1 + 8Q_2(2Q_1 + (1 - \beta)(2Q_2(1 - \beta) - 1))$ is increasing in Q_1, it follows that p_1^I is decreasing in Q_1. For low-to-high arrivals, from the condition (i) in $R^L(\rho)$, we have

$$\frac{\partial p_1^L}{\partial Q_1} = -1 + \frac{Q_2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}} \leq 0\,. \tag{EC.35}$$

Similarly, it follows that π_2^H, π_2^I and π_2^L are decreasing in Q_1. Further, note that $\partial\pi_1^H/\partial Q_1 = \alpha(1-3Q_1)/(4Q_2)+\beta$, which is positive when Q_1 is small. Moreover, $\partial^2\pi_1^H/\partial Q_1^2 = -(2-3Q_1)/(2Q_2) \leq 0$ in $R^H(\rho)$, which shows that π_1^H is concave in Q_1. For independent arrivals, we obtain

$$\frac{\partial \pi_1^I}{\partial Q_1} = \frac{1+4(1+\beta)Q_2}{8Q_2} - \frac{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma + 8Q_1Q_2}{8Q_2\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}}\,. \tag{EC.36}$$

When Q_1 is small, $\partial\pi_1^I/\partial Q_1$ is positive. Further it follows that in $R^I(\rho)$, the numerator in (EC.36) is decreasing in Q_1. Therefore, π_1^I is quasi-concave in Q_1. For low-to-high arrivals, similarly we obtain

$$\frac{\partial \pi_1^L}{\partial Q_1} = \frac{1}{2}\left(1-4Q_1-Q_2+\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta} + \frac{2Q_1Q_2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}}\right)\,, \tag{EC.37}$$

which is positive when Q_1 is small. Further, taking a derivative of $\partial\pi_1^L/\partial Q_1$ with respect to Q_1, one can show that in $R^L(\rho)$,

$$\frac{\partial^2 \pi_1^L}{\partial Q_1^2} = -2 + \frac{2Q_2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}}\left(1-\frac{Q_1Q_2}{(\alpha-(Q_1+Q_2))^2+4\delta}\right) \leq 0\,. \tag{EC.38}$$

Hence π_1^L is concave in Q_1. Further, note that $p_2^H = \rho\alpha/2$ is decreasing in Q_1 and $p_2^I = \rho/2$ does not depend on Q_1. Lastly, from simple algebra, it follows that $p_2^L = \rho(p_1^L + Q_1)$ is increasing in Q_1. □

Proof of Proposition 5: First, note that from the condition (i) in $R^H(\rho)$, we obtain

$$\frac{\partial p_1^H}{\partial Q_2} = 1-\rho-\frac{\alpha^2}{4Q_2^2} \leq 0\,. \tag{EC.39}$$

Taking derivative of p_1^I with respect to Q_2, we then have

$$\frac{\partial p_1^I}{\partial Q_2} = \frac{1-4\beta(1-2Q_1-4(1-\beta)Q_2^2)}{8Q_2^2\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}} - \frac{1-4\beta Q_2}{8Q_2^2}\,. \tag{EC.40}$$

Under the conditions, (i), (ii) and (iii) in $R^I(\rho)$, we find that $\partial p_1^I/\partial Q_2 < 0$. For p_1^L, we obtain

$$\frac{\partial p_1^L}{\partial Q_2} = \frac{1}{2}\left(-1+\frac{-1+2Q_1+5Q_2-4\rho Q_2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}}\right) \leq 0\,, \tag{EC.41}$$

since $-1+2Q_1+5Q_2-4\rho Q_2 \leq \sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}$ in $R^L(\rho)$. Hence the leader's price decreases as Q_2 increases for all three arrival sequences. Further, $\pi_1^s = p_1^s Q_1$, for $s \in \{H, I, L\}$, under the Baseline Leftovers Equilibrium, the leader's profits decrease in Q_2 for all $s \in \{H, I, L\}$. It is straightforward from Table 1 that p_2^H, π_2^H and p_2^I do not depend on Q_2. Taking derivative of S_2^I with respect to Q_2, we obtain

