Fluctuations in Economic Uncertainty and Transmission of Monetary Policy Shocks
Evidence Using Daily Surveys from Brazil *

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Abstract

We use a unique Brazilian dataset on daily survey expectations to obtain direct measures of shocks to central bank target rates and changes in economic uncertainty. Using these measures, we gauge the effect of monetary policy shocks on economic uncertainty, term premia, inflation expectations, and bond yields in Brazil. We find strong evidence that inflation uncertainty is key to transmitting monetary policy shocks to the yield curve via time-varying term premia. Finally, Fed announcements have sizeable spillover effects on the Brazilian bond market, as positive shocks to US yields significantly raise term premia in Brazil through elevated exchange rate risk.

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1 Introduction

How monetary policy shocks get transmitted to the term structure of interest rates is central to economic analysis. A key challenge is that policy shocks affect not only expectations of future inflation but also term premia in proportions that can vary with the underlying state of the economy. Term premia matter because they can create a wedge between short and long-term interest rates, reducing central banks’ ability to control movements in long rates which affect the savings and investment decisions of households and firms. Reflecting this point, recent research has emphasized the importance of variation in term premia for the transmission of monetary policy shocks. ¹

Inflation uncertainty is widely believed to be a key driver of term premia: Inflation erodes the value of nominal bonds, so elevated inflation uncertainty should command a higher nominal term premium.² To the extent that an unanticipated tightening in monetary policy reduces investors' uncertainty about future inflation, it should also decrease nominal term premia.

Although inflation uncertainty can be expected to play an important role in the transmission of monetary policy shocks, its effect has been difficult to document for US data. First, US inflation expectations have been firmly anchored over the past three decades, reducing variation in inflation uncertainty and, hence, the power of tests attempting to link inflation uncertainty with term premia.³ Second, the correlation between real economic activity and inflation in the US has been subject to regime switching, going from negative during the early eighties to positive in the past few decades, rendering the relation between inflation uncertainty and term premia ambiguous.⁴ Third, US monetary policy shocks have been found, empirically, to have similar long-term effects on nominal and real yields, implying small effects on inflation term premia (Nakamura and Steinsson, 2018).

In this paper, we use a unique Brazilian data set that allows us to cleanly and directly measure unanticipated shocks to target rates and their effect on economic uncertainty. Our data set features daily survey expectations of the Brazilian equivalent to the federal funds target rate as reported by key institutional investors and money managers in the Brazilian bond market. The data, which are collected by the Brazilian Central Bank, gives us a measure of the “pure” expectations component of changes to the target rate following meetings of the central bank’s Monetary Policy Committee (COPOM) which sets the target for the overnight rate used in interbank settlements in government

¹Cochrane and Piazzesi (2002), Gürkaynak, Sack and Swanson (2005), Hanson and Stein (2015), and Nakamura and Steinsson (2018) document that US monetary shocks have strong effects on distant forward rates, which is difficult to explain by movements in expected future short rates alone.
²For theoretical work on this point, see Piazzesi et al. (2006), Campbell, Sunderam and Viceira (2009), and Rudebusch and Swanson (2008). Wright (2011) presents empirical evidence that inflation uncertainty is positively correlated with nominal term premia across different countries and time periods, suggesting that changes in inflation uncertainty is an important component of variation in term premia.
³Christensen, Lopez and Rudebusch (2010) extract expected inflation and inflation term premia from US nominal and real yield curves using an affine arbitrage-free model of the term structure. Their empirical results suggest that long-term inflation expectations have been well anchored over the past few years and that the implied risk premium, though volatile, has been close to zero on average.
⁴See, for example, Campbell, Sunderam and Viceira (2009) and David and Veronesi (2013).
securities, known as the COPOM rate. Subtracting the expected value from the actual (COPOM) target rate announced after each meeting gives us a direct measure of the policy surprise which is not contaminated by variation in term premia and is independent of any modeling assumptions.\(^5\) Similarly, comparing inflation survey dispersions before and after the policy announcement gives us a direct measure of how inflation uncertainty is affected by monetary policy shocks.

The Brazilian economy’s history of high and volatile inflation makes it particularly suited for addressing questions concerning the impact of monetary policy shocks on bond markets. During our sample from 2006 to 2017, Brazilian inflation expectations fluctuated in a wide range. Rare among emerging economies, Brazil has large and liquid markets for both nominal and inflation-indexed government bonds, facilitating analysis of how monetary policy shocks affect both nominal and real rates as well as inflation expectations. Our data facilitate sharp estimates of the impact of monetary policy shocks on economic uncertainty (measured by survey dispersion) along with its effect on term structure variables, inflation expectations and inflation term premia.

In addition to our daily survey data on interest rate expectations, we also observe daily expectations of macroeconomic variables such as GDP growth, inflation, and exchange rates. We use these daily survey forecasts of macroeconomic state variables to estimate a rich class of dynamic term structure models with unspanned macro factors introduced by Duffee (2011) and Joslin, Priebsch and Singleton (2014). Unspanned macro risk factors do not affect the current term structure but can help predict term premia and expected future short-term interest rates. Since macroeconomic variables are typically measured infrequently, empirical studies using this methodology have relied on monthly or quarterly data. In contrast, we use our daily survey forecasts of inflation in place of the unobserved daily time series of the underlying inflation rate. Empirically, we find that daily variation in macroeconomic survey forecasts enters with a highly significant effect in the dynamic term structure model for movements in both nominal and real yields.

Combining our direct measures of target rate shocks and economic uncertainty (survey dispersion) with this term structure model, we uncover several pieces of evidence consistent with the hypothesis that an unexpected increase to the central bank’s target rate lowers nominal term premia by reducing inflation uncertainty.

First, regressing changes in the inflation survey dispersion after COPOM announcements against shocks to target rates, we identify a significantly negative effect. Moreover, the effects are long-lived. Combining event study methodology with a vector auto regression as in Gertler and Karadi (2015), we find that tighter target rates reduce inflation uncertainty by a significant amount even after 20 months.

Second, a regression of nominal term premia on the dispersion in inflation survey forecasts shows a strongly positive and significant relation between inflation uncertainty and nominal term premia.

\(^5\)Kuttner (2001), Bernanke and Kuttner (2005), and Hanson and Stein (2015) measure market surprises from interest rate movements on Fed announcement days. Market-based measures introduce endogeneity issues, however, because short-term interest rates are simultaneously influenced by movements in asset prices, see Rigobon and Sack (2004).
This corroborates the finding in Wright (2011) that reduced uncertainty about future inflation is associated with lower nominal term premia. Conversely, dispersion in inflation expectations bears no significant relation to the real term premium. Rather, increased uncertainty about future GDP growth or the Brazilian-US dollar exchange rate has a significantly positive correlation with both nominal and real term premia. These findings suggest that variation in the level of macroeconomic uncertainty, broadly measured, is important for understanding how nominal and real term premia evolve in Brazil.

Third, we show that an unexpected increase in the target rate has a strongly positive effect on long-term real yields while its effect on long-term nominal yields and expected future inflation is close to zero, regardless of whether we use survey data or a model-implied inflation measure. We emphasize that this evidence is completely model free, which makes it more compelling because model-based approaches invariably introduce noise in their separation of movements in expected short rates and term premia. Because the inflation term premium plus the expected inflation must equal the difference between nominal and real yields, our evidence implies that an unexpected increase in the central bank’s target rate has a strongly negative effect on the inflation risk premium. Again, this emphasizes the importance of the inflation uncertainty channel in the transmission of monetary policy shocks in Brazil.

Using our dynamic term structure model, we show that nominal term premia are strongly affected by unanticipated shocks to the COPOM rate. At the three and four-year horizons, a positive 100 bp shock to the COPOM rate is associated with a reduction in the nominal and inflation term premia of 70 and 40 bps, respectively. Even though a positive shock to the COPOM rate significantly increases expected short rates, the effect is almost completely offset by decreases in the nominal term premia and the resulting effect on three and four-year nominal yields is close to zero. We also find that the effect of target rate shocks on the inflation term premium is particularly large when inflation uncertainty — measured by dispersion in inflation survey forecasts — is high.

Inflation prospects are not the only source of uncertainty facing investors in the Brazilian bond market. A substantial portion of Brazilian government bonds is traded and held by foreign investors who are exposed to exchange rate uncertainty because these bonds are denominated in Brazilian Real. A plausible hypothesis is that foreign investors will require higher yields from Brazilian bonds if a shock to foreign monetary policy increases the exchange rate uncertainty.

To explore this point, we investigate the impact of Federal Reserve policy announcements on the Brazilian bond market. We provide estimates of the magnitude of movements in the Brazilian term structure following unanticipated news about U.S. monetary policy and perform a detailed analysis of the mechanism through which such shocks get transmitted. Using data on US and Brazilian interest rate movements on days with Fed announcements, we find that shocks to US Treasury rates have a larger effect on Brazilian nominal yields and affect more maturities in the Brazilian bond market than shocks to the domestic COPOM rate of similar size.

The mechanism for transmission of US interest rate shocks to the Brazilian term structure is,
however, very different than that for domestic interest rate shocks. Higher US yields are associated with higher nominal term premia in Brazil, even for long bond maturities, while they have no effect on expected nominal short rates or expected inflation in Brazil. Conversely, higher-than-expected COPOM rates reduce nominal term premia but also increase expected future nominal short rates in Brazil and these effects largely balance out, leaving Brazilian forward rates and yields unchanged for all but the shortest bond maturities.

The effects of US policy shocks on real yields on Brazilian bonds are similar in magnitude to those associated with COPOM rate shocks, but again the direction is different: Positive shocks to US Treasury rates are associated with higher real term premia in Brazil, whereas positive shocks to the COPOM rate translate into reduced real term premia.

Our daily data on survey dispersion show that higher US Treasury rates are associated with higher dispersion in survey forecasts of the US-Brazilian exchange rate. Unlike the case of domestic interest rate shocks, dispersion in inflation surveys does not increase, however, suggesting that the primary transmission channel for US monetary policy shocks on the Brazilian term structure is through its effect on exchange rate risk. Consistent with this story, we observe significant capital outflows from Brazil following FOMC announcements that indicate a surprise tightening in US monetary policy.

Our analysis of the role played by the uncertainty channel in transmitting monetary policy shocks is closely related to recent research on uncertainty shocks and macroeconomic fluctuations.6 Shocks to various measures of uncertainty have been found to lead to significant changes in inflation, interest rates, output growth and unemployment.7 Our analysis suggests that monetary policy shocks have a strong influence on uncertainty about future inflation and exchange rates which, in turn, may affect economic activity.

The outline of our paper is as follows. Section 2 describes our Brazilian data set and gives a brief review of the institutional background for our analysis. Section 3 introduces our survey-based measure of monetary policy shocks, while Section 4 describes our joint term structure model for nominal and real yields and provides some initial empirical estimates. Section 5 analyzes the impact of domestic monetary policy shocks on economic uncertainty, while Section 6 shows how these shocks affect the Brazilian term structure. Section 7 analyzes the effect of U.S. monetary policy changes on economic uncertainty and interest rates in Brazil, while Section 8 concludes. Additional details are provided in a set of appendices at the end of the paper.

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6Since the seminal work of Bloom (2009), a large number of papers focus on the relation between the macroeconomy and economic uncertainty. See, e.g., Bloom et al. (2018), Bachmann and Bayer (2013), Gilchrist, Sim and Zakrajšek (2014), and Basu and Bundick (2017).

2 Data and the Brazilian Bond Market

We start by introducing the Brazilian bond market data used in our analysis and providing a brief description of the Brazilian interbank market, including the process for setting the Brazilian equivalent to the federal funds rate which plays a critical role in our empirical analysis. Next, we introduce the daily survey forecasts collected by the Brazilian Central Bank (BACEN).

2.1 Markets for Nominal and Real Bonds

Our analysis of the Brazilian term structure uses daily values of yields on government bonds issued by the National Treasury Office of the Ministry of Finance in Brazil. We use both nominal and inflation indexed (real) government bonds. Yield data on real bonds become available in 2006 and we use this as the starting date for our analysis. The data end in September 2017.

We obtain data on the zero-yield term structure from the Brazilian Association of financial and Capital Market Institutions (ANBIMA). ANBIMA collects transaction prices on government bonds from financial and capital market institutions that participate in the market for Brazilian government bonds. This includes trades executed by the main dealers and other major institutions in the government bond market. ANBIMA uses the cubic spline method of Svensson (1994) to construct the yield curve for zero-coupon bonds.\(^8\)

Two types of nominal bonds, denoted LTN and NTN-F, respectively, are used to estimate the nominal zero-coupon yield curve. LTN bonds pay no coupons, have a face value of R1,000 (Brazilian Real) and maturities of 6, 9, 12, 24, 36, and 48 months. NTN-F bonds also pay R1,000 at maturity but in addition pay semiannual coupons at a 10% annual rate. These bonds have maturities of 1, 3, 4, 5, 6, and 10 years.

To estimate the real zero-coupon yield curve, we use data on Brazilian inflation-indexed government bonds. The principal and coupons of these so-called NTN-B bonds are indexed to the IPCA Brazilian consumer price index.\(^9\) Maturities on the NTN-B bonds are 3, 5, 10, 20, 30, and 40 years and the bonds pay semiannual coupons of 6% per year. Table 1 summarizes the different types of Brazilian government bonds.

Our bond data use quoted yields, but we cross-check the data against market yields obtained from the sizeable swap market in Brazilian government bonds. For one-year and two-year bonds, we find a correlation between quoted and market yields of 0.999 and very small differences between the two yield series; see Appendix B for further details.

The market for Brazilian government debt has grown to sizeable proportions in recent years. At the end of September 2017, the total value of the stock of outstanding real bonds (NTN-B)
amounted to 14.1% of the Brazilian GDP while the stock of nominal bonds (LTN and NTN-F) comprised 18.5% of the Brazilian GDP. The bonds also trade in high volume. During September 2017, the average daily trading volume for real bonds (NTN-B) was 11.9 billion Reais ($3.8bn). This volume was about two-thirds of the trading in nominal bonds (LTN and NTN-F) which averaged 18.7 billion Reais ($5.9bn) per day during the same period.\footnote{We restrict our analysis to bonds with maturities up to four years because the prices of bonds with longer maturities are less accurate due to their lower liquidity.}

An important driver of the growth in the Brazilian bond market in recent years was the decision to grant tax exemption to foreign bond investors starting in February, 2006. As of September 2017, 12.6% of the Brazilian government’s real and nominal bonds were held by foreign investors.\footnote{These statistics are obtained from BACEN.}

### 2.2 SELIC and COPOM Rates

The SELIC (Sistema Especial de Liquidacao e Custodia) rate is an average of the interbank rates charged on trades in government securities with a maturity of one day and can be compared to the US Federal funds rate. BACEN keeps the average (effective) SELIC rate close to the SELIC target rate by buying and selling government securities.\footnote{Overnight operations in the Brazilian market for government bonds take the form of purchases with a resale agreement. For example, Banco do Brazil might provide funds to a financial institution against the promised delivery of public securities in an amount equivalent to the loan plus the agreed interest.}

The SELIC target rate—also called the COPOM rate—is determined by the Monetary Policy Committee (COPOM) which consists of eight members of the Brazilian central bank’s board of directors. This rate gets reviewed in meetings between the Brazilian government and COPOM that last two days (Tuesday and Wednesday). The outcome of the meeting, including the new value of the SELIC target, is always announced after the market is closed—typically at 6 p.m. or shortly afterwards—on the second day of the COPOM meeting.\footnote{Until 2005, regular COPOM meetings were held every month. From 2006 onwards, COPOM meetings take place approximately every six weeks, or eight times a year.} In total there were 94 COPOM meetings during our sample from 1/1/2006 to 9/27/2017.

Figure 1 plots the effective SELIC rate and the SELIC target (COPOM) rate along with circles showing the dates when COPOM meetings took place. Differences between the actual and targeted SELIC rate are generally very small—almost always less than 10 basis points. During the sample, the COPOM rate trended down from 18% in 2006 to 7% in 2013, with two notable shifts towards higher rates interrupting this broad downward trend. Between 2013 and 2016, the COPOM rate gradually rose back to 14% before sharply declining to 8% in the autumn of 2017. Brazilian short rates did not hit the zero lower bound in our sample.

Figure 2 shows a histogram depicting the changes in the COPOM rate on the 94 days following a COPOM meeting. Like the Fed funds rate, the COPOM rate follows a step function with changes in multiples of 25 basis points (bps) on days following a COPOM meeting.\footnote{While the Federal Reserve announced a range for the Fed funds rate during parts of our sample, the SELIC}
in the COPOM rate was 150 bps, while the largest rise was 75 bps. The COPOM rate was left unchanged on 29 occasions—a little less than one-third of all meetings—with 17 changes of 50 bps and 15 changes of -50 bps being the second and third most common outcomes.

