Robust New Product Pricing

Benjamin Handel[*]

Kanishka Misra[†]

March 2015

Abstract

We study the pricing decision for a monopolist launching a new innovation. At the time of launch, we assume that the monopolist has incomplete information about the true demand curve. Despite the lack of objective information the firm must set a retail price to maximize total profits. To model this environment, we develop a novel two-period non-Bayesian framework, where the monopolist sets the price in each period based only on a *non-parametric set of all feasible demand curves*. Optimal prices are dynamic as prices in any period allow the firm to learn about demand and improve future pricing decisions. Our main results show that the direction of dynamic introductory prices (versus static) depends on the type of heterogeneity in the market. We find (1) when consumers have homogeneous preferences, introductory dynamic price is higher than the static price (2) when consumers have heterogeneous preferences and the monopolist has no ex-ante information, the introductory dynamic price is the same as the static price and (3) when consumers have heterogeneous preferences and the monopolist has ex-ante information, the introductory dynamic price is lower than the static price. Further, the degree of this initial reduction increases with the amount of heterogeneity in the ex-ante information.

Keywords: Non-Bayesian learning, ambiguity, pricing, new products

[*]Department of Economics, University of California, Berkeley. Contact: handel@berkeley.edu

[†]Ross School of Business, University of Michigan, Ann Arbor. Contact: kanishka@umich.edu

We would like to thank Charles Manski for his advice and support on this project. This research has also benefited from the extensive comments of Igal Hendel, Michael Whinston, Guy Arie, Jeff Ely, Jose Espin, Daniel Garrett, Paul Grieco, Garrett Johnson, Ralf Meisenzahl, Mallesh Pai, S. Sriram, Puneet Manchanda, Eric Schwartz and James Roberts.

"A Bayesian analysis may be "rational" in the weak axiomatic sense, yet be terrible in a practical sense if an inappropriate prior distribution is used."

-Berger [1985] (Statistical Decision Theory and Bayesian Analysis, page 121)

1 Introduction

1.1 Overview

Consider the pricing decision of a monopolist launching a new non-storable product or technology with unit per-period demand. In order to understand the demand curve, the manager conducts market research, this could be in the form of concept testing (Schwartz [1987]), experiments (e.g. Green and Srinivasan [1978]) or surveys (Dolan [1993]) to assess consumers valuations. These provide the manager with some information about the underlying demand curve for their new product. However, it is unrealistic for the manager to have complete information about the demand curve at the time of launch (e.g., Lodish [1980], Besbes and Zeevi [2009], Kahn [2010], Harrison et al. [2012]). In this paper we investigate how a monopolist should set new product prices with limited pre-launch information.

This question relates to a large literature that studies new product introduction. Most papers in this literature assumes firms have complete information about the underlying demand curve (e.g., Robinson and Lakhani [1975], Wernerfelt [1986], Liu and Zhang [2013] please see the online appendix for a more complete list of papers), and derive the optimal firm policy. [1] Researchers have recognized that firms are uncertain about the underlying demand curve for new products (e.g., Rothschild [1974], Lodish [1980], Braden and Oren [1994], Desai et al. [2007], Bonatti [2011], Hitsch [2006], Biyalogorsky and Koenigsberg [2014] please see appendix for a more complete list of papers). However, in solving for optimal firm policy, these papers assume that the firm has a prior (represented as full knowledge or a prior distribution) over the uncertainty in the demand curve. For example, Desai et al. [2007] and Biyalogorsky and Koenigsberg [2014] assume that demand can be in one of two states and firms know precisely the probability of each state; alternatively Braden and Oren [1994] and Hitsch [2006] assume that firms have a prior over the possible uncertainty. With

[1] In this literature the dynamics in new product pricing can be optimal either due to dynamics in demand (e.g., evolving consumer preferences and preference heterogeneity) or supply (e.g., inventory concerns and competition).

this information, the manager can use Bayesian decision theory to pick the policy that maximizes expected profits.

The practical reliability of a Bayesian decision theory analysis depends critically on prior assumption used (Manski [2005], Berger [1985]). A critical input to new product pricing decisions is an accurate demand forecast. Survey studies that have tracked the accuracy of forecasts, suggest very large forecasting errors for new products and technologies. Gartner and Thomas [1993] report new product forecast errors vary from -2900% to +1500% with a mean of -46.9%; and Kahn [2002] report a forecast accuracy for new to market products as 40%. This suggests that firms may not always have appropriate priors. The literature on judgment and decision making suggests that one reason for this is that managers may have behavioral biases when selecting priors for new products (Tyebjee [1987], Forlani et al. [2002], Bolton [2003], Bolton and Reed [2004], Schwartz and Cohen [2004], Lawrence et al. [2006]). Kahn [2010] summarizes "it is very unlikely that new-product forecasting will be free of all biases".

Motivated by the quote by Berger [1985] to start this paper, we develop and investigate a novel dynamic non-Bayesian pricing methodology. Despite the lack of objective information the firm must set a retail price to maximize total profits. To achieve this the retail price must (i) consider current profits and (ii) allow the firm to learn about demand in order to extract higher profits in the future (Rothschild [1974], Grossman et al. [1977], Mirman et al. [1993], more generally labeled as learning by doing Arrow [1962]). Our framework is robust in the sense that the monopolist's price does not depend on subjective information. Instead, at each point in time the monopolist bases her pricing decision only on the *set* of all feasible demand curves. [2] This kind of uncertainty is also known as *Knightian uncertainty* (Knight [2012]) or *ambiguity*. The prices suggested by our proposed methodology can be used by firms who may not be willing to make a prior assumption, or alternatively provide managers information regarding the implications of the subjective assumptions on pricing decisions.

We present a two-period model where the monopolist prices to a unit mass of consumers. Within this framework, we assume each consumer has unit demand with a constant product valuation over time. We consider two environments where consumers either have homogeneous or heterogeneous

[2] For example, if the information available to the firm is that 50% of consumer purchase at $5 and 10% purchase at $10, then any downward sloping demand curve that is consistent with these data is a feasible demand curve.

preferences. We assume that at the time of purchase the monopolist does not have the information to price discriminate and a set a single price in each period. For example, consider pricing at a retail store. These assumptions imply that consumers are *non-strategic* in the sense that they do not have an incentive to misrepresent their valuation to obtain lower future prices. Importantly if the monopolist has full information about consumer valuations, these demand assumptions would not predict dynamic prices. For the heterogeneous preference model, we make one additional distinction regarding ex-ante information available to the firm. We assume that firms have access to some known characteristics about consumers (e.g. segmentation, location, age, or income) based on which they can group consumers. Further, based on the pre-launch market research, firms have partial preference information for each group (labelled ex-ante information). [3] Aggregating this information across groups of consumers allows the firm to partially identify the set of feasible demand curves (see Handel et al. [2013] for econometric identification and estimation methodology).

1.2 Robust Pricing

To study the firm's pricing decisions under ambiguity we develop a novel dynamic non-Bayesian framework that simultaneously considers current profitability and the value of learning. The monopolist's objective function is to maximize aggregate profits. However, she only observes the *set* of all feasible demand curves. Without a subjective prior, the manager cannot integrate over this set to calculate the maximum expected profits. Instead, we assume that the monopolist selects a price in each period using *dynamic minimax regret*, a decision criterion that compares prices based strictly on the set of feasible demand curves. This dynamic decision rule is based on the minimax regret criterion which was introduced by Wald [1950] and has been axiomatized by Milnor [1954] and Stoye [2011]. [4] We discuss alternatives to the dynamic Minimax Regret criterion in section 3.2.4.

Minimax regret has been used to characterize robust decision making in a variety of social choice settings (see e.g. Manski [2005]) as well as in firm decisions (see Bergemann and Schlag [2008], Bergemann and Schlag [2011] and Handel et al. [2013] in Economics and Perakis and Roels

[3] Our notion of observable ex-ante heterogeneity permits a range of pricing environments. Specifically, if all consumers are individually identifiable our setting is equivalent to one in which the firm has panel data.

[4] As summarized by Schlag [2006] "Minimax Regret is the unique criterion that satisfies Ordering, Symmetry, Strong Domination, Continuity, Column Duplication, Convexity, INABA [IIA] and S [Strategic] Independence "

[2010], Ball and Queyrannem [2009], Besbes and Zeevi [2009] and Besbes and Zeevi [2011] in Operations). In the operations literature, Perakis and Roels [2010], find that the minimax regret approach outperforms traditional heuristics used in the literature. Moreover, the notion of Minimax regret is the foundational building block of popular computer science models such as multi-arm bandit problems and machine learning (see Lai [1987], Bubeck and Cesa-Bianchi [2012]). Minimax regret centers around the notion of *regret*, defined in our environment as the profits foregone by the monopolist from not charging the optimal price for the true demand curve. This notion of regret from the statistical decision literature (e.g. Savage [1951]) is completely distinct from the notion of regret discussed in the psychology literature (Janis and Mann [1977]). Specifically, in the absence of ambiguity, the objective function under minimax regret is exactly the same as the standard expected profit maximization.

