Distinguishing Rational and Behavioral Models of Momentum

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Abstract

One of the many challenges facing financial economists is to distinguish the theories explaining momentum. Brav and Heaton (2002) show that it is very difficult to distinguish the "rational" models of structural uncertainty (SU) from "behavioral" models of conservatism (C). In this paper, I reexamine the SU model and the C model proposed by Brav and Heaton (2002) in explaining short run momentum. Based on simulated data, I find that they differ from each other in the relation between agent's earnings forecast revision and the lagged earnings change. This relation is significantly negative for the SU model and significantly positive for the C model. Empirical evidence provides support for the SU model.

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1 Introduction

Financial anomalies such as short term continuation\(^1\) and long term reversal\(^2\) in stock returns have intrigued academics as well as practitioners for a long time. These empirical facts present challenges to the traditional full information rational expectation (RE) models. To explain these empirical puzzles, many theories have been proposed that relax either or both of the two key assumptions of the RE models: investors have complete knowledge of the underlying structure of the economy, and they apply the Bayesian method in forming their expectations. Among these are behavioral theories\(^3\) and rational structural uncertainty (SU) theories\(^4\). Brav and Heaton (2002) propose simplified models to highlight the deviation of behavioral theories and structural uncertainty theories from the RE models. In Brav and Heaton (2002), behavioral models relax the assumption of Bayesian updating and invoke either conservatism bias or representativeness\(^5\) bias to explain the short term continuation and long term reversal evidence respectively. The SU model relaxes the assumption of complete information of the underlying structure but maintains the assumption of Bayesian updating. The SU model can generate both underreaction and overreaction patterns under different situations. Brav and Heaton (2002) claim that the SU model generates overreaction pattern similar to the representativeness model during stable periods and underreaction pattern similar to the conservatism model shortly after a structural change, hence making

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\(^1\) See, for example, Jegadeesh and Titman (1993), Chan et al. (1996).
\(^2\) See, e.g. Lakonishok, Shleifer and Vishny (1994).
\(^3\) See Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), and Hong and Stein (1999).
\(^4\) See, e.g. Brav and Heaton (2002), and Lewellen and Shanken (2002).
\(^5\) Conservatism denotes the agent’s tendency to overweight her prior and/or the old information and underweight the new information in forming her expectation compared to optimal full information Bayesian updating. Representativeness denotes the opposite bias which makes the agent overweight recent information and underweight her prior and old information.
the two theories hard to distinguish. However, this comparison is based on the assumption that the investor can time the structural break. More importantly, this comparison assumes that the behavioral agent is subject to the conservatism bias only shortly after the structural break, but is subject to the representativeness bias during stable periods in forming her expectation. In reality, the investor does not choose which bias she is subject to based on whether the firm is in a stable period or is experiencing a structural change. According to psychological studies, it depends on the trend and consistency of historical information. Especially, when we examine short term continuation instead of long term reversal, it is more natural to compare the SU agent’s expectation to that of a conservatism agent over the whole period instead of only for the period shortly after a structural change. The expectation of a representativeness agent should be compared with that of a SU agent when we study long term reversal evidence.

Adopting the models proposed in Brav and Heaton (2002), this paper attempts to distinguish between the conservatism model and the SU model by examining how recent payoff change affects an agent’s revision in her expectation of future payoffs. The conservatism model is a behavioral model motivated by the conservatism bias, which states that individuals are slow to update their belief when confronted with new evidence. Distinguishing between the conservatism model and the SU model in explaining the momentum evidence has practical relevance. If the momentum evidence is driven by incomplete information, improved information disclosure could help reduce the extent and duration of the momentum profit. Meanwhile, the investment strategy that exploits the short run continuation evidence would be more profitable if applied to firms with higher degree of structural uncertainty, such as small firms and firms with more intangible assets and R&D expenses. However, if
the momentum evidence is driven by behavioral biases, such as conservatism, information disclosure will not help. In addition, if the momentum evidence is driven by incomplete information, the abnormal returns observed in the empirical data may be just a reward for the uncertainty risk induced by incomplete information. This has a significant impact on an investor’s portfolio choices as well as on the asset prices in equilibrium.

Based on the data simulated from the conservatism model and the SU model proposed in Brav and Heaton (2002), I find that for many combinations of the parameter values, the revision in the SU agent’s expectation of future payoff is significantly negatively related to the lagged payoff change, while the same relation is significantly positive for the conservatism agent. The economics behind this result is that although both agents underreact to the new information conditional on a structural change, they respond to the new information in a stable environment differently. The SU agent underreacts to the structural change because of incomplete information of the structure, while the conservatism agent underreacts because of the conservatism bias. Due to incomplete information, the SU agent always considers all the past information in forming her expectation because of concerns of structural stability, and she weighs the new information more than the old information because of concerns of structural instability. This weight-allocation scheme leads to overweight of new information whenever the underlying structure is stable, and underweight of new information whenever there is a structural change. As Brav and Heaton (2002) assume each firm has at most one structural change over its life, at each point of time, the number of firms experiencing a structural change is significantly less than the number of firms in stable periods. Therefore the SU agent’s overreaction to firms not experiencing structural changes dominates her underreaction to firms experiencing structural changes. This leads to the
overall overreaction pattern for the SU agent indicated through the negative relation between her expectation revision and the lagged payoff change. By contrast, the conservatism agent always places more weight on her prior belief compared to the RE agent, and she does not place more weight on the new information than on the old information. Therefore, even during stable periods, the conservatism agent still underreacts to the new information compared to the RE agent. Hence we observe the positive relation between the agent’s expectation revision and the lagged payoff change.

Empirically, I find that the relation between the revision in analyst consensus quarterly earnings estimation and the lagged earnings change is significantly negative. The result remains the same whether the mean analyst estimation or the median analyst estimation is used to compute the revision, and is robust to the choice of the sample period. Hence, this empirical evidence provides support for the SU model.

Another piece of evidence supporting the SU model comes from the relation between the momentum strategy profit and the trend and consistency of past performance over the ranking period. The representativeness-induced underreaction implies higher momentum profit among stocks with consistent paths than among stocks with inconsistent paths. However, Chan, Frankel and Kothari (2004) provide no support for this implication. They use sales growth and net income growth as measures of performance and find that the profit of the momentum strategy does not vary significantly between the consistent-path group and the inconsistent-path group. More importantly, they find that the momentum profit among stocks with consistent paths are often lower than that among stocks with inconsistent paths, contradicting the prediction of the representativeness-induced underreaction.

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6In Brav and Heaton (2002), the payoffs are equivalent to the earnings if all of the earnings are assumed to be distributed to the shareholders.
Simulated data from the SU model show that the profit of the earnings momentum strategy applied to consistent-path stocks is significantly lower\textsuperscript{7} than that applied to inconsistent-path stocks, which is consistent with the results documented in Chan, Frankel and Kothari (2004). Therefore, the empirical findings in Chan et al. (2004) provide another piece of empirical evidence supporting the SU model.

To conclude, the conservatism model and the SU model proposed in Brav and Heaton (2002) are distinguishable in the simulated data if we use the right metrics. Empirical results from my own analysis and those of existing studies provide support for the SU model.

The article proceeds as follows. Section 2 describes the models proposed in Brav and Heaton (2002). Section 3 presents the results based on simulated data. Section 4 discusses the empirical results and Section 5 concludes.

\section{Models}

This section introduces the models proposed in Brav and Heaton (2002). At the beginning of each period $t$, a single, one-period risky asset $A_t$ comes into existence. The asset pays $x_t$ at the end of period $t$. The payoff, $x_t$, is assumed to be normally distributed with mean $\mu_t$ and variance $\sigma^2$. The risk neutral representative agent values the asset at the beginning of period $t$ at its expected payoff $\mu_t$. The agent does not know $\mu_t$ at the beginning of period $t$ but estimates it based on the past $t - 1$ realized payoffs. Let $\hat{\mu}_{t-1}$ denote the estimation of the expected payoff at the end of period $t$ based on the past $t - 1$ realized payoffs, the price of the asset at the beginning of period $t$ is then $\hat{\mu}_{t-1}$ in this economy.

