The Coordination of Intermediation*

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Abstract

We study decentralized trading among financial intermediaries (i.e., dealers), the implied intermediation chains, and fragility in a dynamic model of asset markets. In an equilibrium where inter-dealer trading is active (inactive), the intermediation chain is longer (shorter), dealers provide more (less) liquidity by holding more (less) inventory, and the market is more (less) liquid. A dealer is more likely to provide liquidity to customers if other dealers do so and the market becomes more liquid, leading to coordination motives among dealers. This leads to multiple equilibria, suggesting market fragility in the sense of a shutdown of inter-dealer trading and liquidity drop even without a fundamental shock. A low search friction among dealers effectively reduces intermediation cost, making a dealer's willingness to provide liquidity less sensitive to others' and thereby reducing fragility. Surprisingly, an improvement in customer search technology may instead increase fragility.

KEYWORDS: intermediation, inter-dealer trading, liquidity, coordination, multiple equilibria, fragility

JEL: D02, G01, G11, G12, G21, G23

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1 Introduction

Many major over-the-counter (OTC) asset markets are intermediated in the sense that buyers and sellers cannot trade with each other directly but only through an intermediary (i.e., a dealer). Most intermediaries do not only stand as middlemen but also trade with each other bilaterally in a purely decentralized fashion, such as in corporate bond (Di Maggio, Kermani and Song, 2017), municipal bond (Green, Hollifield and Schurhoff, 2006, Li and Schurhoff, 2018), asset-backed security (Hollifield, Neklyudov and Spatt, 2016), and credit default swap (Eisfeldt, Herskovic, Rajan, and Siriwardane, 2018) markets.

Yet, there is a long-standing debate regarding the bright and dark sides of trading among intermediaries, the implied intermediation chains, and the asset pricing implications. It is well known that trading among intermediaries allows them to manage their inventory more efficiently (Ho and Stoll, 1983, Viswanathan and Wang, 2004). But a longer intermediation chain implied by inter-dealer trading also leads to higher intermediation costs (Gofman, 2014, Philippon, 2015) and higher financial stability risks due to contagion of negative shocks (Allen and Gale, 2000) and counterparty risks (Farboodi, 2017). Moreover, since the past financial crisis, many OTC asset markets have experienced great fluctuation in liquidity or "flash crashes" as intermediaries suddenly stop trading with each other, but without a clear fundamental shock. These issues have triggered critical debates regarding trading among intermediaries as well as scrutiny by key regulators.

In this paper, we reconcile these observations by offering a new strategic perspective to analyze trading among intermediaries, its liquidity implications, and in particular, market fragility. We formally define market fragility as the possibility of a liquidity drop in an asset market without a fundamental shock. In contrast to the modern OTC asset pricing literature (mostly built on the paradigm of Duffie, Garleanu and Pedersen, 2005) that typically considers a unique equilibrium, we show that multiple equilibria and market fragility can generically arise due to a coordination motive in intermediaries' liquidity provision decisions. Also different from the view that longer intermediation chains imply higher financial stability risks, we find that market fragility may decrease rather than increase when it is easier for intermediaries to trade with each other and to form longer intermediation chains.

In doing so, we formulate the endogenous emergence of the inter-dealer market and its potential fragility in a dynamic, search-based intermediated asset market à la Rubinstein and Wolinsky (1987) and Duffie, Garleanu and Pedersen (2005). In our model, buyers and sellers can trade an asset but only through dealers. At any given point of time, any dealer can costly hold a low or a high level of inventory to help provide liquidity, or does not hold any inventory at all, which decisions will be endogenously determined. Also at any time, a fixed mass of buyers and sellers, that is, customers, arrive in the market looking for buying or selling an unit of identical asset. If successfully meets a dealer, a customer negotiates a price with the dealer through bargaining, completes the transaction, and leaves the market. But some customers may also be forced to leave the market before meeting a dealer, capturing a default risk.

In this model, an inter-dealer market may endogenously emerge (or not) in equilibrium. When the fundamental of the asset is high (low) relative to dealers' inventory cost, dealers are more (less) willing to intermediate order flows between buyers and sellers, implying an higher (lower) inventory holding on average and, in particular, a larger (smaller) dispersion of inventory distribution among dealers. Since the gains from a potential inter-dealer trade are positive (negative) when the dispersion of inventory distribution is sufficiently large (small), the inter-dealer market becomes active (inactive), and dealers ultimately provide more (less) liquidity in equilibrium.

The key idea of our paper is that multiple equilibria may generically co-exist in this setting, leading to market fragility. To understand how, we explicitly illustrate the coordination motive among dealers' inventory holding and liquidity provision decisions.

First, inventory cost limits a dealer' ability to provide liquidity, that is, her willingness to take orders from the buyers and sellers. In particular, without an active inter-dealer market, a dealer can only trade with buyers and sellers but not with other dealers. Thus, the only way for an inventory-holding dealer to offload its inventory is to meet and trade with an actively searching buyer. This difficulty in offloading inventory holdings limits a dealer's inventory management capacity, discouraging the dealer's willingness to take an inflow order from an actively searching seller to its balance sheet in the first place.

However, a dealer will be more willing to provide liquidity and hold a higher level of inventory (despite the difficulty in terms of inventory management) if other dealers do so. Formally, this implies a coordination motive in dealers' liquidity provision decision. To see this, consider a certain dealer's willingness to provide liquidity. When other dealers are all willing to hold a higher level of inventory and provide more liquidity, the market becomes more liquid and more customer orders will flow into the intermediation sector. This thereby encourages the given dealer to also provide liquidity to meet the increasing customer demand placed upon the dealer sector.

An alternative way to see this coordination motive is the following. In an intermediated asset market, a dealer's profit essentially comes from the asset being hard to trade, in our model reflected by customers actively searching and thus waiting in equilibrium. When the market becomes more liquid, fewer customers are waiting. Thus, if a dealer gives up an opportunity to serve an already met customer, the chance to be contacted by another random customer in future becomes lower. This in turn justifies the given dealer's higher willingness to serve a customer when other dealers also do so. This coordination motive suggests potential coordination failures in dealers' liquidity provision decisions: they may all provide liquidity, or stop providing liquidity all together.

We relate equilibrium multiplicity to the economic environment, in particular, to the search friction among dealers (recall that most OTC inter-dealer markets are purely decentralized). We show that a lower search friction among dealers can mitigate the concern of coordination failures. Specifically, an active inter-dealer market allows an high-inventory-holding dealer to trade with a low-inventory-holding dealer as they meet, giving high-inventory-holding dealers an alternative way to offload its costly inventory. Hence, this encourages a dealer to take a sell order to its balance sheet in the first place. Importantly, we show that, if the search friction among dealers is sufficiently small in the sense that it is sufficiently easy for dealers to meet and trade with each other, an endogenously emerged inter-dealer market can completely eliminate the strategic complementarity among dealers' inventory holding, thereby eliminate any possible coordination failures which would have happened without the inter-dealer market. On the flip side, as the search friction among dealers becomes higher, it is more likely for the strategic complementarity to emerge even if the inter-dealer market endogenously emerges, implying a higher possibility of coordination failures among dealers.

Regarding asset pricing implications, we show that the equilibrium with (without) an active inter-dealer market, equivalently, with a longer (shorter) intermediation chain, always features higher (lower) liquidity for a given economic environment. When equilibrium multiplicity arises, a switch between the two equilibria may result from non-fundamental reasons as those featured in the bank run literature (e.g., Diamond and Dybvig, 1983). Thus, our model points to the possibility of a relatively abrupt shutdown of the inter-dealer market and consequently a liquidity drop, even if the economic environment does not change.

Our model generates empirically plausible implications along many dimensions. In our comparative statics, we find that market fragility is more likely to happen when the level of dealer inventory cost becomes higher, or when the inventory cost structure becomes more convex. These results suggest adverse effects on asset market liquidity due to certain post-crisis regulatory policies aiming to reduce financial stability risks but effectively increasing intermediaries' inventory costs, for example, the Volcker Rule and the Supplementary Leverage Ratio.[1] This is consistent with various studies that document the relatively abrupt reduction of inter-dealer trading activities in corporate bond markets (e.g., Bessembinder, Maxwell, Jacobsen and Venkataraman, 2017, Choi and Huh, 2017, Schultz, 2017) after the implementation of these policies. Accordingly, Bao, O'Hara and Zhou (2017) and Dick-Nielsen and Rossi (2018) find a significant drop in corporate bond liquidity in this period. During the same time, many other OTC markets have also witnessed reductions of trading activities among intermediaries and "flash crash" events in the absence of a major fundamental shock. For example, inter-dealer positions in the global OTC credit default swap (CDS) markets has declined from \$18 trillion in mid-2011 to \$2 trillion in 2017, accompanied by a significant liquidity drop in the CDS markets.[2]

Interestingly and perhaps surprisingly, we also find that market fragility may be more likely to happen when it is easier for the customers to contact dealers (e.g., due to customer technological improvements). This is surprising because we would have thought a better customer search technology leading to less market fragility. Indeed, faster customer searching implies higher liquidity. But it also implies higher flows into and out of the dealer sector, which in turn intensify the coordination motive among dealers. This is consistent with the observation in Philippon (2015) that a reduction in trading frictions between customers and

[1]See Duffie (2016) and Greenwood, Hanson, Stein, and Sunderam (2017) for more detailed descriptions and assessments of these policies.

[2]See "International Banking and Financial Market Developments," *Bank for International Settlements Quarterly Review*, June 2018.

dealers in the recent decades improve liquidity but may also lead to more market fragility due to the escalated competition among financial intermediaries. Overall, our results suggest that intermediation chains are beneficial to asset market liquidity, but the the process of sustaining an intermediation chain can be fragile. Thus, our paper provides a new perspective to reconcile the debates concerns the costs and benefits of trading among intermediaries.

Contribution and related literature. The main contribution of our paper is to analyze equilibrium multiplicity and the implied fragility of decentralized trading among intermediaries in a standard search-based asset pricing model following Rubinstein and Wolinsky (1987) and Duffie, Garleanu and Pedersen (2005).[3]

This asset pricing literature typically delivers a unique equilibrium,[4] with a few exceptions that feature different economic forces from ours. Di Maggio (2016) builds on Lagos, Rocheteau, and Weill (2011) and considers how speculators in financial markets trade in anticipation of a future random negative shock. He shows that two equilibria, that the speculators may either jointly buy or jointly sell the asset before the shock, may co-exist.[5] Farboodi, Jarosch and Menzio (2018) consider intermediation as a rent extraction activity: they allow traders to acquire a costly commitment technology before trading, and show that there may be multiple equilibria in which different fractions of agents acquire the commitment technology.[6] Gu, Monnet, Nosal, and Wright (2018) explore three different ways of modeling financial intermediaries. They show that when intermediaries derive a positive return from holding the asset (as opposed to incur inventory costs), multiple equilibria may arise because intermediaries may either intermediate or just hoard them.

Several papers in this OTC asset pricing literature consider related but different notions

[3]In the OTC asset pricing literature, some papers feature an explicit but reduced-form (usually modeled as centralized) inter-dealer market to facilitate the analysis of other aspects. For example, Garleanu (2009), Lagos and Rocheteau (2009) and subsequent papers built on them consider unlimited asset holding positions by investors. Babus and Parlatore (2018) consider how strategic traders choose a dealer to trade with.

[4]There are other papers in the broader search literature that feature multiple equilibria, but get equilibrium multiplicity for quite different reasons from ours. For example, Diamond (1982) get equilibrium multiplicity in a production economy by directly assuming increasing returns to matching, while we have constant returns to matching. In Kiyotaki and Wright (1989), multiple equilibria co-exist with different goods acting as money, depending on what the authors call "extrinsic beliefs" that what one agent thinks other agents are accepting as money. Trejos and Wright (2016) show that allowing for non-linear utility functions, as opposed to the linear utility used in the Duffie, Garleanu and Pedersen (2005) paradigm, can also generate multiple equilibria in search models.

[5]Thus, his multiplicity is reminiscent of the notion of "market runs" in Bernardo and Welch (2004).

[6]As the authors noted, this multiplicity does not come from the effect through changes in the composition of searchers, which instead plays a role in our model. Also, after acquiring the commitment technology, their "market equilibrium", which is the counterpart of our trading equilibrium, is always unique.

of market fragility. Weill (2007) and Lagos, Rocheteau, and Weill (2011) consider how dealers provide liquidity during an exogenously specified financial crisis. He and Milbradt (2014) consider the feedback loop between corporate bond default and secondary market illiquidity and show that they reinforce each other. These papers do not have multiple equilibria or trading among intermediaries.