$$\frac{\partial S_2^I}{\partial Q_2} = 1 - 2\beta - \frac{2Q_1 - (1 - 4Q_2(1-\beta))(1-2\beta)}{\sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma}} \geq 0\,, \tag{EC.42}$$

in $R^I(\rho)$. Hence $\pi_2^I = S_2^I \rho/2$ is increasing in Q_2. Next, consider low-to-high arrivals. First, it follows that in $R^L(\rho)$,

$$\frac{\partial p_2^L}{\partial Q_2} = \frac{\rho}{2}\left(-1 + \frac{2Q_1 + 5Q_2 - 4\rho Q_2 - 1}{\sqrt{(\alpha - (Q_1+Q_2))^2 + 4\delta}}\right) \leq 0\,. \tag{EC.43}$$

Lastly, taking derivative of π_2^L with respect to Q_2 and simplifying, we have

$$\frac{\partial \pi_2^L}{\partial Q_2} = \frac{\rho(\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta} - Q_2)}{2}\left(\frac{1 - 2Q_1 - (5-4\rho)Q_2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}} + 1\right). \tag{EC.44}$$

We then obtain that $\partial\pi_2^L/\partial Q_2 \geq 0$ in $R^L(\rho)$. □

Proof of Proposition 6: For part (a), note that $\partial\pi_1^N/\partial\rho = -1/[2(2-\rho)^2] < 0$ and $\partial(\pi_1^N + \pi_2^N)/\partial\rho = -(2+\rho)/[4(2-\rho)^3] < 0$. Hence π_1^N and $\pi_1^N + \pi_2^N$ are decreasing in ρ. In addition, $\partial\pi_2^N/\partial\rho = (2-3\rho)/[4(2-\rho)^3]$, which is increasing in ρ if $\rho \leq 2/3$, and decreasing afterwards. Further, $\partial p_2^N/\partial\rho = 1/2 - 1/(2-\rho)^2$. Thus p_2^N is increasing in ρ if $\rho \leq 2-\sqrt{2}$, and decreasing afterwards.

For part (b), from the expressions in Table 1, it follows that π_1^H and π_1^L decrease as ρ increases. For π_1^I, first, note that

$$\frac{\partial p_1^I}{\partial\rho} = -\frac{Q_2}{2} - \frac{Q_2(4(1-\beta)Q_2 - 1)}{2\sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma}} \leq 0\,. \tag{EC.45}$$

Since $\pi_1^I = p_1^I Q_1$ under the Baseline Leftovers Equilibrium, we obtain that π_1^I is decreasing in ρ. Hence, the leader's profits decrease in ρ for all three arrival sequences. Next, note that $p_2^H = \rho\alpha/2$ and $p_2^I = \rho/2$ are increasing in ρ. In addition, it follows that in $R^L(\rho)$,

$$\frac{\partial p_2^L}{\partial\rho} = \frac{1-Q_2}{2} + \frac{(\alpha-(Q_1+Q_2))^2 + 4\delta - 2\rho Q_2^2}{2\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}} \geq 0\,. \tag{EC.46}$$

Thus, the follower's price increases in ρ for all three arrival sequences. Lastly, note that $S_2^H = \alpha/2$ does not depend on ρ and $S_2^L = (1 + Q_2 - \sqrt{(\alpha-(Q_1+Q_2))^2+4\delta})/2$ increases as ρ increases. Hence, $\pi_2^H = p_2^H S_2^H$ and $\pi_2^L = p_2^L S_2^L$ are increasing in ρ. In addition, we obtain

$$\begin{aligned}\frac{\partial\pi_2^I}{\partial\rho} = \frac{1}{8}\Bigg(&4Q_2(1-(1-2\rho)Q_2) \\ &+ 1 - \sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma} - \frac{\rho Q_2(4Q_2(1-\beta)-1)}{2\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}}\Bigg) \geq 0\,,\end{aligned} \tag{EC.47}$$

under the conditions, (i), (ii) and (iii) in $R^I(\rho)$. Therefore, the follower's profits increase in ρ for all three arrival sequences. For the industry profits under low-to-high arrivals, it follows that

$$\frac{\partial(\pi_1^L+\pi_2^L)}{\partial\rho}=\frac{Q_2}{2}\left(1-2Q_1-(3-4\rho)Q_2+\frac{(\alpha-(Q_1+Q_2))^2+4\delta-2\rho Q_2^2}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}}\right). \tag{EC.48}$$