\textsuperscript{7}For the set of parameter values shown in this paper, the difference of the profits between the consistent-path group and the inconsistent-path group is significant. Other sets of parameter values can induce insignificant positive difference as documented in Chan, Frankel and Kothari (2004).
An important feature of this economy is the stability of $\mu_t$. If $\mu_t$ is constant over time, the structure is stable, otherwise the structure is unstable. For simplicity, Brav and Heaton (2002) only allow for a maximum of one structural change over a firm’s life. The conservatism agent is assumed to have complete knowledge of the stability of $\mu_t$. After observing the past $t$ payoffs, the conservatism agent knows if they are drawn from the same distribution or not. If they are drawn from two different distributions, she knows exactly where the structural change occurs. Payoffs up to and after the change point $r$ are drawn from two normal distributions with the same variances, but different means. In forming her posterior belief, the conservatism agent is assumed to utilize her knowledge of the structural change but does not apply Bayes rule. By contrast, the SU agent is assumed to have incomplete knowledge about the stability of $\mu_t$, but she applies Bayes rule based on her incomplete information.

2.1 Definition of underreaction and overreaction

Before providing detailed descriptions of the models, I would like to explain the definitions of underreaction and overreaction in Brav and Heaton (2002) and in this paper. As stated in Brav and Heaton (2002), overreaction refers to the predictability of good (bad) future returns from bad (good) past performance. Underreaction refers to the predictability of good (bad) future returns from good (bad) past performance. Overreaction can occur when investors put too much weight on recent performance, and underreaction can occur when investors put too little weight on recent performance. Since I am testing the effect of lagged payoff change on the agent’s expectation revision directly, overreaction in this paper refers to the predictability of upward (downward) expectation revision from bad (good) lagged payoff change. Underreaction refers to the predictability of upward (downward) expectation
2.2 Full information rational expectation model

As a benchmark, the full information rational expectation (RE) model assumes that the agent has complete knowledge of the structure and applies the Bayes rule in updating her posterior belief. At the beginning of period $t = n + 1$, the agent can estimate $\mu_{n+1}$ using the observed payoffs of the previous $n$ assets via the Bayes rule. Assuming all of the payoffs are i.i.d., the likelihood function for the past realized $n$ payoffs given $\mu$ and $\sigma$, is normal as the following:

$$l(x_1, ..., x_n | \mu, \sigma) \propto (\sigma^2)^{-\frac{n}{2}} \exp \left( -\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \right).$$

Let $p(\mu, \sigma)$ denote the investor’s prior beliefs. Assuming a simple conjugate setup, these beliefs have the form $p(\mu, \sigma) = p(\mu | \sigma^2).p(\sigma^2)$, where $p(\mu | \sigma^2)$ is conditionally normal and $p(\sigma^2)$ is scaled inverse $\chi^2$:

$$\mu | \sigma^2 \sim N(\mu_0, \frac{\sigma^2}{\kappa_0})$$

$$\sigma^2 \sim Inv - \chi^2(v_0, \sigma_0^2).$$

The marginal distribution of $\mu$ is in the form of a Student’s $t$-distribution. The risk-neutral RE agent values the asset at the mean of this marginal distribution, given by:

$$\hat{\mu}_n = \left( \frac{\kappa_0}{\kappa_0 + n} \right) \mu_0 + \left( \frac{n}{\kappa_0 + n} \right) \bar{x}_n. \quad (1)$$

Note that the estimated mean of the payoff distribution is a weighted average of the prior mean $\mu_0$ and the sample mean $\bar{x}_n$, where the weight is a function of the precision parameter.
and the number of relevant observations $n$. If there is a structural change, Equation (1) will only be applied to payoffs after the change since the RE agent has complete knowledge of the change. Assuming no discount, the price at the beginning of period $n + 1$ is the same as $\hat{\mu}_n$ since there is only one risk neutral representative agent in this economy.

### 2.3 Conservatism model

We denote a behavioral model motivated by the conservatism bias as the conservatism model. Conservatism denotes a psychological heuristic documented in the literature of psychology [e.g., Edwards(1968)] where base rates (prior beliefs and/or older data) are overweighted and new information is underweighted. In Brav and Heaton (2002), the conservatism agent (Beh-C) is assumed to have complete knowledge of the stability of the structure; however, she does not apply the Bayes rule in updating her belief. Specifically, the conservatism agent’s posterior belief is a weighted average of the RE posterior belief and her prior mean as the following:

$$\hat{\mu}_{\text{Beh,C}} = \left( \frac{c}{c + n} \right) \mu_0 + \left( \frac{n}{c + n} \right) \bar{x}_n,$$

(2)

where $c > \kappa_0$ and subscript $C$ denotes conservatism. Although Equation (2) is also a weighted average of the agent’s prior belief $\mu_0$ and the sample mean $\bar{x}_n$, the condition $c > \kappa_0$ ensures that the conservatism agent always places more than optimal weight on her prior belief compared to the RE agent.

In Equation (2), all of the relevant information is treated equally. This reduces the volatility of the estimates since the impact of the extreme realized payoffs is balanced by all of the past relevant payoffs. The weight on the prior, $\frac{c}{c + n}$, decreases with the number of realized payoffs $n$, and the weight on the sample mean, $\frac{n}{c + n}$, increases with $n$. Thus, as the
number of payoffs increases, the estimate gradually converges to the true mean by the law of large numbers. When there is a structural change, only the information after the change will be utilized in the estimation because the conservatism agent has complete knowledge of the change. Hence, the weight on the prior belief shortly after the structural change is very high as the number of relevant payoffs is small. Due to the heavy weight on the prior belief, the conservatism agent exhibits strong underreaction to the information related to a structural change. When there is no structural change, the weight on the prior is still higher than that for the RE agent. Consequently the conservatism agent exhibits an overall underreaction pattern. This predicts a significantly positive relation between the agent’s expectation revision and lagged payoff change.

2.4 Rational structural uncertainty model

In the rational structural uncertainty (SU) model, the agent applies Bayes rule in updating her belief but has incomplete information about the structure. She is not sure if and when a structural change occurs; hence, her posterior belief of the mean of the payoff distribution depends on the posterior probability assigned to the change point. Let \( r \) denote the point after which the change occurs and \( p_0(r) \) denote the agent’s uniform prior probability of the change point \( r \) such that \( \sum_{r=1}^{n} p_0(r) = 1 \). Subscript 0 indicates a prior probability assigned before any payoffs are observed. At time \( t = n \), given the prior probability \( p_0(r) \) and the realized \( n \) payoffs, \( x_1, \ldots, x_n \), the SU agent forms the posterior probability of all the possible locations of the change point \( r \), denoted by \( p_n(r) \), where \( r \) ranges from 1 to \( n \). Since the number of structural changes for a firm is assumed to be no more than one at any time, given

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8The uniform prior is actually an informative one since the prior of no change is only \( 1/n \), while the prior of a structural change is \( (n - 1)/n \).
$r$, the agent assumes that the payoffs, $x_1, \ldots, x_r$, were drawn from a normal distribution with mean $\mu_A$ and the payoffs, $x_{r+1}, \ldots, x_n$, were drawn from a normal distribution with mean $\mu_B$. Both normal distributions are assumed to have the same variance. If $r = n$, it means that the investor believes no structural change has occurred and all of the $n$ payoffs were drawn from the same distribution with mean $\mu_A$.