Our research objective of dynamic robust pricing is similar to that considered in the operations literature (Besbes and Zeevi [2009]) and the computer science literature (Kleinberg and Leighton [2014]). Both areas consider a continuous time model where consumers enter with a random (Poisson) rate where firms can change prices continuously. The proposed solution in operations (Besbes and Zeevi [2009]) is an algorithm with an experimental stage where the firm can learn about demand and then use minimax regret to select a price. Here the experimental stage is divorced from the profit stage, where as in our setup, we consider the potential profit from the learning stages. In the computer science literature, Kleinberg and Leighton [2014] propose a multi-arm bandit solution (parametric approximations to the minimax regret problem) and show that such an algorithm will converge to the first best solution in finite time.

1.3 Contributions

To the best of our knowledge, this is the first work on dynamic pricing from micro-economic foundations that accounts for both (i) setting prices without subjective Bayesian information and (ii) learning about demand through pricing. Our main results show that the monopolist can offer a lower, unchanged or higher introductory price in a dynamic environment (as compared to a static environment) depending on the type of heterogeneity in the market. We find (1) when consumers have homogeneous preferences, introductory dynamic price is higher than the static price (2) when consumers have heterogeneous preferences and the monopolist has no ex-ante information, the introductory dynamic price is the same as the static price and (3) when consumers have heterogeneous preferences and the monopolist has ex-ante information, the introductory dynamic price is lower than the static price. Further the degree of this initial reduction increases with the amount of heterogeneity in the ex-ante information. The difference in results between the homogeneous and heterogeneous preference models is driven by the fact that the homogeneous preference model restricts the set of feasible demand functions to be mass points. Here increasing initial price is attractive as consumer valuations must be either above or below the higher price. If we increase price and consumers do not purchase, then reducing future prices significantly decreases future regret. In the heterogeneous preference model, this is not the case as worst case demand would have consumers with valuations both above and below the first period price. Moreover, under the worst case demand consumers who do not purchase will have lower ex-ante valuations than consumers who do purchase. Here decreasing price allows the firm to bound the maximum regret from these ex-ante lower value consumers and potentially target the ex-ante higher valuation consumers with increasing price in period 2. This result depends critically on ex-ante information. When the firm does not have no ex-ante information, lowering price is no longer valuable.

The remainder of the paper proceeds as follows. Section 2 describes the static model of robust firm pricing. Section 3 presents our main results for the dynamic model of robust firm pricing. Section 4 concludes.

2 Static Monopoly Pricing Under Ambiguity

Before studying the monopolist's dynamic decision problem, we review the benchmark static model for monopoly pricing under ambiguity. This section closely follows previous work by Bergemann and Schlag [2008] and Bergemann and Schlag [2011] on robust monopoly pricing without a subjective prior. [5]

In this section we start with an overview of minimax regret in a static setting. We then solve for the optimal static minimax regret prices in an environment where consumers are homogeneous

[5] Bergemann and Schlag study a case where one consumer is drawn from any feasible probability distribution of consumers over a known support of potential valuations. The monopolist has no subjective information concerning the relative likelihood of these feasible distributions, and selects a random pricing rule to minimize her maximum expected regret over this space of uncertainty. We present a simplified version of this model, where the simplification results in a deterministic pricing rule.

in their valuations. We then consider a model where consumer preferences are heterogeneous. Here we will also allow for heterogeneity in ex-ante information for the firm, whereby firms can have different ex-ante partially identified preferences for consumers.

2.1 Overview of static minimax regret

In a static setting where the monopolist knows that true distribution of consumer valuations F lies within a set of feasible demand curves Ψ. Regret for any chosen price p is defined with respect to each $F \in \Psi$ as defined as:

$$R(p, F) = \text{First Best Profit(F) - Actual Profit}(p\text{,F})$$

Under the First Best Profits, the firm has complete information and will charge each consumer her valuation. [6] Regret here can be interpreted as the measure of the consumer surplus that the firm is unable to capture. For any (p, F) this regret will result from overpricing (underpricing) corresponding to the consumers with valuations below (above) p. In the Bayesian setup, this overpricing and underpricing for each (p, F) pair is weighted by a subjective Bayesian prior over Ψ and regret minimization with respect to this weighting is equivalent to expected profit maximization. With no Bayesian prior to weight the space of feasible demand curves, minimax regret evaluates each possible price by its *maximum* regret over the set Ψ. For any p, maximum regret occurs at a given $F_{wc}(p) \in \Psi$ where actual profits under p are furthest away from the first best profits under $F_{wc}(p)$. Intuitively, maximum regret is the worst possible case of foregone profits over the set of feasible demand curves given p. Once maximum regret is determined for each price, the monopolist solves her problem by selecting the price that minimizes this maximum regret. Thus, minimax regret trades off losses from overpricing with losses from underpricing in a manner that is robust to subjective uncertainty over the set of feasible demand curves. Under the interpretation that regret represents the consumer surplus the firm is unable to capture (or lost consumer surplus), here we can interpret the minimax regret price as price that minimizes the lost surplus for the any distributions of valuations.

[6] We note that in the first-best solution we assume that the firm does have the complete information about F(.) and also observes the consumers' type at the point of purchase. Therefore under the first-best solution the firm does price discrimination.

2.2 Model setup and key informational assumptions

In our model we assume the monopolist sets prices to maximize profits for a zero marginal cost product. In this section we will discuss the information sets and decisions variables we assume for consumers and the monopolist.

The monopolist faces a unit mass of consumers with unit per period demand. We assume each consumer has a stable valuation v_i which is known privately only to the consumer. In any purchase occasion, the consumer observes the price set by the firm p and derives a utility $u_i = v_i - p$. She decides to purchase the good if only if $u_i \geq 0$ or equivalently $v_i \geq p$.

We assume that the firm conducts pre-launch market research that provides *partial information about* v_i. In particular, the only information available to the firm is that v_i lies between v_{iL} and v_{iH}. Where the firm knows v_{iL} and v_{iH} and has no prior information about the likelihoods of valuations in this range. In a related paper, Handel et al. [2013] provide an econometric framework that the monopolist can use to estimate v_{iL} and v_{iH} from pre-launch market research data. For notational convenience, we assume that $v_{iL} = v_L < \frac{v_{iH}}{4}$ for all consumers. [7] Therefore, the information available to the firm about consumer valuations can be summarized as $v_i \in [v_L, v_{iH}] \equiv \delta_i$ for every consumer i. The firm can aggregate these δ_i across consumers to define the set of possible distribution of consumer valuations. We allow for fully non-parametric heterogeneity in preferences across consumers. For example, consider a case where the firm is facing two consumers (A and B) and knows $v_A \in [v_L, v_{AH}]$ and $v_B \in [v_L, v_{BH}]$. The set of feasible aggregate valuations is defined as $\{(v_1, v_2) | v_1 \in [v_L, v_{AH}], v_2 \in [v_L, v_{BH}]\}$.

The type of uncertainty where the decision maker does not have information to place a subjective distribution over the support of possible outcomes is defined as *ambiguity.* Under ambiguity, it is not possible to calculate expected profits. Instead, the monopolist must make a decision based strictly on knowledge of the support of potential valuations. In this paper, we will derive the optimal prices assuming the firm will use the minimax regret criterion. Here, regret is a statistical characterization of the tradeoff between the potential losses from over pricing and the potential losses from under pricing.

[7] Note our results are robust to this assumption, this assumption allows us to simplify notation (see Lemma 1). In our main results (see Leemas 1 and 2, Theorems 1, 2 and 3), we will find that the optimal minimax regret price does not depend on v_L.

2.3 Static Minimax Regret with Homogeneous Preferences

In this section we will assume that all consumer are homogeneous in their preferences (valuations). Under this assumption, we can equivalently consider the monopolist selling to one representative consumer. The firm knows that the consumer has valuation $v \in [v_L, v_H]$ but has no prior information about the likelihoods of these different feasible valuations. The consumer knows her valuation, and will purchase the product if $v \geq p$.