Our paper is also related to a few papers that highlight the bright side of intermediation chains in OTC asset markets by considering the underlying strategic forces. Glode and Opp (2016) argue that longer intermediation chains between a buyer and a seller with market power can help mitigate adverse selection and lead to more efficient trading outcomes. Babus and Hu (2016) argue that intermediation chains and the resulting trading networks can help incentivize better monitoring. Ours differs by offering a more subtle and balanced message: longer intermediation chains are desirable in term of liquidity provision, but the the process of sustaining an intermediation chain can be fragile.

On modeling contributions, our framework features a fully decentralized and two-tier market structure with separate customer-to-dealer and inter-dealer markets, capturing realism. At the same time, we relax the commonly used $\{0, 1\}$ asset holding restrictions in most papers built upon Duffie, Garleanu and Pedersen (2005),[7] aiming to capture various patterns of decentralized trading among intermediaries. Thus, our paper also contributes to a burgeoning branch of literature exploring the endogenous emergence of market structures in OTC asset markets. In this new literature, a wave of papers focuses on the emergence of the core-periphery trading networks observed in various OTC markets, asking why some traders become customers while others become dealers. These models start either from identical traders (Wang, 2017) or heterogenous traders along the dimension of initial asset positions (Afonso and Lagos, 2015), of asset valuations and trading needs (Chang and Zhang, 2016, Shen, Wei and Yan, 2016), of trading technologies (Neklyudov, 2014, Farboodi, Jarosch and Shimer, 2017), or of both trading needs and technologies (Uslu, 2017). Atkeson, Eisfeldt and Weill (2015), Neklyudov and Sambalaibat (2015) and Colliard, Foucault and Hoffmann

[7] Garleanu (2009) and Lagos and Rocheteau (2009) are among the first to allow agents to have unrestricted asset holdings positions but require centralized inter-dealer trading. Afonso and Lagos (2015) and Uslu (2017) consider decentralized trading and allow for a finite N and unrestricted asset holding positions, respectively. But they do not specifically distinguish between customer-to-dealer and inter-dealer trading as we do, and their focuses are also different from ours.

(2018) further combine the OTC search and the network literatures to consider how exogenously specified network structures affect dealers' entry, exit and inventory holding decisions. The closest one to ours is Hugonnier, Lester and Weill (2018), who also use a fully decentralized two-tier market structure but with ex-ante heterogenous dealers to explore endogenous intermediation chains. They also do not focus on multiple equilibria or market fragility. Our work complement these papers in that we have a particular focus on the emergence of multiple equilibria and fragility of the inter-dealer market and the implied intermediation chains. In doing so, we fix the roles of dealers and customers but otherwise use a rich setting where the dealers are ex-ante identical and the trading, pricing, and inventory holding decisions of market participants are all endogenous.

2 The model

Timeline, asset, and preferences. We fix a probability space satisfying the usual conditions. Time is continuous and runs infinitely: $t \in [0, \infty)$. Consider a market for a single indivisible asset. There are three types of agents: buyers, dealers, and sellers. All agents are risk-neutral with time preferences determined by a constant discount rate $r > 0$. The monetary value of a unit of the asset is $\theta > 0$ for a buyer and is normalized as 0 for a seller and for a dealer. We call θ as the fundamental of the asset. We also call buyers and sellers jointly as "customers."

Buyers and sellers (customers). We specify the arrival rate of new buyers and of new sellers to be n.[8] When arriving, a seller brings 1 unit of asset to the market while the buyer has nothing. We focus on intermediated asset markets in the sense that buyers and sellers cannot trade with each other directly but only through dealers. At any time t, there are x_t active buyers and y_t active sellers present on the market searching for dealers, which will be endogenous determined. We denote by $z_t = x_t + y_t$ the total mass of searching customers, or equivalently, the mass of customers who are waiting to be served. A seller or a buyer who has successfully met and transacted with an dealer leaves the market and consumes the monetary value he gets. We also assume that at probability rate $\eta > 0$ a searching customer

[8] The model can be extended to handle different arrival rates of new buyers and sellers without changing the main economic mechanisms, but doing so significantly complicates the calculation.

departs the market without a successful transaction. This departure shock captures that a buyer or seller may lose her trading opportunity if cannot find an intermediary in a timely manner. For example, a junk bond mutual fund running out of cash rushes to find a dealer to sell its illiquid bonds, but it has to default and departs the market if cannot find a dealer timely.

Dealers. Dealers are homogeneous and we normalize the mass of dealers to 1. At any time t, each dealer can hold 0, 1, or 2 units of the asset. As standard in the literature (e.g., Amihud and Mendelson, 1980), we assume a weakly convex inventory cost structure: $C(0) = 0, C(1) = c > 0$ whether c captures the level of inventory costs, and $C(2) = \rho c$ where $\rho \geqslant 2$ captures the the curvature of the cost structure. We denote the endogenous distribution of dealers who hold 0, 1, and 2 units of the asset μ_{0t}, μ_{1t}, and μ_{2t}, respectively, where $\mu_t^0 + \mu_t^1 + \mu_t^2 = 1$. For the ease of exposition, we also call them type-0, type-1, and type-2 dealers, with the type indicating an dealer's inventory holding status.

It is worth noting that we purposefully relax the common assumption in the literature that limits the agents to hold either 0 or 1 unit of the asset. By allowing a dealer to hold 0, 1, or 2 units of the asset, we parsimoniously capture that a dealer may hold no inventory at all, or a low level of inventory, or a high level of inventory. As we show later, this deviation from the literature is important in generating gains from trade between intermediaries and thereby driving our results. As we will also discuss later, our results and the underlying intuition are robust to allowing for more asset holding positions. Thus, we choose the current setting to best flesh out the mechanism.

Searching and matching protocols. At any time t, agents search for counter-parties for a potential trade. To make the model flexible and more realistic, we assume customers and dealers have access to different searching and matching protocols. These matching protocols are consistent with and can be micro-founded by the continuous time random matching framework in Duffie, Qiao and Sun (2017).

First, on the customer-to-dealer market, a customer contacts a random dealer at Poisson rate $\alpha > 0$. Economically, α captures the friction of the customer-to-dealer market: a larger α implies a better technology held by customers and thus a smaller customer-to-dealer search friction. This setting immediately implies that a dealer is contacted by a random customer at endogenous Poisson rate $\beta_t = \alpha(x_t + y_t)$, which depends on the mass of searching customers.

Intuitively, a dealer will be contacted faster when more customers are actively searching.

Second, on the inter-dealer market, a dealer contacts another random dealer according to an independent Poisson process with rate λ, which is also independent to a dealer's inventory holdings. Similarly, λ captures the friction of the inter-dealer market: a larger λ suggests that intermediaries are easier to meet each other.

Bilateral trading. Trading prices are determined through generalized Nash bargaining as standard in the literature. As a benchmark, we assume that all agents, when meet bilaterally, have equal bargaining powers.[9] Hence, agents who meet always equally share the gains from trade, provided the gains from trade are positive.

3 Equilibrium analysis

As standard in the literature, we focus on steady-state (i.e., stationary) trading equilibria, that is, equilibria in which the mass of each type of agents is constant, and trade happens. Thus, we suppress the time argument t in equilibrium variables, and we call a steady-state equilibrium simply an equilibrium below.

Given our focus on trading among intermediaries, we first give a necessary condition for the inter-dealer market being active:

LEMMA **1.** *In any steady-state trading equilibrium, inter-dealer trade happens only if* $\mu_2 > 0$.

Economically, Lemma 1 amounts to saying that inter-dealer trading happens only when the dispersion of dealers' inventory holdings is large enough, that is, only when there are some dealers holding a relatively high level of inventory in equilibrium. Intuitively, if there were only type-0 and type-1 dealers in the market, no inter-dealer trade would happen because of the lack of gains from trade. Instead, a type-2 dealer may potentially trade with a type-0 dealer and both become type-1 dealers, because the gains from such a trade may be positive. Therefore, below we will use whether μ_2 is positive in equilibrium to organize the discussion of the two types of trading equilibria: with or without an active inter-dealer market. Importantly, we will further show that inter-dealer trading must happen when a

[9] The predictions of our model are robust to more sophisticated bargaining protocols that may give rise to unequal relative bargaining powers between the two bargaining parties.

type-2 dealer meets a type-0 dealer. In other words, $\mu_2 > 0$ is also a sufficient condition for the inter-dealer market being active.

We also note a useful result that at any steady state, the inflow-outflow balance implies that the mass of searching buyers must equal that of searching sellers, that is, $x = y$. To see this, notice that inflow-outflow balance of the dealer sector implies $n - \eta x = n - \eta y$. We highlight that this is not an assumption but instead an equilibrium outcome.

3.1 Trading equilibrium with inter-dealer market: $\mu_2 > 0$

In view of Lemma 1, we first characterize a type of equilibria where $\mu_2 > 0$. We characterize the equilibrium conditions and show that the inter-dealer market is active in the sense that when a type-2 dealer meets a type-0 dealer, a transaction must happen and thus both become type-1 dealers. Figure 1 below illustrates this type of equilibria, where solid arrows indicate the flows of assets and dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as dealer's changes in inventory-holding status.

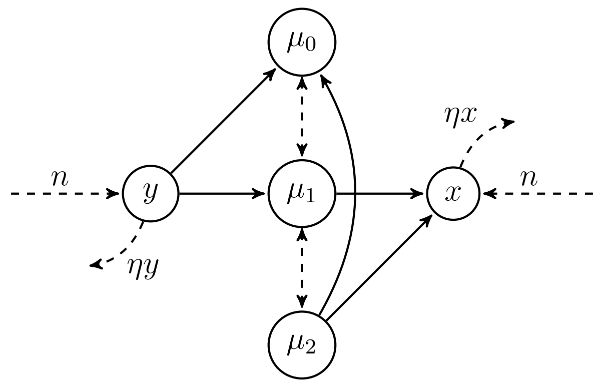

Figure 1: Trading equilibrium with the inter-dealer market

Solid arrows indicate the flows of assets. Dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as dealer's changes in inventory-holding status.

In a steady state, the asset inflows to the buyers must equal the asset outflows from type-1 and type-2 dealers:

$$n - \eta x = x\alpha(\mu_1 + \mu_2)\,, \tag{3.1}$$

and similarly, the asset outflows from the sellers must equal the asset inflows taken by type-0 and type-1 dealers:

$$n - \eta y = y\alpha(\mu_0 + \mu_1)\,. \tag{3.2}$$

Recall that $x = y$ under any steady state, these two accounting identities (3.1) and (3.2) imply that

$$\mu_0 = \mu_2\,, \tag{3.3}$$

that is, the mass of type-0 dealers must equal that of type-2 dealers. In what follows, we define $\mu_I = \mu_0 = \mu_2$ when refer to the equilibrium with an inter-dealer market.

Denote by $V_b^1, V_s^1, V_0^1, V_1^1$, and V_2^1 the equilibrium value functions of an active buyer, seller, type-0 dealer, type-1 dealer, and type-2 dealer, respectively, where the superscript 1 indicates that these value functions are evaluated under the distribution of agents in an equilibrium with an active inter-dealer market. We derive the agents' equilibrium Hamilton-Jacobi-Bellman (HJB) equations, taking into account the endogenous matching and bargaining outcomes:

$$\begin{aligned}
(\eta + r)V_b^1 &= \alpha\left[\mu_2\frac{1}{2}(\theta + V_1^1 - V_2^1 - V_b^1)^+ + \mu_1\frac{1}{2}(\theta + V_0^1 - V_1^1 - V_b^1)^+\right], && (3.4)\\
(\eta + r)V_s^1 &= \alpha\left[\mu_0\frac{1}{2}(V_1^1 - V_0^1 - V_s^1)^+ + \mu_1\frac{1}{2}(V_2^1 - V_1^1 - V_s^1)^+\right], && (3.5)\\
rV_0^1 &= \beta\frac{y}{x+y}\frac{1}{2}(V_1^1 - V_0^1 - V_s^1)^+ + \lambda\mu_2\frac{1}{2}(2V_1^1 - V_0^1 - V_2^1)^+, && (3.6)\\
rV_1^1 &= \beta\left[\frac{x}{x+y}\frac{1}{2}(\theta + V_0^1 - V_1^1 - V_b^1)^+ + \frac{y}{x+y}\frac{1}{2}(V_2^1 - V_1^1 - V_s^1)^+\right] - c, && (3.7)\\
rV_2^1 &= \beta\frac{x}{x+y}\frac{1}{2}(\theta + V_1^1 - V_2^1 - V_b^1)^+ + \lambda\mu_0\frac{1}{2}(2V_1^1 - V_0^1 - V_2^1)^+ - \rho c\,. && (3.8)
\end{aligned}$$

These HJB equations are intuitive. First, since all agents have equal bargaining power, they always equally share the potential gains from trade, if positive. The bargaining process also determines the trading prices in every bilateral trading.