We obtain $(\alpha-(Q_1+Q_2))^2+4\delta-2\rho Q_2^2\geq 0$ in $R^L(\rho)$. Further, condition (v) in $R^L(\rho)$ implies that $2Q_1+(3-4\rho)Q_2\leq 1$. Hence we obtain $\partial(\pi_1^L+\pi_2^L)/\partial\rho\geq 0$. Therefore industry profits increase under low-to-high arrivals. For independent and high-to-low arrivals, we provide a region of parameters where industry profits are non-monotonic in ρ. Under independent arrivals, when $(Q_1,\,Q_2)=(0.5,\,0.4)\in R^I(\rho)$, where $\rho\in[0.75,\,0.9]$, $\pi_1^I+\pi_2^I$ is decreasing and then increasing in ρ afterwards. Under high-to-low arrivals, $\pi_1^H+\pi_2^H$ is decreasing in ρ when $(Q_1,\,Q_2)=(0.5,\,0.4)\in R^H(\rho)$, where $\rho\in[0.75,\,0.83]$, while $\pi_1^H+\pi_2^H$ is increasing in ρ when $(Q_1,\,Q_2)=(0.3,\,0.4)\in R^H(\rho)$, where $\rho\in[0.8,\,0.95]$.

Next, the condition (i) in $R^H(\rho)$ implies $(2-\rho)Q_2(1-(2-\rho)Q_2)\leq 1/4$, from which we obtain $2(2-\rho)^2Q_1Q_2\leq 2(2-\rho)^2Q_2(1-(2-\rho)Q_2)\leq(2-\rho)/2\leq 1$. From this inequality, it follows that

$$\frac{\partial(\pi_1^H-\pi_1^N)}{\partial\rho}=-\frac{2(2-\rho)^2Q_1Q_2-1}{2(2-\rho)^2}\geq 0\,. \tag{EC.49}$$

Further, we obtain that in $R^H(\rho)$

$$\frac{\partial(\pi_2^H-\pi_2^N)}{\partial\rho}=\frac{\alpha^2}{4}-\frac{2-3\rho}{4(2-\rho)^3}\geq 0\,. \tag{EC.50}$$

In addition, for the comparison between independent arrivals and high-to-low arrivals, from $(4Q_2(1-\beta)-1)^2-((1+4Q_2(1+\beta))^2-64Q_2^2\gamma)=-16Q_1Q_2\leq 0$, we obtain

$$\frac{\partial(\pi_1^I-\pi_1^H)}{\partial\rho}=\frac{Q_1Q_2}{2}\left(1-\frac{4Q_2(1-\beta)-1}{\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}}\right)\geq 0\,. \tag{EC.51}$$

Similarly, from the conditions (i), (ii), and (iii) in $R^I(\rho)$, it follows that $\partial(\pi_2^I-\pi_2^H)/\partial\rho\geq 0$. $\square$

Proof of Proposition 7: For part (a), from the expressions in Table 1, first note that

$$r^I = \frac{4Q_2\rho}{4Q_2(1+\beta)+1-\sqrt{(1+4Q_2(1+\beta))^2-64Q_2^2\gamma}}\,, \tag{EC.52}$$

$$r^H = \frac{2\rho\alpha Q_2}{\alpha^2+4\beta Q_2}\,,\quad\text{and} \tag{EC.53}$$

$$r^L = \rho\left(1+\frac{2Q_1}{1-2Q_1-Q_2+\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}}\right). \tag{EC.54}$$

We then obtain that $\rho/2=r^N\leq\rho\leq r^H\leq r^I$ from the conditions (i) and (ii) in $R^I(\rho)$ and $R^H(\rho)$. In addition, it follows that $r^I\leq r^L$ in $R^I(\rho)\cap R^L(\rho)$.