In our framework, the monopolist trades off the losses from not making a sale with the losses from underpricing by choosing a price that minimizes her maximum regret. The monopolist's regret is her *first-best* profit, denoted π^* given the resolution of ambiguity minus the profit actually earned in that scenario under the chosen minimax regret price. In our setting, the monopolist's first best profit given true valuation v, is v. The actual profit earned, denoted $\pi(p, v)$ is p if $v \geq p$ and 0 otherwise. Thus, regret conditional on p and true valuation v is:

$$R(p, v) = \begin{cases} v - p & \text{if } v \geq p \\ v & \text{if } v < p \end{cases}$$

If the monopolist sells the product at p, regret is how much more she could have earned if charging the consumer's true value, this is the regret of under-pricing. If the monopolist does not sell the product, her regret equals the consumer's value v, the difference between the zero profit she earned and the maximum amount that she could have earned, this is the regret of over-pricing. The monopolist trades off these two different types of losses by choosing p to minimize her maximum regret over the entire space of ambiguity. She solves the general problem:

$$MMR = \min_{p} \max_{v \in [v_L, v_H]} R(p, v) = \min_{p} \max_{v \in [v_L, v_H]} v - p\mathbf{1}(v \geq p)$$

The following lemma restates the static optimal pricing rule found in Bergemann and Schlag (2006) under deterministic pricing:

Lemma 1. *The monopolist's static minimax regret price* $p^* = \frac{v_H}{2}$

Proof. The regret from under-pricing $(v - p; v \geq p)$ is maximized when $v = v_H$. While the regret from over-pricing $(v; v < p)$ is maximized at $v = p - \epsilon$ for $\epsilon \to 0$. Thus, for a given p, the maximum regret [8], $MR(p) = \max[v_H - p, p - \epsilon]$ $\epsilon \to 0$. Since $v_H - p$ is decreasing in p, this implies that minimax regret is attained when the regret from over-pricing with that from under-pricing are equal, when $v_H - p = p$ or $p^* = \frac{v_H}{2}$.

Note: If we remove the assumption that $v_L \leq \frac{v_H}{4}$, then the monopolist's solution, $p^* = \max[\frac{v_H}{2}, v_L]$. □

2.4 Static Minimax Regret with Heterogeneous Preferences

The firm in this setting prices to a continuum of consumers, each of whom is known to have a valuation $v_i \in [v_L, v_{iH}]$. Define $v_{H+} = \text{Max}(v_{iH})$, or the highest upper bound across consumers. Define $v_{H-} = \text{Min}(v_{iH})$, or be the lowest upper bound across consumers. To simplify exposition, we assume $v_{H-} > \frac{v_{H+}}{2}$. We assume, that the distribution of v_{iH} can be described by the continuously differentiable distribution $G(v_H)$ with bounded density $g(v_H)$. Since we assume the firm knows v_{iH} for every consumer i, therefore the firm knows $G(v_H)$ perfectly.

We interpret the ex-ante heterogeneity in preferences bounds as arising from a situation where the monopolist has some demographic information to characterize the population of consumers while having only partial information about preferences conditional on known demographic information. We assume that at the time of purchase the firm cannot distinguish between consumers and therefore does not have the required information to price discriminate.

In order to solve its pricing problem, the firm must contend with ambiguity over the set of feasible demand curves, which can be derived from knowledge about possible sets of valuations for each consumer. Each potential demand curve, $F(p)$ describes the proportion of buyers who will buy the product at a given price p. Each $F(p)$ is a weakly decreasing function mapping the space of feasible valuations to $[0, 1]$. We define the set of feasible demand curves, Ψ, as the set of weakly decreasing functions that satisfy the following restrictions:

$$\Psi \equiv F(p) : \begin{cases} F(p) = 1 & \text{if } p < v_L \\ F(p) \leq 1 - G(p) & \text{if } v_L \leq p \leq v_{H+} \\ F(p) = 0 & \text{if } p > v_{H+} \end{cases}$$

Ψ is the set of all possible true demand curves that the monopolist could be facing given his

[8] Please note that while this is labeled maximum regret, this is more formally the supremum of regret.

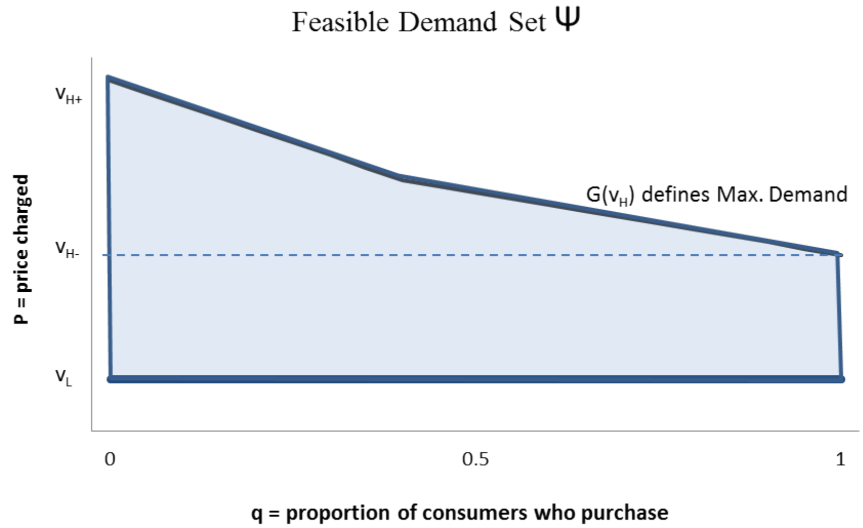

Figure 1: The set of feasible demand curves is every possible downward sloping demand curve that lies in the blue region.

knowledge of the distribution of consumer valuation supports. For each price p, as many as $1-G(p)$ consumers could have actual values more than p while it is possible for up to all consumers to have value less than p so long as $p > v_L$. Ψ is depicted in Figure 1.

Now, the monopolist chooses p to minimize her maximum regret relative to the set of feasible demand distributions. $\pi^*(F)$ is the first best profit given complete information about $F(.)$. This equates to charging all consumers their valuations

$$\pi^*(F) = \int_{v_L}^{v_{H+}} v \; dF(v)$$

The monopolist's actual profit given price p and demand distribution $F(.)$ is:

$$\pi(p, F) = pF(p)$$

Thus, the monopolist's regret for charging price p with true demand $F(.)$ is:

$$\begin{aligned} R(p, F) &= \pi^*(F) - \pi(p, F) \\ &= \int_{v_L}^{v_{H+}} v \; dF(v) - pF(p) \end{aligned}$$

For a given price $\hat{p}$, we define the monopolist's *worst case demand* as the potential demand curve $F(.) \in \Psi$ that will yield maximum regret. For each price $\hat{p}$ charged, a different $F(.)$ can yield maximum possible regret, making worst case regret. More formally:

$$F_{wc}(\hat{p}) = arg \max_{F(.)\in\Psi} R(\hat{p}, F)$$
$$MR(\hat{p}) = R(\hat{p}, F_{wc}(\hat{p}))$$

The price that the monopolist chooses under ambiguity given the set of feasible demand curves Ψ solves the minimax regret problem:

$$\min_{p} \max_{\Psi} R(p, F) \equiv \min_{\hat{p}} R(\hat{p}, F_{wc}(\hat{p}))$$

The monopolist's minimax regret problem is equivalent to the problem where the monopolist minimizes regret under $F_{wc}(\hat{p})$ with respect to $\hat{p}$. Intuitively, for each possible price the monopolist could charge, he considers the worst-case outcome conditional on that price within the set of feasible demand functions. The minimax regret price is the price that minimizes regret under its worst-case demand. Given the constant lower bound v_L for all δ_i, $F_{wc}(\hat{p})$ can be found using a threshold value assignment rule that depends on v_{iH}. $v_{wc}(v_{iH}, \hat{p})$ is the worst case valuation for specific consumer i given price $\hat{p}$. The following lemma shows (A) how we can determine v_{wc} as a function of p and (B) the solution to the monopolist's minimax max regret problem $\min_p MR(p, v_{wc})$

Lemma 2. *A) For $\hat{p} \leq v_{H-}$; $F_{wc}(\hat{p})$ is composed from potential individual valuations using the cutoff rule:*

$$v_{wc}(v_{iH}, \hat{p}) = \begin{cases} \hat{p} - \epsilon & \text{if } v_{iH} < 2\hat{p} \\ v_{iH} & \text{if } v_{iH} \geq 2\hat{p} \end{cases}$$

For $\hat{p} > v_{H-}$; we have $v_{wc}(v_{iH}, \hat{p}) = max(v_{iH}, \hat{p} - \epsilon)$

B) The monopolist's static regret minimizing price is $\frac{Med(v_H)}{2}$

Proof in the Appendix.

3 Dynamic Monopoly Pricing Under Ambiguity

To study the monopolist's multi-period pricing problem we use a dynamic version of minimax regret that extends the static criterion to account for the value of learning (see Hayashi [2011] for axiomatic foundations). We structure the learning dynamics assuming the monopolist is forward looking and non-Bayesian. Specifically, the monopolist understands how her set of feasible demand curves could be narrowed in each period conditional on **(i)** the price that she charges and **(ii)** the possible purchase quantities she could observe given that price.

In each period, the monopolist evaluates multi-period regret by computing foregone profits over *all* remaining periods for a given price, feasible demand curve, *and* future price. [9] The future price the monopolist considers depends on her decision-making dynamics. We structure decision-making dynamics with the assumption that the monopolist is *sequentially rational*. In our context, sequential rationality means that in each period the monopolist **(i)** dynamically minimizes maximum regret given her current information set and **(ii)** knows that in all subsequent periods she will do the same.[10] This assumption corresponds to the scenario where the monopolist's management team meets each period to determine current prices and cannot credibly commit to future prices. [11] Thus, when the monopolist computes multi-period maximum regret for a given current price, she endogenizes her future pricing behavior at each possible contingent information set. After computing multi-period maximum regret in this way for each possible price, the monopolist selects the price that minimizes this object.