Assigning informative prior to $\mu_A$ and $\mu_B$ and a prior that they are independent conditional on $\sigma^2$, Brav and Heaton (2002) show that, given $n$ observed payoffs, the posterior distribution of the change point $r$ is

$$p_n(r) = \frac{p(x_1, \ldots, x_n|r)p_0(r)}{\sum_{r=1}^n p(x_1, \ldots, x_n|r)p_0(r)},$$

where

$$p(x_1, \ldots, x_n|r) = \int_{\mu_A, \mu_B, \sigma} p(x_1, \ldots, x_n|r, \mu_A, \mu_B, \sigma)$$

$$\times p_0(\mu_A|\sigma)p_0(\mu_B|\sigma)p_0(\sigma)d_\mu_A d_\mu_B d_\sigma.$$

The posterior probability of the change point $r \in \{1, \ldots, n\}$ is obtained by integrating the joint posterior probability over the unknown parameters $\mu_A$, $\mu_B$, and $\sigma^2$, and the result is:

$$p_n(r = 1, \ldots, n-1) \propto \{(\kappa_0 + r)(\kappa_0 + n - r)\}^{-\frac{1}{2}}$$

$$\times \left\{ \sum_{i=1}^{r} (x_i - \bar{x}_r)^2 + \sum_{i=r+1}^{n} (x_i - \bar{x}_{n-r})^2 \right\}^{-\frac{1}{2}}$$

$$+ \kappa_0 \left( \frac{r}{\kappa_0 + r}(\bar{x}_r - \mu_0)^2 + \frac{n - r}{\kappa_0 + n - r}(\bar{x}_{n-r} - \mu_0)^2 \right)$$

$$+ \nu_0 \sigma_0^2 \right\}^{-\frac{n + \nu_0}{2} - \frac{\nu_0}{2}}.$$
\[ p_n (r = n) \propto (\kappa_0 + n)^{-\frac{1}{2}} \times \left\{ \sum_{i=1}^{n} (x_i - \bar{x}_n)^2 + \kappa_0 \left( \frac{n}{\kappa_0 + n} (\bar{x}_n - \mu_0)^2 \right) + v_0 \sigma_0^2 \right\}^{-\frac{n+v_0}{2}}, \]

where \( \bar{x}_r = r^{-1} \cdot \sum_{i=1}^{r} x_i, \bar{x}_{n-r} = (n-r)^{-1} \cdot \sum_{i=r+1}^{n} x_i, \) and \( \bar{x}_n = n^{-1} \cdot \sum_{i=1}^{n} x_i. \) The marginal distributions for \( \mu_A \) and \( \mu_B \) are as the following:

\[ p_n(\mu_i) = \sum_{r=1}^{n} p_n(\mu_i|r)p_n(r) \quad (i = A, B). \]

The marginal distribution for \( \mu_n \) is then:

\[ p_n(\mu_n) = \sum_{r=1}^{n-1} p_n(\mu_B|r)p_n(r) + p_n(\mu_A|r = n)p_n(n), \]

where the first part indicates that the SU agent considers the possibility of a structural change occurring after one of the first \( n - 1 \) periods. The second part results from the scenario where no change has occurred over the last \( n \) periods. Brav and Heaton (2002) show that the SU agent’s estimate of \( \mu_n \) is given by:

\[ \hat{\mu}_n = \sum_{i=1}^{n-1} p_n(i) \left[ \frac{\kappa_0}{\kappa_0 + (n-i)} \mu_0 + \frac{(n-i)}{\kappa_0 + (n-i)} \bar{x}_{n-i} \right] + p_n(n) \left[ \frac{\kappa_0}{\kappa_0 + n} \mu_0 + \frac{n}{\kappa_0 + n} \bar{x}_n \right], \quad (3) \]

where \( \bar{x}_{n-i} \) is the sample mean of the \( n - i \) most recent payoffs. Equation (3) can be viewed as a weighted average of the conditional Bayesian estimates, where the weights are given by the posterior probabilities of the possible locations of the change point.

\(^9\)See detailed derivation of the marginal distributions in Smith (1975).
Note that the SU agent’s estimate nests the RE agent’s estimate. If the SU agent has complete information about the exact location of the change point, the SU estimate reduces to the RE estimate. For example, with complete information, if the structure is stable, \( p_n(n) \) would be equal to 1. In that case, Equation (3) would be equivalent to Equation (1). However, since the SU agent has incomplete information, she needs to consider all the possible locations of the change point, including the possibility of no change. Hence, she has to take into account all the past information in her estimates, even if the information before the change should be discarded when there is a change. This leads to underreaction to the structural change since she has to allocate some weight to the irrelevant data due to incomplete information.

In addition to the underreaction feature conditional on a structural change, the SU agent always puts more weight on more recent payoffs due to concerns of instability. To see this, we collect the items from Equation (3) and get the weight placed on the observation \( j \in \{2, \ldots, n\} \) in the estimate \( \hat{\mu}_n \) as

\[
\sum_{i=1}^{j-1} p_n(i) \left( \frac{1}{n-i} \right) \left( \frac{n-i}{\kappa_0 + (n-i)} \right) + p_n(n) \left( \frac{1}{n} \right) \left( \frac{n}{n + \kappa_0} \right),
\]

which obviously increases with \( j \). The weight placed on the first observation is simply

\[
p_n(n) \left( \frac{1}{n} \right) \left( \frac{n}{n + \kappa_0} \right).
\]

The intuition of the above relation is conditional on the location of the change point, only the payoffs after the perceived change point enter the conditional Bayesian estimate. Therefore, given \( n \) observed payoffs, the \( i^{th} \) payoff is relevant whenever the perceived change point \( r \)
is less than or equal to \(i\). For example, the \(n^{th}\) payoff will be counted \(n\) times since it is relevant to the estimation over all the possible locations of the change point \(r = 1, \cdots, n\). Hence the more recent the payoff is, the more times it is counted in the posterior mean and the more weight it receives.

This feature leads to overreaction to new information when the structure is stable because if the SU agent had known that the structure was stable, she would have treated all the past payoffs equally instead of putting more weight on more recent payoffs. Since the model only allows for a maximum of one structural change over a firm’s life, the number of firms experiencing structural changes is expected to be lower than the number of firms in stable periods. This conjecture is confirmed in the simulated data. On average, at any time, only about 5% of the firms in the top and bottom payoff change deciles experience structural changes. Therefore the SU agent’s overreaction to firms in stable periods dominates her underreaction to firms experiencing structural changes. As a result, we shall observe an overall overreaction pattern for the SU agent cross-sectionally. This implies a significantly negative relation between the SU agent’s expectation revision and the lagged payoff change, which is confirmed in Section 3.

3 Simulation Results

This section replicates the sample path of estimated mean as shown in Figure 2 of Brav and Heaton (2002) and examines the difference between the SU model and the conservatism model using simulated data. I study the relation between the agent’s expectation revision and the lagged payoff change through three ways: the cross-sectional average of the ex-
pectation revisions for stocks with extreme lagged payoff changes, the Pearson’s correlation
coefficient between those two, and the cross-sectional regression coefficient on the lagged
payoff change. The results all confirm that this relation is significantly negative for the SU
agent and significantly positive for the conservatism agent. The results are also robust to the
choice of parameter values, such as the location of the change point, the direction and the
magnitude of the change, and the volatility of the true payoff distribution. These evidence
shows that indeed the two models can be distinguished from each other through this rela-
tion. Furthermore, following Chan, Frankel and Kothari (2004), I apply a path-dependent
momentum strategy to the data simulated from the SU model, and I find that the momentum
profit for the consistent-path stocks is lower than that for the inconsistent-path stocks. This
finding contradicts the prediction of the behavioral model motivated by representativeness
and conservatism biases. However it is consistent with the empirical evidence documented
in Chan, Frankel and Kothari (2004). Hence this coincidence provides support for the SU
model.