For part (b), first note that $r^N = \rho/2$ is increasing in ρ, Q_1 and Q_2. Next, for high-to-low arrivals, p_1^H is decreasing in ρ, whereas p_2^H is increasing in ρ. Consequently, r^H is increasing in ρ. Further, from the condition (i) in $R^H(\rho)$, we obtain

$$\frac{\partial r^H}{\partial Q_1} = \frac{2\rho Q_2\left(\alpha^2 - 4\beta Q_2\right)}{\left(\alpha^2 + 4\beta Q_2\right)^2} \geq 0\,. \tag{EC.55}$$

Note that p_2^H in Table 1 does not depend on Q_2 and that p_1^H is decreasing in Q_2 from (EC.39). As a result, r^H is increasing in Q_2. Next, for independent arrivals, p_1^I is decreasing in ρ from (EC.45) and $p_2^I = \rho/2$ is increasing in ρ. Thus $r^I = p_2^I/p_1^I$ is increasing in ρ. Observe that p_1^I is decreasing in Q_1. Further, p_1^I is decreasing in Q_2 from Proposition 5. $p_2^I = \rho/2$ does not depend on Q_1 or Q_2. Therefore $r^I = p_2^I/p_1^I$ is increasing in Q_1 and Q_2. Lastly, for low-to-high arrivals, p_1^L is decreasing in ρ. Further, from (EC.46), we have $\partial p_2^L/\partial\rho \geq 0$. Hence we obtain $\partial r^L/\partial\rho \geq 0$. From (EC.35), p_1^L is decreasing in Q_1. In addition, p_2^L is increasing in Q_1. Hence, r^L is increasing in Q_1. Finally, in $R^L(\rho)$, $(Q_1 + Q_2 - \alpha)^2 \leq (Q_1 + Q_2 + 4\beta Q_2 - \alpha)^2 \leq (Q_1 + Q_2 - \alpha)^2 + 4\delta$, from which we obtain

$$\frac{\partial r^L}{\partial Q_2} = \frac{2\rho Q_1\left(\alpha - (Q_1+Q_2) - 4\beta Q_2 + \sqrt{(\alpha-(Q_1+Q_2))^2 + 4\delta}\right)}{\sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}\left(\alpha - (Q_1+Q_2) + \sqrt{(\alpha-(Q_1+Q_2))^2+4\delta}\right)} \geq 0\,. \;\square \tag{EC.56}$$

Proofs for the Remaining Comparative Statics in Table 1: The remaining comparative statics results in Table 1 that we have not proved are $\partial\pi_T^s/\partial Q_1$ and $\partial\pi_T^s/\partial Q_2$ for $s \in \{H, I, L\}$.

For high-to-low arrivals, note that π_1^H is quasi-concave in Q_1, and π_2^H is decreasing in Q_1. As a result, $\pi_T^H = \pi_1^H + \pi_2^H$ is quasi-concave in Q_1. Furthermore, from $\partial\pi_1^H/\partial Q_2 < 0$ and $\partial\pi_2^H/\partial Q_2 = 0$, we obtain $\partial\pi_T/\partial Q_2 < 0$.

For independent arrivals, similarly note that π_1^I is quasi-concave in Q_1, and π_2^I is decreasing in Q_1. As a result, $\pi_T^I = \pi_1^I + \pi_2^I$ is quasi-concave in Q_1. In addition, taking a derivative of π_T^I with respect to Q_2, we obtain

$$\begin{aligned}\frac{\partial\pi_T^I}{\partial Q_2} &= \frac{\rho Q_2(1+4(1-\beta)Q_2) + Q_1(1+4(1+\beta)Q_2) - (Q_1+\rho Q_2)\sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma}}{8Q_2} \\ &\quad\times \frac{A + B\sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma}}{8Q_2^2\sqrt{(1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma}}\,,\end{aligned} \tag{EC.57}$$