Second, a buyer may buy a unit of asset either from a type-2 or a type-1 dealers, while a seller may sell to either a type-0 or type 2 dealer. This is reflected in the buyer's and seller's HJB equations (3.4) and (3.5).

Finally, for the three types of dealers, they all have their unique pattern of trading, reflected by (3.6), (3.7), and (3.8). A type-0 dealer can either buy a unit of asset from a seller or from a type-2 dealer, a type-1 dealer can either buy from a seller or sell to a buyer, while a type-2 dealer can either sell to a buyer or to a type-0 dealer. Consistent with our earlier argument, only the type-0 and type-2 dealers are potential candidates for an inter-dealer trade.

Analyzing these value functions, we first show that an inter-dealer trade must happen as long as a type-0 dealer meets a type-2 dealer.

LEMMA **2.** *In any equilibrium with* $\mu_2 > 0$*,* $2V_1^1 - V_0^1 - V_2^1 > 0$ *holds. That is, the gains from trade between a type-0 and a type-2 dealer are always strictly positive and thus the inter-dealer market is active.*

Lemma 2 implies that a trade must happen between a type-0 and a type-2 dealer. In the proof for Lemma 2, we show that the gains from trade from a potential trade between a type-0 and a type-2 dealer can be expressed as

$$2V_1^1 - V_0^1 - V_2^1 = \frac{(\rho - 2)c + \kappa_1 \beta}{\kappa_2},$$

where $\kappa_1 > 0$ and $\kappa_2 > 0$ are two strictly positive constants that are determined in equilibrium. It shows that the potential gains from trade are captured by two independent terms. The first term captures an instantaneous effect: it shows that an inter-dealer trade allows the two dealers to jointly save their inventory costs, consistent with the classical view of the inventory sharing. The second term captures a continuation effect: it implies that an inter-dealer trade allows the two dealers, who subsequently become two type-1 dealers, to jointly handle more order flows from customers. Since β is always strictly positive in any equilibrium, an inter-dealer trade happens as long as the second effect dominates, even if the inventory cost is not convex. In other words, the inventory cost structure being weakly convex is one (weak and economically relevant) sufficient condition for the inter-dealer market being active, but not a necessary condition.

So far, Lemmas 1 and 2 show that $\mu_2 > 0$ is both a sufficient and a necessary condition for the inter-dealer market being active. This verifies our equilibrium categorization: the equilibrium features an active inter-dealer market when $\mu_2 > 0$ while the inter-dealer market is not active when $\mu_2 = 0$.

A natural and important question is when an equilibrium with an active inter-dealer market exists. In principle, the existence of such an equilibrium requires all the five relevant trades as illustrated by the five solid arrows in Figure 1 happen. In other words, the gains from trade as shown in the right hand sides of value functions (3.4), (3.5), (3.6), (3.7), and (3.8) must be weakly positive. It is not surprising, however, that many of these constraints will be slack in equilibrium. The following proposition shows that the trade between a type-1 dealer and a seller is sufficient to determine the equilibrium existence:

Proposition **1.** *A trading equilibrium with an active inter-dealer market exists if*

$$V_2^1 - V_1^1 - V_s^1 \geqslant 0\,, \tag{3.9}$$

where V_2^1, V_1^1 and V_s^1 satisfy the value functions (3.4), (3.5), (3.6), (3.7), and (3.8).

Proposition 1 implies that the only criterion for the emergence of the inter-dealer market is that a type-1 dealer is willing to take a sell order from a seller, given the corresponding distribution of agents. In other words, other relevant trades must happen as the trade between a type-1 dealer and a seller happens.

The intuition is as follows. First, holding more inventories is more costly to a dealer due to the weakly convex inventory cost structure. Thus, that a type-1 dealer, who already hold a low level of inventory, find it profitable to increase her inventory implies that a type-0 dealer must find it profitable to increase her inventory. This implies that the trade between a type-0 dealer and a seller must happen.

Second, dealers do not have any ultimate interest in holding the asset. Thus, that a dealer finds it profitable to buy from a seller implies that she must also find it profitable to sell to a buyer later, regardless of her type. This implies that the trade between a type-1 dealer and a buyer and that between a type-2 dealer and a buyer must happen.

Finally, notice that Lemma 2 already guarantees that the trade between a type-0 and a type-2 dealer, that is, inter-dealer trading, must happen as $\mu_2 > 0$.

Following Proposition 1, we may further formulate the equilibrium criterion (3.9) with respect to the fundamental of the asset θ:

Corollary 1. *There exists a lower threshold $\underline{\theta}$ such that for all $\theta \geqslant \underline{\theta}$, a trading equilibrium with an active inter-dealer market exists.*

Corollary 1 implies that the inter-dealer market will endogenously emerge when the asset fundamental is high enough given other model parameters. Note that corollary 1 can be also stated with a higher threshold of inventory cost $\bar{c}$ in the sense that a trading equilibrium with an active inter-dealer market exists when the inventory cost c is lower than the threshold. Intuitively, when the asset fundamental is high enough relative to dealers' inventory cost, dealers are more likely to hold a high level of inventory in equilibrium and accordingly trade with each other to better manage their inventory holdings.

It is straightforward to formulate two empirically relevant equilibrium outcomes: the average length of intermediation chains and the aggregate inventory held by the dealer sector. In our framework, we define the length of intermediation chains l as the number of dealers through which an asset is intermediated from a seller to a buyer. Note that l is a random variable under equilibrium. We define the aggregate inventory v as the amount of assets held by all the types of dealers. Given the trading pattern in Figure 1, we have the following result, where the superscript 1 indicates that these variables are evaluated under the distribution of agents in an equilibrium with an active inter-dealer market.

Corollary 2. *In a trading equilibrium with an active inter-dealer market, the average length of intermediation chains is strictly larger than 1, that is, $\mathbb{E}[l^1] > 1$, and the aggregate inventory is $v^1 = 1$.*

In Corollary 2, $\mathbb{E}[l^1] > 1$ is a straightforward implication of the inter-dealer market being active, and $v^1 = 1$ follows from the inflow-outflow balance, in particular, (3.3). Later on, we will compare them to their counterparts in the equilibrium without an inter-dealer market. Note that our goal is not to directly mapping these equilibrium outcomes to data but rather to qualitatively compare them across the two types of equilibria.

3.2 Trading equilibrium without inter-dealer market: $\mu_2 = 0$

Next, we characterize a type of equilibria where $\mu_2 = 0$. In view of Lemmas 1 and 2, we have already known that $\mu_2 > 0$ is a sufficient and necessary condition for the inter-dealer market being active. Equivalently, $\mu_2 = 0$ is also a sufficient and necessary condition for the inter-dealer market being inactive. Figure 2 below illustrates this type of equilibria.

Under the equilibrium without an active inter-dealer market, inflow-outflow balance implies

$$n - \eta x = x\alpha\mu_1 \, , \tag{3.10}$$

$$n - \eta x = y\alpha\mu_0 \, . \tag{3.11}$$

Recall that $x = y$ under any equilibrium, (3.10) and (3.11) together imply

$$\mu_0 = \mu_1 = \frac{1}{2} \, , \tag{3.12}$$

that is, the mass of type-0 dealers must equal that of type-1 dealers under the equilibrium without an inter-dealer market.

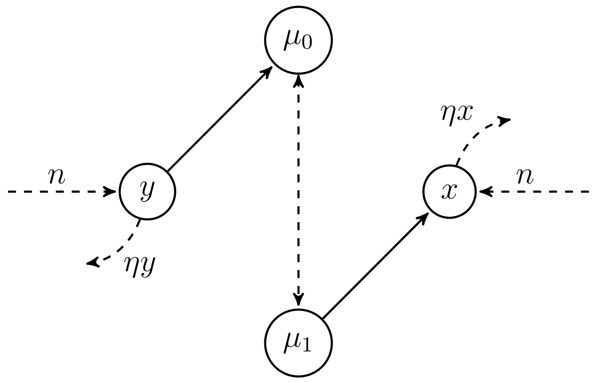

Figure 2: Equilibrium without the inter-dealer market

Solid arrows indicate the flows of assets. Dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as dealer's changes in inventory-holding status.

Denote by V_b^0, V_s^0, V_0^0, and V_1^0 the equilibrium value functions of an active buyer, seller, type-0 dealer, and type-1 dealer, respectively, where the superscript 0 indicates that these value functions are evaluated under the distribution of agents in an equilibrium without an inter-dealer market. We can similarly derive the agents' equilibrium HJB equations, taking into account the endogenous matching and bargaining outcomes:

$$(\eta + r)V_b^0 = \alpha\mu_1 \frac{1}{2}(\theta + V_0^0 - V_1^0 - V_b^0)^+ , \quad (3.13)$$

$$(\eta + r)V_s^0 = \alpha\mu_0 \frac{1}{2}(V_1^0 - V_0^0 - V_s^0)^+ , \quad (3.14)$$

$$rV_0^0 = \beta \frac{y}{x+y}\frac{1}{2}(V_1^0 - V_0^0 - V_s^0)^+ , \quad (3.15)$$

$$rV_1^0 = \beta \frac{x}{x+y}\frac{1}{2}(\theta + V_0^0 - V_1^0 - V_b^0)^+ - c \,. \quad (3.16)$$

The following proposition characterizes the conditions under which an equilibrium without an active inter-dealer market exists:

PROPOSITION **2.** *A trading equilibrium without an inter-dealer market exists if $V_0^0 \geqslant 0$ and*

$$V_2^0 - V_1^0 - V_s^0 \leqslant 0 \,, \quad (3.17)$$

where the hypothetical value function V_2^0 is given by

$$rV_2^0 = \beta \frac{x}{x+y}\frac{1}{2}(\theta + V_1^0 - V_2^0 - V_b^0)^+ + \lambda\mu_0 \frac{1}{2}(2V_1^0 - V_0^0 - V_2^0)^+ - \rho c \,, \quad (3.18)$$

and V_1s and V_s satisfy the value functions (3.13), (3.14), (3.15), and (3.16).

Proposition 2 specifies two conditions. First, the value function of a type-0 dealer under the corresponding distribution of agents, V_0^0, must be weakly positive. To see the intuition, recall that we need to verify the gains from trade for all relevant trades to be positive, that is, all relevant trades in Figure 2 must happen. When $V_0^0 \geqslant 0$, it must be that a type-0 dealer is willing to take a sell order from a seller since this is the only trade that a type-0 dealer can conduct. Then, because dealers do not have any ultimate interest in holding the asset, that a type-0 dealer finds it profitable to buy from a seller implies that she must also find it profitable to sell to a buyer later (i.e., when she becomes a type-1 dealer). This implies that the trade between a type-1 dealer and a buyer must happen.

Second, we need to further verify that a type-1 dealer would not find it profitable if it were to buy from an active seller. In other words, a type-1 dealer would never deviate from such an equilibrium by taking a sell order from a seller. This is captured by the additional condition

(3.18), where the hypothetical value functional for a type-2 dealer is instead evaluated under the distribution without an active inter-dealer market.

We highlight that conditions (3.9) and (3.17) regarding the trade between a type-1 dealer and a seller are not mutually exclusive because these value functions are evaluated under different distributions. In other words, conditions (3.9) and (3.17) may both hold, or are both violated. This observation has direct implication on potential equilibrium multiplicity, which we elaborate below.

Similar to Corollary 1, we may formulate the equilibrium criterion (3.17) with respect to the asset fundamental:

Corollary **3.** *Suppose* $V_0^0 \geqslant 0$*. There exists a upper threshold* $\overline{\theta}$ *such that for all* $\theta \leqslant \overline{\theta}$*, a trading equilibrium without an active inter-dealer market exists.*

Also similar to Corollay 2, we calculate the length of intermediation chains and aggregate inventory under equilibrium, where the superscript 0 indicates that these variables are evaluated under the distribution of agents in an equilibrium without an inter-dealer market.

Corollary **4.** *In a trading equilibrium with an active inter-dealer market, the length of intermediation chains is* $l^0 = 1$*, and the aggregate inventory is* $v^0 = \frac{1}{2}$*.*

Compared to Corollary 2, Corollary 4 thus suggests that the intermediation chain is strictly shorter and dealers holder a lower level of inventory on average under an equilibrium without an inter-dealer market.