3.1 Sample path of estimated mean

Before examining the difference between the two models, I replicate the sample paths of
the estimated means from the RE model, the SU model and the conservatism model as
shown in Figure 2 of Brav and Heaton (2002). The replication serves two purposes: confirm
the generality of the patterns shown in Figure 2 of Brav and Heaton (2002); validate the
simulation method used here. Brav and Heaton (2002) claims that it is hard to distinguish
the SU model and the conservatism model since their sample paths of estimated mean exhibit
similar patterns shortly after the structural change. To replicate the sample paths, I choose
the same parameter values\textsuperscript{10} as used in Figure 2 of Brav and Heaton (2002).

Figure 1 shows the sample paths of the estimates from the RE model, the SU model and the conservatism (Beh-C) model. The true mean of the payoff distribution jumps from 11 to .3 after the 20\textsuperscript{th} payoff. The estimates from the RE model reflect both complete structural information and rational information processing. The SU estimates reflect uncertainty of the stability of the structure, while the conservatism estimates reflect the conservatism bias. Although both the SU model and the conservatism model generate underreaction to the structural change as shown in the slow convergence of the estimated mean shortly after the change, the SU agent only underreacts in the periods shortly after the structural change. During the other time periods when the structure is stable, the SU agent’s estimates fluctuate a lot with the realized payoffs because she puts more weight on more recent payoffs. Consequently the SU agent overreacts to the new information during stable periods. The conservatism agent also underreacts to the structural change, but in the stable periods, her expectation is relatively smooth and does not fluctuate with recent payoffs as much as the SU agent does since recent payoffs are smoothed out through the sample mean.

Brav and Heaton (2002) argue that it is possible to make the two paths closer to each other during stable periods by allowing the conservatism agent to allocate unequal weight to the relevant payoffs instead of equal weight as in Equation (2). However, in that case, the nature of conservatism bias would demand more weight on old payoffs than on new payoffs since the conservatism bias denotes an individual’s tendency to underweigh new information and overweight the base rate (the prior belief and/or the older data). This new allocation of weights would be totally opposite to what the SU agent does. Therefore I conjecture this

\textsuperscript{10}The parameter values used in Figure 2 of Brav and Heaton (2002) are: $\mu_0 = 10, \kappa_0 = 1, v_0 = 40, \sigma_0 = 15, \mu_A = 11, \mu_B = .3, \sigma = 13.7, c = \kappa_0 + 5$. 

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relaxation would induce even more underreaction to the new data for the conservatism agent and sharper difference between the two models during stable periods.

### 3.2 Relation between lagged payoff change and expectation revision

To verify the predicted difference between the SU model and the conservatism model in terms of the relation between the lagged payoff change and the agent’s expectation revision, I simulate 3000 independent sequences of 50 payoffs each and compute the agent’s expectations for the conservatism agent and the SU agent using Equation (2) and Equation (3) respectively. Note that negative estimated means make the computed expectation revision, the percentage change in estimated means, meaningless. Therefore, instead of using the same parameter values as used in Brav and Heaton (2002), I decrease the magnitude of the structural change and the standard deviation of the underlying payoff distributions to avoid those negative estimated means shown in Figure 1. Specifically, the mean of the true payoff distribution switches between 11 and 7 if a structural change occurs with the direction of the change determined by a discrete uniform distribution. The volatility of the true payoff distribution, $\sigma$, is reduced to 2. The prior belief $\mu_0$ is still 10 as in Brav and Heaton (2002). Due to the reduced volatility, I focus more on the sign of the relation than on the magnitude.

To increase the diversity of the simulated data, each firm has a randomly assigned location of the change point between 1 and 50 drawn from a uniform distribution. If the change point is 50, then the firm does not experience any structural change over the 50 periods. If the change point is less than 50, the payoffs before the change point are independently drawn from one normal distribution, and the payoffs after the change point are independently drawn
from another normal distribution with a different mean and the same variance. The direction of the change, i.e., whether the mean of the payoff distribution increases or decreases after the change, also follows a uniform distribution. Hence, at each time period, there are some firms experiencing good structural changes, some firms experiencing bad structural changes and some firms in stable periods. The random location of the change point and the random direction of the change ensure that the diversity of the simulated data resembles that of the empirical data.

I then examine this relation using the simulated data set from three aspects: the cross-sectional average expectation revisions for firms with extreme lagged payoff changes; the Pearson’s correlation coefficient between the lagged payoff change and the agent’s expectation revision; the regression coefficient on the lagged payoff change. The results are detailed as follows.

3.2.1 Cross-sectional average of the expectation revisions for firms with extreme lagged payoff changes

To verify the predicted difference as explained before, I first examine the cross-sectional average of the expectation revisions for firms with extreme lagged payoff changes. Since the assets in Brav and Heaton (2002) only exist for one period, the payoffs are equivalent to earnings if we assume all the earnings are distributed to the shareholders. Hence I use the payoff change and the earnings change interchangeably hereafter.

At the end of period $t + 1$, the revision in the agent’s expectation of future payoff for firm $i$ is defined as

$$\text{Expectation Revision}_{i,t+1} = \frac{\hat{\mu}_{i,t+1}}{\mu_{i,t}} - 1.$$
The standardized earnings change$^{11}$ (SEC) for firm $i$ at the end of period $t$ is defined as

$$SEC_{it} = \frac{x_{it} - x_{it-1}}{\sigma_{it}},$$

where $\sigma_{it}$ is the standard deviation of the last four $x_{it} - x_{it-1}$. At the beginning of each time period, I sort the 3000 firms into deciles based on the firms’ lagged SECs. The decile with the highest (lowest) SECs is called the winner (loser) group. The revision in the agent’s expectation of each firm’s future payoff is computed and averaged within the winner group and the loser group.

Figure 2 shows the cross-sectional average of the SU agent’s expectation revisions for the winner group, the loser group and the difference between the two groups over time. One striking observation is that the average expectation revisions for the winner group lie below the loser group almost everywhere. As shown in Table 1, the time series means of the cross-sectional average revisions for the winner group and the loser group are $-0.15\%$ and $0.26\%$ respectively. The mean revision for the winner-minus-loser group is $-0.41\%$. The average revisions for the three groups are all statistically significant at the 5% level. This result confirms the prediction that the SU agent tends to overreact to the SEC measure on average.

One potential explanation of the above findings is that there are two types of firms in the winner and the loser SEC deciles. One type consists of firms experiencing structural changes, and the other type consists of firms with lucky or unlucky random draws, but that

$^{11}$In fact, the standardization does not affect the results qualitatively. The relation between the expectation revision and the lagged payoff change, $x_{it} - x_{it-1}$, is also negative for the SU model and positive for the conservatism model. The difference between the two models is actually magnified using raw payoff changes instead of standardized earnings changes.
are in stable periods. Furthermore, the number of firms in stable periods exceeds that of firms experiencing structural changes\(^\text{12}\). For firms in stable periods, the SU agent tends to overreact since she overweighs the new information due to concerns of instability. However, for firms experiencing structural changes, the SU agent tends to underreact to the new information because she also allocates weight on the irrelevant data before the structural change due to concerns of stability. Both overreaction and underreaction occur for the same reason – lack of knowledge of the structure, but they apply to different types of firms. Since firms in stable periods outnumber firms experiencing structural changes, the SU agent exhibits overreaction to the SEC measure on average. Two factors contribute to the domination of firms in stable periods in the extreme SEC deciles. One is the model assumption in Brav and Heaton (2002) that each firm has at most one structural change over its life. The other is that the SEC measure does not contain much information about a structural change.