Compared with Fed funds rate data over the same period, the fluctuations observed in our data are large. The combination of large policy shifts and high levels of uncertainty about future inflation makes for a particularly interesting data set when it comes to assessing the effect of monetary policy shifts on the bond market and on economic uncertainty.

2.3 Survey Forecasts

Our sample of survey forecasts is obtained from BACEN. The survey uses an online tool, the Market Expectations System, which can be accessed at www.bcb.gov.br/expectativa. At present, more than 100 institutions are active in the system, including the largest banks, brokers, asset managers, consultants and other non-financial entities. The survey can be accessed and updated at any point in time. However, data entered after the financial markets close at 5 p.m. are used to compute statistics for the next business day. BACEN publishes consolidated statistics from the survey such as the median, average, standard deviation, maximum and minimum values. 15

Every business day during our sample, BACEN collects from its participating institutions 13 monthly forecasts (one through thirteen months ahead) and five annual forecasts (one through five years ahead) of the COPOM rate, the consumer price index, IPCA, and the US dollar exchange rate. 16 Similarly, BACEN collects five quarterly (one through five quarters ahead) and five annual forecasts of GDP growth.

Monthly and annual COPOM rate forecasts are constructed quite differently. The dates used in the monthly forecasts coincide with the next 13 COPOM meetings and participants are asked to predict what the target rate will be on the dates following a COPOM meeting. For the annual forecasts, survey participants are asked to predict the average daily value of the COPOM rate in individual calendar years. Table 2 summarizes the format of the survey forecasts.

The daily surveys use a fixed event date format, i.e., participants are asked on day \( t \) to predict the value of a variable on certain event dates, \( \tau_1(t), \tau_2(t), ..., \tau_k(t) \), where \( \tau_0(t) \leq t < \tau_1(t) < \tau_2(t) < ... < \tau_k(t) \). 17 For the inflation survey, the event date is the release date for the IPCA which generally occurs at the end of the second week of the month. Similarly, for GDP growth the event date coincides with the date of the first release of a quarterly GDP figure. For the exchange rate target is stated as a single rate throughout our sample.

15 The median value is the statistic most closely monitored in the Focus–Market Readout which gets published every Monday at 8:30 a.m., using data collected up to 5 p.m. on the previous Friday. Individual forecasts in the Market Expectations System are confidential and only accessible to members of the COPOM and managers of the Market Expectations system.

16 Most active logins update their forecasts multiple times each month for the IPCA (headline inflation) survey. If an institution does not update its forecasts within a 30-day period, to avoid stale data the system automatically disregards forecasts from this institution when calculating the daily statistics.

17 \( \tau_0(t) \) is the starting date of the survey conducted on day \( t \).
surveys, the release (event) date is the last business day of each month. Finally, for the COPOM rate, the event date coincides with the COPOM meetings.

To align the survey forecasts with our fixed-maturity term structure data, we convert the fixed event forecasts into rolling window forecasts with a constant horizon. Appendix A explains how we produce daily one-year-ahead survey forecasts of the COPOM rate, \( \bar{r}_{t,1yr} \), inflation, \( \hat{\pi}_{t,1yr} \), GDP growth, \( \hat{g}_{t,1yr} \), and exchange rate movements, \( E\bar{X}_{t,1yr} \). We undertake similar calculations to get one-year-ahead survey dispersion measures.

The two top panels in Figure 3 plot daily survey forecasts of the one-year-ahead inflation rate and the depreciation in the $US-Brazilian real (R$) exchange rate, respectively. Unsurprisingly, the inflation forecasts (top panel) are highly persistent at the daily horizon. On average, during our sample from 2006 to 2017, survey participants expected inflation of 5.2% per year but we observe large variation around this figure: inflation forecasts follow a broad upward trend from 3% in 2007 to 7% in 2016, interrupted by a decline in 2009 following the global financial crisis. Between early 2016 and September 2017, inflation expectations decline from 7% to 4%. The predicted depreciation in the R$ against the $US (second panel) is also quite persistent, with the R$ expected to depreciate against the US dollar by about 5% per year since 2014. From 2009-2010, and again during parts of 2011 and 2012, the real was expected to appreciate against the dollar.

Daily dispersions in the survey forecasts of inflation and exchange rate depreciation are plotted in the bottom two panels of Figure 3. We see clear evidence of elevated uncertainty levels during periods such as the global financial crisis (fall of 2008) and during the recession in 2016.

3 A Survey-based Measure of Monetary Policy Shocks

Analysis of the transmission mechanism for monetary policy shocks critically depends on having an accurate measure of policy shocks. Shifts in central bank rates are largely anticipated and only unanticipated changes should have an impact on markets. Because monetary policy shocks are typically not directly observable, proxies must be used in their place. For example, Kuttner (2001) and Bernanke and Kuttner (2005) use changes in federal funds rate futures contracts on days with announcements by the Federal Open Market Committee to measure policy surprises. Hanson and Stein (2015) use changes in the yield on nominal two-year Treasury bonds on such days to measure target rate surprises. However, bond yields can change either because of shifts in expected future short rates or because of shifts in risk premia, confounding the interpretation of market-based measures of monetary policy shocks.\(^{18}\)

Our survey forecasts provide a direct measure of the pure expectation shifts about future spot rates because they are not contaminated by risk premium (pricing) effects and other endogeneity.

\(^{18}\)Monetary policy shocks can also affect term premia by changing the demand of yield oriented investors which counteracts the effect of inflation uncertainty on term premia; Hanson and Stein (2015) suggest that the demand effect is important for US treasury bond yields.
issues. Let $r_{t+1|t}^e$ be the day-$t$ survey forecast of the COPOM rate on day $t + 1$. This forecast uses information obtained prior to 5 p.m. on day $t$, the last day of the COPOM meeting. Although the COPOM rate is set on day $t$, the new rate only gets announced after markets have closed and so day $t + 1$ is the “event date” for measuring the impact of the day-$t$ announcement.

We can decompose the actual post-meeting COPOM rate ($r_{t+1}^{\text{COPOM}}$) into its expected value, $r_{t+1|t}^e$ (observed from the surveys) and the (residual) unexpected component, $r_{t+1}^u : r_{t+1}^{\text{COPOM}} = r_{t+1|t}^e + r_{t+1}^u$. Subtracting the pre-announcement COPOM rate, $r_t^{\text{COPOM}}$, from both sides and defining $\Delta r_{t+1}^{\text{COPOM}} = r_{t+1}^{\text{COPOM}} - r_t^{\text{COPOM}}$ and $\Delta r_{t+1|t}^e = r_{t+1|t}^e - r_t^{\text{COPOM}}$, we get the following relation between the actual one-day change in the COPOM rate, $\Delta r_{t+1}^{\text{COPOM}}$, and the expected change, $\Delta r_{t+1|t}^e$:

$$\Delta r_{t+1}^{\text{COPOM}} = \Delta r_{t+1|t}^e + r_{t+1}^u.$$  

(1)

Applying event study methodology to the 94 days where the Brazilian central bank committee (COPOM) met, we can use this decomposition to estimate the one-day response of term structure and inflation rate measures to both expected and unexpected changes in the COPOM rate.\(^{19}\)

Other papers have used survey data to measure the impact of policy news on market expectations, but there are notable differences between our analysis and the existing literature. Previous studies of how monetary policy affects the term structure have mostly used survey data observed at the monthly or quarterly horizon.\(^{20}\) For example, Kim and Orphanides (2012) and Nakamura and Steinsson (2018) use Blue Chip survey data measured at the monthly frequency while Chernov and Mueller (2012) use monthly, quarterly and semi-annual survey data.\(^{21}\) However, large amounts of information get released during monthly or quarterly intervals and this can introduce considerable noise in survey expectations from the perspective of measuring the impact of individual news releases.

### 3.1 Quality of the Survey-based Measure

Before engaging in the analysis of our survey-based measure of monetary policy shocks, it is important to evaluate the quality of the COPOM rate forecasts right before central bank announcements. Studies such as Coibion and Gorodnichenko (2012) and Andrade et al. (2016) point out the importance of accounting for staleness in survey forecasts which may result from information rigidities. Concerns related to infrequent updating of forecasts is an important issue when interpreting survey

\(^{19}\)Gagnon et al. (2010) and Krishnamurthy and Vissing-Jorgensen (2011) also use event study methods to evaluate the impact of unconventional monetary policy measures.

\(^{20}\)Gürkaynak, Sack and Swanson (2005) use weekly surveys of expectations on a range of macroeconomic variables to estimate how news about such variables affect interest rates. They do not, however, use survey data to measure monetary policy surprises, using federal funds futures data for this purpose instead.

\(^{21}\)Chernov and Mueller (2012) show that survey data can be used to take advantage of the richer information sets available to survey participants compared to the information embedded in the past history of individual variables alone. Kim and Orphanides (2012) argue that inference in small samples with persistent term structure factors can be improved by including survey data.
To check if agents update their forecasts on the days before and after the COPOM meetings, we conduct Mincer-Zarnowitz efficiency tests, regressing $\Delta r_{t+1}^{COPOM}$ on an intercept and the predicted change, $\Delta r_{t+1|t}^e = r_{t+1|t}^e - r_{COPOM}^t$ obtained from the survey data:

$$\Delta r_{t+1}^{COPOM} = \alpha + \beta \Delta r_{t+1|t}^e + \epsilon_{t+1}. \quad (2)$$

Under the null of unbiasedness in survey expectations, $\alpha = 0$ and $\beta = 1$. If survey participants report “stale” forecasts, we would expect to see an estimate of $\beta$ significantly smaller than unity. We also would not expect it to have strong predictive power over the actual change in the COPOM rate, $\Delta r_{t+1}^{COPOM}$. Conversely, an estimate of $\beta$ at or above unity is suggestive of survey forecasts that are updated in a timely manner leading up to the COPOM meeting.

Figure 4 presents a scatter plot of actual versus predicted daily changes in the COPOM rate. Our estimates of the parameters in (2) are $\hat{\alpha} = -0.007$ and $\hat{\beta} = 1.057$ which is very close to the values we would expect if forecasts are unbiased. Moreover, it is clear from both the estimates and the plot that the predicted changes closely match up with actual changes to the COPOM rate. The $R^2$ of the regression in (2) reflects how good survey participants were at forecasting COPOM rate changes. For our data, $R^2 = 0.938$, suggesting that the surveys were very accurate in predicting changes in monetary policy over the sample and get updated quickly around the COPOM meeting dates.

An alternative method for evaluating the accuracy of our survey-based measure of monetary policy shocks is to examine how market rates respond to the corresponding expected and unexpected components of the target rate change. In efficiently functioning markets, bond prices should incorporate investors’ current expectations of future COPOM rate changes and so we should not expect to find a significant relation between the expected change in the COPOM rate, $\Delta r_{t+1|t}^e$, (known on day $t$) and future changes in interest rates on the announcement date (day $t+1$). Conversely, unexpected shocks to the COPOM rate ($r_{t+1}^u$) could well affect market rates on day $t+1$.

As an informal test of this implication, Figure 5 presents a series of scatter plots relating changes in the nominal yields of bonds expiring 1, 2, 3, and 4 years from now to expected changes in the COPOM rate ($\Delta r_{t+1|t}^e$, left column plots) and unexpected changes ($r_{t+1}^u$, right column). While the estimated slopes are positive in all the plots, they are notably smaller for the expected changes than for the unexpected changes, consistent with unexpected shifts in the COPOM rate having a larger effect on changes in yields than shifts that are anticipated.

We conduct a more formal analysis of the relation between term structure variables and the expected and unexpected COPOM rate changes in Sections 5 and 6. Reassuringly, we do not find a single instance in which the expected change in the COPOM rate is significantly related to future changes in interest rates or term premia. In sharp contrast, we find very strong evidence that
unanticipated changes in the COPOM rate matter to these measures.

4 Modeling the Brazilian term structure

This section introduces the dynamic term structure model which we use to analyze movements in Brazilian nominal and real bond yields, explains how we generalize the model to include the daily survey forecasts as unspanned macro factors, and provides details of how we estimate the parameters of the model. Finally, we use the term structure model to produce estimates of expected short rates, expected inflation and term premia in Brazil.

4.1 A Joint Term Structure Model for Nominal and Real Yields

Our term structure analysis applies the affine, arbitrage-free dynamic Gaussian term structure approach proposed by Joslin, Priebsch and Singleton (2014) (JPS, henceforth) which extends standard homoskedastic Gaussian dynamic term structure models (GDTSMs) to include information from unpriced macroeconomic state variables. Because we are interested in jointly modeling the dynamics in nominal and real yields, we adopt a specification similar to that proposed by Christensen, Lopez and Rudebusch (2010). However, we deviate from the assumption in Christensen, Lopez and Rudebusch (2010) of a common slope factor for nominal and real yields. This modification is necessary because the slopes of the nominal and real yield curves are significantly less correlated in the Brazilian data than in US data.

First some notations. Let $P_t^N(\tau)$ and $P_t^R(\tau)$ denote the prices, at time $t$, of nominal ($N$) and real ($R$) zero-coupon bonds with unit payoff, expiring $\tau$ periods from now, i.e., at time $t+\tau$. Nominal and real bond yields are given by $y_t^N(\tau) = -\frac{\ln(P_t^N(\tau))}{\tau}$ and $y_t^R(\tau) = -\frac{\ln(P_t^R(\tau))}{\tau}$. Further, define nominal and real stochastic discount factors (SDFs), $M_t^N$ and $M_t^R$. From the absence of arbitrage,

\begin{equation}
P_t^N(\tau) = \mathbb{E}_t^P \left( \frac{M_{t+\tau}^N}{M_t^N} \right),
\end{equation}

\begin{equation}
P_t^R(\tau) = \mathbb{E}_t^P \left( \frac{M_{t+\tau}^R}{M_t^R} \right),
\end{equation}

where $\mathbb{E}^P(.)$ is the expectation operator computed under the physical measure ($\mathbb{P}$).

In common with standard affine Gaussian term structure models, we assume that the nominal SDF depends on the instantaneous nominal risk-free rate, $r_t^N$, as well as the cross-product of a vector of risk prices, $\lambda_t$, and shocks to a Wiener process, $dW_t$:

\begin{equation}
\frac{dM_t^N}{M_t^N} = -r_t^N \, dt - \lambda_t^N \, dW_t.
\end{equation}
Following studies such as Christensen, Lopez and Rudebusch (2010), Abrahams, Adrian, Crump, Moench and Yu (2016) and Chernov and Mueller (2012), we assume that the real SDF is affected by the same weighted average of shocks, $\lambda_t' dW_t$:

$$\frac{dM^R_t}{M^R_t} = -r^R_t dt - \lambda_t' dW_t,$$  \hspace{1cm} (6)

where $r^R_t$ is the instantaneous real risk-free rate. As shown by Christensen, Lopez and Rudebusch (2010), the assumption that the risk exposures of the nominal and real SDFs are identical implies the following Fisher equation as the length of the time interval shrinks towards zero:

$$\pi_t = (r^N_t - r^R_t).$$  \hspace{1cm} (7)

The real and nominal pricing kernels implicitly define the price level, $Q_t = M^R_t/M^N_t$, and the instantaneous inflation rate $\pi_t dt$ equals the change in the price level, $dQ_t/Q_t$.

Short rates and prices of risk are assumed to be linear functions of an $n$-dimensional vector of state variables, $X_t$:

$$r^N_t = \delta^N_0 + \delta^N_1 X_t,$$  \hspace{1cm} (8)

$$r^R_t = \delta^R_0 + \delta^R_1 X_t,$$  \hspace{1cm} (9)

$$\lambda_t = \lambda_0 + \Lambda_1 X_t,$$  \hspace{1cm} (10)

where $X_t$ follows a multivariate Ornstein-Uhlenbeck process under the $\mathbb{P}$ measure:

$$dX_t = K (\mu - X_t) dt + \Sigma dW_t.$$  \hspace{1cm} (11)

Similarly, under the risk neutral measure ($\mathbb{Q}$), $X_t$ is governed by the process

$$dX_t = K^* (\mu^* - X_t) dt + \Sigma dW^*_t,$$  \hspace{1cm} (12)

where $dW^*_t$ are increments to the following Wiener process (under the $\mathbb{Q}$ measure)

$$dW^*_t = dW_t + (\lambda_0 + \Lambda_1 X_t),$$

and

$$K^* = K + \Sigma \Lambda_1,$$  \hspace{1cm} (13)

$$\mu^* = (K^*)^{-1} (K \mu - \Sigma \lambda_0).$$

As shown by Duffie and Kan (1996), the log-prices of zero-coupon bonds, $p^N_t(\tau), p^R_t(\tau)$ are
affine functions of the state variables $X_t$:

$$p_t^N(\tau) = A^N(\tau) + B^N(\tau)' X_t,$$

(14)

$$p_t^R(\tau) = A^R(\tau) + B^R(\tau)' X_t,$$

(15)

where $A^N(\tau), B^N(\tau)$ are functions of $K^*, \mu^*, \delta_0^N$ and $\delta_1^N$. Similarly, $A^R(\tau), B^R(\tau)$ are functions of $K^*, \mu^*, \delta_0^R$ and $\delta_1^R$.