4 Coordination of intermediation

Having characterized all the possible types of trading equilibria, we consider the potential for equilibrium multiplicity and the underlying strategic interactions. Given our focus on trading among intermediaries, below we focus on the parameter regions where $V_0^0 \geqslant 0$ is satisfied, with the formal parametric condition given in the Appendix.

The analysis of equilibrium multiplicity hinges on two important observations from Section 3. First, Propositions 1 and 2 suggest that whether a type-1 dealer is willing to take an order from a seller and to effectively increase her inventory, captured by $V_2 - V_1 - V_s$, is the sole criterion to determine the equilibrium, given the distribution of other agents in the corresponding equilibrium. Second, in turn, the distribution of other agents is solely determined by whether other type-1 dealers take orders from sellers and to effectively increase their inventories. Since we focus on steady-state equilibria, these observations allows us to use a two-strategy game among type-1 dealers to represent the strategic interaction embedded in the dynamic framework.

4.1 Equilibrium multiplicity

We formally characterize under what conditions multiple equilibria may emerge, that is, when the equilibria with and without an inter-dealer market may co-exist.

PROPOSITION **3.** *For a given set of model parameters, if*

$$\begin{cases} V_2^1 - V_1^1 - V_s^1 > 0 & (4.19) \\ V_2^0 - V_1^0 - V_s^0 < 0\,, & (4.20) \end{cases}$$

then an equilibrium with $\mu_2 > 0$ *and an equilibrium with* $\mu_2 = 0$ *co-exist.*

Proposition 3 is intuitive and follows from Propositions 1 and 2. Condition (4.19) implies that a type-1 dealer is willing to take an order from a seller and to become a type-2 dealer when other type-1 dealers also take orders from sellers (and thus the trading pattern and distribution of agents follow from Figure 1). According to Proposition 1, this implies that an equilibrium with an inter-dealer market exists. In parallel, Condition (4.20) implies that a type-1 dealer is not willing to take an order from a seller and to become a type-2 dealer when other type-1 dealers also refuse to trade with sellers (and thus the trading pattern and distribution of agents follow from Figure 2). According to Proposition 2, this implies that an equilibrium without an inter-dealer market exists. When (4.19) and (4.20) are both satisfied, both equilibria co-exist accordingly.

The idea underlying Proposition 3 resembles the classic notion of coordination in complete information static games. Whether a type-1 dealer trades with a seller and becomes a type-2 dealer would depend on her belief about other type-1 dealers' strategies. Thus, we can focus on type-1 dealer's this given strategy, and below we intuitively call it type-1 dealers' liquidity provision strategy, that is, whether a type-1 dealer provide liquidity or not.

As usual in standard coordination games, we allow the agents to play mixed strategies. Specifically, we allow the probability at which a trade between a type-1 dealer and a seller happens to be $\phi \in (0, 1)$. In a mixed-strategy equilibrium, a given type-1 dealer has probability ϕ to trade with a seller when other type-1 dealers also trade with sellers conditional on a meeting with probability ϕ.[10] In this case, an inter-dealer market emerges with probability ϕ, which is determined by

$$V_2^{\phi} - V_1^{\phi} - V_s^{\phi} = 0 \tag{4.21}$$

as well as other value functions as prescribed by (3.4), (3.5), (3.6), (3.7), and (3.8). Appendix C provides a detailed micro-foundation from which ϕ can be constructed from the staged bargaining game imbedded in our setting, and Appendix D provides the formal procedure to solve for the mixed-strategy equilibria, if any.

We provide an numerical example to illustrate Proposition 3. In this example, we choose $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, $\rho = 2.5$, $\alpha = 5$, and $\lambda = 0.01$.

The top panel of Figure 3 plots $V_2^1 - V_1^1 - V_s^1$ and $V_2^0 - V_1^0 - V_s^0$, the payoff gains of a type-1 dealer trading with a seller given other type-1 dealers' strategy, against different values of θ. It shows that in the given economic environment, type-1 dealers' liquidity provision decisions exhibit coordination motives for intermediate levels of θ. In particular, for intermediate levels of θ, $V_2^1 - V_1^1 - V_s^1$ is positive while $V_2^0 - V_1^0 - V_s^0$ is negative. Therefore, Proposition 3 suggests equilibrium multiplicity within that intermediate range of θ. This is confirmed by the bottom panel of Figure 3, which plots the equilibrium emergence of the inter-dealer market given different values of θ. Intuitively, when θ is sufficiently large (small), the inter-dealer market endogenously emerges (closes). When θ is intermediate, whether the inter-dealer market will emerges depends on a type-1 dealer's belief about other dealers' liquidity provision decisions.

Where do type-1 dealers' coordination motives come from? To address this question intuitively, consider a certain type-1 dealer's willingness to provide liquidity when λ is small. We can show that as $\lambda \to 0$:

$$\lim_{\lambda \to 0} x = n \left(\alpha \left(1 - \frac{1}{2 + \phi} \right) + \eta \right)^{-1},$$

[10] Note that this description of mixed-strategy equilibria also accommodate the pure-strategy equilibria we already consider above.

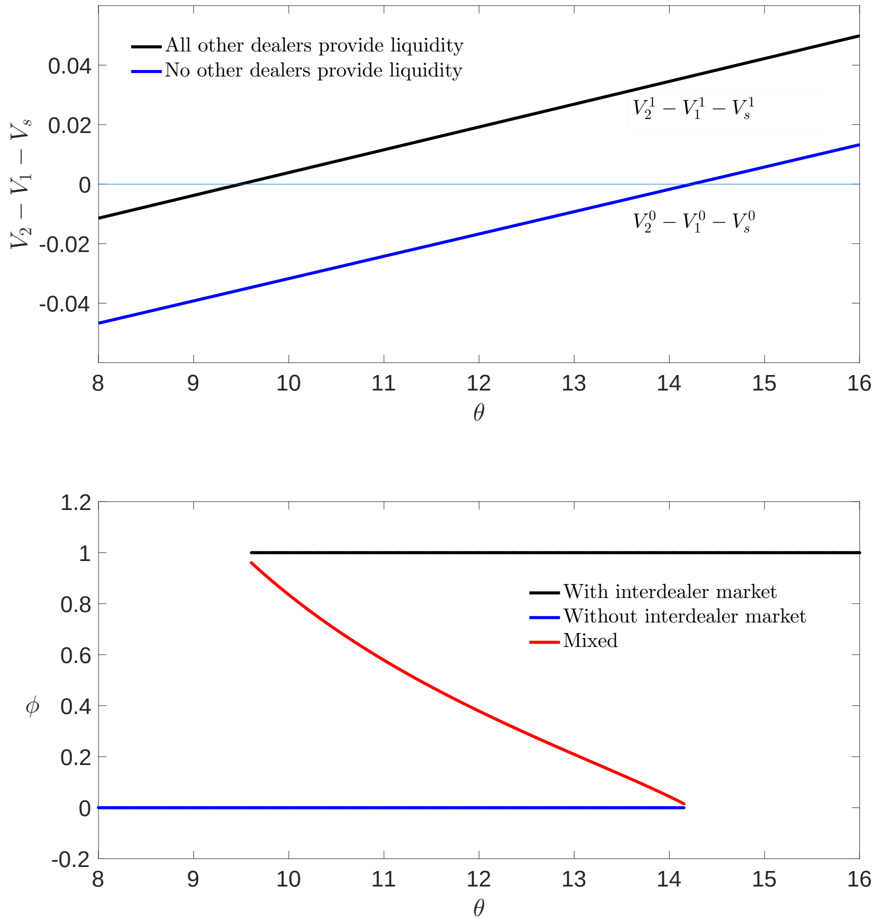

Figure 3: The correspondence of payoff gains and equilibrium multiplicity

The top panel plots the gains from trade between a type-1 dealer and a seller against asset fundamental θ, under the distribution when all other type-1 dealers trade with sellers (black) and under the distribution when all other type-1 dealers do not trade with sellers (blue). The bottom panel plots the equilibrium probability of the trade between a type-1 dealer and a seller against asset fundamental θ. Parameters: $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, $\rho = 2.5$, $\alpha = 5$, and $\lambda = 0.01$.

which is decreasing in ϕ. Economically, this means that when other type-1 dealers are more willing to hold a higher level of inventory and provide more liquidity, that is, when ϕ becomes larger, more customer orders will flow into the dealer sector. This thereby encourages the given type-1 dealer to also provide liquidity to meet the increasing customer demand placed upon the dealer sector.

4.2 Liquidity implications

A natural question is how much liquidity the dealers provide to the customers. We formally define the notion of liquidity in our economy as follows.

DEFINITION 1. *The equilibrium liquidity of the asset market is defined by the asset intermediated per unit of time:*

$$L = n - \eta x \,. \tag{4.22}$$

This definition implies that we can also use the mass of waiting customers x or y to measure equilibrium asset illiquidity. Recall that the rate at which a dealer is contacted by a random customer is endogenously determined as $\beta = \alpha(x + y)$, this immediately implies:

LEMMA 3. *In any equilibrium, β is lower when L is higher.*

The following formal result suggests that trading among intermediaries unambiguously improve liquidity:

PROPOSITION 4. *When multiplicity happens, L is higher in an equilibrium with inter-dealer trading than the comparable equilibrium without inter-dealer trading.*

Proposition 4 suggests that an equilibrium with the inter-dealer market is unambiguously better than a comparable equilibrium without the inter-dealer market in terms of liquidity provision. This allows us to rank the two equilibria over the dimension of asset market liquidity. It also helps us to uncover the intuition underlying market fragility, as we formulate below.

Lemma 3 and Proposition 4 also provide an alternative way to see the source of type-1 dealers' coordination motives in liquidity provision. In an intermediated asset market, a dealer's profit essentially comes from the asset being hard to trade, in our model reflected by customers actively searching and thus waiting in equilibrium. When the market becomes more liquid, fewer customers are waiting. Thus, if a type-1 dealer gives up an opportunity to serve an already met customer, the chance to be contacted by another random customer in future becomes lower. This in turn justifies the given type-1 dealer's higher willingness to serve a customer when other dealers also do so. This coordination motive suggests potential coordination failures in type-1 dealers' liquidity provision decisions: they may all provide liquidity, or stop providing liquidity all together.

5 Market fragility

A natural question is how does market fragility relate to the friction of the inter-dealer market: when it becomes easier for the intermediaries to meet each other, would it become more or less likely for multiple equilibria to occur?

For this purpose, we explicitly separate fundamental characteristics of our economy, including potential gains from trade θ and inventory costs c and ρ, from non-fundamental characteristics, in particular, the meeting rate of the inter-dealer market λ. We formally define market fragility as follows.

Definition 2. *For a family of economies parameterized by θ, c and ρ, it is fragile if there exists a non-zero subset $\{\theta, c, \rho\}$ of the space $\mathbb{R}^2_+ \times (1, +\infty)$ in which multiple equilibria occur.*

Economically, the definition captures the idea that if market liquidity may suddenly jump even when fundamental varies continuously, the market is fragile.

5.1 A frictionless benchmark

We first consider a benchmark under which dealers can meet each other frictionlessly. Concretely, we consider the possibility of market fragility when $\lambda \to \infty$. This resembles the modeling of a centralized inter-dealer market in the literature, for example, Garleanu (2009) and Lagos and Rocheteau (2009). The following proposition suggests that market fragility never happens under this frictionless benchmark.

Proposition 5. *For any family of economies parameterized by θ, c and ρ, these exists a threshold $\bar{\lambda}$ such that the family of economies is not fragile when $\lambda > \bar{\lambda}$.*

Proposition 5 suggests that the equilibrium with an inter-dealer market is the unique equilibrium regardless of the fundamentals θ, c and ρ, as long as the friction of the inter-dealer market is sufficiently small. Intuitively, a more efficient inter-dealer market implies that it is easier for dealers to offload their inventory holdings. This makes every dealer more willing to hold more inventory regardless of other dealers' strategy, eventually yielding a unique equilibrium.

5.2 Market friction and market fragility

We then explore the case when the inter-dealer market friction becomes large. To help illustrate the mechanism, we consider what happens when it is extremely hard for dealers to meet each other, that is, λ is close to 0. The following proposition characterizes when market fragility happens.