The results documented in Chan (2003) can actually be explained by the above results for the SU model. Chan (2003) separates firms into two groups: one with news (news group) and the other without news (no-news group). He then applies price momentum strategy to the two groups separately and finds that momentum profit only exists in the news group. For the no-news group, he finds short term reversal instead of short term drift predicted by conservatism model. The extreme price movements accompanied by news reflect structural changes more than extreme random draws, while the extreme price movements not accompanied by news, similar to the SEC measure, reflect extreme random draws more than real structural changes. If that is the case, the SU agent’s overreaction to firms in stable periods and underreaction to firms experiencing structural changes explains why the

\(^{12}\) The conjecture is verified in the simulated data, as on average only about 5\% of the firms in the two extreme SEC deciles are experiencing structural changes.
momentum strategy only works in the news group but not in the no-news group.

The cross-sectional average of the expectation revision for the winner group, the loser group and the winner-minus-loser group for the conservatism model is shown in Figure 3. Contrary to the pattern in Figure 2 for the SU model, the mean revisions of the winner group always lie above those of the loser group. As shown in Table 1, the time series means of the cross-sectional average revisions for the winner group and the loser group are 0.07% and −0.17% respectively. The mean revision for the winner-minus-loser group is 0.23%. The average revisions for the three groups are all statistically significant at the 10% level. These results indicate that overall the conservatism agent underreacts to the SEC measure.

The explanation for the above results is that since the conservatism agent has complete information of the structure, she knows if the firms in the extreme deciles are experiencing structural changes or not. If the firm is in stable periods, she will not overreact to its recent payoff. Since all of the relevant data are weighed equally in her estimate, her response to the most recent extreme payoffs is moderated by all the past data. This reduces the effect of the recent extreme payoff on the her expectation. In fact, her weight on the new information is still low compared to the RE agent during the stable periods. For firms experiencing structural changes, the conservatism agent discards the irrelevant information before the change just as the RE agent does. However, she dogmatically places more weight on her prior belief, leading to underreaction to firms experiencing structural changes. Hence, the conservatism agent overall exhibits underreaction to recent extreme performance.

In sum, this experiment shows that the SU model can be successfully differentiated from the conservatism model\(^{13}\).

\(^{13}\)There are other combinations of parameter values that can generate the above difference between the SU model and the conservatism model. They are available from the author upon request.
3.2.2 Pearson’s correlation coefficient

In addition to examining the stocks in the extreme SEC deciles, I also examine if the difference between the SU model and the conservatism model shown in Section 3.2.2 is also supported by the cross-sectional Pearson’s correlation coefficient between the agent’s expectation revision and the lagged SEC. I find the time series average of the cross-sectional Pearson’s correlation coefficients is negative for the SU model and positive for the conservatism model, which is consistent with the findings in the extreme SEC deciles. As shown in Figure 4, most of the cross-sectional correlation coefficients lie below zero for the SU model and above zero for the conservatism model. The time series averages of the cross-sectional correlation coefficients are −0.03 and 0.014 for the SU model and the conservatism model respectively. Notice that the sign of the coefficient is more important than the level due to the reduced volatility of the simulated data in order to avoid negative estimates, and nonlinearity may reduce the level of the correlation coefficient as well.

3.2.3 Fama-MacBeth regression

I also apply Fama-MacBeth\textsuperscript{14} method to the simulated data to see if the difference is supported by the regression method. Indeed the regression method confirms the difference between the two models. At each time period, I run a cross-sectional regression of the agent’s expectation revisions for the 3000 firms on the firms’ last period’s SECs. The coefficient on the lagged SEC is significantly positive for the conservatism model and significantly negative for the SU model. The time series mean of the coefficients on the lagged SEC is −.0008 for

\textsuperscript{14}Since the simulated sequences are independent of each other and the payoffs are i.i.d., Fama-MacBeth method is used here not to correct for the cross-sectional correlation. However, in the empirical test detailed in Section 4, Fama-MacBeth method is needed to correct for cross-sectional correlation.
the SU model and 0.0004 for the conservatism model. The t-statistics are −6.35 for the SU model and 5.52 for the conservatism model as reported in Table 2. Same as the correlation coefficient, the signs of the regression coefficients are more relevant than the level.

### 3.2.4 Robust check

All the three methods detailed in previous sections show that the two models are distinguishable through the relation between the agent’s expectation revision and the lagged payoff change. To check the robustness of the difference, I perform Monte Carlo simulation. Specifically, I simulate 100 data sets using the set of parameter values that was used to generate the results shown in previous sections. It turns out that every one of the 100 data sets produces the predicted difference between the SU model and the conservatism model. The mean and t-stat of the 100 average expectation revisions for the winner-minus-loser group are −0.4% and −36 respectively for the SU model, and are 0.2% and 24.7 respectively for the conservatism model. The regression method supports this sharp difference as well. The Monte Carlo simulation results for other sets of parameter values are qualitatively the same, i.e., every one of the 100 data sets produces sharp difference between the two models in terms of the relation between the agent’s expectation revision and the lagged payoff change.

In sum, the robust check shows that the combination of the agent’s expectation revision and the lagged payoff change highlights the difference between the SU model and the conservatism model and serves as a good metric to differentiate the two models. The agent’s expectation revision directly relates to the models and provides a direct way to test how the agent’s form her expectations. The payoff change is independent of the type of the agent.

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15 The Monte Carlo simulation results for the other sets of parameter values are available upon request.
and is easy to measure empirically.

### 3.3 Path-dependent momentum profit

In this section, I show that the SU model and the conservatism model also differ in terms of the relation between the trend and consistency of past performance and the earnings momentum strategy profit. The path-dependent momentum strategy picks the winner and loser stocks from the groups with consistent paths and from the groups with inconsistent paths separately.

Some behavioral models ascribe the underreaction evidence to the representativeness bias, which states that individuals overweigh recent information and underweigh the base rate. One prediction of the representativeness bias is that bad news following a series of consistent good news should induce more underreaction than the same bad news following a series of inconsistent good news. The argument is that individuals subject to the representativeness bias tend to extrapolate too much into the future based on the trend and consistency of the past performance. Therefore, after a series of consistent interim good performances, investors tend to believe more firmly that the company is indeed in a better stage than if they observe a similar performance over the same time period, but with an inconsistent path of interim performances. Hence the agent underreacts more to the same bad news if it follows a series of consistent good performances than if it follows a series of inconsistent good performances. Therefore the conservatism model predicts higher momentum profit among stocks with consistent paths than among stocks with inconsistent paths following path-breaking news.

However, Chan, Frankel and Kothari (2004) provide opposite findings. They use sales
growth and net income growth as measures of performance and find that the profit of momentum strategy does not vary significantly between the consistent-path group and the inconsistent-path group. More importantly, they find that the momentum profit among stocks with consistent paths are often lower than that among stocks with inconsistent paths. This finding contradicts the prediction of the representativeness induced underreaction.

In the simulated data from the SU model, I find the profit of the momentum strategy applied to stocks with consistent paths is significantly lower than that applied to stocks with inconsistent paths. The finding is contrary to the prediction of the representativeness-induced underreaction and is consistent with the results documented in Chan et al. (2004). Therefore this provides another piece of evidence supporting the SU model.