The dynamic minimax regret solution for an N period problem (or N period continuation problem) can be found recursively using backwards induction. In the final period, this criterion reduces to static minimax regret. In the two-period framework, the monopolist solves its static minimax regret pricing problem for all potential second period information sets, and incorporates this information into its dynamic problem in the first period. We assume that there is no discount

[9]Though the monopolist may learn over time that an initially feasible demand curve is not true demand, from the current perspective the regret calculation with respect to any feasible demand curve presumes that that curve *is* true demand and, as a result, does not need to consider removal of that demand function over time.

[10]Sequential rationality in this single agent problem is similar to the concept of sub-game perfection in dynamic multi-player games.

[11]If the firm commits to a full sequence of prices it will not be able to take advantage of its ability to learn as it will evaluate the entire sequence of prices with respect to only the initially information available. Specifically, it will minimize multi-period maximum regret in period 1, but, in period 2, will be forcing itself to choose a potentially sub-optimal price. As a result, we view the assumption that the firm does not commit, and acts sequentially rational, as an appropriate way to set up dynamic decision-making and learning with the minimax regret criterion.

factor, i.e. both period profits are weighted equally in period one. The analytical challenge in analyzing this situation is that the worst case distribution (demand curve that causes the most regret) must be consistent with decision making in each time period.

We will start with the analysis of a homogeneous preference model and then consider the heterogeneous preference model.

3.1 Dynamic Minimax Regret with Homogeneous Preferences

We introduce dynamic minimax regret in a two period model where consumer have homogeneous preferences, here we consider the monopolist selling to one representative consumer. Here the consumer's first period purchase decision provides information to the monopolist that she will incorporate into her second period pricing rule. As in the static problem, we assume at the beginning of the first period that the monopolist knows that the consumer has valuation $v \in [v_L, v_H]$. We will denote the firm's first period information set as δ_1. After the monopolist sets a first period price p_1, she observes whether or not the consumer purchases, and updates her range of possible valuations. If the consumer purchases, the monopolist knows that the consumer's valuation must be at least p_1. If the consumer does not purchase, the monopolist knows that the consumer's valuation is lower than p_1. The monopolist's second period information sets, δ_2, under these contingencies are:

$$\delta_2 = [v_L, p_1] \text{ if } v < p_1$$

$$\delta_2 = [p_1, v_H] \text{ if } v \geq p_1$$

Once the monopolist has an updated information δ_2 at the beginning of the second period, she chooses the second period price $p_2^*(\delta_2)$ that minimizes maximum regret as described in the static model in section 2.3. In the first period the monopolist minimizes maximum regret over a state space composed of all possible valuations δ_1 *and* incorporates the way it will price in the second period conditional on the information set it must have at that point in time, contingent on $v \in \delta_1$ and p_1. For any given $v \in \delta_1$ and price p_1, there is *only one purchase history* that could be consistent with v. In other words, if v is the *true* valuation the monopolist knows in the first period, the information set she will have in period two. The monopolist's multiperiod regret will be the difference between her ideal profit (twice the consumer's actual valuation) and her actual profit summed over both periods. However, the space of uncertainty which she is concerned with in the first period will include only feasible second period behavior/information if v is in fact the true valuation. More formally, the firm's multiperiod regret as a function of p_1 and true valuation v is:

$$R(p_1, v) = 2v - p_1\mathbf{1}[v \geq p_1] - p_2(\delta_2(p_1, v))\mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]$$

The ability to learn enters this formulation through the ability of the monopolist to impact the second period information set with its choice of p_1. When choosing p_1 the monopolist considers the impact that this choice will have on δ_2, $p_2(\delta_2)$, and first period regret. The sequential optimality assumption implies that the monopolist cannot commit to p_2 in period one but does know what she will choose in that period conditional on her information set.

Maximum regret as a function of p_1 is:

$$MR(p_1) = \max_{v\in\delta_1} R(p_1, v) = \max_{v\in\delta_1} 2v - p_1\mathbf{1}[v \geq p_1] - p_2^*(\delta_2(p_1, v))\mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]$$

Given maximum regret conditional on p_1, the monopolist selects the first period price p_1^* that minimizes this maximum regret:

$$p_1^* = \arg\min_{p_1}\max_{\delta_1} R(p_1, v) = \arg\min_{p_1}\max_{\delta_1} 2v - p_1\mathbf{1}[v \geq p_1] - p_2^*(\delta_2(p_1, v))\mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]$$

Given this setting we can solve for the monopolists optimal solution. Our main result stated in Theorem 1, where we find that learning will lead the monopolist to increase her introductory price.

Theorem 1. Higher introductory prices when consumers with homogeneous preferences *The monopolist facing consumers with homogeneous preferences will set a higher introductory price in a dynamic setting relative to the static setting.* $p_1^* = \frac{4v_H}{7} > p^* = \frac{v_H}{2}$. *With* $MR(p_1^*) = \frac{6v_H}{7} = \frac{6}{7}MR(p^*)$.

Proof in the Appendix.

When we make the model dynamic to incorporate the value of learning under ambiguity, in the one consumer model the monopolist always chooses a first period price that is higher than the optimal price in the static model. If the consumer purchases the price remains at the higher level in the second period, if the consumer does not purchase the price is lowered. In the dynamic framework, the monopolist's minimax regret is $\frac{1}{7}$ less than it would be applying static minimax regret in a multiperiod setting, incorporating learning in both settings.

A firm lowering initial price would want to price in that direction only if information learned from the consumer purchasing could later reduce regret coming from high valuation consumers (otherwise aggregate regret could not be lower than repeating the static solution twice). However in this case, lowering initial price provides *no information to change* the second period regret. Consider the case that the consumer has valuation a $v = v_H$, if $p_1^* < \frac{v_H}{2}$ then the consumer will purchase and $\delta_2 \equiv [p_1^*, v_H]$. By lemma 1 $p_2^*(\delta_2) = p^*(\delta_1) = \frac{v_H}{2}$, the static minimax regret pricing rule. The regret in the second period will be $v_H - p_2^*(\delta_2) = \frac{v_H}{2}$. This is exactly the same as the static maximum regret.

Conversely, when the firm increases initial price, it learns valuable information that it can use in the contingency of the consumer having a valuation just below the price charged. [12] The ability to learn this information gives the firm flexibility to extract more of the profit if the consumer has a high valuation. Because it knows if the consumer has a low valuation, it can re-optimize in the second period and significantly lower regret. In effect, because there is less total value to be lost from the consumer having a low valuation. Here the firm can first try to extract profits assuming the consumer has a high valuation and if that fails, the firm can lower the second period price.

3.2 Dynamic Minimax Regret with Heterogeneous Preferences

In this section, we extend the analysis to consider dynamic minimax regret to the case where consumers have heterogeneous preferences. Here each consumer's first period purchase decision provides information to the monopolist that she will incorporate into her second period pricing rule. As in the static problem, we assume at the beginning of the first period that the monopolist knows that each consumer i has valuation $v_i \in [v_L, v_{iH}]$. We will define $\delta_{i1} \equiv [v_L, v_{iH}]$ as the identified set for consumer i in time period 1. We describe the first period purchase decisions of each consumer with the binary variable $b_i = \mathbf{1}[v_i \geq p_1]$. The monopolist's second period information on each

[12] In the static case, the regret from overpricing is highest when the consumers valuation is just below the price charged

consumer, δ_{i2} is obtained by narrowing δ_{i1} for each consumer after the first period conditional on b_i

$$\delta_{i2}(p_1, b_i) \equiv [v_L, p_1] \text{ if } b_i = 0$$
$$\delta_{i2}(p_1, b_i) \equiv [p_1, v_{iH}] \text{ if } b_i = 1$$

Once the monopolist has updated information δ_{i2} for each i, she determines the space of feasible demand curves in period two and sets p_2 as described in the static model of pricing under ambiguity. In order to derive the set of feasible demand curves contingent on consumer first period purchase decisions, we need to introduce some additional notation. Denote by q the proportion of consumers that purchased the product in the first period. Define $G(v_H|b=1)$ as the distribution of valuation upper bounds in second period conditional on consumers having purchased in the first period. For consumers that purchase, their (a) lower bound of preferences will be p_1 and (b) upper bound will remain the same in period two (when $p_1 < v_{H-}$, this will be true from the assumption that $v_{H-} > \frac{v_{H+}}{2}$). Since consumers with different v_{iH} are identifiable from one another, the monopolist derives the distribution $G(\cdot|b=1)$ as follows:

$$G(v_H|b=1) = \frac{\int_{v_{H-}}^{v_H} g(s|b=1)ds}{q}$$

A decision not to purchase the product in the first period reduces the upper bound of feasible valuations for consumer i from v_{iH} to p_1, implying that $G(v_H|b=0)$ is degenerate with all mass at value p_1. The firm will maintain the lower bound of v_L for these consumers. Using these properties, we derive the set of feasible second period demand curves, Ψ_2, as the set of weakly decreasing functions satisfying the following restrictions (graphically shown in figure 2):

$$\Psi_2 \equiv F_2(p) : \begin{cases} F_2(p) = 1 & \text{if } p < v_L \\ F_2(p) \geq q & \text{if } v_L \leq p < p_1 \\ F_2(p) = q & \text{if } p = p_1 \\ F_2(p) \leq q(1 - G(p|b=1)) & \text{if } p_1 < p \leq v_{H+} \\ F_2(p) = 0 & \text{if } p > v_{H+} \end{cases}$$