PROPOSITION **6.** *For any family of economies parameterized by θ, c and ρ, these exists a threshold $\underline{\lambda}$ such that the family of economies is fragile when $\lambda < \underline{\lambda}$.*

The economic intuition underlying Proposition 6 is the coordination motives in dealers' inventory holding and liquidity provision decisions. When the inter-dealer market is less efficient, holding more inventory becomes more costly. Dealers' inventory holding decisions thus exhibit strategic complementarity: when other dealers hold more inventory and intermediate more order flows, less order flows go to the dealer in question, dampening its ability to make profits. Thus, the dealer in question also become more willing to hold more inventory despite the effectively higher cost. This strategic complementarity is stronger when the inter-dealer market is less efficient, leading to market fragility.

6 Comparative statics

To provide more visual guidance and intuition regarding market fragility, we provide several numerical comparative statics to illustrate under what conditions market fragility is more likely to occur, and what are the impact on market liquidity. To focus on trading among intermediaries, we restrict our attention to the parameter region where dealers are willing to participate under any equilibrium. We fix the following common parameters: $\bar{\mu} = 1$, $n = 1$, and $r = 1$.

6.1 Inter-dealer market friction and market fragility

One of the key ideas of our model is that equilibrium multiplicity may arise due to the coordination motives of dealers in inventory holding and liquidity provision decisions. The coordination motives are in turn affected by the friction of the inter-dealer market: when dealers are easier to meet and thus trade with each other, one dealer's inventory holding and liquidity provision decisions depend less on other dealers', thereby weaken the strategic complementarity and eventually reduces potential market fragility. The following Figure 4 illustrates this idea, using the inventory cost concavity parameter $\rho = 2.5$, search friction parameter $\alpha = 5$, and exit shock parameter $\eta = 1$.

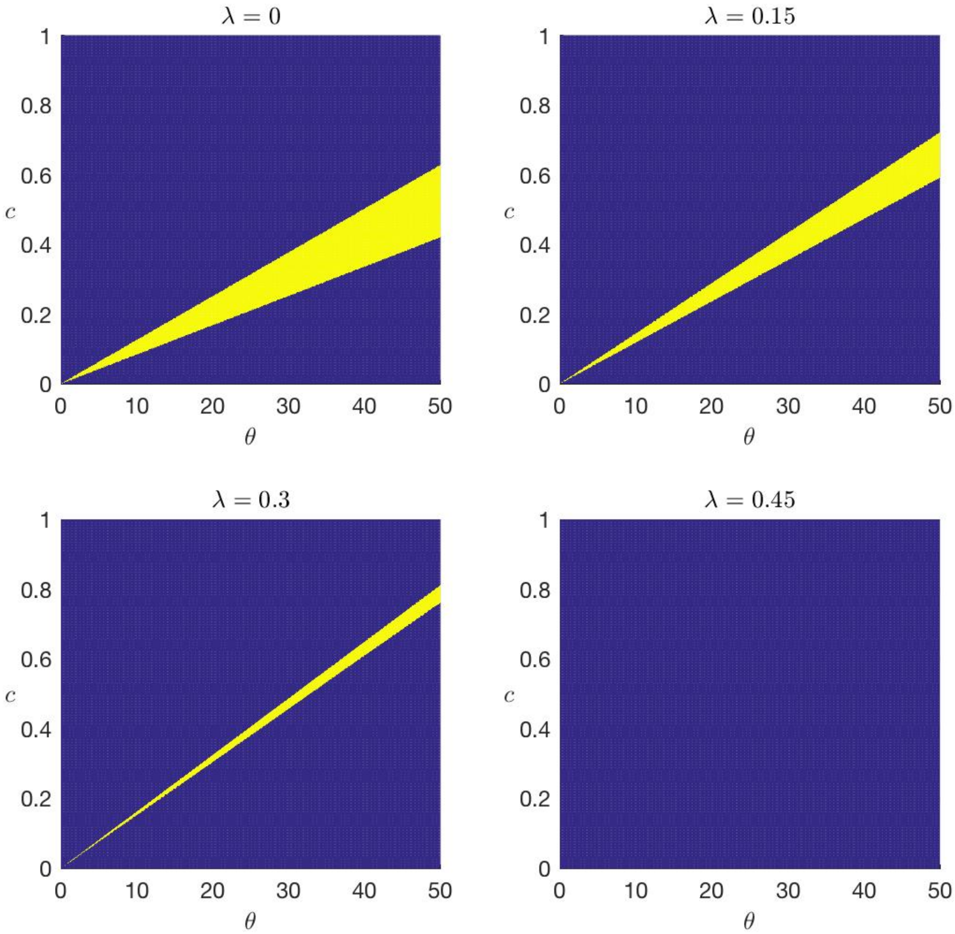

Figure 4: Inter-dealer market friction and market fragility

Each panel plots the parameter region where equilibrium multiplicity arises over the (θ, c)-parameter space. Common parameters: $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$.

The four panels in Figure 4 show that, as the friction of the inter-dealer market becomes less severe, captured by a reduced λ, the area where multiplicity happens over the $(\theta, c)-$region becomes smaller. Specifically, as λ takes values from 0 to 0.15 and then to 0.3, the family of economics is always fragile, but becomes less fragile in the sense that multiplicity is less likely to happen. When the inter-dealer market becomes sufficiently efficient as $\lambda = 0.45$, the strategy complementarity among dealer' inventory holding decisions is completely offset, and thus multiplicity cannot happen for any admissible parameters.

There are two important take-aways from Figure 4. First, the possibility of market fragility, measured by the area in the $(\theta, c)-$region where multiplicity happens, strictly decreases as the friction of the inter-dealer market becomes smaller. This is consistent with the theoretical predictions of Propositions 5 and 6.

Second, when the family of economies is fragile for a given level of inter-dealer market friction, equilibrium multiplicity always happens when the ratio of the potential gains from trade to the inventory cost lies in an intermediate range. Only in such cases, dealers' inventory holding strategy exhibits strategic complementarity. This is consistent with the theoretical predictions of Propositions 1 and 3.

6.2 Inventory cost, market fragility, and liquidity

We also explore the impacts of inventory costs on market fragility and liquidity across different parameter regions. The results are illustrated in Figures 5 and 6 below.

First, Figure 5 show the results when the inter-dealer market friction λ and the level of inventory cost c vary, under the same inventory cost concavity $\rho = 2.5$, search friction parameter $\alpha = 5$, and exit shock friction $\eta = 1$. Specifically, the left three panels in Figure 5 features a family of economics where fragility happens. Here, the inter-dealer market is extremely inefficient in the sense that dealers cannot meet with each other at all (i.e., $\lambda = 0$). As the level of inventory cost c becomes higher from 0.1 to 0.2 and then to 0.4, the equilibrium without an active inter-dealer market is more likely to happen. In particular, multiplicity is more likely to happen as the inventory cost becomes higher. Interestingly, when the inventory cost is higher, multiplicity is more likely to happen when the potential gain from trade per unit of asset is higher.

In contrast, the right three panels in Figure 5 features a family of economics where fragility does not happen. Here, the inter-dealer market is efficient in the sense that dealers can meet with each other at a sufficiently large rate (i.e., $\lambda = 0.4$). As the inventory cost becomes higher from 0.1 to 0.2 and then to 0.4, the equilibrium without an active inter-dealer market is more likely to happen. However, multiplicity never happens. These results are

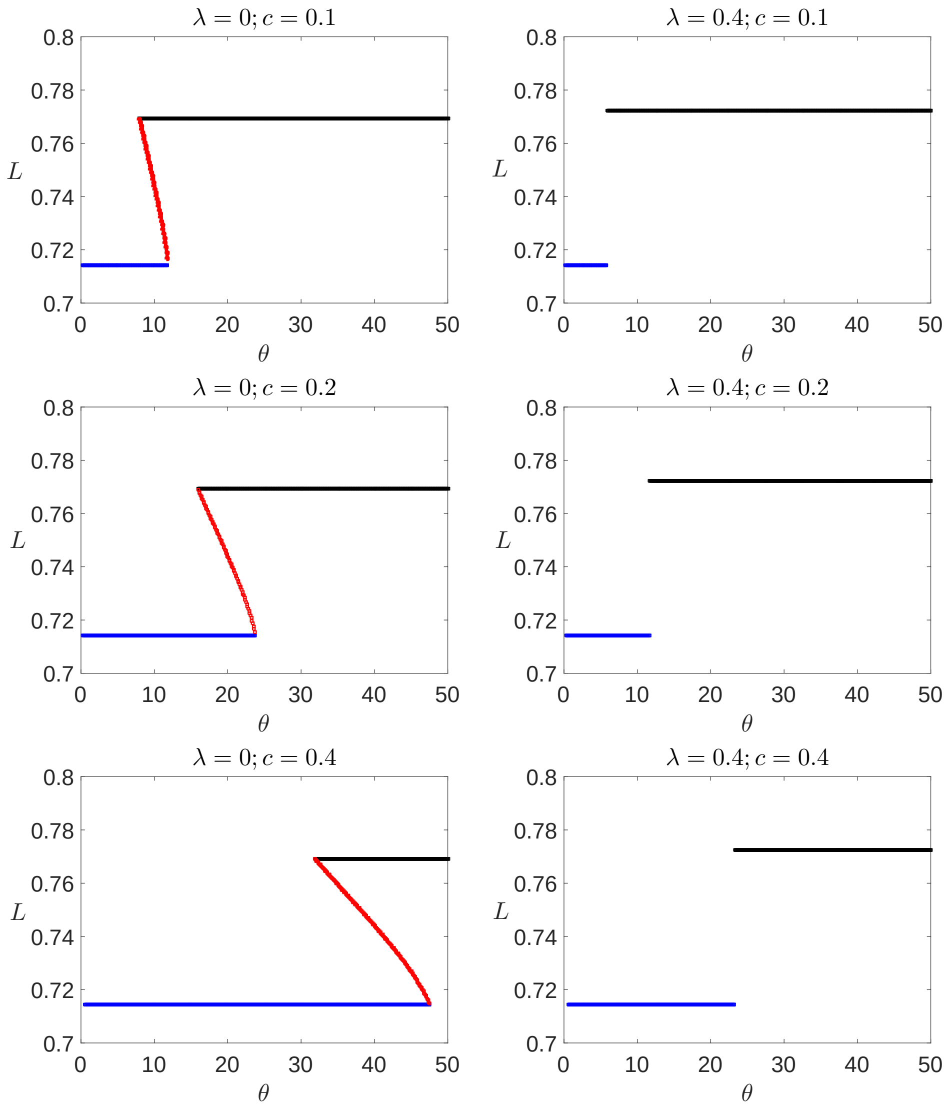

Figure 5: Equilibrium liquidity and fragility when λ and c vary

Each panel plots equilibrium liquidity L against the asset fundamental θ. Equilibrium multiplicity arises in the left panels. Common parameters: $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$.

consistent with the theoretical predictions of Propositions 5 and 6.