The outline of the path-dependent momentum strategy is illustrated clearly in Figure 5 which is adopted from Chan, Frankel and Kothari (2004). Specifically, at the beginning of each time period \( t \), I rank firms into quintiles in ascending order according to the cumulative four period earnings growth defined as

\[
\frac{(x_t + x_{t-1} + x_{t-2} + x_{t-3}) - (x_{t-4} + x_{t-5} + x_{t-6} + x_{t-7})}{|x_{t-4} + x_{t-5} + x_{t-6} + x_{t-7}|}.
\]

In both the low growth and the high growth quintiles, I further separate the firms into a consistent category and an inconsistent category. The firm in the low (high) growth quintile has a consistent path if each of its past four earnings news\(^{16}\) over the periods from \( t - 3 \) to \( t \) is below (above) the median level. If two or less of the past four news are below (above) the

\(^{16}\)Earnings news at period \( t \) is defined as the following:

\[
\text{Earnings News}_{it} = \frac{x_{it} - \tilde{\mu}_{it-1}}{\sigma_{it}},
\]

where \( \sigma_{it} \) is the standard deviation of the last four \( x_{it} - \tilde{\mu}_{it-1} \).
median, it has an inconsistent path. I then define the stocks as having disconfirming news if the news after the ranking period are above (below) the median for the low (high) growth quintile. The portfolio constructed among the consistent groups with disconfirming news involves buying stocks in the high growth quintile with consistent paths and disconfirming news following the paths and selling stocks in the low growth quintile with consistent paths and disconfirming news following the paths. The same strategy applies to the portfolio constructed among the inconsistent groups with disconfirming news. The hedge strategy is long the portfolio formed among the consistent groups and short the portfolio formed among the inconsistent group. Notice that the disconfirming news following the high (low) growth path is bad (good); hence, the momentum profit in either group is expected to be negative since I essentially buy losers and sell winners. The representativeness-induced underreaction theory predicts the loss from the consistent group should be larger than that from the inconsistent group. Therefore the net profit should be negative as well since the investor long the portfolio with larger loss and short the portfolio with a smaller loss.

In contrast, the above strategy applied to the simulated data from the SU model shows a significantly positive difference between the one-period returns of the consistent group and the one-period returns of the inconsistent group, although the momentum portfolio for the consistent group actually generates positive returns\textsuperscript{17}. As shown in Figure 6, the difference between the consistent group and the inconsistent group is significantly positive with a mean of 0.2 and a t-statistic of 10.9. Chan et al. (2004) find the difference to be insignificant, but the sign is often positive as well, hence their findings also provide some indirect support for

\textsuperscript{17}This seemingly lack of underreaction in the consistent group actually confirms the explanation for the SU model. It means when the path is consistent, the uncertainty surrounding the structural change is actually reduced to such a degree that no underreaction is observed. In addition, this reversal in the sign is not a general pattern.
the SU model.

I conjecture the reason behind the above findings is that when the path of good performance preceding the bad news is inconsistent, it is harder for the SU agent to accurately pin down the location of the change point. This may lead him to assign a higher posterior probability of the wrong change point than when the path is consistent. Since the posterior probability of the change point determines the weight on the data after the perceived change point, she places more weight on the irrelevant data before the structural change than when the path is consistent. This leads to less weight on the payoffs after the real change and more underreaction to the new information. Therefore, the SU model predicts that the underreaction among stocks with consistent path should be less than that among stocks with inconsistent path. This is totally opposite to what the behavioral theory predicts.

In summary, the SU model can be successfully distinguished from the conservatism model using simulated data. The SU model exhibits a significantly negative relation between the agent’s expectation revision and the lagged payoff change, while the conservatism model exhibits a significantly positive relation. In addition, the findings related to the path-dependent momentum profit for the SU model is more in line with the empirical evidence.

4 Empirical Tests

4.1 Data and methodology

In this section, I examine the relation between the agent’s expectation revision and the lagged earnings change using empirical data by regression methods. I use the revision in analyst consensus quarterly earnings estimation as the empirical proxy for the representative
agent’s expectation revision in the simulation. The empirical counterpart of the SEC measure is computed as the standardized difference between two consecutive reported quarterly earnings,

$$\text{SEC}_{iq}^{\text{emp}} = \frac{e_{iq} - e_{iq-1}}{\sigma_{iq}},$$

where $\sigma_{iq}$ is the standard deviation of the last four $e_{iq} - e_{iq-1}$, and “emp” denotes “empirical”. Like the SEC measure computed using simulated data, the empirical SEC measure is more closely related to random draws than to structural changes. This is a reasonable conjecture since we typically assume earnings follow a seasonal random walk and use standardized seasonal difference in earnings to measure earnings news. In the simulated data, I find a significantly positive relation for the conservatism model and a significantly negative relation for the SU model. Hence a negative relation between the analyst consensus estimation revision and the lagged SEC implies that the SU model is consistent with the empirical data. On the other hand, a positive relation is more supportive of the conservatism model.

I consider all of the domestic, primary stocks covered in the Institutional Brokers Estimate System (I/B/E/S) summary file. Only firms with enough data to compute the standardized earnings change are included. I also delete records with obvious data entry error such as records with reported earnings per share greater than $20. Thus the final sample covers the analyst consensus estimations from May 1987 to February 2005 since I/B/E/S starts reporting analyst quarterly earnings estimation in October 1983.

For each fiscal quarter, firms are required to file an earnings report with the Securities and Exchanges Commission within 90 days from the end of the fiscal quarter. In the middle of each month, the I/B/E/S summary file reports the analyst consensus estimation for the unreported fiscal quarter. Therefore for each fiscal quarter, there is a consensus estimation
in each month before the firm reports the actual quarterly earnings. If the firm reports quarterly earnings at the end of a month, there would be a consensus estimation in the middle of the same month. To reflect the immediate effect of the last reported earnings on the analyst consensus estimation for the next fiscal quarter, the earliest consensus estimation for each fiscal quarter is extracted from the database.

I scale the difference in analyst consensus estimations of the next quarterly earnings by the closing price on the last consensus estimation date. I call it the earnings estimation revision. In the simulation, I can choose the parameter values such that the agent’s expectations are all positive to make the computation of expectation revision meaningful. However, in empirical data, the analyst consensus estimation can be negative. That is why I scale the difference in two consensus estimations by the closing price on the previous consensus estimation date. To compute the empirical measure equivalent to the SEC measure in the simulation, I scale the actual quarterly earnings change by the standard deviation of the previous eight earnings changes, excluding the current earnings change.

In each consensus estimation month, I run a cross-sectional regression of the revision in analyst consensus estimation against the lagged standardized actual earnings change (Lag_SEC). As firms have different fiscal quarters and report earnings in different months, I have 214 consensus estimation months over the sample period. The time series average of the cross-sectional coefficients on the lagged standardized earnings change is \(-0.00046\) and the t-statistic is \(-2.6\) as shown in Table 3. I also run a multiple regression of the earnings estimation revision against the lagged SEC and the current SEC at each consensus estimation month. The coefficient on the lagged SEC is still significantly negative with a mean of \(-0.00031\) and a t-statistic of \(-1.89\). Both the simple regression and the multiple
regression show that the revision in the analyst consensus earnings estimation is negatively correlated to the lagged SEC. In Table 3, the earnings revision is computed using the mean analyst consensus estimation. I also used median analyst consensus estimation to compute the earnings estimation revision. The regression results are almost identical to the results shown in Table 3. I also run the regression using subsamples by dividing the whole sample into two halves. The first half covers from May 1987 to December 1997 and the second half covers thereafter. The results are qualitatively the same. The coefficients on the lagged SEC are significantly negative in both subperiods with the second half exhibiting a much stronger overreaction as shown in Table 418.

The significantly negative relation between the revision in analyst consensus quarterly earnings estimation and the lagged standardized earnings change in empirical data coincides with the findings of the SU model using simulated data. It shows that on average an analyst tends to overreact to the most recent earnings data when forming her estimation of the next quarterly earnings. This empirical evidence provides support for the SU model.

5 Conclusion

This paper shows that the SU model and the conservatism model proposed in Brav and Heaton (2002) can be distinguished from each other using simulated data. Empirical evidence provides more support for the SU model. In the simulated data, the relation between the agent’s expectation revision and the lagged payoff change is significantly negative for the SU model and significantly positive for the conservatism model. Using empirical data, I

18 The statistical significance of the negative coefficient on the lagged SEC for the first half subperiod is sensitive to the cutoff year. However, the sign of the coefficient is always negative and is not sensitive to the cutoff year.
find that the revision in the analyst consensus earnings estimation is significantly negatively related to the lagged earnings change. This result supports the SU model. The empirical results documented in Chan (2003) supports the SU model as well since he finds short run momentum only in stocks with news but short run reversal in stocks without news. In addition, the simulated relation between the momentum profit and the trend and consistency of the path for the SU model is also in line with the empirical results documented in Chan et al. (2004). Hence it provides support for the SU model as well.