In addition to being a description of the set of feasible demand curves in the second period,

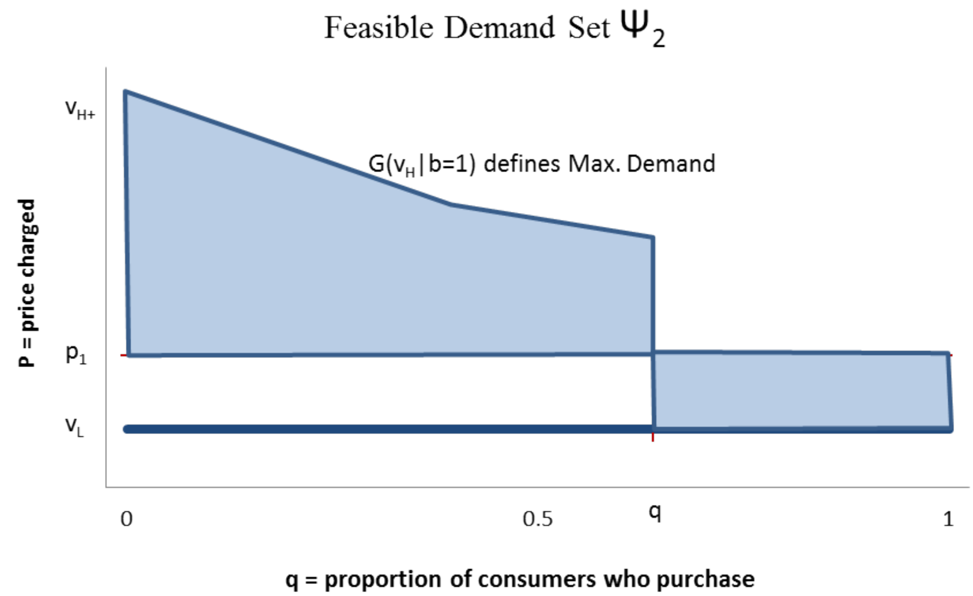

Figure 2: A representation of Ψ_2. The shaded region represents the space in which all downward sloping demand curves are feasible in period 2. Notice that all demand curves must go through point (p_1, q) which is the exact price and quantity sold in period one.

we note that Ψ_2 implicitly describes the first period purchase history of consumers. That is, that knowledge of Ψ_2 implies knowledge of q and $G(v_H|b = 1)$.

In the heterogeneous preference setting the structure of the second period information set is different from that in the static model therefore with dynamic consistency we solve this in two steps. In section 3.2.1 we study second period pricing problem conditional on the information structure Ψ_2, and use the results from that to study the first period pricing problem in section 3.2.2 where we will derive our main results.

Before presenting our main results we add two regularity assumptions on the distribution $G(v_H)$. These assumptions are conservative and ensure that there are no sections of $G(v_H)$ where there is low density $g(v_H)$ relative to $1 - G(v_H)$, similar to a monotone likelihood ratio assumption. These relationships for the median of v_H of truncated distributions of $G(v_H)$ hold for all standard distributions that occur on a bounded interval $G(\cdot)$ (truncated normal, uniform, etc.). Condition (I) states that the median of successive truncations of G does not change too quickly as q changes. Condition (II) states that the median of successive truncations of G does not change too quickly as the truncation threshold x changes. [13]

[13] These two assumptions can be mapped directly into one another given a specified relationship between values and

Assumption 1 (Median Regularity).

$$\text{(I)} \qquad \frac{\partial MED(v_H | v_H > Q_{1-q})}{\partial q} > -\frac{MED(v_H | v_H > Q_{1-q})}{q} \; \textit{for } q > 0$$

$$\text{(II)} \qquad \frac{\partial MED(v_H | v_H > x)}{\partial x} < 1$$

Where Q_x denotes the x quantile of $G(v_H)$.

3.2.1 Second Period Pricing

The second period monopoly pricing problem is conceptually identical to the static pricing problem under ambiguity described in section 2.4. However, learning from prior period gives the space of feasible demand curves Ψ_2 a different structure in the second period problem. We solve the monopolist's second period problem, considering the two cases of selecting $p_2 > p_1$ or $p_2 \leq p_1$. This is a useful framework because, conditional on choosing $p_2 > p_1$, the monopolist essentially ignores consumers who did not purchase in the first period, as the maximum regret for consumers who did not purchase in the first period will be p_1 which does not depend on p_2. Therefore the prices in the second period are set to minimize maximum second period regret among only consumers who purchased in the first period. Conversely, if $p_2 < p_1$, the monopolist sells for sure to all first period purchasers in the second period. However the regret from first period purchasers increases relative to first period as a result of the reduced price. On the other hand, lowering p_2 can only reduce regret for consumers who did not purchase in the first period.

Lemma 3 describes the monopolist's second period solution, conditional on p_2 being either larger or smaller than p_1. For each feasible Ψ_2 and corresponding q that could arise conditional on p_1, the monopolist will choose p_2 to minimize maximum regret across the restricted solutions described in Lemma 3. Specifically, If the minimax regret solution conditional on $p_2 > p_1$ yields lower (higher) maximum regret than the solution conditional on $p_2 \leq p_1$, the monopolist will choose the p_2^* and face maximum regret found in the solution restricting $p_2 > p_1$ ($p_2 \leq p_1$). In the solution to the full model, we use these properties of the second period solution as inputs into finding the dynamic

quantiles of G. Instead of unifying these assumptions with such a mapping, for exposition and clarity we state both. Lastly, while condition (II) says that the change in the median relative to changes in the truncation value cannot be too large in absolute value, condition (I) makes sure that this derivative with respect to quantiles does not jump too quickly relative to the actual median normalized by the fraction of individuals purchasing.

minimax regret solution.

Lemma 3. *I) Conditional on selecting second period price $p_2 > p_1$, the monopolist's second period minimax regret price p_2^* and maximum regret given that price are:*

$$\begin{aligned} p_2^* &= \frac{Med(v_H|b=1)}{2} \\ MR_2 &= (1-q)p_1 + q(p_2G(2p_2|b=1) + \int_{2p_2^*}^{v_{H+}} v_H - p_2 \; dG(v_H|b=1)) \end{aligned}$$

II) Conditional on selecting second period price $p_2 \leq p_1$, the monopolist's second period minimax regret price p_2^* and maximum regret given that price are:

$$\begin{aligned} p_2^* &= \begin{cases} \frac{p_1}{2} & \text{if } q < 0.5 \\ \in [\frac{p_1}{2}, p_1] & \text{if } q = 0.5 \\ p_1 & \text{if } q > 0.5 \end{cases} \\ MR_2 &= (1-q)p_2^* + [\int_{p_1}^{v_{H+}} (v_H - p_2) dG(v_H|b=1)]q \end{aligned}$$

3.2.2 First Period Pricing

When setting a dynamic minimax regret first period price, the monopolist knows what second period price she will charge contingent on any information set Ψ_2. For each p_1 the monopolist could charge, she knows which Ψ_2 are *possible* in the second period. Furthermore, the monopolist knows that the demand function $F(.)$ is the same for both periods. This has two main implications for our solution. First, since the worst-case demand function must be the same for both periods, the two periods need to be solved jointly. Second, when evaluating dynamic regret from a given $F(.) \in \Psi_1$ for a specific p_1, the monopolist knows exactly what consumer purchase decisions in the first period she will observe under this demand function.

The monopolist's first period problem dynamic minimax regret problem is:

$$\min_{p_1} \max_{F(.) \in \Psi_1} 2\pi^* - \pi_1(p_1, F) - \pi_2(p_2^*(\Psi_2(p_1, F)), F)$$

The monopolist's ideal profit $2\pi^*$ is still the single-period first best profit given known demand $F(.)$ earned in each period. The first period profit π_1 depends on the first period price and true $F(.)$. The second period profit π_2 depends on the second period price, set contingent on the information learned from first period purchase decisions, and true demand. The monopolist minimizes maximum regret dynamically by selecting the price that yields the lowest possible maximum aggregate regret over all feasible demand curves. The knowledge of *how* she will set the second period price impacts the multi-period maximum regret calculation for each first period price. For instance, if the monopolist sets a high initial price and nobody purchases, she knows that she will respond by setting a much lower price in the second period. Therefore from a *first period perspective* maximum multi-period regret from setting a high price may not come from a feasible demand curve that leads to no consumers initially purchasing because second period maximum regret will be low in this contingency.