### 4.2 Unspanned Macro Risk Factors

Several studies, including Duffee (2011), Joslin, Priebsch and Singleton (2014), Ludvigson and Ng (2009), and Wright (2011) consider the possibility that some of the factors in the Gaussian term structure model could be unspanned. To explore this possibility, we partition the vector of observed state variables into $X_t = (X_t^N, X_t^R, X_t^M)'$. $X_t^N$ consists of information from the nominal term structure such as principal components extracted from the cross-section of nominal yields of different bond maturities; $X_t^R$ comprises information from the real term structure, and $X_t^M$ contains other types of information, including macroeconomic state variables.

To ensure that $X_t^N$ spans all the information in the nominal yield curve, we impose two restrictions on the coefficients of our term structure model. First, we restrict $\delta_1^N$ in the expression for the nominal short rate in (8) such that $r_t^N$ has zero loadings on $(X_t^R, X_t^M)'$. Second, we restrict $K^*$ in (12) such that the expectation of $dX_t^N$ has zero loadings on $(X_t^R, X_t^M)$. Under these restrictions, it follows that the elements of $B^N(\tau)$ in (14) that capture the effect of $(X_t^R, X_t^M)$ on nominal bond prices equal zero and so the log-nominal bond prices in (14) simplify to

$$p_t^N(\tau) = A^N(\tau) + B^N(\tau)' X_t^N.$$  

(16)

These restrictions imply that variation in $(X_t^R, X_t^M)$ does not affect nominal bond yields, $y_t^N(\tau)$, although they can contain information that could be useful for forecasting future nominal interest rates.

We can further restrict $\delta_1^R$ in (9) such that the real short rate to have zero loadings on $(X_t^N, X_t^M)$ and restrict $K^*$ so that $dX_t^R$ has zero loadings on $(X_t^N, X_t^M)$ in (12). Imposing this, the elements of $B^R(\tau)$ that capture the effect of $(X_t^R, X_t^M)'$ on bond prices will equal zero and log-real bond prices in (15) become

$$p_t^R(\tau) = A^R(\tau) + B^R(\tau)' X_t^R.$$  

(17)

When these restrictions hold, $(X_t^R, X_t^M)$ will be unspanned factors for the nominal yields while $(X_t^N, X_t^M)$ are unspanned factors for the real yields. $X_t^M$ serve as common unspanned factors for the joint model and plausibly include macroeconomic state variables that are important for forecasting future interest rates but may not be needed to fit the cross-section of yields.
In summary, the vector of state variables in our joint model for nominal and real bond yields takes the form \( X_t = \left( X_t^N, X_t^R, X_t^M \right) \). The model imposes a set of tight constraints: \( (X_t^R, X_t^M) \) are not spanned by the nominal yield factors, \( X_t^N \), but may contain information about the expected value of \( X_t^N \). Similarly, \( (X_t^N, X_t^M) \) are not spanned by the real yield factors, \( X_t^R \), but can contain information about the expected value of \( X_t^R \).

These properties are reflected in the following constraints on equations (8) and (9):

\[
\begin{align*}
    r_t^N &= \delta_0^N + \delta_1^N X_t^N, \\
    r_t^R &= \delta_0^R + \delta_1^R X_t^R,
\end{align*}
\]

while \( K^* \) in equation (13) takes the form

\[
K^* = \begin{pmatrix}
    K^N & 0 & 0 \\
    0 & K^R & 0 \\
    K_M N & K_M R & K_M M
\end{pmatrix},
\]

where \( K^N \) and \( K^R \) are \( 3 \times 3 \) matrices governing the dynamics of the nominal and real yield factors, respectively, under the \( Q \)-measure.

Following Christensen, Diebold and Rudebusch (2011), we further constrain \( K^* \) and \( \mu^* \) and equations (8) and (9) to ensure that the nominal and real yields, \( y_t^N(\tau), y_t^R(\tau) \), are priced by the Nelson-Siegel function plus a constant\(^{22}\)

\[
\begin{align*}
    y_t^N(\tau) &= \frac{-p_t^N(\tau)}{\tau} = L_t^N + \frac{1 - e^{-\lambda_N \tau}}{\lambda_N \tau} S_t^N + \left[ \frac{1 - e^{-\lambda_N \tau}}{\lambda_N \tau} - e^{-\lambda_N \tau} \right] C_t^N + \frac{A^N(\tau)}{\tau}, \\
    y_t^R(\tau) &= \frac{-p_t^R(\tau)}{\tau} = L_t^R + \frac{1 - e^{-\lambda_R \tau}}{\lambda_R \tau} S_t^R + \left[ \frac{1 - e^{-\lambda_R \tau}}{\lambda_R \tau} - e^{-\lambda_R \tau} \right] C_t^R + \frac{A^R(\tau)}{\tau}.
\end{align*}
\]

Expressions for the yield adjustment terms \( A^N(\tau), A^R(\tau) \) can be found in Christensen, Diebold and Rudebusch (2011).

### 4.3 Estimation

While the bond pricing model is set in continuous time, our data are observed at discrete points in time, with a fixed interval \( (h) \) of one day. As Christensen, Diebold and Rudebusch (2011) point

\(^{22}\)The additional constraints are \( \delta_0^N = \delta_0^R = 0, \delta_1^N = \delta_1^R = (1, 1, 0), \mu^* = 0 \) and

\[
K^N = \begin{pmatrix}
    0 & 0 & 0 \\
    0 & \lambda_N & 0 \\
    0 & -\lambda_N & \lambda_N
\end{pmatrix}, \quad K^R = \begin{pmatrix}
    0 & 0 & 0 \\
    0 & \lambda_R & 0 \\
    0 & -\lambda_R & \lambda_R
\end{pmatrix}.
\]
out, discrete and fixed-interval observations generated by an Ornstein-Uhlenbeck process have the same likelihood function as a VAR(1) process in discrete time, i.e.,

\[ X_t = \mu + \Phi (X_{t-h} - \mu) + \varepsilon_t, \]

where \( \Phi = e^{-Kh} \), \( \varepsilon_t \sim N(0, \Xi) \) with covariance matrix \( \Xi = \int_0^h e^{-Ks}\Sigma\Sigma' e^{-K's}ds \). When the time interval \( h \) is small, we can use the approximation \( \Phi \approx I - Kh \) and \( \Xi \approx h\Sigma\Sigma' \).

Building on a literature stretching back to Nelson and Siegel (1987), we use three-factor models to fit the nominal and real yield curves. Letting \( L, S, C \) refer to level, slope and curvature factors, respectively, we set \( X_t^N = (L_t^N, S_t^N, C_t^N)' \) for the nominal yield curve and \( X_t^R = (L_t^R, S_t^R, C_t^R)' \) for the real yield curve. Following the discussion above, the nominal yield curve is spanned by the state variables in \( X_t^N \) while the real yield curve is spanned by \( X_t^R \):

\[ p_t^N(\tau) = A^N(\tau) + B^N(\tau)'X_t^N, \]

\[ p_t^R(\tau) = A^R(\tau) + B^R(\tau)'X_t^R. \]

To estimate the affine, arbitrage-free term structure model with unspanned macro factors, we follow the minimum chi-square approach of Hamilton and Wu (2012). This estimation approach is asymptotically equivalent to maximum likelihood estimation and much easier to compute.

Following Joslin, Priebsch and Singleton (2014), we assume that the first three principal components extracted from the nominal yields, \( \tilde{X}_t^N \equiv (PC_{1,t}^N, PC_{2,t}^N, PC_{3,t}^N)' \), and from the real yields, \( \tilde{X}_t^R \equiv (PC_{1,t}^R, PC_{2,t}^R, PC_{3,t}^R)' \), are priced without error. Combining this assumption with equations (21) and (22), there exists a one-to-one linear mapping from \( \tilde{X}_t^N \) to \( X_t^N \) and from \( \tilde{X}_t^R \) to \( X_t^R \). Hence, there is a one-to-one linear mapping between \( X_t \) and \( \tilde{X}_t \equiv (\tilde{X}_t^N, \tilde{X}_t^R, X_{Mt})' \) and \( \tilde{X}_t \) also follows a VAR(1) process

\[ \tilde{X}_t = \tilde{\mu} + \tilde{\Phi} \tilde{X}_{t-h} + \tilde{\varepsilon}_t, \quad \tilde{\varepsilon}_t \sim N(0, \tilde{\Xi}). \]

The principal components other than \( \tilde{X}_t^N \) and \( \tilde{X}_t^R \) are allowed to be affected by pricing errors, \( \nu_{t}^{e,N} \) and \( \nu_{t}^{e,R} \):

\[ PC_{t}^{e,N} = \eta^N + \Psi^N \tilde{X}_t^N + \nu_{t}^{e,N}, \]

\[ PC_{t}^{e,R} = \eta^R + \Psi^R \tilde{X}_t^R + \nu_{t}^{e,R}. \]

Here \( PC_{t}^{e,N} \) (\( PC_{t}^{e,R} \)) denote the principal components of the nominal (real) yields other than \( \tilde{X}_t^N \) (\( \tilde{X}_t^R \)). Hence, \( (\tilde{X}_t, PC_{t}^{e,N'}, PC_{t}^{e,R'})' \) follows a restricted VAR(1) whose coefficients are nonlinear functions of \( \mu, K, \Sigma, \lambda^N \) and \( \lambda^R \). Hamilton and Wu (2012) propose to first estimate \( \tilde{\mu}, \tilde{\Phi}, \eta^N, \Psi^N, \eta^R, \Psi^R \) and the covariance matrix of the innovations by OLS and find structural parameter values
such that the implied reduced-form parameters are close to the OLS estimates.\textsuperscript{23}

Bauer, Rudebusch and Wu (2012) show that OLS estimates tend to underestimate the persistence of $\tilde{X}_t$, introducing a bias that can be economically significant. We correct for this bias in the OLS estimates using a method proposed in Kilian (1998).

### 4.4 Brazilian Yield Curve Estimates

We next summarize some of the key estimates of the term structure model fitted to the Brazilian data. Further details are provided in Appendix B.

Following conventional practice, we extract principal components (PCs) from our daily panel of zero-coupon bond yields with maturities ranging from six months to five years. The first three PCs explain 95%, 4.4%, and 0.4%, respectively, of the daily variation in nominal yields. Thus, in line with results for the U.S. bond market, three principal components explain almost all of the variation in daily bond yields. For the real yields, the first three PCs explain 95%, 4%, and 0.5% of the daily variation.\textsuperscript{24} All PCs are highly persistent, with one-day autocorrelations exceeding 0.99 and one-month autocorrelations above 0.70.

The unique availability of daily macroeconomic forecasts allows us to estimate our models with unspanned macro factors at the daily frequency. Inflation expectations, $\hat{\pi}_t$, and the predicted exchange rate depreciation, $\hat{E}X_t$, are obvious candidates as predictors of future short-term rates for a relatively open economy such as Brazil. Hence, we augment the vector of PCs with a pair of unspanned macro risk factors, $X_t^M = (\hat{\pi}_t, \hat{E}X_t)'$, obtaining a state vector

$$\tilde{X}_t = (PC_{1t}^N, PC_{2t}^N, PC_{3t}^N, PC_{1t}^R, PC_{2t}^R, PC_{3t}^R, \hat{\pi}_t, \hat{E}X_t).$$

A key argument for including unspanned factors in the term structure model is that they can help improve forecasts of future interest rates. It seems appropriate to use forward-looking survey forecasts and so incorporate not just current information but agents’ expectations of future outcomes.

To formally test the importance of unspanned macro factors, Table 3 reports the outcome of a set of Wald tests that capture the individual and joint statistical significance of the inflation and exchange rate survey forecasts. For comparison with existing studies, we report not only daily but also monthly results. Panel A shows that the inflation and exchange rate forecasts are individually as well as jointly significant at the 1% critical level at both the daily and monthly horizons, when serving as unspanned risk factors in the nominal yield model. For the real yields (Panel B), exchange rate expectations are statistically significant at the 10% level on their own.

\textsuperscript{23}These estimates are as efficient as maximum likelihood estimates. In fact, following Joslin, Singleton and Zhu (2011) and Hamilton and Wu (2012), the resulting estimates of $\tilde{\mu}$ and $\tilde{\Phi}$ are identical to the OLS estimates. Because there is a one-to-one mapping from $\{\mu, \Phi\}$ to $\{\tilde{\mu}, \tilde{\Phi}\}$, it suffices to report estimates of the latter.

\textsuperscript{24}Yield data on real bonds with maturities shorter than 18 months are unreliable during a few points in our sample. We use an EM algorithm to replace these (few) data points, see Appendix C.
as well as in combination with the inflation forecasts. This holds both for the daily and for the monthly data. Inflation expectations are far less useful for predicting changes in real yields than for predicting changes in nominal yields, which seems economically plausible\textsuperscript{25}

\section*{4.5 Expected Short Rates, Inflation, and Term Premia}

We next use our affine term structure model to derive expressions for the components shifting the yield curve. Under the physical probability measure, $E^P_t[\cdot]$, the $\tau$-period-ahead expected short interest rate is computed as

$$E^P_t (r^N_{t+\tau}) = \delta^N_0 + \delta^N_1 \bar{X}^N_t,$$

(23)

The integral of the expected value of $X_{t+\tau}$ between $t + p$ and $t + p + q$ is computed as

$$E^P_t \left( \int_p^{p+q} X_{t+\tau} d\tau \right) = K^{-1} \left( e^{-K(p+q)} - e^{-Kp} \right) (X_t - \mu) + q\mu.$$

(24)

Hence, the average nominal short rates from $t + p$ to $t + p + q$, $\bar{r}^N_t(p,q)$, can be computed as

$$E^P_t (\bar{r}^N_t(p,q)) = \delta^N_0 + \delta^N_1 \mu + \frac{1}{q} \delta^N_1 K^{-1} \left( e^{-K(p+q)} - e^{-Kp} \right) (X_t - \mu).$$

(25)

Using the estimated parameters from our term structure model, the top panel in Figure 6 plots the one-year nominal rate, $y^N_t(1)$, and the average expected short interest rate over the next year, $E^P_t \left( \int_0^1 r^N_{t+s} ds \right)$. The two series are very similar as both are affected by a broad downward trend from 2006 to 2013, interrupted by the Global Financial crisis before trending upwards between 2013 and the end of 2015 and finally trending downwards during 2016 and 2017.

The second panel in Figure 6 plots the nominal term premium constructed from the difference between the nominal forward rate and the average expected short rate from period $t + p$ to period $t + p + q$:

$$TP^N_t(p,q) = f^N_t(p,q) - E^P_t (\bar{r}^N_t(p,q)),$$

(26)

where the “in-$p$-years for-$q$-years” nominal forward rate at time $t$, $f^N_t(p,q)$, is computed as $f^N_t(p,q) = [(p+q)y^N_t(p+q) - py^N_t(p)] / q$.

The nominal term premium fluctuates between -1% and 3% over the course of our sample, peaking at the height of the Global Financial crisis in the fall of 2008 and, again, during the onset of the Brazilian recession in 2016.\textsuperscript{26}

\textsuperscript{25} A set of Wald tests show that once inflation and exchange rate survey expectations are included in the model for bond yields, other survey forecast variables are no longer significant predictors of variation in the term structure.