Another potential way to distinguish the two models is to examine whether the magnitude of underreaction varies with the level of structural uncertainty. Since the conservatism agent’s underreaction is solely caused by the conservatism bias, we should not expect the magnitude of underreaction to be related to the level of structural uncertainty. For the SU agent, the two should be positively related since the underreaction is caused by structural uncertainty. From this perspective, many empirical evidence related to momentum strategy can be explained by the SU model. For example, Hong et. al. (2000) find that momentum strategy generates higher profit among small firms than among large firms. It is a natural assumption that small firms involve more structural uncertainty than large firms. The reason could be the number of analysts following large firms is larger, hence facilitating the correct interpretation of the information related to large firms. This helps reduce the structural uncertainty and the level of underreaction, hence the momentum profit.
References


Table 1. Average Revision of Standardized Earnings Change Groups

<table>
<thead>
<tr>
<th></th>
<th>SU Model</th>
<th>Conservatism Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean  t-stat</td>
<td>Mean  t-stat</td>
</tr>
<tr>
<td>Winner</td>
<td>-0.15 -3.28</td>
<td>0.07  1.75</td>
</tr>
<tr>
<td>Loser</td>
<td>0.26 -6.03</td>
<td>-0.17 -5.46</td>
</tr>
<tr>
<td>Winner-Loser</td>
<td>-0.41 -5.33</td>
<td>0.23  5.40</td>
</tr>
</tbody>
</table>

Table 1: This table reports the time series mean (in percentage) and the t-statistics of the cross-sectional average expectation revision of the winner group, the loser group and the winner-minus-loser group. I simulate 3000 independent firms with 50 payoffs each. Each firm has a randomly assigned change point that is uniformly distributed between 1 and 50. If a structural change occurs, the mean of the payoff distribution switches between 11 and 7. The direction of the change is also uniformly distributed for each firm. The agent’s expectations are computed using the conservatism model. At the beginning of each period, the 3000 simulated firms are ranked into deciles in ascending order based on the firms’ previous standardized earnings changes (SEC). The winner (loser) portfolio consists of stocks in the bottom (top) SEC decile. The portfolios are held for one period after the ranking. The agent’s revision in expectation in period \( t \) is computed as \( \hat{\mu}_t / \hat{\mu}_{t-1} - 1 \), and the standardized earnings change for firm \( i \) in period \( t \) is defined as \( SEC_{it} = \frac{x_{it} - x_{it-1}}{\sigma_{it}} \) where \( \sigma_{it} \) is the standard deviation of the last four \( x_{it} \). In order to ensure positive estimates, the parameter values used in the simulation are: \( \mu_0 = 10, \kappa_0 = 1, v_0 = 40, \sigma_0 = 15, \mu_A = 11, \mu_B = 7, \sigma = 2, c = \kappa_0 + 5 \).
Table 2: Simulated Cross-Sectional Regression of Expectation Revision on Lagged SEC

<table>
<thead>
<tr>
<th></th>
<th>SU Model Mean</th>
<th>t-stat</th>
<th>Conservatism Model Mean</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.6</td>
<td>6.04</td>
<td>0.1</td>
<td>0.57</td>
</tr>
<tr>
<td>SEC</td>
<td>−0.8</td>
<td>−6.35</td>
<td>0.4</td>
<td>5.52</td>
</tr>
</tbody>
</table>

Table 2: A cross-sectional regression is estimated each period of the expectation revisions of the individual firms on the firms’ lagged standardized earnings change (SEC). The reported statistics are the means (x1000) and the t-statistics of the time series of the coefficients from the period-by-period regressions. To get the simulated data set, I simulate 3000 independent firms with 50 payoffs each. Each firm has a randomly assigned change point that is uniformly distributed between 1 and 50. If a structural change occurs, the mean of the payoff distribution switches between 11 and 7. The direction of the change is also uniformly distributed for each firm. The agent’s expectations are computed using the conservatism model. At the beginning of each period, the 3000 simulated firms are ranked into deciles in ascending order based on the firms’ previous standardized earnings changes (SEC). The winner (loser) portfolio consists of stocks in the bottom (top) SEC decile. The portfolios are held for one period after the ranking. This figure shows the time series of the average earnings expectation revision after the ranking for the winner portfolio, the loser portfolio and the winner-minus-loser portfolio for the SU model. The agent’s revision in expectation in period t is computed as $\hat{\mu}_t/\hat{\mu}_{t-1} - 1$, and the standardized earnings change for firm i in period t is defined as $SEC_{it} = \frac{x_{it} - x_{it-1}}{\sigma_{it}}$ where $\sigma_{it}$ is the standard deviation of the last four $x_{it} - x_{it-1}$. In order to ensure positive estimates, the parameter values used in the simulation are: $\mu_0 = 10, \kappa_0 = 1, v_0 = 40, \sigma_0 = 15, \mu_A = 11, \mu_B = 7, \sigma = 2, c = \kappa_0 + 5$. 

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Table 3: Empirical Cross-Sectional Regression of Analyst Consensus Earnings Estimation Revision on SEC

<table>
<thead>
<tr>
<th></th>
<th>Simple Regression</th>
<th>Multiple Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean t-stat</td>
<td>Mean t-stat</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.75 2.26</td>
<td>0.83 2.41</td>
</tr>
<tr>
<td>Lag_SEC</td>
<td>-0.46 -2.6</td>
<td>-0.31 -1.86</td>
</tr>
<tr>
<td>Current_SEC</td>
<td>0.16 0.61</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: A cross-sectional regression is estimated each consensus estimation month of the analyst consensus earnings estimation revision on the lagged standardized earnings change (SEC) in the simple regression. The multiple regression includes the firm’s current standardized earnings change as well. The reported statistics are the means (x1000) and the t-statistics of the time series of coefficients from the month-by-month regressions. The sample covers 214 consensus estimation months during the period of May 1987 to February 2005. The actual earnings and the analyst consensus earnings estimation are obtained from the I/B/E/S summary file. Notice that for each fiscal quarter, there is a consensus estimation in each month before the actual earnings is reported. To reflect the immediate effect of the earnings announcement on the analyst consensus estimation of the next quarterly earnings, only the consensus estimation immediately following the earnings report are used to compute the change in the analyst consensus earnings estimation. The estimation revision is computed as the change in the mean analyst consensus estimation scaled by the closing price on the previous consensus estimation date. The standardized earnings change (SEC) is computed as the actual quarterly earnings change standardized by the volatility of the previous eight earnings changes.
Table 4: Subperiod Empirical Cross-Sectional Regression of Analyst Consensus Earnings Estimation Revision on SEC

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>t-stat</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.34</td>
<td>0.95</td>
</tr>
<tr>
<td>Lag_SEC</td>
<td>−0.23</td>
<td>−1.7</td>
</tr>
</tbody>
</table>