where

$$A = 8Q_1^2 Q_2 + 4\rho(1-2\beta)Q_2^2 + (1 - 4(1+2\rho Q_2)Q_2 + 16\beta(1-\beta)Q_2^2)Q_1\,, \tag{EC.58}$$

and

$$B = 4\beta Q_1 Q_2 - Q_1 + 4\rho(1-2\beta)Q_2^2\,. \tag{EC.59}$$

In $R^H(\rho)$, $B \leq 0$. Further one can show that $A^2 \leq ((1+4Q_2(1+\beta))^2 - 64Q_2^2\gamma)B^2$ in $R^H(\rho)$. This proves that $\partial\pi_T^I/\partial Q_2 \leq 0$.

For low-to-high arrivals, similarly note that π_1^L is quasi-concave in Q_1, and π_2^L is decreasing in Q_1. As a result, $\pi_T^L = \pi_1^L + \pi_2^L$ is quasi-concave in Q_1. In addition, π_T^L is decreasing in Q_2 at $(Q_1, Q_2) = (0.5, 0.49)$ with $\rho = 0.6$ whereas π_T^L is increasing in Q_2 at $(Q_1, Q_2) = (0.5, 0.46)$ with $\rho = 0.8$. The Baseline Leftovers Equilibrium arises under both cases. Hence $\partial\pi_T^L/\partial Q_2$ is indeterminate. □

Proof for the Unlimited Capacities Equilibrium : We first obtain the follower's best response given the leader's price p_1;

$$p_2^*(p_1) = \begin{cases} \frac{\rho p_1}{2} & \text{if} \quad p_1 \leq \frac{2(1-\rho)}{2-\rho}\,, \\ p_1 - (1-\rho) & \text{if} \quad \frac{2(1-\rho)}{2-\rho} \leq p_1 \leq 1 - \frac{\rho}{2}\,, \\ \frac{\rho}{2} & \text{if} \quad 1 - \frac{\rho}{2} \leq p_1\,. \end{cases} \tag{EC.60}$$

Plugging (EC.60) into the leader's profit maximization problem and optimizing, we obtain p_1^N. Then it follows that $p_2^N = p_2^*(p_1^N)$. □

Numerical Examples for Robustness: Leftovers equilibria can emerge in other production cost settings, such as when unit production costs are not proportional to quality. Let c_i denote the unit production cost for Firm i, so that the cost to firm i of selling the quantity x is $c_i x$, where, of course, $x \leq Q_i$. These represent costs of exercising the firm's capacity and differ from the firm's unit capacity cost. If $c_1 = 0.1$, $c_2 = 0.064$ and $\rho = 0.8$, so the unit production costs are not proportional to ρ, then a Baseline Leftovers Equilibrium arises under independent arrivals when $Q_1 = 0.5$ and $Q_2 = 0.42$. The equilibrium outcome is $p_1^I = 0.305$ and $p_2^I = 0.432$, with the leader leaving 0.195 in leftovers to the follower, who sells 0.129 units.

Leftovers equilibria can also emerge when production costs are not linear in quantity. For example, if Firm i's cost to produce x units is $C_i(x) = c_i x^2$ for $(c_1, c_2) \in \{(0.1, 0.08), (0.1, 0.064), (0.1, 0.01)\}$, a Leftovers Equilibrium arises with $Q_1 = 0.5$, $Q_2 = 0.42$ and $\rho = 0.8$ under independent arrivals. In addition, if $C_i(x) = c_i\sqrt{x}$, a Leftovers Equilibrium also arises under the same parameter set.

Lastly, Leftovers Equilibria can also arise when the lower quality firm prices first. If the leader has quality 0.8, the follower has quality 1 (so the lower quality firm goes first), and the capacities are 0.42 and 0.50 for the leader and the follower, respectively, then, under independent arrivals, the lower quality leader leaves 0.40 in leftovers for the high quality follower to exploit. The (lower quality) leader's equilibrium price is 0.144 and the (high quality) follower's equilibrium price is 0.5. The leader's demand is 0.82 leaving 0.4 in leftovers and the follower satisfies only 0.24 of those leftovers in equilibrium.