\textsuperscript{26} The plots in Figure 6 use the JPS model with unspanned macro factors. However, the term premium calculated under a GDTSM model without macro factors follows the same broad trend and the two term premium series have a correlation of 0.96.
The third panel in Figure 6 shows that the one-year real bond yield trended down from 12% in 2006 to 2% in 2013. Between 2013 and early 2015, the real rate rises to 7% before declining sharply at the end of the sample. We can compute an estimate of the real term premium from

\[ TP_t^R(p,q) = f_t^R(p,q) - E_t^P(\hat{r}_t^R(p,q)) , \]

where \( f_t^R(p,q) \) is the real forward rate from period \( t + p \) to period \( t + p + q \). The bottom panel in Figure 6 shows that the real term premium fluctuates around 1.5% from 2006 to 2010 and is very volatile between mid-2008 and mid-2009. During the relatively benign economic environment between 2010 and 2013, the real term premium drops from about 1.5% to a value just below zero where it stays until early 2016. During the 2016-17 recession the term premium rises to 1%.

Using the Fisher equation in (7), we can construct a measure of the average expected inflation rate from \( t + p \) to \( t + p + q \), \( \bar{\pi}_t(p,q) = \left( \int_{p}^{p+q} \pi_{s+t} ds \right) / q \),

\[ E_t^P[\bar{\pi}_t(p,q)] = E_t^P[\bar{\pi}_t^N(p,q)] - E_t^P[\bar{\pi}_t^R(p,q)]. \]

Finally, the inflation term premium can be expressed as the difference between the break-even rate and the expected inflation:

\[ TP_t^\pi(p,q) = f_t^N(p,q) - f_t^R(p,q) - E_t^P[\bar{\pi}_t(p,q)]. \]

We use these formulas to extract information on expected short rates, inflation and term premia from the yield curves.

To get a better sense of these relations, the top panel in Figure 7 plots the actual inflation rate against the expected inflation rate generated by our term structure model, in both cases using a one-year horizon. The two series clearly follow a common trend. The inflation term premium, shown in the bottom panel of Figure 7, is quite volatile and peaks at the beginning of 2016, shortly after Brazil had experienced inflation of 15%.

5 Monetary Policy Shocks and Economic Uncertainty

This section uses our new survey-based measures to provide more details on the effect of target rate shocks on economic uncertainty and term premia. Piazzesi et al. (2006), Rudebusch and Swanson (2008) and Wright (2011) argue that uncertainty about future inflation is a key driver of bond term premia.\(^{27}\) Having direct evidence on the uncertainty channel in the transmission of monetary policy shocks is, therefore, important.

\(^{27}\)Wright (2011) examines the evolution in term premia across ten international bond markets and finds that term premia are positively related to the dispersion in survey forecasts of inflation, suggesting that reduced uncertainty about inflation is a key reason for the secular decline in term premia observed in these markets.
5.1 Inflation Uncertainty and Term Premia

Our daily data on dispersion in survey forecasts of GDP growth, inflation, and exchange rates allow us to undertake a broad analysis of the relation between macroeconomic uncertainty and variation in nominal and real term premia. As we next demonstrate, our results offer new insights into the broader economic determinants of real and nominal term premia.

We begin by regressing our daily values of real and nominal term premia on survey dispersion in inflation forecasts, GDP growth forecasts and exchange rate forecasts, all computed using a one-year forecast horizon. Table 4 shows results from these regressions. First, consider the results for the nominal term premium (Panel A). Consistent with Wright (2011), we find a strongly positive relation between dispersion in survey forecasts of inflation and the nominal term premium establishing that higher inflation uncertainty is associated with a higher nominal term premium. Moreover, daily variation in the inflation survey dispersion explains about 10% of the variation in the nominal term premium.

Survey dispersion in GDP growth also has a significantly positive correlation with the nominal term premium and the explanatory power of this variable ($R^2 = 0.195$) is nearly twice as high as that of the inflation rate survey dispersion. Including both dispersion measures simultaneously increases the $R^2$ to 0.282, with both measures having a highly significant effect.

Finally, dispersion in survey forecasts of the Brazilian exchange rate is significantly positively associated with nominal term premia. Once all three dispersion measures are included in the model, only the survey dispersion for GDP growth and inflation remain statistically significant, however.

These results suggest that variation in the nominal term premium is driven not only by inflation uncertainty but gets affected by uncertainty about GDP growth as well so that macroeconomic uncertainty more broadly affects nominal risk premia.

Turning to the real term premium (Panel B), we find a positive relation between the real term premium and the survey dispersion in forecasts of GDP growth and the dollar-real exchange rate. In both cases the relation is statistically significant at the 1% level and the explanatory power is quite high with $R^2$ values of 0.171 and 0.118, respectively. Conversely, the relation between dispersion in inflation forecasts and the real term premium is statistically insignificant with very little explanatory power ($R^2 = 0.015$).

These findings are in line with what we might expect from economic theory: real term premia should not be affected by inflation uncertainty and instead be driven by uncertainty about determinants of real economic growth such as GDP growth and, for an open economy such as the Brazilian one, exchange rate movements.

5.2 Survey Dispersion and Monetary Policy Shocks

We next explore how inflation uncertainty is affected by monetary policy shocks, separately addressing short-run and long-run effects.
5.2.1 Short-run Effects

We first use our daily measures of dispersion in the one-year-ahead inflation survey forecasts to estimate the short-run effect of a higher-than-expected target rate on inflation uncertainty. Specifically, we regress \( h \)-day log-changes in the inflation survey dispersion following central bank announcements on the unexpected change in the target rate and a vector of control variables:

\[
\log \sigma_{t+h}^\pi - \log \sigma_{t-1}^\pi = \beta_h r_{t-1}^u + \text{control}_{t-1} + \epsilon_{t-1,t+h}, \tag{30}
\]

where the control variables are observed on the day before the announcement date (day \( t \)). Our control variables include the short-term swap rate (\( \text{swap}_t \)), survey forecasts of GDP growth (\( \hat{\text{GDP}}_t \)), inflation (\( \hat{\pi}_t \)) and the exchange rate (\( \hat{E}_t \)), and log survey dispersions in GDP growth, inflation and exchange rate forecasts, \( \log \hat{\sigma}_g^t \), \( \log \hat{\sigma}_\pi^t \), \( \log \hat{\sigma}_{ex}^t \), respectively.

Using the regression in (30), Figure 8 plots the effect of a 1% unexpected increase in the COPOM target rate on the log dispersion in the inflation survey. The estimated coefficient \( \beta_h \) is negative, displays a downward trend, and becomes statistically significant after three weeks. The economic magnitude of the policy shock on inflation dispersion is also large as we observe a 20% decrease in the inflation survey dispersion after three weeks.

5.2.2 Long-run Effects

To measure the effect of a monetary policy shock on inflation uncertainty at longer horizons such as a few months, we use an approach that combines a VAR model and our event study approach as in Gertler and Karadi (2015). Let \( Y_t \) denote the vector of state variables

\[
Y_t = (\text{COPOM}_t, \text{swap}_t, \text{GDP}_t, \hat{\pi}_t, \hat{E}_t, \log \hat{\sigma}_g^t, \log \hat{\sigma}_\pi^t, \log \hat{\sigma}_{ex}^t)^\prime.
\]

Because revisions to daily survey data can be sluggish on days without big macro economic news and the Central Bank target rate only changes when COPOM meetings are held, we build our VAR model using data recorded at intervals of 20 business days (one month) rather than at the daily horizon. To this end, consider the VAR for \( Y_t \):

\[
Y_t = A + BY_{t-20} + \nu_{t-20,t}. \tag{31}
\]

Assuming that \( I - B \) is invertible, \( Y_t \) can be expressed as

\[
Y_t = (I - B)^{-1}A + C(L^{20})\nu_{t-20,t}, \tag{32}
\]

where \( L^{20} \) is the lag operator \( L^{20}(\epsilon_{t-20,t}) = \epsilon_{t-40,t-20} \) and \( C(L) = C_0 + C_1L + C_2L^2 + \cdots \).
The reduced form shocks, $\nu_{t-20,t}$, can be expressed as a function of the structural shocks, $\xi_{t-20,t}$:

$$\nu_{t-20,t} = \Theta_0 \xi_{t-20,t},$$ \hspace{1cm} (33)

where the first element of $\xi_{t-20,t}$ is a monetary shock that is uncorrelated with other shocks. $\xi_{t-20,t}$ also includes shocks to other macroeconomic variables such as economic growth and inflation which might explain changes in the COPOM rate. Our proxy for shocks to these unobserved macroeconomic state variables is the innovations to the survey forecasts of GDP growth, inflation and exchange rates.

Stock and Watson (2018) show that the $h$-period-ahead dynamic causal effect of a monetary policy shock can be estimated consistently provided that $\Theta_0$ is nonsingular. This invertibility condition is plausible if the Brazilian Central bank determines future target rates based on forecasts of future inflation, GDP growth and exchange rates. To assess if the assumption of invertibility is valid for our VAR, we use a parametric bootstrap to conduct a Hausman test on the difference between $h = 6, 12, \text{and} 18$-month impulse response functions estimated from a structural VAR versus from local projections. With $p$-values of 0.15, 0.45 and 0.24 at the three horizons, the Hausman test fails to reject the null hypothesis of invertibility.

The $h$-period impulse response of a one-unit shock to the first variable in the VAR is

$$\hat{\beta}_h = \hat{C}_h \hat{\Theta}_{0,1},$$

where $\Theta_{0,1}$ is the first column of $\Theta_0$. Under the conditions outlined by Stock and Watson (2018), this is also a dynamic causal effect.

As in Stock and Watson (2018), we set the first element of $\Theta_{0,1}$ equal to unity. The remaining elements in $\Theta_{0,1}$ characterize the effect of unexpected shocks to the COPOM rate on the penultimate elements in $Y_t$, which can be estimated from daily data as in Gertler and Karadi (2015).

Following Gertler and Karadi (2015), we assume that the relationship between $\nu_{t-d,t}$ and its structural counterpart is the same for $d \leq 20$:

$$\nu_{t-d,t} = \Theta_0 \xi_{t-d,t}.$$

Our survey data provides us with a measure of the unexpected (shock) component of the change in the COPOM rate on central bank meeting days. We treat this as an instrument for the monetary policy shock and assume that this is independent of other macroeconomic shocks:

$$E(\xi_{t,(1)} u_{t-1}) = E((r_{t-1}^u)^2),$$

$$E(\xi_{t,(2:n)} u_{t-1} | \text{control}_{t-1}) = 0.$$
Under this instrumental variables assumption, the $i^{th}$ element of $\Theta_{0,1}$ satisfies

$$\Theta_{0,i1} = \frac{E(Y_{i,t+d-1}r_{t-1}^{u}|Y_{t-1})}{E((r_{t-1}^{u})^2)},$$

which can be estimated by regressing $Y_{i,t+d-1}$ on $r_{t-1}^{u}$ and $Y_{t-1}$. We measure the response of swap rates using a daily window ($d = 1$) and measure the response of survey variables and survey dispersions over five business days ($d = 5$) in order to allow for possible delays in survey revisions.

Figure 9 plots the estimated impulse response function. Impulse responses are negative and significant for horizons lasting between two and twenty four months, with effects that start at -0.1, increasing to -0.05 at the two-year horizon. This evidence suggests, first, that tighter monetary policy reduces inflation uncertainty and, second, that monetary policy shocks have a long-run effect on inflation uncertainty as perceived in our survey of inflation forecasts.

6 Monetary Policy Shocks and the Term Structure

Section 5 documented a positive correlation between nominal term premia and inflation uncertainty and showed that positive shocks to monetary policy (i.e., a tightening) have a strong and persistent negative effect on inflation uncertainty. These findings indicate that monetary policy shocks influence nominal term premia and inflation term premia through their effect on inflation uncertainty. We next test this hypothesis by examining the effect of monetary policy shocks on the nominal and real yield curves and their components.

6.1 Policy Shocks, Inflation Expectations and Inflation Term Premia

Inflation term premia compensate for the risk associated with uncertain future inflation rates. By locking in the nominal forward rate from $t+p$ to $t+p+q$ and shorting the real forward rate from $t+p$ to $t+p+q$, investors can enter an interest rate swap whose real payoff is negatively correlated with inflation from $t+p$ to $t+p+q$. The inflation term premium is the difference between the break-even inflation rate and the expected inflation:

$$TP_t^\pi(p,q) = f_t^N(p,q) - f_t^R(p,q) - \mathbb{E}[\bar{\pi}_t(p,q)].$$

The two forward rates in (34) are observable and the expected inflation rate can either be measured using our survey data or be derived from our term structure model.

Table 5 reports results from applying these two approaches. Regressing changes in the survey-based or model-implied expected inflation rate on the expected and unexpected shifts in the COPOM rate. We find little evidence of a systematic relation between inflation expectations and either expected or unexpected changes in the COPOM rate (Panel A). If anything, there is a small negative effect of unexpected COPOM rate shocks at the three- and four-year horizons where
a positive 100 bp shock to the COPOM rate is associated with a 9 bp reduction in the inflation rate.

In contrast, we find a significantly negative and economically large effect of unexpected changes to the COPOM rate on the inflation term premium (Panel B). The effects of an unexpected 100 bp rise in the COPOM rate on the model-implied inflation term premia are -33, -34, -42, -43 bps, respectively, for one-year bonds expiring 1, 2, 3, and 4 years from now. The effect on the survey-based inflation term premium is very similar (-26 bps) for one-year bonds expiring a year from now. Thus, tighter-than-expected monetary policies are associated with large reductions in inflation term premia, with slightly larger effects at the long horizons.

To further explore the “uncertainty channel” hypothesis, we add to our regression for the change in the inflation term premium the cross-product of the unexpected change in the COPOM rate \( r_{t-1}^u \) and the normalized (mean zero, unit standard deviation) survey dispersion in inflation expectations, \( \tilde{\sigma}_t^{\pi} \), measured on the first day of the COPOM meeting:

\[
\Delta TP_t^{\pi}(p,q) = \beta_1 \Delta r_{t-1}^e + \beta_2 r_{t-1}^u + \beta_3 \tilde{\sigma}_t^{\pi} r_{t-1}^u + \varepsilon_t. \tag{35}
\]

Estimates of \( \beta_3 \), presented in Panel C of Table 5, allow us to test whether the effect of monetary policy shocks on inflation term premia depends on the level of inflation uncertainty at the time of the shock. We find significantly negative estimates of \( \beta_3 \) ranging from -0.16 for bonds expiring one year from now to -0.33 for bonds expiring in four years.\(^{28}\)

The magnitude of \( \beta_3 \) is greater than half of the unconditional effect of unexpected target rate changes, as measured by \( \beta_2 \). This suggests that the effect of monetary policy shocks is 50-100% higher when inflation uncertainty is one or two standard deviations above its average. Specifically, when inflation survey dispersion is at its mean, a 100 bp shock to the target rate is associated with 28 and 34 bp changes in the term premia at one- and four-year horizons, respectively. Raising the inflation survey dispersion by one standard deviation, the corresponding effects are increased to 47 and 67 bps, respectively. Finally, if inflation survey dispersion is two standard deviations above its normal level, the effects of a 100 bp shock are 59 bps and 100 bps, respectively. Our evidence is, thus, consistent with the inflation term premium being reduced substantially more by a positive shock to the COPOM rate in states with high uncertainty about inflation as captured by our survey dispersion measure.

We conclude from this evidence that tighter-than-expected monetary policies reduce the inflation risk premium embedded in Brazilian bond prices, consistent with the reduced inflation uncertainty documented in Section 5. Moreover, the effect is stronger in states with high inflation uncertainty. On the other hand, monetary policy shocks have little effect on expected inflation, highlighting the importance of risk for transmitting monetary policy shocks.

\(^{28}\)Reassuringly, the one-year estimate of \( \beta_3 \) based on the survey data (-0.20) is very similar to the model-based estimate.
6.2 Effect of Policy Shocks on Nominal and Real Yields

We next show that the strong response of the inflation term premium, along with the weak response of the expected inflation rate, produces stark differences between the impact of monetary policy shocks on nominal and real yields in Brazil. We report results using regressions of the form

$$\Delta y_t = \beta_1 \Delta r^e_{t|t-1} + \beta_2 \tau^u_t + \varepsilon_t,$$

where $\Delta y_t$ now tracks changes to yields, nominal and real expected future short rates, or nominal and real term premia.

Panel A of Table 6 shows that an unexpected 100 bp increase in the Brazilian target rate raises the one-year nominal yield by a statistically significant 60 bps. This is broadly similar to estimates for the US such as Cochrane and Piazzesi (2002) (62 bps) and Kuttner (2001) (72 bps).

COPOM rate shocks have a much smaller effect on the medium and long end of the nominal term structure in Brazil: At the three and four-year horizons, unexpected 100 bps shocks to the target rate are associated with statistically insignificant 23 and 12 bp changes in the nominal bond yield. In contrast, Kuttner (2001), Cochrane and Piazzesi (2002), Beechey and Wright (2009), Hanson and Stein (2015) and Nakamura and Steinsson (2018) find that monetary policy shocks in the US have a significant effect on ten-year nominal rates, ranging from 32-50 bps for a 100 bp shock to the Fed target rate.