Table 4: A cross-sectional regression is estimated each consensus estimation month of the analyst consensus earnings estimation revision on the lagged standardized earnings change (SEC) in the simple regression. The multiple regression includes the firm’s current standardized earnings change as well. The reported statistics are the means (x1000) and the t-statistics of the time series of coefficients from the month-by-month regressions. The whole sample covers 214 consensus estimation months during the period of May 1987 to February 2005. The actual earnings and the analyst consensus earnings estimation are obtained from the I/B/E/S summary file. Notice that for each fiscal quarter, there is a consensus estimation in each month before the actual earnings is reported. To reflect the immediate effect of the earnings announcement on the analyst consensus estimation of the next quarterly earnings, only the consensus estimation immediately following the earnings report are used to compute the change in the analyst consensus earnings estimation. The estimation revision is computed as the change in the mean analyst consensus estimation scaled by the closing price on the previous consensus estimation date. The standardized earnings change (SEC) is computed as the actual quarterly earnings change standardized by the volatility of the previous eight earnings changes.
Figure 1: This figure shows the sample paths of the estimated means from the three models: the full information rational expectation (RE) model, the rational structural uncertainty (SU) model and the conservatism (Beh-C) model. The structural change occurs after the 20th payoff. The parameters are adapted from Fig.2 in Brav and Heaton (2002): $\mu_0 = 10, \kappa_0 = 1, v_0 = 40, \sigma_0 = 15, \mu_A = 11, \mu_B = .3, \sigma = 13.7, c = \kappa_0 + 5.$
Figure 2: I simulate 3000 independent firms with 50 payoffs each. Each firm has a randomly assigned change point that is uniformly distributed between 1 and 50. If a structural change occurs, the mean of the payoff distribution switches between 11 and 7. The direction of the change is also uniformly distributed for each firm. The agent’s expectations are computed using the SU model. At the beginning of each period, the 3000 simulated firms are ranked into deciles in ascending order based on the firms’ previous standardized earnings changes (SEC). The winner (loser) portfolio consists of stocks in the bottom (top) SEC decile. The portfolios are held for one period after the ranking. This figure shows the time series of the average earnings expectation revision after the ranking for the winner portfolio, the loser portfolio and the winner-minus-loser portfolio for the SU model. The agent’s revision in expectation in period $t$ is computed as $\hat{\mu}_t/\hat{\mu}_{t-1} - 1$, and the standardized earnings change for firm $i$ in period $t$ is defined as

$$\text{SEC}_{it} = \frac{x_{it} - x_{it-1}}{\sigma_{it}},$$

where $\sigma_{it}$ is the standard deviation of the last four $x_{it} - x_{it-1}$. In order to ensure positive estimates so that the computed expectation revision is meaningful, the parameter values used in the simulation are: $\mu_0 = 10, \kappa_0 = 1, v_0 = 40, \sigma_0 = 15, \mu_A = 11, \mu_B = 7, \sigma = 2, c = \kappa_0 + 5$. 

39
Figure 3: I simulate 3000 independent firms with 50 payoffs each. Each firm has a randomly assigned change point that is uniformly distributed between 1 and 50. If a structural change occurs, the mean of the payoff distribution switches between 11 and 7. The direction of the change is also uniformly distributed for each firm. The agent’s expectations are computed using the conservatism model. At the beginning of each period, the 3000 simulated firms are ranked into deciles in ascending order based on the firms’ previous standardized earnings changes (SEC). The winner (loser) portfolio consists of stocks in the bottom (top) SEC decile. The portfolios are held for one period after the ranking. This figure shows the time series of the average earnings expectation revision after the ranking for the winner portfolio, the loser portfolio and the winner-minus-loser portfolio for the SU model. The agent’s revision in expectation in period $t$ is computed as $\hat{\mu}_t/\hat{\mu}_{t-1} - 1$, and the standardized earnings change for firm $i$ in period $t$ is defined as

$$\text{SEC}_{it} = \frac{x_{it} - x_{it-1}}{\sigma_{it}},$$

where $\sigma_{it}$ is the standard deviation of the last four $x_{it} - x_{it-1}$. In order to ensure positive estimates, the parameter values used in the simulation are: $\mu_0 = 10, \kappa_0 = 1, v_0 = 40, \sigma_0 = 15, \mu_A = 11, \mu_B = 7, \sigma = 2, c = \kappa_0 + 5$. 

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Figure 4: I simulate 3000 independent firms with 50 payoffs each. Each firm has a randomly assigned change point that is uniformly distributed between 1 and 50. If a structural change occurs, the mean of the payoff distribution switches between 11 and 7. The direction of the change is also uniformly distributed for each firm. This figure shows the time series of the cross-sectional Pearson correlation coefficients between the agent’s expectation revision and the lagged standardized earnings change (SEC) for both the SU model and the conservatism model. The agent’s revision in expectation in period $t$ is computed as $\hat{\mu}_t / \hat{\mu}_{t-1} - 1$, and the standardized earnings change for firm $i$ in period $t$ is defined as

$$SEC_{it} = \frac{x_{it} - x_{it-1}}{\sigma_{it}},$$

where $\sigma_{it}$ is the standard deviation of the last four $x_{it} - x_{it-1}$. In order to ensure positive estimates, the parameter values used in the simulation are: $\mu_0 = 10, \kappa_0 = 1, v_0 = 40, \sigma_0 = 15, \mu_A = 11, \mu_B = 7, \sigma = 2, c = \kappa_0 + 5$. 

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Figure 5: This figure is adopted from Chan et al. (2004). It illustrates the path dependent momentum strategy applied to the simulated data from the SU model. At the beginning of each period $t$, I rank the 3000 firms into quintiles in ascending order according to the cumulative four period earnings growth defined as

$$\frac{(x_t + x_{t-1} + x_{t-2} + x_{t-3}) - (x_{t-4} + x_{t-5} + x_{t-6} + x_{t-7})}{|x_{t-4} + x_{t-5} + x_{t-6} + x_{t-7}|}.$$ 

In the top (low growth) quintile and the bottom (high growth) quintile, I further separate firms into the consistent category and the inconsistent category. The firm in the low (high) growth quintile has a consistent path if all of its past four earnings news over the periods $t, t-1, t-2$ and $t-3$ are below (above) the median level. If two or less of the past four earnings news are below (above) the median level, it has an inconsistent path. The earnings news is defined as

$$\text{Earnings News}_{it} = \frac{x_{it} - \hat{\mu}_{it-1}}{\sigma_{it}}.$$ 

I then define the stocks as having disconfirming news if the news after the ranking period are below (above) the median level for the high (low) growth quintile. Portfolio A buys stocks in the high growth quintile with consistent paths and disconfirming news after the paths and sells stocks in the low growth quintile with consistent paths and disconfirming news after the paths. Portfolio B is constructed in a similar way except that it is created among the stocks with inconsistent paths.
Figure 6: At the beginning of each period \( t \), I rank the 3000 firms into quintiles in ascending order according to the cumulative four period earnings growth defined as

\[
(\sum_{i=0}^{t-4} \hat{x}_t) - (\sum_{i=0}^{t-3} \hat{x}_t)\
\frac{|\sum_{i=0}^{t-4} \hat{x}_t + \hat{x}_{t-5} + \hat{x}_{t-6} + \hat{x}_{t-7}|}{|\sum_{i=0}^{t-4} \hat{x}_t + \hat{x}_{t-5} + \hat{x}_{t-6} + \hat{x}_{t-7}|}
\]

In the top (low growth) quintile and the bottom (high growth) quintile, I separate firms into consistent category and inconsistent category. The firm in the low (high) growth quintile has a consistent path if all of its past four earnings news over the periods \( t, t-1, t-2 \) and \( t-3 \) are below (above) the median level. If less than or equal to two of the past four earnings news are below (above) the median level, it has an inconsistent path. The earnings news is defined as

\[
\text{Earnings News}_{it} = \frac{x_{it} - \hat{\mu}_{it-1}}{\sigma_{it}}
\]

I then define the stocks as having disconfirming news if the news after the ranking period are below (above) the median level for the bottom (top) quintile. The strategy applied to the consistent group buys stocks in the high growth quintile with consistent paths and disconfirming news after the paths and sells stocks in the low growth quintile with consistent paths and disconfirming news after the paths. Similar strategy is applied to the inconsistent group. This figure shows the returns to the strategy applied to the consistent group and inconsistent group. The same simulated data set is used as in all the previous figures.