In a similar fashion, Figure 6 shows the results when the inter-dealer market friction λ and the inventory cost concavity ρ vary, under the same level of inventory cost $c = 0.1$ and search friction $\eta = 1$. As the inventory cost concavity parameter ρ becomes higher from 2 to 4 and then to 6, the equilibrium without an active inter-dealer market is more likely to happen, but market fragility happens only when the inter-dealer market friction is higher (i.e., $\lambda = 0$). In particular, multiplicity is more likely to happen as the inventory

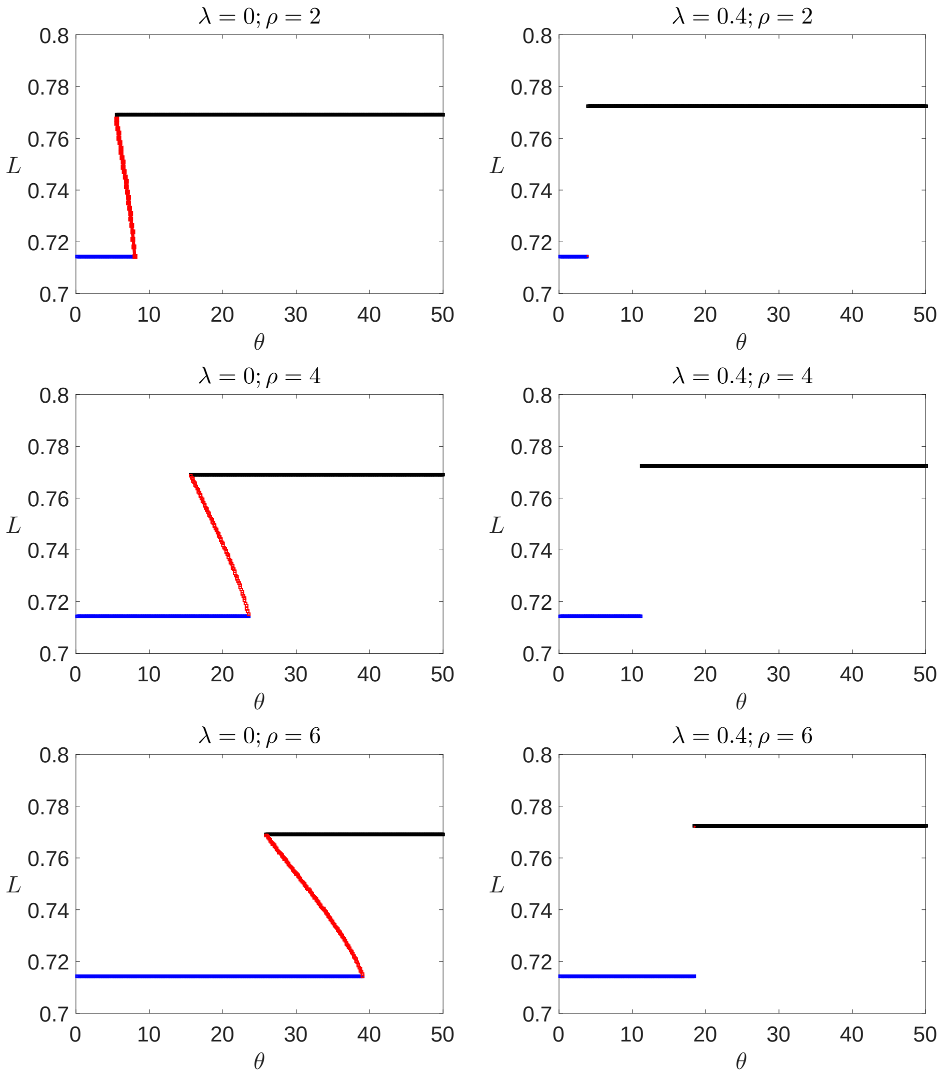

Figure 6: Equilibrium liquidity and fragility when λ and ρ vary

Each panel plots equilibrium liquidity L against the asset fundamental θ. Equilibrium multiplicity arises in the left panels. Common parameters: $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, and $\alpha = 5$.

cost structure becomes more concave. These results are again consistent with the theoretical predictions of Propositions 5 and 6.

6.3 Customer search friction, market fragility, and liquidity

We next explore the impacts of the search friction between customers and dealers on market fragility and liquidity across different parameter regions. The results are illustrated in Figure 7 below.

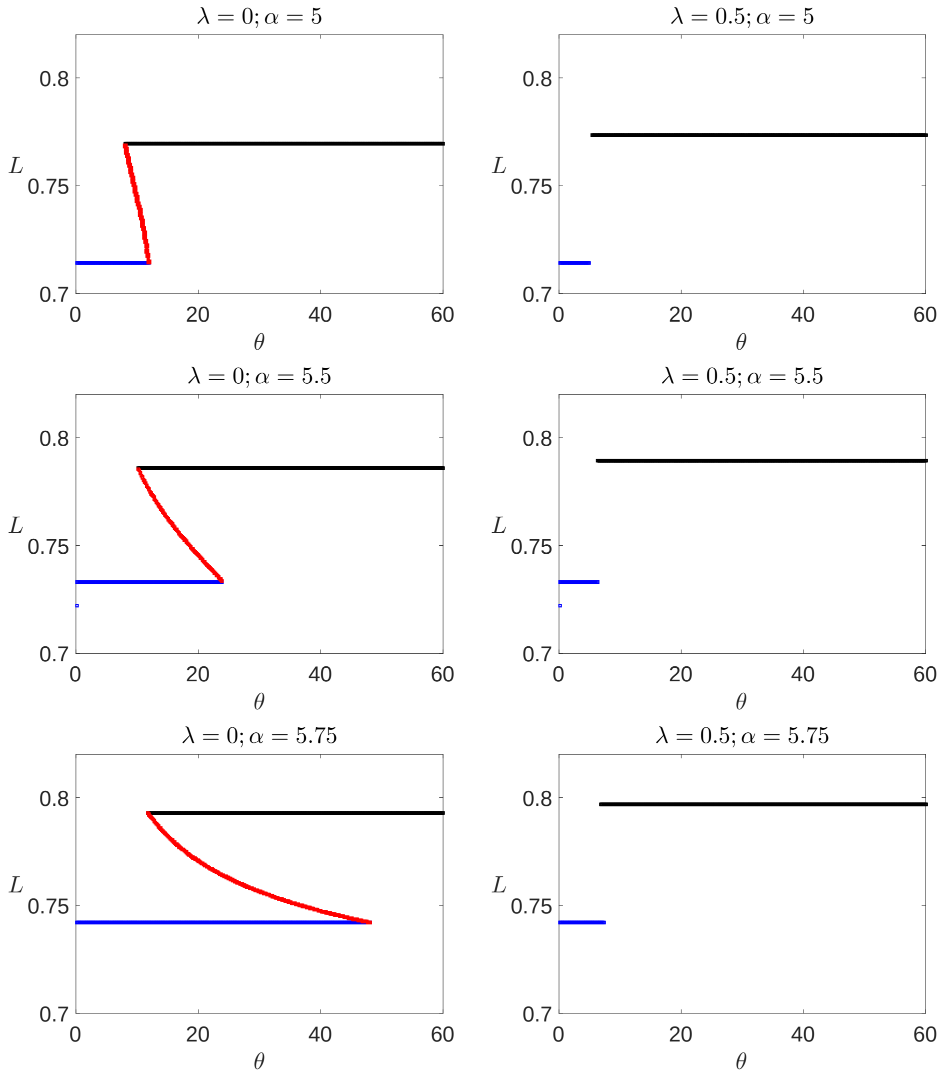

Figure 7: Equilibrium liquidity and fragility when λ and α vary

Each panel plots equilibrium liquidity L against the asset fundamental θ. Equilibrium multiplicity arises in the left panels. Common parameters: $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, and $\rho = 2.5$.

Here, we consider varying the inter-dealer market friction λ and the search friction parameter k between customers and dealers, under the same inventory structure $c = 0.1$ and $\rho = 2.5$, and the exit shock parameter $\eta = 1$. Similar, market fragility happens only when the inter-dealer market friction is higher (i.e., $\lambda = 0$). As it becomes easier for customers to contact dealers, that is, as the search friction becomes higher from 5 to 5.5 and then to 5.75, the equilibrium evolution has two additional interesting features.

First, equilibrium market liquidity becomes universally higher regardless the type of equilibrium. This is economically intuitive because a lower search friction makes it easier for customers to contact dealers, improving market liquidity.

However, perhaps surprisingly, market fragility in the sense of the possibility of multiplicity (happens when λ is close to 0) becomes higher as the search friction between customers and dealers becomes lower. However, this result is still economically intuitive due to the endogenous competition and coordination motives among dealers in our framework. When it becomes easier for customers to contact the dealers, dealers complete more fiercely for order flows. Thus, the coordination motives among dealers to provide liquidity become higher, under the same level of inter-dealer market friction. This is broadly consistent with the pattern that a decline in trading frictions between customers and dealers in the recent decades may instead lead to a higher market fragility due to the escalated competition among financial intermediaries (e.g., Philippon, 2015).

6.4 Departure shocks, market fragility, and liquidity

We finally explore the impacts of customers' departure shocks on market fragility and liquidity across different parameter regions. The results are illustrated in Figure 8 below.

Here, we vary the inter-dealer market friction λ and the exit shock parameter η between customers and dealers, under the same inventory cost structure $c = 0.1$ and $\rho = 2.5$, and search friction parameter $\alpha = 5$. Similar, market fragility happens only when the inter-dealer market friction is higher (i.e., $\lambda = 0$).

As the exit shock parameter becomes higher from 0.7 to 1 and then to 1.3, equilibrium market liquidity becomes universally lower regardless the type of equilibrium due to the higher possibility of customer exiting without successfully finding a dealer. More interestingly, market fragility in the sense of the possibility of multiplicity (happens when λ is close to 0) becomes lower as the exit shock becomes larger. The intuition still comes from the endogenous competition and coordination motives among dealers in our framework. When the customers are able to attempt contacting the dealers for a longer time, dealers complete more fiercely for order flows. Thus, the coordination motives among dealers to provide liquidity become higher, under the same level of inter-dealer market friction.

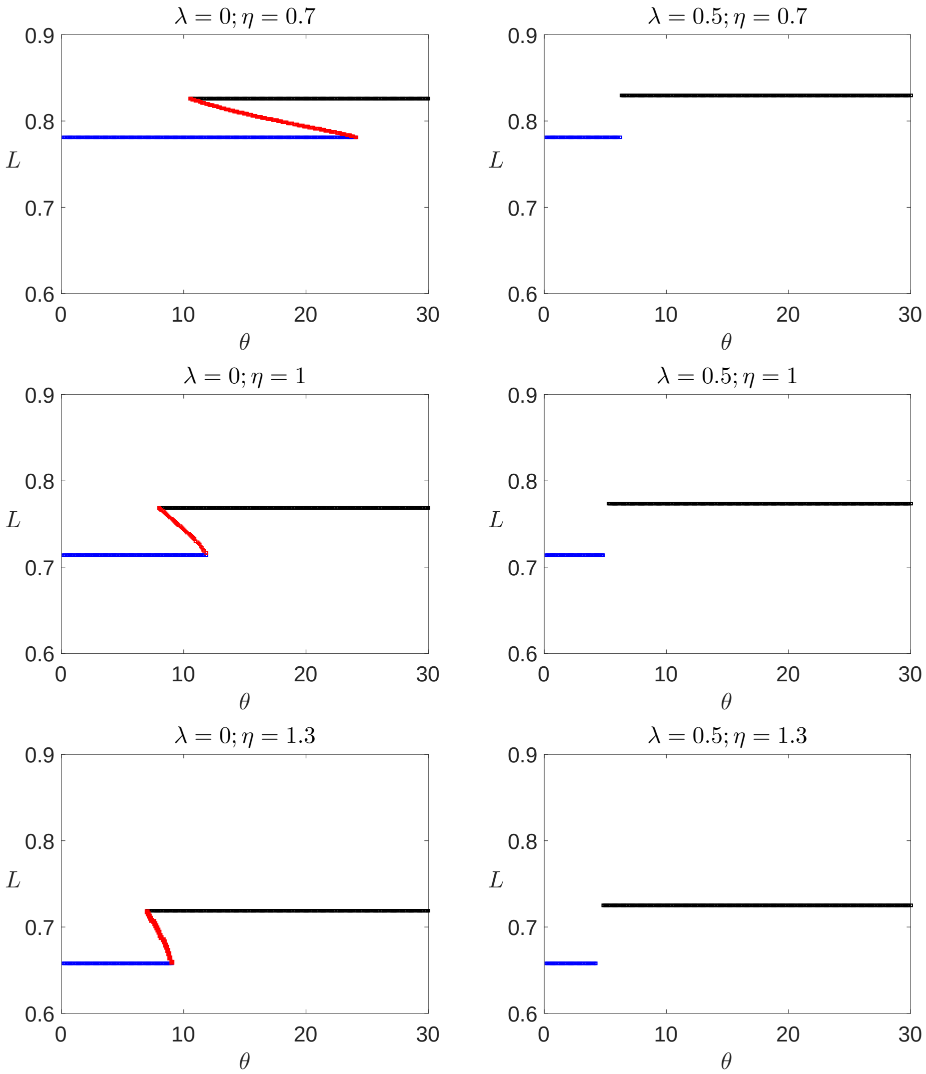

Figure 8: Equilibrium liquidity and fragility when λ and η vary

Each panel plots equilibrium liquidity L against the asset fundamental θ. Equilibrium multiplicity arises in the left panels. Common parameters: $n = 1$, $r = 1$, $c = 0.2$, $\rho = 2.5$, and $\alpha = 5$.