The small medium- and long-term effects of COPOM rate shocks on nominal yields are mirrored in insignificant, even negative, estimates of the effects on nominal forward rates at the two, three, and four-year horizons (Panel B). These small effects reflect economically large and statistically significant coefficients for the underlying components which largely cancel out: COPOM rate shocks have a positive and significant effect on expected future short rates (declining from 82 bps to 54 bps for a 100 bp shock as the horizon is extended; Panel C) but a negative and significant effect on nominal term premia at the three and four-year horizons (rising to 75 bps; Panel D).

Essentially, monetary policy shocks only affect nominal yields at the shortest end of the Brazilian term structure because of the large negative term premium effect observed at the longer three- and four-year horizons.

Shocks to the COPOM rate have a longer-lasting effect on real yields and forward rates than on their nominal counterparts (Panels E and F). For example, a 100 bp shock to the COPOM rate is associated with a significant increase of 51 bps in the four-year Brazilian real yield. Our estimates are comparable with estimates for the US term structure reported by Hanson and Stein (2015) (five- and 10-year year rates increase by 65 and 42 bps, respectively for a 100 bp shock to the two-year Treasury yield) and Nakamura and Steinsson (2018) (a 0.64 unit increase in the five-year real yield per unit of change in the one-year nominal rate), though a bit smaller than the estimates in Beechey and Wright (2009) (increases of 162 bps and 50 bps in the five- and ten-year real bond yield for each 100 bp increase in the Fed funds futures rate).
The significant medium-to-long run effect of COPOM rate shocks on real yields is reflected in these shocks’ large effect on real forward rates even at the four-year horizon (Panel F). In turn, the large effects of monetary policy shocks on real forward rates is a result of large effects on expected real short rates ranging from 57-71 bps for an unexpected 100 bp rise in the COPOM rate (Panel G). Although real term premia tend to be reduced at the longer maturities (three and four years), the effects are smaller in magnitude (10-30 bps; Panel H) and so, unlike in the nominal case, do not cancel out against the effect on expected real short rates.

We conclude that monetary policy shocks have a significant effect on the real yields of all bond maturities studied here. In contrast, their effect on nominal yields is much weaker—particularly at the longer horizons—as their negative effect on nominal term premia largely offsets the positive effect on the expected path of future short rates.

6.3 Importance of Unspanned Macro Factors

Table 3 shows that unspanned macro factors make a significant contribution to the dynamic term structure models fitted to the Brazilian bond data. Moreover, information from the unspanned macro factors is especially relevant on COPOM announcement days when there is an influx of macroeconomic news: The variances of the daily changes in the average inflation and exchange rate survey forecasts are 40% and 133% higher, respectively, on announcement versus non-announcement days.

As an alternative way to illustrate the empirical importance of including unspanned macro factors in our term structure model, we can compare one-day responses of expected nominal short rates computed using term structure models with and without unspanned factors.29

Figure 10 undertakes this comparison, plotting the effect of unexpected changes in the COPOM rate on expected nominal short rates for both models. The effect of an unexpected change in the COPOM rate is very similar at the shortest one- and two-year horizons. Conversely, the two models produce very different results for the expected path of short rates for bonds with three or four years of maturity with the impact of an unanticipated change in the COPOM rate estimated to be much smaller under the simple model that excludes unspanned factors.

Given that unspanned macro factors can help predict short rates, these plots illustrate how incorporating such factors into the term structure model helps to significantly reduce biases in estimates of the effects of monetary policy shocks on expected short rates and term premia.

6.4 Alternative Measure of Policy Shocks

Our survey measure of the unanticipated change in the COPOM rate could be affected by measurement errors due to staleness in survey participants’ updates to their entries, bias in the composition

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29 Term structure studies that use the earlier class of GDTSMs include Dai and Singleton (2002), Ang and Piazzesi (2003), Cochrane and Piazzesi (2009), and Joslin, Singleton and Zhu (2011).
of participants, etc. We therefore compare our survey measure to the more conventional measure—the change in the two-year government bond yield on policy announcement days—used by Hanson and Stein (2015) and Abrahams et al. (2016).

To this end, we first regress our survey-based measure of monetary policy shocks on changes in two-year bond yields on the day following the 94 COPOM meetings. We find a statistically significant slope of 0.40 and an $R^2$ of 0.12. Hence, while changes in market rates are clearly correlated with the surprise component identified in the survey data, the relationship is surprisingly weak. As argued earlier, this may reflect simultaneous movements in expected future short rates and term premia, both of which affect the two-year Treasury rate.

Next, we undertake a series of regressions similar to those in Table 6, for the nominal term premia and inflation risk premia. These regressions include the expected and unexpected components of the survey forecasts as well as changes in the two-year government bond yield, orthogonalized with respect to the unexpected change in the COPOM rate calculated from the survey forecasts:

$$\Delta y_t = \beta_1 r^e_{t-1} + \beta_2 r^u_t + \beta_3 \Delta \text{yield}^\perp_{(0,2)} + \varepsilon_t.$$  

Our analysis focuses on risk premium effects because we expect the survey-based forecasts to provide a less noisy estimate of the policy shock that more directly reveals the effect of policy changes on risk premia than market rates which confound changes in expectations of future short rates and term premium effects.

Table 7 shows the results from these regressions. Unexpected COPOM rate changes extracted from the survey forecast data continue to have a highly significant and negative correlation with nominal and inflation term premia even after including (orthogonalized) changes to the two-year bond rate. In contrast, orthogonalized changes in the two-year Treasury rate are not significantly associated with these term premia. In this sense, the conventional market rate measure of monetary policy shocks is dominated by our survey-based measure.

These results illustrate the advantage of using our direct measure of the unexpected change in the COPOM rate, rather than a conventional market-based measure such as the change in the two-year government bond yield.

7 Fed Policy Shocks and the Brazilian Term Structure

The U.S. Federal Reserve undertook a set of important policy measures during our sample which includes the global financial crisis and its aftermath. For example, the Fed engaged in a series of unconventional quantitative easing policies such as setting the target range of the federal funds rate between zero and 25 basis points (the “zero lower bound”) and implementing large-scale asset purchases (LSAPs) of Treasury securities and other types of bonds. Prior to this period, but still included in our sample, there were many regular FOMC meetings.
Actions by the Federal Reserve might have an impact on the Brazilian term structure. Most obviously, expansionary monetary policies in the US could affect flows in and out of Brazilian bonds through the concomitant changes in global liquidity and through changes in the dollar-real exchange rate. A substantial amount of Brazilian nominal and real bonds are held by foreign investors and these investors will command higher term premia if monetary policy shocks in the US increase exchange rate uncertainty. In this section, we examine the role of economic uncertainty in transmitting US monetary shocks to Brazil yield curves.

7.1 Fed Policy Shocks and Economic Uncertainty in Brazil

We start by analyzing how Fed policy shocks affect economic uncertainty in Brazil. Tighter-than-anticipated Brazilian monetary policy is likely to reduce domestic inflation uncertainty, but the effect of tighter US monetary policy shocks on economic uncertainty in Brazil is less clear.

Our analysis merges the dates of FOMC meetings during our sample from 2006-2017 with the dates from 11/25/2008 through 9/21/2011 identified by Wright (2012) as days on which the Federal Reserve announced monetary policy measures during the zero lower bound period. This gives us a set of 93 dates with Fed policy announcements.

To measure monetary policy surprises, we follow Hanson and Stein (2015) and Abrahams et al. (2016) and use the one-day change in the two-year nominal yield on the 93 Fed event days in our sample with announcements by the FOMC. We use these changes as indications of the unanticipated effects of the Fed’s actions on the US bond market.

Next, we regress log-changes in the cumulative $h$-day-ahead survey dispersion ($\sigma_{t+h}$) on changes to the two-year US Treasury yield, $\Delta y_{t,(0,2)}$, and a vector of control variables:

$$\log(\sigma_{t+h}) - \log(\sigma_{t-1}) = \beta_h \Delta y_{t,(0,2)} + \text{control}_{t-1} + \epsilon_{t,t+h}. \tag{37}$$

As control variables we use the variables listed below equation (30) and also include the lagged value of the two-year US Treasury yield.

Figure 11 plots the outcome of this regression for different values of $h$ using survey dispersion for the exchange rate, inflation, and GDP growth rate variables. Positive shocks to US interest rates are associated with greater uncertainty about the dollar-real exchange rate (top panel) and the effect is increasing in the horizon, becoming statistically significant after three weeks. A 1% increase in the two-year US T-bill rate is associated with a 40% increase in the exchange rate survey dispersion one month after the announcement. Hence, US monetary policy shocks have

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30 The Federal Reserve’s policy decisions are interesting also from another perspective: While concerns about endogeneity cannot be dispelled in the case of BACEN’s decisions on the COPOM rate, changes to the US Federal Reserve’s policy are unlikely to have been influenced by the state of the economy in Brazil and, thus, can plausibly be treated as exogenous shocks.

31 These dates include days with announcements of Fed asset purchases, days with speeches given by Ben Bernanke, and FOMC meeting days.
economically important and fairly long-lasting effects on exchange rate uncertainty. In contrast, we fail to find significant effects on survey dispersion for inflation and GDP growth (middle and bottom panels).

Our results show that positive shocks to US nominal bond yields increase the riskiness of Brazilian bonds. In particular, higher US interest rates resulting from a tightening in US monetary policy are associated with significantly higher dispersion in survey expectations of the $US-R$ exchange rate, indicating that Brazilian bonds are perceived as riskier to investors.

### 7.2 US Interest Rate Shocks and the Brazilian Term Structure

We next explore how the Fed’s monetary policy decisions affect the Brazilian term structure. A simple regression of daily changes in Brazilian nominal bond yields on changes in the two-year US yield on the 93 FOMC meeting days (reported in Panel A of Table 8) shows that 100 bp increases in the US yield are associated with increases of 33, 55, 67 and 73 bps in the yields of Brazilian bonds expiring one, two, three and four years from now, respectively. Higher U.S. interest rates are, thus, associated with significantly higher Brazilian bond yields and the effect of US monetary policy shocks is strongest at the longer end of the Brazilian yield curve. This is in marked contrast to our findings for Brazilian COPOM rate shocks which do not impact Brazilian yields for bond maturities beyond one year (Panel A in Table 6).

These regressions do not, of course, show whether the spillover on Brazilian bond yields reflects higher short-term interest rate expectations or movements in the Brazilian term premium. To address this issue, we conduct a similar analysis as that undertaken in Section 6. We find that US policy shocks have a negligible effect on expected nominal short rates (Panel C) but identify a large and highly significant positive effect on nominal term premia in the Brazilian bond market (Panel D). Interestingly, the effect is positive and grows with the bond maturity: a 100 bp shock to the two-year US yield is associated with a 31 bp increase in the one-year-ahead nominal term premium, a 69 bp increase in the two-year-ahead term premium and increases around 100 bps in Brazilian term premia at the three and four-year horizons.

Further analysis shows that changes to US Treasury rates associated with Fed policy announcements have little or no effect on Brazilian inflation expectations (Panel E) but have a sizeable positive impact on the inflation term premium (Panel F) ranging from 26 to 52 bps for each 100 bp change in the two-year US Treasury rate for the three longest bond maturities.

Our results suggest that whereas higher-than-expected COPOM rates lead to a lower one-year bond yield in Brazil but have no effect on bonds with longer maturities, higher-than-expected US interest rates have the opposite effect, i.e., they increase Brazilian bond yields for both short and long maturities. In contrast to domestic shocks, US interest rate shocks do not affect expected nominal short rates in Brazil. However, they have a large effect on nominal and inflation term premia in Brazil and so US monetary policy shocks affect Brazilian interest rates primarily through
a risk (term) premium channel.

We also identify a significant relation between shocks to the two-year US Treasury rate and real yields (Panel G), real forward rates (Panel H), and real term premia (Panel J) although, as for the nominal bond yields, expected real short rates appear not to be affected by such changes (Panel I). Specifically, an unanticipated 100 bp increase in the two-year US Treasury yield raises real yields in Brazil by 20-40 bps, with the largest effect seen for the two longest maturities. Because shocks to US interest rates have no effect on expected real short rates in Brazil, such shocks affect real yields in Brazil entirely through higher real term premia. Consistent with this, Panel J shows that a 100 bp increase in US interest rates is associated with increases of about 50 bps in Brazilian real term premia for the three longest bond maturities.

In conclusion, our results show that unanticipated shocks to US monetary policy do not have any notable effect on expected nominal and real short rates or on the expected inflation rate in Brazil. However, they have a large and statistically significant effect on both nominal and real term premia in the Brazilian bond market and, therefore, on yields and forward rates.

Moreover, we identify an interesting asymmetry in the effect of monetary policy shocks depending on whether they originate from Brazil or from the US. While higher-than-expected COPOM rates set by the Brazilian central bank have an unambiguously negative effect on nominal and real term premia in Brazil, higher-than-expected US rates have a positive effect on both nominal and real term premia in the Brazilian bond market.

These findings are related to results by Albagli et al. (2019) who use a panel data set to study spillover effects from US monetary policy to bond markets in developed and emerging market countries. Consistent with our evidence, Albagli et al. (2019) find that higher US interest rates result in higher nominal bond yields also in emerging markets. They attribute this to a demand effect as yield-seeking investors shift their demand away from emerging bond markets when US interest rates increase, leading to higher bond rates also in emerging markets.

Our evidence suggests that an alternative risk-based effect may also be at work: Foreign investors shun away from R$-denominated assets when faced with elevated exchange rate uncertainty following shocks indicating a tightening of US monetary policies.

### 7.3 Portfolio Flows

This finding reflects the complex effect that a policy tightening in the US has on the Brazilian bond market. Higher US rates are likely to affect the Brazilian bond market—as well as the broader economy—both through the dollar-real exchange rate which affects trade patterns and through capital (portfolio) flows.

Higher US interest rates, if not matched by the Brazilian central bank, are likely to lead to capital flows out of Brazil and towards the US. To examine the importance of this channel, we regress equity flows into Brazil after FOMC announcement on the change in the two-year US
nominal yield on the 93 days with Fed-related news. We find that capital flow into Brazil are significantly correlated with US monetary shocks. Specifically, a 1% increase in the two-year US yield is associated with a daily equity outflow from Brazil averaging US$ 497m in the three-day window after FOMC announcements. This effect is statistically significant and larger than twice the size of the daily standard deviation of equity flows to Brazil (US$ 232 million.)

8 Conclusion

We use a unique data set containing daily survey forecasts of economic variables in Brazil to obtain direct measures of unanticipated changes in monetary policy and variation in economic uncertainty following central bank announcements.

Our analysis identifies fluctuation in inflation uncertainty as key to the transmission of monetary policy shocks in Brazil, with both inflation uncertainty and inflation term premia falling by a significant amount following a surprise tightening in policy. We document that shifts in economic uncertainty drive variation in term premia and demonstrate that this helps explain why the effect of monetary policy shocks on medium- and long-term nominal bond yields is far smaller in Brazil than in the US. Effectively, the strong term premium effects observed in the Brazilian bond market nullify the central bank’s control of long-term nominal yields.

We also identify strong spillover effects on the Brazilian term structure from shocks to US Treasury rates on days with Federal Reserve announcements. Using our data on survey dispersion, we identify variation in exchange rate uncertainty as being more important for transmitting US target rate shocks to the Brazilian term structure than variation in inflation uncertainty. Tighter-than-expected Fed policies—reflected in higher US Treasury yields—significantly raise both nominal and real term premia in Brazil and are associated with higher Brazilian bond yields of all maturities. Hence, Fed policy shocks have a very different effect on the Brazilian term structure than domestic policy shocks, with higher US interest rates raising Brazilian nominal and real term premia while higher Brazilian rates reduce them.

Our analysis uncovers complex interactions between economic uncertainty and monetary policy shocks. Monetary policy shocks are themselves a key source of economic uncertainty. Our findings for the Brazilian economy suggest that central banks’ control over long-term nominal yields is attenuated when inflation uncertainty is high and so complements recent findings that monetary policy is less powerful in recessions (Tenreyro and Thwaites, 2016). Economic uncertainty is, thus, an important state variable to be accounted for both when analyzing the monetary policy transmission mechanism and when designing stabilization policies.
References


Gagnon, Joseph, Matthew Raskin, Julie Remache and Brian P Sack. 2010. “Large-scale asset purchases by the Federal Reserve: did they work?” *FRB of New York Staff Report* (441).