In order to solve the monopolist's first period dynamic minimax regret pricing problem, we need to find what feasible demand curve will yield the highest maximum regret for a given p_1 conditional on sequentially optimal behavior in the second period. We define worst-case demand in period t conditional on p_t:

$$F_{wc}(p_t) = arg \max_{F(.)\in\Psi_t} R(p_t, F)$$

In the previous subsection we studied the monopolist's second period solution, which depends on $F_{wc}(p_2)$. $F_{wc}(p_2)$ is determined within the context of the second period static minimax regret problem. In this section, we will characterize properties of $F_{wc}(p_1)$ in order to find the dynamic minimax regret pricing solution p_1^*. Determining $F_{wc}(p_1)$ is much more challenging than $F_{wc}(p_2)$ because it must account for the dynamic price and outcome path engendered by p_1, not just static outcomes. To analyze $F_{wc}(p_1)$ we must know (i) $F_{wc}(p_2)$ for each feasible Ψ_2 and p_2 (ii) p_2^* conditional on each value of (p_1, Ψ_1, F) and (iii) what multi-period maximum regret will be for p_1 for each $F(.)$ given the learning and second period pricing that will occur. Once we know $F_{wc}(p_1)$ for each p_1, the monopolist selects the first period price p_1 that minimizes multi-period maximum regret with respect to $F_{wc}(p_1)$. In the previous subsection we discussed how to characterize (i) and (ii), while this section uses those results to address (iii) and characterize the dynamic minimax regret first period solution.

We prove our main results in the following steps:

1. **Characterize the worst case demand function.** We identify a subset of demand curves

within Ψ_1 that could be worst case demand, conditional on p_1. We consider the two cases $p_1 < p_2^*$ and $p_1 \geq p_2^*$ separately and for each consider what functions could be the worst case demand functions. This step simplifies our problem by determining the effect of the two possible sequential minimax regret pricing rules on $F_{wc}(p_1)$. Restricting p_2 relative to p_1 allows us to narrow the space of feasible $F_{wc}(p_1)$ to two demand curves conditional on p_1, which we use to determine which p_2 yields maximum regret. Specifically here we establish the worst case demand curve does not have any weight on valuations who purchase in only one period. Our result characterized formally by the following claim.

Claim 1. *I) When $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) > p_1^*$, $F_{wc}(p_1)$ does not contain consumers who purchase only in the first period. II) When $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) \leq p_1^*$, there exists $F_{wc}(p_1)$ that does not contain consumers who purchase only in the second period. Proof in the Online Appendix.*

2. **Showing dynamic consistency.** Using the results from section 3.2.1, we show dynamic consistency along the dynamic minimax regret price path. This shows that along the optimal price path, the monopolist prefers the same p_2^* (i.e. same pricing direction relative to p_1) before and after learning the worst case demand in period one. Our result characterized formally by the following lemma.

   Lemma 4. *Conditional on p_1 and on Ψ_2 consistent with $F_{wc}(p_1)$ occurring, along the dynamic minimax regret price path, the monopolist would choose $p_2 = p_2(\Psi_2(F_{wc}(p_1), p_1))$ ex ante. Proof in the Appendix.*

3. **Optimal pricing** We use these results as inputs into the main theorem where we solve for optimal first period prices.

   Theorem 2. *When G is non-degenerate $p_1^* < p^*$, i.e. the monopolist will lower introductory price in a dynamic setting relative to the static setting. Further, for any Ψ_1, $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) > p_1^*$ along the dynamic minimax regret price, i.e. if a purchase history consistent with worst-case demand is observed, the monopolist will price upwards over time. When G is degenerate, $p_1^* = p^* = p_2(\Psi_2(F_{wc}(p_1, p), p_1))$.*
   Proof in the Appendix.

This result characterizes two important features of the monopolist's dynamic minimax regret pricing rule in a setting with arbitrary heterogeneity in preferences across consumers. First, the ability to learn about demand causes the monopolist to lower introductory price relative to a static setting. Second, after the introductory price, if the worst possible partial realization of uncertainty occurs through learning from first period purchases, the monopolist will increase her price in the second period.

Intuitively, both these results are true because of the way information learned through first period purchases impacts dynamic maximum regret in an environment with preference heterogeneity. For any p_1, the monopolist's worst-case demand will have some consumers who do not purchase with values just below p_1, and some consumers that do purchase and have values equal to their maximum valuations. Crucially, under worst-case demand, consumers who do not purchase will have low v_H relative to those who do pruchase. Setting $p_2 > p_1$ allows the monopolist to establish low maximum regret levels from consumers who do not purchase over both periods (low v_H), which lets the monopolist then focus exclusively on minimizing maximum regret for high valuation consumers in the second period. Setting $p_2 < p_1$ does not similarly bound maximum regret from consumers that purchase. When the monopolist lowers price over time she can restrict the valuation of someone who purchases to above p_1, but when lowering price deterministically increases maximum regret from this set of consumers by $p_1 - p_2$ for each consumer. This result depends critically on ex-ante information heterogeneity. We have shown that lower initial price are valuable when ex-ante heterogeneity information exists in a dynamic setting with ambiguity and that the monopolist will decrease initial prices. However, where there is no ex-ante heterogeneity information (G is degenerate), lowering initial prices is not valuable because there are no consumers with relatively low v_H to target with a lower first period price.

We derive the dynamic MMR result with uniform ex-ante heterogeneity information in the following sections.

3.2.3 Dynamic Pricing with Uniform ex-ante Information Heterogeneity

We illustrate the solution to the heterogeneous preference model by explicitly solving the monopolist's dynamic minimax regret problem when pricing to a mass of consumers with heterogeneous preferences (v_i). The firm has ex-ante information to bound each consumers preferences ($v_i \in [v_L, v_{Hi}]$). In this section we add the assumption that the distribution of v_{Hi} ($G(v_H)$) is uniform, or $G(v_H) \to U[v_{H-}, v_{H+}]$. This form of demand ambiguity could arise in a setting where the monopolist observes a uniform distribution of an important demographic variable in its target population, and believes that the most someone could value their product is linked directly to that variable.

Given the result of Theorem 2, we know that the monopolist will price charge a lower introductory price when consumers have heterogeneous preferences and that, under a first period purchase history consistent with worst case demand, the monopolist will choose a higher second period price. The following theorem solves explicitly for the monopolist's solution.

Theorem 3. *The dynamic minimax regret first period pricing rule for the monopolist with uniform ex-ante heterogeneity information is* $p_1^* = \frac{23v_{H+}+49v_{H-}}{144} < \frac{v_{H-}+v_{H+}}{4} = p^*$. $p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*)) = \frac{714v_{H+}+294v_{H-}}{2016} > p_{MMR}$.

Proof in the Appendix.

Corollary 1 describes how the extent of introductory price reduction relates to the degree of ex-ante information heterogeneity in the population. We find as the distribution of ex-ante information heterogeneity becomes more dispersed, the monopolist's introductory price decreases. Likewise, as the distribution of ex-ante information heterogeneity becomes less disperse, the monopolist's introductory price increases. In the limiting case, as $v_{H+} - v_{H-} \to 0$, we find $p_1^* \to p^*$ from below. This is consistent with the ex-ante homogenous case (no ex-ante information, or $G(.)$ is degenerate), where the first period dynamic price (p_1^*) is exactly equal to the static price (p^*) (see Theorem 2).

Corollary 1. *For* $G(\cdot) \to U[v_{H-}, v_{H+}]$ *and* $G'(\cdot) \to U[v_{H-} - \alpha, v_{H+} + \alpha]$, $p_1^*(\Psi_1(G)) > p_1^*(\Psi_1(G'))$ *for* $\alpha > 0$.

Proof: Follows directly from $p_1^* = \frac{23v_{H+}+49v_{H-}}{144}$.

3.2.4 Alternatives to minimax regret with Uniform ex-ante Information Heterogeneity

An alternative to minimax regret in the decision theory literature, is to use the maxmin criterion, or set a price that maximizes the firm's minimum possible payoff over the range of possible valuations. Formally $p^{MM} = \arg\max_p \min_{F(.)\in\Psi} \pi(p, F)$. In our setting the maxmin price is set to v_L for the static, dynamic first and second period cases. To see this, consider a distribution where all consumers are of type v_L (a feasible distribution), any price larger that v_L will yield zero profit. Therefore the minimum profits for any price higher than v_L is 0, while the minimum profits for a price v_L is v_L. We believe the maxmin criterion is less appealing that minimax regret in a dynamic pricing context for two reasons. First, maxmin focuses only on the potential losses from not selling the product (overpricing) and does not consider the potential losses from foregone profits by selling at a price far lower than the consumer's valuation (underpricing). In contract, minimax regret explicitly considers the tradeoff between underpricing and from overpricing. Second, unlike dynamic minimax regret, there is no price experimentation or learning with dynamic maxmin.