7 Conclusion

We propose a search-based asset pricing model to study decentralized and endogenous trading among financial intermediaries and the implied intermediation chains. In particular, we focus on the emergence of multiple equilibria and the implied market fragility, which is formally defined as the possibility of a liquidity drop in an asset market without a fundamental shock. In an equilibrium where inter-dealer trading is active (inactive), the intermediation chain is longer (shorter), dealers provide more (less) liquidity by holding more (less) inventory, and the market is more (less) liquid. Importantly, a dealer is more likely to provide liquidity if other dealers do so and the market becomes more liquid, leading to coordination motives among dealers. The coordination motives in turn lead to multiple equilibria and suggest market fragility. Market fragility is more likely to happen when the level of dealer inventory cost becomes higher, when the inventory cost structure becomes more convex, and when it is easier for the customers to contact dealers (e.g., due to customer technological improvements). A low search friction among dealers effectively reduces dealers' intermediation cost, making a dealer's willingness to provide liquidity less sensitive to other dealers' decisions and thereby reducing market fragility.

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Appendix

A Additional results

We first prove an additional result that implying an equilibrium restriction on the distribution of agents. This result will be repeatedly used in other proofs.

LEMMA 4. *In any trading equilibrium with $\mu_2 > 0$, there must be that $0 < \mu_I \leqslant \frac{1}{3}$. In other words, $\mu_1 \geqslant \frac{1}{3}$.*

PROOF OF LEMMA 4. Notice that, in any steady state equilibrium, the amount of type-0 dealers that take a sell order and become type-1 dealers must equal the amount of type-1 dealers that take a buy order and become type-0 dealers:

$$\mu_0(\beta\frac{y}{x+y} + \lambda\frac{\mu_2}{\mu}) = \mu_1\beta\frac{x}{x+y}\,.$$

Because $x = y$ in any equilibrium, it follows that

$$\frac{\beta}{2}\mu_0 + \lambda\frac{\mu_0\mu_2}{\mu} = \frac{\beta}{2}\mu_1\,. \tag{A.1}$$

Then by equation (3.3), condition (A.1) thus becomes

$$\lambda\mu_I^2 + \frac{3}{2}\beta\mu_I - \frac{\beta}{2} = 0\,. \tag{A.2}$$

Solving (A.2) and picking the positive root yields:

$$\mu_I = \frac{-\frac{3}{2}\beta + \sqrt{\frac{9}{4}\beta^2 + 2\lambda\beta}}{2\lambda} \in \left[0, \frac{1}{3}\right]\,,$$

completing the proof.

In particular, Lemma 4 implies

$$\begin{cases} \mu_I = 0 & \text{if } \lambda \to \infty\,, \\ \mu_I = \frac{1}{3} & \text{if } \lambda = 0\,. \end{cases}$$

B Proofs

Proof of Lemma 2. We manipulate the HJB equations. First, (3.7) minus (3.6) gives

$$\frac{1}{2}(V_1^1 - V_0^1)(2r + \beta + \lambda\mu_I) = \frac{\beta}{4}(\theta - V_b) + \frac{1}{2}(V_2^1 - V_1^1)\left(\frac{\beta}{2} + \lambda\mu_I\right) - c\,, \tag{B.1}$$

and (3.8) minus (3.7) gives

$$\frac{1}{2}(V_2^1 - V_1^1)(2r + \beta + \lambda\mu_I) = \frac{\beta}{4}V_s + \frac{1}{2}(V_1^1 - V_0^1)\left(\frac{\beta}{2} + \lambda\mu_I\right) - (\rho - 1)c\,. \tag{B.2}$$

Combining (3.4) and (B.1) yields

$$\begin{aligned}&\frac{1}{2}(V_1^1 - V_0^1)(2r + \beta + \lambda\mu_I)\\ =\;& \frac{\beta}{4}\frac{(\eta + r)\theta + \alpha\mu_I\frac{1}{2}(V_2^1 - V_1^1) + \alpha(1 - 2\mu_I)\frac{1}{2}(V_1^1 - V_0^1)}{\eta + r + \frac{\alpha}{2}(1 - \mu_I)} + \frac{1}{2}(V_2^1 - V_1^1)\left(\frac{\beta}{2} + \lambda\mu_I\right) - c\,.\end{aligned}$$

Define

$$B = \beta + \lambda\mu_I + \frac{\beta}{2}\frac{\alpha\mu_I}{2(\eta + r) + \alpha(1 - \mu_I)}\,,$$

and

$$Q = \frac{\beta(\eta + r)}{2(\eta + r) + \alpha(1 - \mu_I)}\,.$$

It follows that

$$\frac{1}{2}(V_1^1 - V_0^1)(2r + B + Q) = \frac{1}{2}(V_2 - V_1)B + \frac{\theta}{2}Q - c\,. \tag{B.3}$$

Similarly, combining (3.5) and (B.2) yields

$$\begin{aligned}&\frac{1}{2}(V_2^1 - V_1^1)\left(2r + \beta + \lambda\mu_I - \frac{\beta}{2}\frac{\alpha(1 - 2\mu_I)}{2(\eta + r) + \alpha(1 - \mu_I)}\right)\\ =\;& \frac{1}{2}(V_1^1 - V_0^1)\left(\frac{\beta}{2} + \lambda\mu_I + \frac{\beta}{2}\frac{\alpha\mu_I}{2(\eta + r) + \alpha(1 - \mu_I)}\right) - (\rho - 1)c\,,\end{aligned}$$

that is,

$$\frac{1}{2}(V_2^1 - V_1^1)(2r + B + Q) = \frac{1}{2}(V_1^1 - V_0^1)B - (\rho - 1)c\,. \tag{B.4}$$

Hence, (B.3) minus (B.4) implies

$$\frac{1}{2}(2V_1^1 - V_2^1 - V_0^1)(2r + 2B + Q) = \frac{\theta}{2}Q + \underbrace{(\rho - 2)c}_{\geqslant 0}\,. \tag{B.5}$$

Because $B > 0$ and $Q > 0$, this implies that $2V_1^1 - V_2^1 - V_0^1 > 0$. Therefore, inter-dealer trading must happen as $\mu_2 > 0$, concluding the proof.

Proof of Proposition 1. We manipulate the HJBs. Following the proof for Lemma 2, (B.4) and (B.5) imply

$$\frac{1}{2}(V_2^1 - V_1^1) = (2r + Q)^{-1}\left(\frac{B}{2r + 2B + Q}\left(\frac{Q}{2} + (\rho - 2)c\right) - (\rho - 1)c\right). \tag{B.6}$$

Conditions (B.6) and (B.3) then imply

$$\frac{1}{2}(V_1^1 - V_0^1)(2r + B + Q) = B\left(\frac{1}{2}(V_1^1 - V_0^1) - \frac{\frac{\theta}{2}Q + (\rho - 2)c}{2r + 2B + Q}\right) + \frac{\theta}{2}Q - c\,. \tag{B.7}$$

Therefore,

$$\frac{1}{2}(V_1^1 - V_0^1) = (2r + Q)^{-1}\frac{(2r + B + Q)(\frac{\theta}{2}Q - c) - B(\rho - 1)c}{2r + 2B + Q}\,. \tag{B.8}$$

Similarly, (B.6) can be rewritten as

$$\frac{1}{2}(V_2^1 - V_1^1) = (2r + Q)^{-1}\frac{B(\frac{\theta}{2}Q - c) - (2r + B + Q)(\rho - 1)c}{2r + 2B + Q}\,. \tag{B.9}$$

The symmetry between (B.8) and (B.9) implies

$$\begin{aligned}
&\alpha(1 - 2t)\frac{1}{2}(V_2^1 - V_1^1 - V_s^1) \\
= &\left(\eta + r + \frac{\alpha\mu_I}{2}\right)V_s^1 - \alpha\mu_I\frac{1}{2}(V_1^1 - V_0^1) \\
= &\frac{2(\eta + r) + \alpha\mu_I}{2(\eta + r) + \alpha(1 - \mu_I)}\left(\alpha\mu_I\frac{1}{2}(V_1^1 - V_0^1) + \alpha(1 - 2\mu_I)\frac{1}{2}(V_2^1 - V_1^1)\right) - \alpha\mu_I\frac{1}{2}(V_1^1 - V_0^1) \\
= &\frac{-\alpha(1 - 2\mu_I)\alpha\mu_I\frac{1}{2}(V_1^1 - V_0^1) + (2(\eta + r) + \alpha\mu_I)\,\alpha(1 - 2\mu_I)\frac{1}{2}(V_2^1 - V_1^1)}{2(\eta + r) + \alpha(1 - \mu_I)}\,,
\end{aligned}$$

completing the proof.

Proof of Corollary 1. The equilibrium with an inter-dealer market exists when

$$V_2^1 - V_1^1 - V_s^1 \geqslant 0\,. \tag{B.10}$$

Note that Lemma 4 implies that $\mu_I \in \left[0, \frac{1}{3}\right]$. Therefore, $\alpha(1-2\mu_I) > 0$ and

$$2(\eta + r) + \alpha(1-\mu_I) > 0\,.$$

Hence, (B.10) holds if and only if

$$2(\eta + r)\frac{1}{2}(V_2^1 - V_1^1) \geqslant \alpha\mu_I\frac{1}{2}(2V_1^1 - V_2^1 - V_0^1), \tag{B.11}$$

and by (B.9) and (B.5), we have

$$\frac{2(\eta+r)}{2r+Q}\frac{B\left(\frac{\theta}{2}Q - c\right) - (2r+B+Q)(\rho-1)c}{2r+2B+Q} \geqslant \frac{\alpha\mu_I\left(\frac{\theta}{2}Q + (\rho-2)c\right)}{2r+2B+Q}\,.$$

Since $B > 0$ and $Q > 0$, we also have

$$2(\eta+r)\left(B\left(\frac{\theta}{2}Q - c\right) - (2r+B+Q)(\rho-1)c\right) \geqslant (2r+Q)\alpha\mu_I\left(\frac{\theta}{2}Q + (\rho-2)c\right)\,. \tag{B.12}$$

Note that (B.12) holds if and only if

$$\left(\frac{\theta}{2}Q + (\rho-2)c\right)(2(\eta+r)B - (2r+Q)\alpha\mu_I) \geqslant 2(\eta+r)(\rho-1)c(2r+2B+Q)\,, \tag{B.13}$$

implying

$$2(\eta+r)B - (2r+Q)\alpha\mu_I > 0\,. \tag{B.14}$$

Plugging the expression of B and Q into (B.14), we have

$$2(\eta+r)\left(\frac{\beta}{2} + \lambda\mu_I + \frac{\beta}{2}\frac{\alpha\mu_I}{2(\eta+r)+\alpha(1-\mu_I)}\right) > \alpha\mu_I\left(2r + \frac{\beta(\eta+r)}{2(\eta+r)+\alpha(1-\mu_I)}\right)\,,$$

that is,

$$\left(\frac{2r}{\eta + r}\alpha - 2\lambda\right)\mu_I < \beta\,. \tag{B.15}$$

Note that

$$(\beta + 2\lambda\mu_I)(2r + Q) - 2rB > 0\,, \tag{B.16}$$

therefore, there exists $\bar{\theta}$ such that $V_0 \geqslant 0$ for all $\theta \geqslant \bar{\theta}$, where $\bar{\theta}$ is defined such that $rV_0^1 = 0$. This completes the proof.