### Table 1: Types and Characteristics of Brazilian government bonds

<table>
<thead>
<tr>
<th>Bond</th>
<th>Payoff index</th>
<th>Annual Coupons</th>
<th>Bond Maturities</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTN</td>
<td>Fixed notional</td>
<td>0%</td>
<td>6, 12, 24, 36 and 48 months</td>
</tr>
<tr>
<td>NTN-F</td>
<td>Fixed notional</td>
<td>10%</td>
<td>1, 3, 4, 5, 6 and 10 years</td>
</tr>
<tr>
<td>NTN-B</td>
<td>IPCA</td>
<td>6%</td>
<td>3, 5, 10, 20, 30 and 40 years</td>
</tr>
</tbody>
</table>

**Notes:** For each of the three types of Brazilian government bonds listed in the first column, the table shows whether the bond payments are fixed or linked to a price index. IPCA is the Índice Nacional de Preços ao Consumidor Amplo consumer price index. The table also shows the annual coupon rate along with the available bond maturities.

### Table 2: Survey variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Frequencies</th>
<th>Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation (IPCA)</td>
<td>Annual, monthly</td>
<td>1-5 years/1-13 months</td>
</tr>
<tr>
<td>COPOM rate(^1)</td>
<td>Annual, monthly</td>
<td>1-5 years/1-13 months</td>
</tr>
<tr>
<td>GDP</td>
<td>Annual, quarterly</td>
<td>1-5 years/1-5 quarters</td>
</tr>
<tr>
<td>Exchange Rate(^1) (Real/US Dollar)</td>
<td>Annual, monthly</td>
<td>1-5 years/1-13 months</td>
</tr>
</tbody>
</table>

**Notes:** This table lists the variables for which daily survey forecasts are recorded, along with the frequency of the variable, the number of daily forecasts for each event (target) date and the forecast horizons covered by the surveys. Each survey reports the mean, median, maximum, and minimum forecast along with the standard deviation computed across survey participants.

\(^1\) For this variable the surveys ask both for the end-of-period value (i.e., the value after a COPOM meeting) and the average value during a particular year.
Table 3: Wald Tests for significance of unspanned macro survey factors

<table>
<thead>
<tr>
<th>Panel A - Nominal Factor</th>
<th>Panel B - Real Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Daily Data</strong></td>
<td><strong>Monthly Data</strong></td>
</tr>
<tr>
<td>$\hat{\pi}_{t+1yr</td>
<td>t}$</td>
</tr>
<tr>
<td>$EX_{t+1yr</td>
<td>t}$</td>
</tr>
<tr>
<td>Joint</td>
<td>Joint</td>
</tr>
<tr>
<td>6.225***</td>
<td>4.269***</td>
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<tr>
<td>( 0.000)</td>
<td>( 0.007)</td>
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<tr>
<td>3.545**</td>
<td>4.657***</td>
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<tr>
<td>( 0.014)</td>
<td>( 0.005)</td>
</tr>
<tr>
<td>5.687***</td>
<td>3.009**</td>
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<tr>
<td>( 0.001)</td>
<td>( 0.029)</td>
</tr>
<tr>
<td></td>
<td>( 0.000)</td>
</tr>
</tbody>
</table>

| **Daily Data**           | **Monthly Data**      |
| $\hat{\pi}_{t+1yr|t}$  | $\hat{\pi}_{t+1yr|t}$ |
| $EX_{t+1yr|t}$           | $EX_{t+1yr|t}$        |
| Joint                    | Joint                 |
| 2.069                    | 1.256                 |
| ( 0.102)                 | ( 0.295)              |
| 2.228*                   | 2.251*                |
| ( 0.083)                 | ( 0.088)              |
| 1.772                    | 1.931                 |
| ( 0.150)                 | ( 0.122)              |
|                         | ( 0.062)              |

Notes: This table reports Wald test statistics for the significance of including inflation survey and exchange rate survey variables as unspanned factors in the term structure model. Panel A tests the null hypothesis that the loadings of the nominal factors on the survey forecasts equal zero, while Panel B tests the null hypothesis that the loadings of the real factors on the survey forecasts equal zero. Left panels use daily data while right panels use monthly data. The first row of each sub panel reports Wald statistics for testing that the loadings of the nominal or real bond factors on inflation forecasts equal zero. The second row of each sub panel reports Wald statistics for testing that the loadings of the nominal or real bond factors on the predicted exchange rate depreciation equals zero. The third row of each sub panel reports Wald statistics for testing the joint null hypothesis that the loadings of the nominal or real bond factors on both survey forecasts equal zero. P-values are reported in brackets. Our data cover the period from 2006/01/02 to 2017/09/27.
### Table 4: Nominal and real term premia and dispersion in survey forecasts

#### Panel A - Nominal term premium

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Inflation</th>
<th>GDP</th>
<th>Exchange Rate</th>
<th>Recession</th>
<th>R²</th>
<th>Observations</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>0.623***</td>
<td>0.571***</td>
<td>1.095***</td>
<td>0.105</td>
<td>2949</td>
</tr>
<tr>
<td></td>
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<td>0.420***</td>
<td>0.597***</td>
<td>0.457***</td>
<td>0.944***</td>
<td>0.195</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>-1.070**</td>
<td>0.247*</td>
<td>0.456***</td>
<td>0.357***</td>
<td>0.538</td>
<td>0.163</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>-2.159***</td>
<td>0.349***</td>
<td>0.524***</td>
<td>0.165</td>
<td>0.517</td>
<td>0.282</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>-1.789</td>
<td>0.312</td>
<td>0.703***</td>
<td>-0.111</td>
<td>0.652</td>
<td>0.187</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>-1.292***</td>
<td>0.336</td>
<td>0.400***</td>
<td>-0.002</td>
<td>0.394</td>
<td>0.244</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>-2.339***</td>
<td>0.595</td>
<td>0.400***</td>
<td>-0.002</td>
<td>0.336</td>
<td>0.290</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>-1.877***</td>
<td></td>
<td></td>
<td></td>
<td>0.595</td>
<td>0.401</td>
<td></td>
</tr>
</tbody>
</table>

#### Panel B - Real term premium

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Inflation</th>
<th>GDP</th>
<th>Exchange Rate</th>
<th>Recession</th>
<th>R²</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.341***</td>
<td>0.379</td>
<td>0.379***</td>
<td>0.314***</td>
<td>0.613***</td>
<td>0.015</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>0.944***</td>
<td>0.155</td>
<td>0.373***</td>
<td>0.332***</td>
<td></td>
<td>0.171</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>0.538</td>
<td>0.350</td>
<td>0.297***</td>
<td>0.175</td>
<td>0.086</td>
<td>0.118</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>0.517</td>
<td>0.407</td>
<td>0.300***</td>
<td>0.165*</td>
<td></td>
<td>0.181</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>0.652</td>
<td>0.469</td>
<td>0.400***</td>
<td>0.010</td>
<td>0.081</td>
<td>0.119</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>0.394</td>
<td>0.317</td>
<td></td>
<td></td>
<td>0.089</td>
<td>0.200</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>0.336</td>
<td>0.446</td>
<td></td>
<td></td>
<td>0.089</td>
<td>0.200</td>
<td>2949</td>
</tr>
<tr>
<td></td>
<td>0.595</td>
<td>0.393</td>
<td></td>
<td></td>
<td>0.090</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** This table reports results from time-series regressions of daily values of the annualized one-year term premia on daily values of the dispersion in survey forecasts of one-year inflation, GDP growth, and the US dollar-Brazilian real exchange rate. We also include a Brazilian recession dummy indicator obtained from the OECD. Nominal and real forward term premia are backed out from a three-factor Gaussian term structure model augmented with unspanned macro factors, as proposed by Joslin et al. (2014). The term premium is computed as the difference between the forward rate under the risk neutral probability measure and the expected short rate, averaged over the next year, under the physical probability measure. The results use daily data over the period 1/1/2006 - 9/27/2017. Heteroskedasticity and autocorrelation consistent standard errors are reported in brackets. *: significant at the 5% level; **: significant at the 1% level.
Table 5: One-day responses of interest rates, term premia, and inflation expectations to expected and unexpected changes in the COPOM rate following Central Bank meetings

<table>
<thead>
<tr>
<th>Panel A - Expected Inflation</th>
<th>$E^P_{\text{survey}}(\pi_{0,1})$</th>
<th>$E^P(\pi_{0,1})$</th>
<th>$E^P(\pi_{1,1})$</th>
<th>$E^P(\pi_{2,1})$</th>
<th>$E^P(\pi_{3,1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Change</td>
<td>0.026**</td>
<td>-0.005</td>
<td>0.001</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>-0.059</td>
<td>-0.070</td>
<td>0.007</td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.303</td>
<td>0.026</td>
<td>0.008</td>
<td>0.088</td>
<td>0.183</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B - Inflation term premia</th>
<th>$TP_{\pi,\text{survey}}(\pi_{0,1})$</th>
<th>$TP_{\pi}(\pi_{0,1})$</th>
<th>$TP_{\pi}(\pi_{1,1})$</th>
<th>$TP_{\pi}(\pi_{2,1})$</th>
<th>$TP_{\pi}(\pi_{3,1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Change</td>
<td>-0.002</td>
<td>0.017</td>
<td>0.008</td>
<td>-0.003</td>
<td></td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>-0.211*</td>
<td>-0.281***</td>
<td>-0.332***</td>
<td>-0.366***</td>
<td>-0.343***</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.028</td>
<td>0.094</td>
<td>0.080</td>
<td>0.048</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C - Inflation term premia</th>
<th>$TP_{\pi,\text{survey}}(\pi_{0,1})$</th>
<th>$TP_{\pi}(\pi_{0,1})$</th>
<th>$TP_{\pi}(\pi_{1,1})$</th>
<th>$TP_{\pi}(\pi_{2,1})$</th>
<th>$TP_{\pi}(\pi_{3,1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Change</td>
<td>-0.012</td>
<td>0.017</td>
<td>0.008</td>
<td>-0.003</td>
<td></td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>-0.205**</td>
<td>-0.156*</td>
<td>-0.164</td>
<td>-0.269**</td>
<td>-0.329**</td>
</tr>
<tr>
<td>Cross product</td>
<td>0.086</td>
<td>0.112</td>
<td>0.120</td>
<td>0.134</td>
<td>0.099</td>
</tr>
</tbody>
</table>

Notes: This table reports one-day changes in expected inflation and the inflation term premium following changes to the COPOM rate announced after a COPOM meeting. Expected future short rates and term premia are computed from a dynamic Gaussian term structure model that uses daily survey forecasts of inflation and the exchange rate as unspanned macro risk factors. Short rates, term premia and inflation rates start at times 0, 1, 2, or 3 (measured in years) and have a one-year horizon. We decompose changes in the COPOM rate into an expected change, observed from the daily COPOM rate surveys, and an unexpected component, or shock. The survey-based inflation term premium is calculated from

$$TP_{\pi,\text{survey}}(\pi_{p,q}) = E^P_{\text{survey}}[\pi_t(p,q)] - f^{N_t}(p,q) - E^P_{\text{survey}}[\pi_t(p,q)],$$

where $E^P_{\text{survey}}[\pi_t(p,q)]$ represents survey-based measure of expected inflation rate. The first columns of each panel reports statistics of survey-based measure of expected inflation or inflation term premium. The column 2-5 of each panel reports statistics of model implied expected inflation (Panel A) or inflation term premium (Panel B and Panel C). Panel A and B reports coefficient estimates based on regressions of changes in the dependent variable, $\Delta y_t$, on the expected change in the COPOM rate, $\Delta r_{e,t-1}$, and the surprise to the COPOM rate, $r_{ut}$, on the expected change in the COPOM rate, $\Delta y_t = \beta_1 \Delta r_{e,t-1} + \beta_2 r_{ut} + \epsilon_t$. Panel C includes the cross-product of the normalized (mean zero, unit standard deviation) inflation survey dispersion times the unexpected change in the COPOM rate, $\sigma^2_{r_{t-1}}$, in the regression

$$\Delta y_t = \beta_1 \Delta r_{e,t-1} + \beta_2 r_{ut} + \beta_3 \sigma^2_{r_{t-1}} + \epsilon_t.$$

All coefficients are estimated using daily data from 94 COPOM meetings. Heteroskedasticity and autocorrelation consistent $t$-statistics are reported in brackets. *: significant at the 10% level; **: significant at the 5% level; ***: significant at the 1% level.
Table 6: One-day responses of interest rates, term premia, and inflation expectations to expected and unexpected changes in the COPOM rate following Central Bank meetings

<table>
<thead>
<tr>
<th>Panel A - Nominal Yields</th>
<th>Panel B - Nominal Forward Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^N(1)$</td>
<td>$f_t^N(0,1)$</td>
</tr>
<tr>
<td>$y_t^N(2)$</td>
<td>$f_t^N(1,1)$</td>
</tr>
<tr>
<td>$y_t^N(3)$</td>
<td>$f_t^N(2,1)$</td>
</tr>
<tr>
<td>$y_t^N(4)$</td>
<td>$f_t^N(3,1)$</td>
</tr>
<tr>
<td>Expected Change</td>
<td>0.042</td>
</tr>
<tr>
<td>( 0.029)</td>
<td>( 0.032)</td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>0.608***</td>
</tr>
<tr>
<td>( 0.147)</td>
<td>( 0.153)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.256</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C - Expected nominal short rates</th>
<th>Panel D - Nominal term premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^r_t(r_{0,1}^n)$</td>
<td>$T_f^p(0,1)$</td>
</tr>
<tr>
<td>$E^r_t(r_{1,1}^n)$</td>
<td>$T_f^p(1,1)$</td>
</tr>
<tr>
<td>$E^r_t(r_{2,1}^n)$</td>
<td>$T_f^p(2,1)$</td>
</tr>
<tr>
<td>$E^r_t(r_{3,1}^n)$</td>
<td>$T_f^p(3,1)$</td>
</tr>
<tr>
<td>Expected Change</td>
<td>0.027</td>
</tr>
<tr>
<td>( 0.035)</td>
<td>( 0.049)</td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>0.815***</td>
</tr>
<tr>
<td>( 0.123)</td>
<td>( 0.215)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.261</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel E - Real Yields</th>
<th>Panel F - Real Forward Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^R(1)$</td>
<td>$f_t^R(0,1)$</td>
</tr>
<tr>
<td>$y_t^R(2)$</td>
<td>$f_t^R(1,1)$</td>
</tr>
<tr>
<td>$y_t^R(3)$</td>
<td>$f_t^R(2,1)$</td>
</tr>
<tr>
<td>$y_t^R(4)$</td>
<td>$f_t^R(3,1)$</td>
</tr>
<tr>
<td>Expected Change</td>
<td>0.022</td>
</tr>
<tr>
<td>( 0.030)</td>
<td>( 0.030)</td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>0.697***</td>
</tr>
<tr>
<td>( 0.192)</td>
<td>( 0.217)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.259</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel G - Expected short rates</th>
<th>Panel H - Real term premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^f_t(r_{0,1}^n)$</td>
<td>$T_f^p(0,1)$</td>
</tr>
<tr>
<td>$E^f_t(r_{1,1}^n)$</td>
<td>$T_f^p(1,1)$</td>
</tr>
<tr>
<td>$E^f_t(r_{2,1}^n)$</td>
<td>$T_f^p(2,1)$</td>
</tr>
<tr>
<td>$E^f_t(r_{3,1}^n)$</td>
<td>$T_f^p(3,1)$</td>
</tr>
<tr>
<td>Expected Change</td>
<td>0.032</td>
</tr>
<tr>
<td>( 0.037)</td>
<td>( 0.043)</td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>0.574***</td>
</tr>
<tr>
<td>( 0.161)</td>
<td>( 0.142)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.161</td>
</tr>
</tbody>
</table>

Notes: This table reports one-day changes in expected nominal and real interest rates and term premia following changes to the COPOM rate announced after a COPOM meeting. Expected future short rates and term premia are computed from a dynamic Gaussian term structure model that uses daily survey forecasts of inflation and the exchange rate as unspanned macro risk factors. Short rates, term premia and inflation rates start at times 0, 1, 2, or 3 (measured in years) and have a one-year horizon. We decompose changes in the COPOM rate into an expected change, observed from the daily COPOM rate surveys, and an unexpected component, or shock. Coefficient estimates in Panels A-H are based on regressions of changes in the dependent variable, $\Delta y_t$, on the expected change in the COPOM rate, $\Delta r_{t+1}^e$, and the surprise to the COPOM rate, $r_t^e$:

$$\Delta y_t = \beta_1 \Delta r_{t+1}^e + \beta_2 r_t^e + \epsilon_t.$$ 

The coefficients are estimated using daily data from 94 COPOM meetings. Heteroskedasticity and autocorrelation consistent t-statistics are reported in brackets. *: significant at the 10% level; **: significant at the 5% level; ***: significant at the 1% level.
Table 7: Response of nominal term premia and inflation term premia to COPOM rate shocks and changes in the 2-year (orthogonalized) bond yield