In appendix section 8, we derive the optimal Bayesian price assuming an uninformative prior. Explicitly here the decision maker assumes, based on a *subjective prior*, that each consumers valuations $v_i \sim U[v_L, v_{Hi}]$. In a dynamic setting the Bayesian decision maker can update her information set based on the price charged (p) and purchase decision of the consumer. If consumer i purchases at a price p, the monopolist will update her information to $v_i \sim U[p, v_{Hi}]$. If consumer i does not purchases at a price p, the monopolist will update her information to $v_i \sim U[v_L, p]$. We derive the uninformative prior Bayesian price as:

$$p^B = \frac{1}{2}\left(v_L + \frac{v_{H+} - v_{H-}}{log(v_{H+} - v_L) - log(v_{H-} - v_L)}\right)$$

We find that the optimal Bayesian price depends on v_L, v_{H+} and v_{H-}, whereas the dynamic minimax regret price depends only on v_{H+} and v_{H-}. [14] Moreover, we show that $\frac{\partial p^B}{\partial v_L} < 0$ (Appendix section 8). Therefore if the firm gets new data that increases v_L for all consumers, the Bayesian price derived p^B will decrease (till $p^B = v_L$). Consider the two time period pricing decisions for a decision maker using Bayesian updating. The implication here is that if the firm sets a price in the first time period (p_1^B) and all consumers purchase, the second period price (p_2^B) will be equal to p_1^B. This is unlike the dynamic minimax regret solution where p_2^{MMR} will be higher than p_1^{MMR} if all consumer purchase in the first time period.

We will evaluate the implications of these decision criterion on firm profits in the next section.

[14] One of the implications here is that adding heterogeneity in v_L across consumers will impact the Bayesian price but not the minimax regret price

3.2.5 Numerical Example with Uniform ex-ante Information Heterogeneity

We present a numerical example that illustrates the solution with uniform ex-ante information heterogeneity. Consider a new product launch for a non-storable good with considerable ambiguity. Assume, the monopolist knows that all consumers could have the same minimal value $v_L = 2$ and the highest possible valuations v_{iH} are distributed $U[10, 18]$ ($v_{H-} = 10$, $v_{H+} = 18$). The static minimax regret price (p^*_{MMR}), the first period dynamic minimax regret price p^*_1), the maximin price (p^{MM}) and Bayesian price with an uninformative prior (p^B) are given below

$$
\begin{aligned}
p^{MMR} &= \frac{median(v_H)}{2} = \frac{v_{H+} + v_{H-}}{4} = 7.0 \\
p_1^{MMR} &= \frac{23v_{H+} + 49v_{H-}}{144} = 6.3 \\
p^{MM} &= v_L = 2.0 \\
p^B &= \frac{1}{2}\left(v_L + \frac{v_{H+} - v_{H-}}{log(v_{H+} - v_L) - log(v_{H-} - v_L)}\right) = 6.8
\end{aligned}
$$

To understand the profit implications of the Minimax regret price, we simulate true preferences from a parametric distribution and compute the resultant firm profits. Note importantly, consistent with our model we assume that the firm only knows that $v_i \in [v_L, v_{Hi}]$, and does not know the distribution within this set. We generate consumers' true valuations v_i from a Beta distribution with parameters α and β between v_L and v_{Hi}. [15] We compare the profit results for the following six prices: (1) static minimax regret price ($p^{MMR} = 7.0$); (2) first period dynamic minimax regret price ($p_1^{MMR} = 6.3$); (3) second period dynamic minimax regret price (p_2^{MMR}, derived numerically as a function of p_1^{MMR} and first period purchase decisions); (4) maxmin price, there is a single solution for the static and both periods in the dynamic model ($p^{MM} = 2.0$); (5) static Bayesian price assuming an uninformative prior ($p^B = 6.8$); and (6) second period Bayesian updating with an uninformative prior (p_2^B, derived numerically as a function of p^B and first period purchase decisions). We simulate outcomes for three sets of parameters (α and β), to see how the shape of the true preferences impacts the resultant profits. For each set of parameters, we simulate preferences for 500,000 consumers.

The results of the simulation are shown in figure 3. The rows of the figure represent the three simulations (labeled A though C) with different parameter (α,β) values to generate the true

[15] We thank the associate editor for suggesting this simulation.

preferences ($v_i \sim$ Beta(α, β) between v_L and v_{Hi}) [16]. The first column of the figure plots the distribution of valuations across the population. The second column plots the prices charged based on the different pricing methods described above. The third column plots the realized profits. We will discuss the results for each simulation in detail.

- In simulation A ($\alpha = 2$, $\beta = 9$), true preferences are skewed to the left. In a static setting, the maxmin price ($p^{MM} = 2.0$) ensures all consumers purchase and results in higher profits than the both the minimax regret price (p^{MMR}) and the uninformed Bayesian price (p^B). In dynamic setting, under minimax regret the firm observes only 9% of consumers purchase in the first period and lowers price in the second period ($p_2^{MMR} = \frac{p_1^{MMR}}{2} = 3.1$, as in Lemma 3 part II under the condition $q < 0.5$). We find this results in higher profits under the second period dynamic minimax regret price than the maxmin price. With Bayesian updating, the firm will also lower price in the second period (p_2^B). We find that the dynamic minimax regret prices result higher profits that the Bayesian updating prices in both time period.

- In simulation B ($\alpha = 2$, $\beta = 2$), true preferences are symmetric. In a static setting, both the minimax regret price (p^{MMR}) and the uninformed Bayesian price (p^B) result in higher profits that the conservative maxmin price (p^{MM}). In the dynamic setting, under minimax regret the firm observes 69% of consumers purchase in the first period and will not change the price in the second period ($p_2^{MMR} = p_1^{MMR} = 6.3$, as in Lemma 3 part II under the condition $q > 0.5$). With Bayesian updating, the firm will also maintain the same price in the second period (p_2^B). We find that both dynamic minimax regret prices and Bayesian updating prices result in similar profits in both time period.

- In simulation C ($\alpha = 9$, $\beta = 2$), true preferences are skewed to the right. In a static setting, the minimax regret price (p^{MMR}) results in higher profits that the uninformed Bayesian price (p^B) and the conservative maxmin price (p^{MM}). In the dynamic setting, under minimax regret the firm observes nearly all consumers purchase in the first period and will increase the price in the second period ($p_2^{MMR} = \frac{Med(v_H|b=1)}{2} = 7.0$, as in Lemma 3 part I). This is unlike the case with Bayesian updating, where despite high demand, the firm will maintain the same

[16] In addition to the simulations discussed below we added a simulation with U shaped preferences ($\alpha = 0.5$, $\beta = 0.5$, the results are similar to the simulations shown here.