Proof of Proposition 2. We manipulate the HJB equations. To begin, conditions (3.13) and (3.16) imply

$$rV_1^0 = \frac{\beta(\eta + r)V_b^0}{\alpha} - c\,. \tag{B.17}$$

and conditions (3.14) and (3.15) imply

$$rV_0^0 = \frac{\beta(\eta + r)V_s^0}{\alpha}\,. \tag{B.18}$$

Moreover, (3.16) minus (3.15) implies

$$\begin{aligned} r(V_1^0 - V_0^0) &= -\beta\frac{1}{2}(V_1^0 - V_0^0) + \frac{\beta}{2}\left(\frac{1}{2}(\theta - V_b^0) + \frac{1}{2}V_s^0\right) - c \\ &= -\beta\frac{1}{2}(V_1^0 - V_0^0) + \frac{\beta}{4}\left(\theta + (V_2^0 - V_b^0)\right) - c\,, \end{aligned} \tag{B.19}$$

implying

$$r(V_1^0 - V_0^0) = \frac{\beta}{\alpha}(\eta + r)(V_b^0 - V_s^0) - c\,.$$

Therefore,

$$V_s^0 - V_b^0 = \frac{\alpha}{\beta(\eta + r)}\left(-r(V_1^0 - V_0^0) - c\right)\,. \tag{B.20}$$

Then, (B.19) and (B.20) imply

$$r(V_1^0 - V_0^0) = -\frac{\beta}{2}(V_1^0 - V_0^0) + \frac{\beta}{4}\theta + \frac{\alpha}{4(\eta + r)}\big(-r(V_1^0 - V_0^0) - c\big) - c\,.$$

Therefore,

$$V_1^0 - V_0^0 = \left(r + \frac{\beta}{2} + \frac{r\alpha}{4(\eta + r)}\right)^{-1} \left(\frac{\beta}{4}\theta - (c_1 - c_0)(1 + \frac{\alpha}{4(\eta + r)})\right). \tag{B.21}$$

On the other hand, (B.18) implies

$$V_s^0 = \frac{\alpha}{\beta(\eta + r)} r V_0^0 \,. \tag{B.22}$$

Also note that (B.22) and (3.15) imply

$$rV_0^0 = \frac{\beta}{4}(V_1^0 - V_0^0) - \frac{\beta}{4}V_s^0 = \frac{\beta}{4}(V_1^0 - V_0^0) - \frac{\alpha}{4(\eta + r)} r V_0^0 \,.$$

Therefore,

$$r\left(1 + \frac{\alpha}{4(\eta + r)}\right) V_0^0 = \frac{\beta}{4}(V_1^0 - V_0^0)\,. \tag{B.23}$$

Define

$$D = 1 + \frac{\alpha}{4(\eta + r)}\,.$$

Then (B.21) becomes

$$V_1^0 - V_0^0 = \frac{\frac{\beta}{4}\theta - Dc}{\frac{\beta}{2} + rD}\,. \tag{B.24}$$

Conditions (B.23) and (B.24) imply

$$rDV_0^0 = \frac{\beta}{4}\frac{\frac{\beta}{4}\theta - Dc}{\frac{\beta}{2} + rD}\,. \tag{B.25}$$

As a final requirement for the equilibrium, we need $V_0^0 \geqslant 0$. By (B.25) and the fact that $D > 0$, we have

$$\theta \geqslant 4\beta^{-1} Dc\,. \tag{B.26}$$

completing the proof.

Proof of Proposition 4. Denote by L^ϕ and x^ϕ, $\phi \in \{0, 1\}$, the equilibrium liquidity and mass of buyers under the two types of equilibria (without or with an inter-dealer market). Consider the equilibrium with an inter-dealer market first. The inflow-outflow balance condition (3.1) implies that

$$n - \eta x^1 = \alpha x^1 (1 - \mu_I)\,. \tag{B.27}$$

On the other hand, under the equilibrium without an inter-dealer market, inflow-outflow balance (3.10) implies that

$$n - \eta x^0 = \frac{1}{2}\alpha x^0\,. \tag{B.28}$$

Notice that Lemma 4 implies that $\mu_I \leqslant \frac{1}{3}$, that is, $1 - \mu_I \geqslant \frac{2}{3} > \frac{1}{2}$. Hence, conditions (B.27) and (B.28) jointly imply that $x^1 < x^0$. This immediately implies $L^1 > L^0$ by definition, concluding the proof.

Proof of Proposition 5. We consider the limit as $\lambda \to \infty$. First, under the equilibrium with an active inter-dealer market, the inflow-outflow balance implies

$$x = \frac{n}{\alpha + \eta}\,. \tag{B.29}$$

It follows that

$$\beta = 2\alpha x = \frac{2\alpha n}{\alpha + \eta}\,. \tag{B.30}$$

At the same time,

$$\lim_{\lambda \to \infty} \lambda \mu_I = \infty\,,$$

suggesting that the inter-dealer market is active with the equilibrium mass of type-0 and type-2 dealers being 0. Intuitively, because dealers can contact each other infinitely quickly, any type-2 dealer will immediately trade with a type-0 dealer and then both become type-1 dealers.

Recall the definition of B and Q in the proof for Proposition 2. Direct calculation yields:

$$\lim_{\lambda \to \infty} B = \infty\,, \tag{B.31}$$

$$\lim_{\lambda \to \infty} Q = \frac{2\alpha n (r + \eta)}{(\alpha + \eta)\,(2(r + \eta) + \alpha)}\,. \tag{B.32}$$

Following the argument in the the proof for Proposition 2,

$$\lim_{\lambda\to\infty} V_2^1 - V_1^1 - V_s^1 \geqslant 0$$

if and only if

$$\theta \geqslant \underline{\theta} = \frac{2\rho c(\alpha+\eta)\left(2(r+\eta)+\alpha\right)}{2n\alpha(r+\eta)}\,, \tag{B.33}$$

suggesting that an equilibrium with an active inter-dealer market must exist when the asset fundamental is high enough.

Then, consider the equilibrium without an inter-dealer market. The inflow-outflow balance implies

$$x = \frac{2n}{\alpha+2\eta}\,. \tag{B.34}$$

It follows that

$$\beta = 2\alpha x = \frac{4\alpha n}{\alpha+2\eta}\,. \tag{B.35}$$

Similar calculation following the argument in the the proof for Proposition 2 shows that

$$\lim_{\lambda\to\infty} V_2^0 - V_1^0 - V_s^0 > 0\,, \tag{B.36}$$

regardless of θ. Thus, an equilibrium without an inter-dealer market never exists. By continuity, this concludes the proof.

Proof of Proposition 6. We consider the limit as $\lambda \to 0$. Under the equilibrium with an active inter-dealer market, the inflow-outflow balance implies

$$x = \frac{3n}{2\alpha+3\eta}\,. \tag{B.37}$$

It follows that

$$\beta = 2\alpha x = \frac{6\alpha n}{2\alpha+3\eta}\,. \tag{B.38}$$

Recall the definition of B and Q in the proof for Proposition 2. Direct calculation yields:

$$\lim_{\lambda\to 0} B = \frac{3\alpha n}{2\alpha+3\eta}\frac{6(r+\eta)+3\alpha}{6(r+\eta)+2\alpha}\,, \tag{B.39}$$

$$\lim_{\lambda \to 0} Q = \frac{3\alpha n}{2\alpha + 3\eta} \frac{3(r+\eta)}{6(r+\eta)+2\alpha}. \tag{B.40}$$

Hence, $V_2^1 - V_1^1 - V_s^1$ is increasing in θ if and only if

$$3n > \left(\frac{2}{3}\alpha + \eta\right) \frac{2r}{\eta + r}, \tag{B.41}$$

completing the proof by continuity.

C Strategy representation of the bargaining game

This appendix presents a strategy representation of the staged bargaining game embedded in our dynamic model. Since the game played by agents in our framework is essentially a complete information dynamic bargaining game, it is clear that at any steady-state equilibrium, the sub-game played by two meeting agents j and k can be summarized by the following two-strategy (sub-)game:

		Agent k: *Accept*	Agent k: *Reject*
Agent j	*Accept*	$\frac{G(\{V_i\}_i)}{2}, \frac{G(\{V_i\}_i)}{2}$	$0, 0$
	Reject	$0, 0$	$0, 0$

where $G(\{V_i\}_i)$ denotes the potential gains from trade between the two meeting agents j and k, which are in turn determined endogenously by all the agents' value functions given the steady state of the dynamic game as well as agents' rational expectations of achieving the corresponding steady state. Intuitively, only when the two meeting agents both choose "Accept", trade will happen. In turn, only when the potential gains from trade are positive, the two meetings agents will choose "Accept" simultaneously. On the flip side, at least one agent will choose "Reject" when the potential gains from trade are negative, and thus a trade will not happen. When the potential gains from trade are zero, agents may play mixed strategies, where their equilibrium mixed strategies will be determined by the steady-state distribution of the mass of agents as well as their value functions.

Below we focus on the staged bargaining game played by a type-1 dealer and a seller conditional on a meeting. First, note that Propositions 1 and 2 suggest that whether a type-

1 dealer is willing to take an order from an active seller and to effectively increase its inventory is the solely important criterion to determine which type of equilibria is sustainable, given the equilibrium distribution and values of other agents in the corresponding equilibrium. To formulate type-1 dealers' strategy as well as their willingness to trade requires us to analyze the bargaining (sub-)game between an type-1 dealer and an active seller, when they meet each other:

		Seller: *Accept*	Seller: *Reject*
Type-1 Dealer	*Accept*	$\frac{V_2-V_1-V_s}{2}, \frac{V_2-V_1-V_s}{2}$	$0, 0$
	Reject	$0, 0$	$0, 0$

As suggested by the bargaining (sub-)game above, the type-1 dealer's inventory decision of whether or not to increase its inventory holding by taking an order from a seller is solely determined by whether $V_2-V_1-V_s$ is positive or negative, given the endogenously determined value functions in the corresponding equilibrium.

We explicitly show how the trading probability ϕ between a type-1 dealer and a seller, conditional on a meeting, can be constructed from the staged bargaining (sub-)game. Specifically, a type-1 dealer's strategy in the above bargaining (sub-)game is $(p, 1-p)$ while a seller's strategy is $(q, 1-q)$, whenever they meet each other. In this case, a trade between a type-1 dealer and a seller happens with probability $\phi = pq$ when they meet. Notice that the bargaining (sub-)game itself is not sufficient to determine the equilibrium profile (p, q). Rather, the equilibrium probability pq at which a trade between a type-1 dealer and a seller happens will be determined by the condition $V_2 - V_1 - V_s = 0$ as well as other value functions as prescribed by (3.4), (3.5), (3.6), (3.7), and (3.8).

D Derivation of the mixed-strategy equilibria

This appendix explicitly derive the mixed-strategy equilibria we consider in the main text. In any candidate mixed-strategy equilibria with $\phi \in (0, 1)$,

Under a mixed-strategy equilibrium, the inflow-outflow balance of the dealer sector implies

$$x = y\,, \tag{D.1}$$

$$x\alpha(\mu_1 + \mu_2) = n - \eta y\,, \tag{D.2}$$

and

$$y\alpha(\phi\mu_1 + \mu_0) = n - \eta y\,. \tag{D.3}$$

Conditions (D.1), (D.2), and (D.3) together imply

$$\mu_1 + \mu_2 = \phi\mu_1 + \mu_0\,. \tag{D.4}$$

At the same time, the inflow-outflow balance of type-0 dealers suggests

$$\mu_0\left(\beta\frac{y}{x+y} + \lambda\mu_2\right) = \mu_1\beta\frac{x}{x+y}\,, \tag{D.5}$$

while the inflow-outflow balance of type-2 dealers suggests

$$\mu_2\left(\beta\frac{x}{x+y} + \lambda\mu_0\right) = \mu_1\beta\frac{\phi y}{x+y}\,. \tag{D.6}$$

First, condition (D.4) implies $\phi\mu_1 + \mu_0 = 1 - \mu_0$, that is,

$$\mu_1 = \phi^{-1}(1 - 2\mu_0) \tag{D.7}$$

and consequently,

$$\mu_2 = 1 - \mu_0 - \mu_1 = 1 - \mu_0 - \phi^{-1}(1 - 2\mu_0)\,. \tag{D.8}$$

Thus, conditions (D.1), (D.5), (D.7), and (D.8) jointly imply

$$(2-\phi)\lambda\mu_0^2 + \left(\left(1 + \frac{\phi}{2}\right)\beta - (1-\phi)\lambda\right)\mu_0 - \frac{\beta}{2} = 0\,, \tag{D.9}$$

which determine μ_0 under a mixed-strategy equilibrium with ϕ.

To check (D.9) has meaningful solutions, define

$$\begin{aligned}\Delta &= \left(\left(1+\frac{\phi}{2}\right)\beta-(1-\phi)\lambda\right)^2+2\beta(2-\phi)\lambda \\ &= \left(1+\frac{\phi}{2}\right)^2\beta^2+(1-\phi)^2\lambda^2-2\left(1+\frac{\phi}{2}\right)\beta(1-\phi)\lambda+2\beta(2-\phi)\lambda,\end{aligned}$$

where the sum of the last two terms

$$\begin{aligned}&-2\left(1+\frac{\phi}{2}\right)\beta(1-\phi)\lambda+2\beta(2-\phi)\lambda \\ =\ &2\beta\lambda\left(\left(1-\frac{\phi}{4}\right)^2+\frac{7}{16}\phi^2\right)>0.\end{aligned}$$

Therefore, $\Delta > 0$ and

$$\mu_0 = \frac{(1-\phi)\lambda-\left(1+\frac{\phi}{2}\right)\beta+\sqrt{\left(1+\frac{\phi}{2}\right)^2\beta^2+(1-\phi)^2\lambda^2+2\beta\lambda\left(\frac{\phi^2}{2}-\frac{\phi}{2}+1\right)}}{2(2-\phi)\lambda}. \tag{D.10}$$

Note that the negative solution is dropped since $\mu_0 \in [0,1]$. By (D.7) and (D.8), this fully pins down the distribution of the dealer sector, and further of the customers by (D.5) and (D.6).