<table>
<thead>
<tr>
<th>Panel A - Nominal term premia</th>
<th>$TP^n_{0,1}$</th>
<th>$TP^n_{1,1}$</th>
<th>$TP^n_{2,1}$</th>
<th>$TP^n_{3,1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Change</td>
<td>0.015</td>
<td>0.054</td>
<td>0.048</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.050)</td>
<td>(0.053)</td>
<td>(0.054)</td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>-0.209**</td>
<td>-0.473*</td>
<td>-0.688***</td>
<td>-0.788***</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.267)</td>
<td>(0.221)</td>
<td>(0.213)</td>
</tr>
<tr>
<td>$\Delta yield^1_{0,2}$</td>
<td>0.157</td>
<td>0.275</td>
<td>0.509**</td>
<td>0.397</td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.235)</td>
<td>(0.244)</td>
<td>(0.254)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.101</td>
<td>0.070</td>
<td>0.145</td>
<td>0.124</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B - Inflation term premia</th>
<th>$TP^\pi_{0,1}$</th>
<th>$TP^\pi_{1,1}$</th>
<th>$TP^\pi_{2,1}$</th>
<th>$TP^\pi_{3,1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Change</td>
<td>0.031</td>
<td>0.030</td>
<td>0.023</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.023)</td>
<td>(0.029)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Unexpected Change</td>
<td>-0.421***</td>
<td>-0.407***</td>
<td>-0.503***</td>
<td>-0.545***</td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td>(0.115)</td>
<td>(0.145)</td>
<td>(0.167)</td>
</tr>
<tr>
<td>$\Delta yield^1_{0,2}$</td>
<td>-0.082</td>
<td>0.044</td>
<td>0.095</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.116)</td>
<td>(0.144)</td>
<td>(0.161)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.116</td>
<td>0.127</td>
<td>0.120</td>
<td>0.098</td>
</tr>
</tbody>
</table>

Notes: This table reports one-day changes in the nominal term premium and the inflation term premium following changes to the COPOM rate announced after a central bank meeting. We decompose changes in the COPOM rate into an expected change, observed from the daily COPOM rate surveys, and an unexpected component, or surprise. Coefficient estimates come from regressions of changes in the dependent variable, $\Delta y_t$, on the expected change in the COPOM rate, $\Delta r^e_t|_{t-1}$, the surprise to the COPOM rate, $r^u_t$, and changes to the 2-year Brazilian nominal yield, $\Delta yield^1_{0,2}$, orthogonalized so it is uncorrelated with expected and unexpected changes to the COPOM rate:

$$\Delta y_t = \beta_1 \Delta r^e_t|_{t-1} + \beta_2 r^u_t + \beta_3 \Delta yield^1_{0,2} + \epsilon_t.$$  

The coefficients are estimated using daily data from 94 COPOM meetings that took place over the period 1/1/2006 - 9/27/2017. Heteroskedasticity and autocorrelation consistent t-statistics are reported in brackets. *: significant at the 10% level; **: significant at the 5% level; ***: significant at the 1% level.
Table 8: One-day responses of Brazilian interest rates, term premia, and inflation expectations to changes in the two-year US Treasury yield following Fed announcements

<table>
<thead>
<tr>
<th>Panel A - Nominal yield</th>
<th>Panel B - Nominal forward rates</th>
<th>Panel C - Expected nominal short rates</th>
<th>Panel D - Nominal term premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^N_t(1)$</td>
<td>$f_t^N(0,1)$</td>
<td>$\Delta y^N_{t(0,2)}$</td>
<td>$TP^n_{0,1}$</td>
</tr>
<tr>
<td>$y^N_t(2)$</td>
<td>$f_t^N(1,1)$</td>
<td>(0.333)**</td>
<td>$TP^n_{1,1}$</td>
</tr>
<tr>
<td>$y^N_t(3)$</td>
<td>$f_t^N(2,1)$</td>
<td>(0.148)</td>
<td>$TP^n_{2,1}$</td>
</tr>
<tr>
<td>$y^N_t(4)$</td>
<td>$f_t^N(3,1)$</td>
<td>(0.074)</td>
<td>$TP^n_{3,1}$</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>(0.022)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel E - Expected inflation</th>
<th>Panel F - Inflation term premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y^E_{t(0,2)}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta y^E_{t(0,2)}$</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel G - Real yield</th>
<th>Panel H - Real forward rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^R_t(1)$</td>
<td>$f_t^R(0,1)$</td>
</tr>
<tr>
<td>$y^R_t(2)$</td>
<td>$f_t^R(1,1)$</td>
</tr>
<tr>
<td>$y^R_t(3)$</td>
<td>$f_t^R(2,1)$</td>
</tr>
<tr>
<td>$y^R_t(4)$</td>
<td>$f_t^R(3,1)$</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel I - Expected real short rates</th>
<th>Panel J - Real term premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y^U_{t(0,2)}$</td>
<td>$TP^U_{0,1}$</td>
</tr>
<tr>
<td>$\Delta y^U_{t(0,2)}$</td>
<td>$TP^U_{1,1}$</td>
</tr>
<tr>
<td>$\Delta y^U_{t(0,2)}$</td>
<td>$TP^U_{2,1}$</td>
</tr>
<tr>
<td>$\Delta y^U_{t(0,2)}$</td>
<td>$TP^U_{3,1}$</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports one-day changes in expected nominal and real interest rates, term premia, and expected inflation in Brazil following announcements by the US Federal Reserve. Expected future short rates and term premia are computed from a dynamic Gaussian term structure model that uses daily survey forecasts of the inflation and US dollar-real exchange rate as unspanned macro risk factors. Short rates, term premia and expected inflation rates start at times 0, 1, 2, or 3 (measured in years) and have a one-year horizon. We use daily changes to the two-year US Treasury yield as our proxy for the market surprise of the policy announcement. Coefficient estimates are based on regressions of changes in the dependent variable, $\Delta y_t$, on changes in the two-year US Treasury yield, $\Delta y^U_{t(0,2)}$:

$$\Delta y_t = \alpha + \beta \Delta y^U_{t(0,2)} + \epsilon_t.$$  

The coefficients are estimated using daily data from 93 FOMC meetings that took place over the period 1/1/2006 - 9/27/2017. Heteroskedasticity and autocorrelation consistent t-statistics are reported in brackets. *: significant at the 10% level; **: significant at the 5% level; ***: significant at the 1% level.
**Figure 1:** Evolution in the COPOM rate over time
The figure plots the time series of the COPOM rate (step function) along with the effective SELIC market rate (solid line) over the period 1/1/2006 through 9/27/2017.

**Figure 2:** Distribution of changes to the COPOM rate
This figure shows the distribution of changes to the COPOM rate over the sample 1/1/2006-9/27/2017. All changes are in units of 25 basis points.
**Figure 3:** Time series of (a) survey forecasts and (b) dispersion in survey forecasts

The top graphs show time series of daily survey forecasts of inflation (top chart) and the change in the real per dollar exchange rate (second chart). The forecasts use the mean of the daily surveys and assume a one-year forecast horizon. The bottom graphs show time series of the dispersion in the daily survey forecasts of inflation (third chart) and exchange rate (bottom chart). Survey dispersion is measured as the cross-sectional standard deviation of the daily survey forecasts.

**(a) Survey Forecast**

![Inflation Forecast](image)

![Dollar exchange rate depreciation](image)

**(b) Survey Dispersion**

![Inflation Dispersion](image)

![Dollar exchange rate depreciation](image)
Figure 4: Actual versus predicted changes in the COPOM rate
This figure plots actual changes in the COPOM rate against survey forecasts of the change in the COPOM rate on 94 meetings of the Brazilian Central Bank committee that sets the interest rate target.

\[ y = -0.007 + 1.057x \]
\[ (0.012) \quad (0.027) \]

\[ R^2 = 0.938 \]

Expected Change in COPOM rate (%)
Actual change in COPOM rate (%)
Figure 5: One-day responses of nominal yields to expected and unexpected changes in the COPOM rate

Effect of expected changes

Effect of unexpected changes

These graphs show one-day changes in nominal yields following changes in the COPOM rate on any of the 94 COPOM meeting dates during our sample. The vertical axis represents changes in yields, measured in percent. The left (right) diagrams plot the change in the expected nominal rates against the expected (unexpected) components of changes to the COPOM target rate, along with the best fitting linear model.
Figure 6: Time series of short rates and term premia 1/1/2006 - 9/27/2017.

(a) Nominal rates and term premia

(b) Real rates and term premia
Figure 7: Time series of actual inflation, expected inflation and one-year inflation term premia
Figure 8: Cumulative log-changes in inflation survey dispersion in response to an unexpected shock of 1% to the COPOM rate

The figure plots the response of $h$-day log-changes in the inflation survey dispersion following a 1% unexpected increase in the COPOM rate. Specifically, we plot the estimated value of $\beta_h$ from the regression $\ln(\sigma_{t+h}^{\pi}) - \ln(\sigma_{t-1}^{\pi}) = \beta_h r_{t-1}^{lu} + \text{control}_{t-1} + \varepsilon_{t-1,t+h}$. Control variables are observed on the day before the announcement date (day $t$) and include short-term swap rates, survey forecasts of GDP, inflation and exchange rates and lagged values of log-survey dispersion in GDP, inflation and exchange rate forecasts.
Figure 9: Effect on the dispersion of inflation survey forecasts of an unexpected shock of 1% to the COPOM rate

This figure plots the response of the log-dispersion of inflation survey forecasts following an unexpected 1% increase in the COPOM rate. The dotted red lines show bootstrapped 95% confidence intervals.
Figure 10: One-day responses of short rate expectations to unexpected changes in the COPOM rate: Comparing models with and without unspanned macro risk factors

These graphs plot one-day changes in expected future nominal rates against the surprise component of the changes in the COPOM rate announced on the 94 COPOM meeting dates during our sample. The red circles and red lines are based on a dynamic Gaussian term structure model without unspanned macro risk factors while the blue asterics and blue lines use a model that accounts for unspanned macro risk factors.
Figure 11: Response of cumulative changes to log-dispersion of survey forecasts of the Brazilian-US exchange rate (top panel), inflation (middle panel), and GDP growth (bottom panel) following a change of 1% in the 2-year US nominal yield on FOMC meeting days

(a) Survey dispersion in exchange rate change

(b) Survey dispersion in inflation

(c) Survey dispersion in GDP growth

This figure plots the estimated value of $\beta_h$ from the regression $\log(\sigma_{t+h}) - \log(\sigma_{t-1}) = \beta_h \Delta y^{US}_{t,(0,2)} + \text{control}_{t-1} + \epsilon_{t,t+h}$. Control variables are observed the day before the announcement date (day $t$) and include lagged values of the 2-year US Treasury yield, short-term swap rates, log survey forecasts of GDP, inflation and exchange rates, and survey dispersion in GDP, inflation and exchange rate forecasts.
Appendix A  Converting Fixed-event Survey Forecasts into Fixed Horizon Forecasts

This appendix explains how we convert the fixed-event survey forecasts into forecasts with a fixed horizon such as one year.

Let $\hat{\pi}_{t}^{\tau_1}$ be the (average) survey forecast of inflation for the nearest (current) event date (e.g., April 20) on day \( t \) (e.g., March 25th). Similarly, let $\hat{\pi}_{t}^{\tau_2}$ be the day \( t \) (March 25) survey forecast for the next release (e.g., May 20), and so on. Hence, the superscript \( \tau_i \) refers to the \( i \)th nearest future event date with \( \tau_1 \) denoting the closest (current) release date, \( \tau_0 \) denotes the most recent past event date, while the subscript \( t \) refers to the current date. Figure A1 illustrates these dating conventions and the relation between fixed versus rolling event forecasts.

Further, let $days_{1t}$ be the number of days that already occurred in the current event window up to day \( t \) while $ndays_{1t} = \tau_1(t) - \tau_0(t) \geq days_{1t}$ is the total number of days in this event window. Using the month-on-month inflation forecast \( i \) months (events) ahead on day \( t \), $\hat{\pi}_{t}^{\tau_i}$, BACEN computes the 12 month-ahead rolling window inflation survey forecast on day \( t \) as

$$
\hat{\pi}_{t, 1 yr} = \left[ \prod_{i=1}^{12} \left( 1 + \hat{\pi}_{t}^{\tau_i} \right) \times \left( \frac{1 + \hat{\pi}_{t}^{\tau_{13}}}{1 + \hat{\pi}_{t}^{\tau_1}} \right)^{days_{1t}/ndays_{1t}} \right] - 1. \quad (A.1)
$$

Following this practice, and to account for the fact that survey forecasts do not always overlap with forecast horizons such as one year, we compute one-year-ahead forecasts as follows. Let $\hat{r}_{t}^{\tau_j}$ be the survey forecast of the average COPOM rate for the \( j \)-period-ahead event \( (\tau_j) \). To compute the one-year-ahead forecast of the COPOM rate from the yearly forecasts, we use the following formula:

$$
\hat{r}_{t, 1 yr} = \left[ (1 + \hat{r}_{t}^{\tau_1}) \times \left( \frac{1 + \hat{r}_{t}^{\tau_{12}}}{1 + \hat{r}_{t}^{\tau_1}} \right)^{days_{1t}/ndays_{1t}} \right] - 1. \quad (A.2)
$$

The BACEN survey also provides forecasts of quarterly GDP growth, computed relative to the same quarter of the previous year and denoted $\hat{g}_{t}^{\tau_j}$, for horizons up to five quarters, \( j = 1, \ldots, 5 \). We convert this into a rolling four-quarter-ahead forecast using the formula

$$
\hat{g}_{t, 1 yr} = \left[ (1 + \hat{g}_{t}^{\tau_4}) \times \left( \frac{1 + \hat{g}_{t}^{\tau_{12}}}{1 + \hat{g}_{t}^{\tau_4}} \right)^{days_{1t}/ndays_{1t}} \right] - 1, \quad (A.3)
$$

where $\hat{g}_{t}^{\tau_j}$ is the growth rate for GDP in quarter \( j \) relative to the same quarter during the previous year.

To compute the year-on-year percentage change in the exchange rate, measured as the number of Reals per US dollar relative to the exchange rate at the end of the previous month, $EX_{t}^{\tau_0}$, we
use the formula

$$\Delta \hat{E}X_{t,1yr} = \left[ \left( \frac{\hat{E}X_{t}^{\tau_12}}{EX_{t}^{\tau_0}} \right) \times \left( \frac{\hat{E}X_{t}^{\tau_13}}{EX_{t}^{\tau_1}} \times \frac{EX_{t}^{\tau_0}}{EX_{t}^{\tau_1}} \right)^{(day_{11}/ndays_{1t})} \right] - 1,$$  \hspace{1cm} (A.4)

where $\hat{E}X_{t}^{\tau_i}$ is the average forecast of the exchange rate level for release $\tau_i$ and $EX_{t}^{\tau_0}$ denotes the (known) exchange rate on the past event date.

BACEN provides the cross-sectional standard deviation of the one-year-ahead forecasts for inflation but not for GDP growth or the exchange rate. To obtain fixed-horizon estimates of the standard deviations for the COPOM rate, GDP growth and the Real/$US exchange rate, we use the following formulae

$$\hat{\sigma}^{r,\tau_i}_{t,1yr} = \left[ \left( 1 + \hat{\sigma}^{r,\tau_12}_t \right) \times \left( \frac{1 + \hat{\sigma}^{r,\tau_24}_t}{1 + \hat{\sigma}^{r,\tau_12}_t} \right)^{(day_{11}/ndays_{1t})} \right] - 1,$$  \hspace{1cm} (A.5)

$$\hat{\sigma}^{g,\tau_i}_{t,1yr} = \left[ \left( 1 + \hat{\sigma}^{g,\tau_4}_t \right) \times \left( \frac{1 + \hat{\sigma}^{g,\tau_5}_t}{1 + \hat{\sigma}^{g,\tau_4}_t} \right)^{(day_{11}/ndays_{1t})} \right] - 1,$$  \hspace{1cm} (A.6)

$$\hat{\sigma}^{ex,\tau_i}_{t,1yr} = \left[ \left( \frac{\hat{\sigma}^{ex,\tau_12}_t}{EX_{t}^{\tau_0}} \right) \times \left( \frac{\hat{\sigma}^{ex,\tau_13}_t \times EX_{t}^{\tau_0}}{EX_{t}^{\tau_1}} \right)^{(day_{11}/ndays_{1t})} \right],$$  \hspace{1cm} (A.7)

where $\hat{\sigma}^{r,\tau_i}$, $\hat{\sigma}^{g,\tau_i}$, $\hat{\sigma}^{ex,\tau_i}$ are the dispersion (standard deviation) across survey participants’ forecasts of the COPOM rate, GDP growth and the Real exchange rate for event horizon $\tau_i$ on day $t$.

**Appendix B Yield Curve Estimates**

Table A1 presents summary statistics for the Brazilian bond yields, while Table A2 presents estimates of the principal components (PCs) extracted from daily panels of zero-coupon bond yields using nominal (Panel A) and real (Panel B) yields, respectively. The standard deviation of the first PC, at 0.054 for the nominal yields and 0.054 for the real yields, is one and two orders of magnitude greater than that of the second PC (0.012 and 0.011, respectively) and the third PC (0.003 and 0.004, respectively.) The estimates in Table A2 show that three principal components are sufficient to capture almost all variation in our yield data.

Figure A2 shows snapshots of the term structure of nominal (top panel) and real (bottom panel) yields for a few days in our sample. We see both upward sloping (2008 and 2009) and downward sloping (2007) nominal and real yield curves in our sample.

Figure A3 plots time-series of the three PCs extracted from the nominal yields (top window) and the real yields (bottom window). Analysis of the loadings on the PCs (shown in Figure
A4) shows that the loadings on the first PC measured across different bond maturities are flat, meaning that changes in the first PC correspond to parallel shifts in the yield curve. This PC can therefore be interpreted as a level factor. Loadings on the second PC decrease along the maturity dimension, so that a change in the second PC rotates the yield curve and thus represents a slope factor. Loadings on the third principal component are hump shaped with the hump occurring for intermediate maturities. The third PC therefore affects the curvature of the yield curve and so is labelled a curvature factor. This interpretation of the first three PCs is parallel to empirical findings for the US term structure.

Table A3 reports bias-adjusted estimates of $\tilde{\mu}, \tilde{\Phi}$ (in the first eight rows) and minimum $\chi^2$ estimates of $\lambda^N, \lambda^R$. We also report the average value of the implied instantaneous short rate in our sample, $r^N$ and $r^R$. All yield factors and unspanned macro factors are highly significant. Moreover, lagged values of the third PC in the nominal term structure affect future values of the first nominal PC and the lagged inflation survey data affect future values of the third nominal PC. For the real PCs, we find that lags of real and nominal PCs load on to future value of real PCs. In addition, lagged exchange rate forecasts predict the first real PC and lagged inflation forecasts predict the third nominal PC and the second real PC.

Our main analysis of the Brazilian interest rates uses quoted bond prices to compute yields. To see if these quoted prices differ from market yields, we obtained data on one-day interbank deposit futures contracts whose underlying asset is the cumulative daily rates between the time of trading and the contract maturity. Each futures contract is on a nominal R$ 100,000 bond and the contract is similar to a zero coupon bond, except with daily margin adjustments so that the daily cash flow is calculated as the difference between the settlement prices on the current and previous days.

Table A4 shows summary statistics for these daily swap yields from interbank borrowing/lending in the short term market for one- and two-year nominal government bonds. The mean of the quoted prices is on average a little higher (typically around 10 basis points) than the market yields, but the two series are very strongly correlated with correlations of 0.999. A time-series plot of the two series (Figure A5) reveals that, except for a slightly different mean, they are essentially identical.

Appendix C  EM Filter Estimates of Missing Yield Data on Inflation-indexed Bonds

Our data on the real yields of inflation-indexed bonds with a maturity less than or equal to 18 months appear to be unreliable during three spells in our sample, namely 05/01/2013 - 06/30/2013, 08/01/2013 - 11/30/2013 and 05/01/2015 - 10/31/2015.

Interpretation of the principal components in terms of level, slope, and curvature goes back at least to Litterman and Scheinkman (1991).
We deal with this issue by treating these yield observations as missing data which we replace based on the following procedure which uses a Kalman filter and EM algorithm.

Step one estimates the weight for the first three principal component using real yields obtained from the data sample without missing yields. Step two uses OLS to estimate $\tilde{\mu}$, $\tilde{\Phi}$ and the parameters of the covariance matrix of the error term $Q$, again using data without missing yields. Step three regresses the real yields for all maturities against the first three principal components

$$P_t = A + B\tilde{X}_{1,t} + v_t,$$  \hfill (C.1)

again using the sample with complete yield data. We also estimate the covariance matrix of $v_t$, denoted $R$.

Step four, sets up a state space model, with

$$\tilde{X}_t = \tilde{\mu} + \tilde{\Phi}\tilde{X}_{t-1} + w_t.$$  \hfill (C.2)

as the state equation and (C.1) as the observation equation, where $w_t \sim N(0,Q)$ and $v_t \sim N(0,R)$. Using this state space model, we apply Kalman filtering to obtain the entire sequence of $\tilde{X}_t$.

Step five uses the filtered $\tilde{X}_t$ from step four to reestimate $\tilde{\mu}$, $\tilde{\Phi}$ in the full sample by OLS and using the residuals outside the intervals with missing data to reestimate $Q$.

Step six iterates on steps four and five until the estimated values of $\tilde{\mu}$, $\tilde{\Phi}$ differ from the previous estimates by a sufficiently small value to indicate convergence.

Finally, we take the estimates $\tilde{\mu}$, $\tilde{\Phi}$ as given after previous steps. Then, we apply the procedure in Hamilton and Wu (2012) to estimate $\Sigma$ and $\lambda$ using data outside the intervals with missing yields.
<table>
<thead>
<tr>
<th></th>
<th>6 months</th>
<th>9 months</th>
<th>12 months</th>
<th>18 months</th>
<th>24 months</th>
<th>36 months</th>
<th>48 months</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A - Nominal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.114</td>
<td>0.115</td>
<td>0.116</td>
<td>0.118</td>
<td>0.119</td>
<td>0.122</td>
<td>0.123</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.023</td>
<td>0.022</td>
<td>0.022</td>
<td>0.021</td>
<td>0.020</td>
<td>0.019</td>
<td>0.018</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.997</td>
<td>0.997</td>
<td>0.996</td>
<td>0.996</td>
</tr>
<tr>
<td>$\rho(21)$</td>
<td>0.952</td>
<td>0.948</td>
<td>0.944</td>
<td>0.937</td>
<td>0.930</td>
<td>0.917</td>
<td>0.908</td>
</tr>
<tr>
<td>$\rho(63)$</td>
<td>0.826</td>
<td>0.818</td>
<td>0.808</td>
<td>0.789</td>
<td>0.772</td>
<td>0.743</td>
<td>0.724</td>
</tr>
<tr>
<td><strong>Panel B - Real</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.056</td>
<td>0.057</td>
<td>0.058</td>
<td>0.060</td>
<td>0.061</td>
<td>0.064</td>
<td>0.064</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.024</td>
<td>0.032</td>
<td>0.022</td>
<td>0.020</td>
<td>0.019</td>
<td>0.018</td>
<td>0.017</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>$\rho(21)$</td>
<td>0.923</td>
<td>0.945</td>
<td>0.950</td>
<td>0.955</td>
<td>0.955</td>
<td>0.955</td>
<td>0.955</td>
</tr>
<tr>
<td>$\rho(63)$</td>
<td>0.805</td>
<td>0.847</td>
<td>0.839</td>
<td>0.864</td>
<td>0.868</td>
<td>0.867</td>
<td>0.861</td>
</tr>
</tbody>
</table>

**Notes:** This table reports summary statistics such as the mean, standard deviation and serial correlation of nominal and real yields on Brazilian zero-coupon bonds sampled at the daily horizon. All statistics are based on the sample 01/01/2006 - 09/27/2017.
Table A.2: Properties of the principal components extracted from Brazilian bond yields: 01/01/2006-09/27/2017

<table>
<thead>
<tr>
<th></th>
<th>Panel A - Nominal yields</th>
<th>Panel B - Real yields</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PC 1</td>
<td>PC 2</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>29.261</td>
<td>1.349</td>
</tr>
<tr>
<td>Variance Explained</td>
<td>95.230</td>
<td>4.390</td>
</tr>
<tr>
<td>Mean</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.054</td>
<td>0.012</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>0.997</td>
<td>0.994</td>
</tr>
<tr>
<td>$\rho(21)$</td>
<td>0.940</td>
<td>0.863</td>
</tr>
<tr>
<td>$\rho(63)$</td>
<td>0.798</td>
<td>0.594</td>
</tr>
</tbody>
</table>

Notes: This table reports eigenvalues along with the proportion of the variance of bond yields explained by the first three principal components extracted from cross-sections of Brazilian zero-coupon bond yields. We also report the standard deviation along with $i$th order serial correlation estimates, $\rho(i)$, for the principal components. Panel A shows results for nominal bond yields recorded at the daily horizon, while Panel B shows results for real bond yields recorded at the daily horizon. The data cover the period 01/01/2006 through 09/27/2017.
Table A.3: Minimum Chi-square estimates of the term structure model with unspanned macro risk factors

<table>
<thead>
<tr>
<th></th>
<th>( \mu )</th>
<th>( \hat{\Phi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( PC_{1}^{N} )</td>
<td>( PC_{2}^{N} )</td>
</tr>
<tr>
<td>( PC_{1}^{N} )</td>
<td>0.0015***</td>
<td>1.0026***</td>
</tr>
<tr>
<td>( PC_{2}^{N} )</td>
<td>0.0006***</td>
<td>0.0130</td>
</tr>
<tr>
<td>( PC_{3}^{N} )</td>
<td>-0.0001</td>
<td>0.0006</td>
</tr>
<tr>
<td>( PC_{1}^{R} )</td>
<td>0.0014</td>
<td>0.0022</td>
</tr>
<tr>
<td>( PC_{2}^{R} )</td>
<td>0.0007***</td>
<td>0.0010</td>
</tr>
<tr>
<td>( PC_{3}^{R} )</td>
<td>-0.0001</td>
<td>0.0002</td>
</tr>
<tr>
<td>( \Pi_{t+1yr</td>
<td>t} )</td>
<td>0.0002***</td>
</tr>
<tr>
<td>( EX_{t+1yr</td>
<td>t} )</td>
<td>-0.0008</td>
</tr>
<tr>
<td>( \lambda^{N} )</td>
<td>1.38</td>
<td>(0.000)</td>
</tr>
<tr>
<td>( \tau^{N} )</td>
<td>11.22%</td>
<td></td>
</tr>
<tr>
<td>( \lambda^{R} )</td>
<td>1.06</td>
<td>(0.000)</td>
</tr>
<tr>
<td>( \tau^{R} )</td>
<td>5.12%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports bias-adjusted minimum Chi-square estimates of the parameters of a term structure model that includes three principal components (denoted \( PC_{1}^{N} \), \( PC_{2}^{N} \) and \( PC_{3}^{N} \)) extracted from Brazilian discount nominal bonds with maturities of 6, 9, 12, 18, 24, 36 and 48 months, three principal components (denoted \( PC_{1}^{R} \), \( PC_{2}^{R} \) and \( PC_{3}^{R} \)) extracted from Brazilian discount real bonds with maturities of 6, 9, 12, 18, 24, 36 and 48 months, along with one-year-ahead survey forecasts of inflation, \( \Pi_{t+1yr|t} \), and the US dollar-real exchange rate, \( EX_{t+1yr|t} \). Defining \( \tilde{X}_{t} = (PC_{1}^{N}, PC_{2}^{N}, PC_{3}^{N}, PC_{1}^{R}, PC_{2}^{R}, PC_{3}^{R}, \Pi_{t+1yr|t}, EX_{t+1yr|t}) \), the estimated model takes the form

\[ \tilde{X}_{t+1} = \hat{\mu} + \hat{\Phi}\tilde{X}_{t} + \tilde{\xi}_{t+1}. \]

Following Christensen, Diebold and Rudebusch (2011), the model is reparameterized with a latent state vector, \( F_{t} = (L_{t}^{N}, S_{t}^{N}, C_{t}^{N}, L_{t}^{R}, S_{t}^{R}, C_{t}^{R}, \Pi_{t+1yr|t}, EX_{t+1yr|t}) \), that follows a VAR such that

\[
\begin{align*}
\gamma_{t}^{N}(\tau) &= -p_{t}^{N}(\tau) = L_{t}^{N} + \frac{1 - e^{-\lambda^{N}\tau}}{\lambda^{N}\tau} S_{t}^{N} + \left[ \frac{1 - e^{-\lambda^{N}\tau}}{\lambda^{N}\tau} - e^{-\lambda^{N}\tau} \right] C_{t}^{N} + \frac{A^{N}(\tau)}{\tau} \\
\gamma_{t}^{R}(\tau) &= -p_{t}^{R}(\tau) = L_{t}^{R} + \frac{1 - e^{-\lambda^{R}\tau}}{\lambda^{R}\tau} S_{t}^{R} + \left[ \frac{1 - e^{-\lambda^{R}\tau}}{\lambda^{R}\tau} - e^{-\lambda^{R}\tau} \right] C_{t}^{R} + \frac{A^{R}(\tau)}{\tau},
\end{align*}
\]

where, \( \gamma_{t}^{N}(\tau) \) is the yield at time \( t \) that matures at \( t + \tau \). Short-term nominal interest rates in this model take the form \( r_{t}^{N} = L_{t}^{N} + S_{t}^{N} \). Short-term real interest rates in this model take the form \( r^{R} = L_{t}^{R} + S_{t}^{R} \). We report \( \tau^{N} \) and \( \tau^{R} \) as the sample average of short nominal and real rates. The parameters are estimated using the minimum Chi-square method proposed in Hamilton and Wu (2012). The bias in the VAR coefficients is corrected using the bootstrap method in Killian (1998). Coefficient estimates are reported in the main lines with standard errors in brackets underneath. *: significant at the 10% level; **: significant at the 5% level; ***: significant at the 1% level.
Table A.4: Summary statistics for quoted (ANBIMA) and market-based (swap) yields

<table>
<thead>
<tr>
<th></th>
<th>1 year yield</th>
<th>2 years yields</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quoted</td>
<td>Market</td>
</tr>
<tr>
<td>Mean</td>
<td>11.40</td>
<td>11.32</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>2.27</td>
<td>2.26</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.999</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Notes: This table shows summary statistics for daily market (swap) yields and quoted (ANBIMA) yields from interbank borrowing/lending in the short term market for one- and two-year nominal government bonds.

Figure A.1: Fixed event versus rolling event forecasts

This figure compares the fixed event forecasting format (bottom diagram) to the more conventional format with fixed horizon forecasts (top diagram).  \( \hat{y}_{t+1} \) refers to the period-1 forecast of \( y \) at the first event following time \( t \). In the figure, this event occurs after time \( t+1 \) as shown by the red lines. Hence the forecast horizon for \( \hat{y}_{t+1} \) is one period shorter than the forecast horizon for \( \hat{y}_{t+1|t} \). Event dates refer to the dates of the COPOM meetings for setting the target rate and so are fixed. In the top diagram the forecast horizon is fixed at two (green arrows) or three (purple arrows) periods.


**Figure A.2:** Yield curves for nominal and real bonds
The graphs show snapshots of the term structure of nominal (top panel) and real (bottom panel) yields on zero-coupon bonds at individual days included in our sample. Maturities range from 6 through 48 months.

**Nominal Yields**

![Graphs showing nominal yield curves for different dates.]

**Real Yields**

![Graphs showing real yield curves for different dates.]

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Figure A.3: Time series of principal components extracted from nominal and real yields
The graphs plot the first three principal components extracted from daily yields on Brazilian zero-coupon nominal bonds (top graph) and real bonds (bottom graph) over the sample 1/1/2006 - 09/27/2017.

Figure A.4: Loadings on principal components (01/01/2006-09/27/2017)
The figure plots the loadings of different bond maturities on principal components (PCs) extracted from nominal yields (left window) and real yields (right window) using Brazilian government bonds with maturities ranging from 6 through 48 months.
Figure A.5: Time series of yields based on quotes (ANBIMA) and market prices (SWAP)
These graphs show the daily market-based (swap) yields and quoted yields from interbank borrowing/lending in the short term market for one- and two-year nominal government bonds.

1 year yields

2 years yields