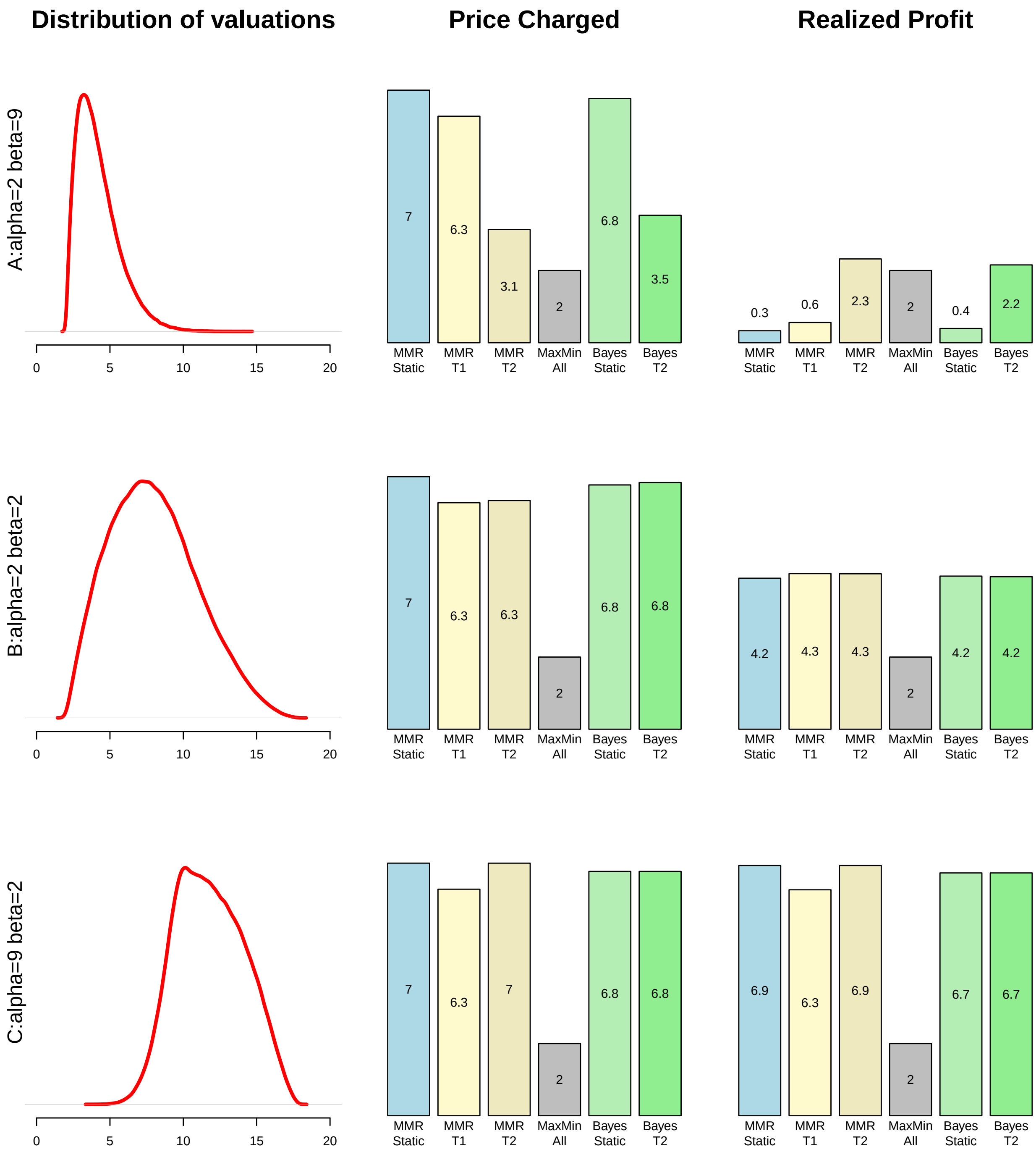

Figure 3: Simulation experiment with the following setting $v_L = 2$; $v_{Hi} \sim U[10, 18]$; $v_i \sim \text{Beta}(\alpha, \beta)$ between v_L and v_{Hi}. We consider three sets of parameters (α and β) labeled A through C (rows of the figure above). In each simulation we simulate the true preferences for 500,000 consumers from the assumed distribution. The charts in the first column represents the true distribution of preferences for the simulation. The charts in the second column and third column represent the prices charged and the corresponding realized profits.

price in the second period (p_2^B) (as discussed in 3.2.4). We find that the dynamic minimax regret second period price result higher profits than the Bayesian updating price.

In summary, these simulations highlight the way dynamic minimax regret can learn about demand from the first period and adjust prices in the second period. Further, these simulations illustrate that ability to learn results in comparable or higher second period profits.

3.3 Summary of Results

Overall our results suggest that the difference between the first period dynamic price and the static price depends critically on consumer heterogeneity. The main results are characterized in the Table 1. The results show that the role of leaning information varies with heterogeneity in preferences and ex-ante information heterogeneity. In a setting where all consumers have homogeneous preferences, the monopolist will set high (compared to the static model) period 1 prices, with lowering the price in period 2 if consumers do not purchase. While in a setting with heterogeneity in preferences and ex-ante information, the monopolist will set low (compared to the static model) period 1 prices, and if the worst case demand is realized then will increase prices in period 2. The main difference lies in the fact that the homogeneous preference model restricts the set of feasible demand functions to be mass points, which alters the intuition in the heterogeneous preference model. In the homogeneous consumers model, increasing initial price is attractive as consumer valuations must be either above or below the higher price. If we increase price and consumers do not purchase, then reducing future prices significantly decreases future regret. In the heterogeneous preference model, this would not be the case because worst case demand would have consumers with valuations both above and below p_1. Moreover, under the worst case demand consumers who do not purchase will have lower ex-ante valuations than consumers who do purchase. Here decreasing price allows the firm to bound the maximum regret from these ex-ante lower value consumers and potentially target the ex-ante higher valuation consumers with increasing price in period 2. This result depends critically on ex-ante information heterogeneity. When the firm does not have information about ex-ante heterogeneity, lowering price is no longer valuable.

Assumption about heterogeneity		Main Result		
	Ex-ante	Static	Dynamic	
Preferences	Information	p^*	p_1^*	p_2^* (under F_{wc})
No		$\frac{v_H}{2}$	$> p^*$	p_1^* or $\frac{p_1^*}{2}$
Yes	No	$\frac{Median(v_H)}{2}$	$= p^*$	$= p_1^*$
Yes	Yes	$\frac{Median(v_H)}{2}$	$< p^*$	$> p_1^*$

Table 1: Main Results for Robust New Products Prices

4 Conclusion

This paper focuses on how monopolists will price when they face ambiguity about demand, but can reduce that ambiguity over time as they acquire information from consumer purchase decisions. The manger will make pricing decisions in a manner that optimally trades off information learned about demand with current profits in an environment. We show that incorporating learning causes the monopolist to reduce introductory prices and then adaptively price based on the information that is learned. When the first period purchase outcomes are as bad as possible from the monopolist's perspective, she will price upwards over time. We present a novel non-Bayesian framework for studying dynamic decision making. Marketers introducing new products face significant ambiguity when pricing new products. The current literature assumes that the manager uses a Bayesian prior to price in early periods. In contrast, we assert that the manager knows only that each consumer's valuation lies with a range of possible valuations and, has no subjective information on which of these are more likely. An alternative view of our results is to understand the implications of subjective beliefs on pricing decisions.

Our main result show that the monopolists can offer a lower, unchanged or higher introductory price in a dynamic environment (as compared to a static environment) depending on the type of heterogeneity in the market. We find (1) when consumers have homogeneous preferences, introductory dynamic price is higher than the static price (2) when consumers have heterogeneous preferences and the monopolist has no ex-ante information, the introductory dynamic price is the same as the static price and (3) when consumers have heterogeneous preferences and the monopolist has ex-ante information, the introductory dynamic price is lower than the static price. Further the degree of this initial reduction increases with the amount of heterogeneity in the ex-ante information. The extant literature shows that dynamics in pricing are optimal either due to dynamics in demand (e.g., evolving preference) or supply (e.g., inventory constraints). We add a demand learning objective for the firm that can also lead to dynamic optimal prices.

In this paper, we acknowledge that we study a stylized problem with three main limitations. First, while we study a monopoly pricing problem, situations where pricing under ambiguity is relevant might involve multiple firms pricing strategically. Second, our model incorporates firm side learning, while consumer valuations are static over time. In new product settings, consumers might also exhibit preference formation or learning (e.g., Erdem and Keane [1996]). Third, we are restricted to non-storable goods. Considering storable goods a research might consider goods where consumers have incentives to stockpile (Hendel and Nevo [2006]) and goods (e.g., technology) where firms might consider temporal price discrimination (e.g., Nair [2007]).

While the focus of our paper is on pricing, the framework presented here can be applied to other marketing mix instruments (e.g., advertising). In many settings, Marketers have incomplete information about consumers preferences and/or responses to marketing mix instruments. Considering ambiguity provides an alternative for managers unwilling to make subjective prior assumptions. We hope future empirical and analytical research in Marketing will explore robust marketing mix decisions.

References

K. J. Arrow. The economic implications of learning by doing. *The Review of Economic Studies*, 29 (3):155–173, 1962.

M. O. Ball and M. Queyrannem. Toward robust revenue management: Competitive analysis of online booking. *Operations Research*, pages 950–963, 2009.

D. Bergemann and K. Schlag. Pricing without priors. *Journal of the European Economic Association*, 6(2-3):560–569, 2008.

D. Bergemann and K. Schlag. Robust monopoly pricing. *Journal of Economic Theory*, 146(6): 25272543, 2011.

J. O. Berger. *Statistical decision theory and Bayesian analysis*. Springer, 1985.

O. Besbes and A. Zeevi. Dynamic pricing without knowing the demand function: Risk bounds and near-optimal algorithms. *Operations Research*, 57(6):1407–1420, 2009.

O. Besbes and A. Zeevi. On the minimax complexity of pricing in a changing environment. *Operations Research*, 59(1):66–79, 2011.

E. Biyalogorsky and O. Koenigsberg. The design and introduction of product lines when consumer valuations are uncertain. *Production and Operations Management*, 2014.

L. E. Bolton. Stickier priors: The effects of nonanalytic versus analytic thinking in new product forecasting. *Journal of Marketing Research*, 40(1):pp. 65–79, 2003.

L. E. Bolton and A. Reed. Sticky priors: The perseverance of identity effects on judgment. *Journal of Marketing Research*, 41(4):397–410, 2004.

A. Bonatti. Menu pricing and learning. *American Economic Journal: Microeconomics*, 3(3):124–163, 2011.

D. J. Braden and S. S. Oren. Nonlinear pricing to produce information. *Marketing Science*, 13(3): pp. 310–326, 1994.

S. Bubeck and N. Cesa-Bianchi. Regret analysis of stochastic and nonstochastic multi-armed bandit problems. *Foundations and Trends in Machine Learning*, 5(1), 2012.

P. S. Desai, O. Koenigsberg, and D. Purohit. The role of production lead time and demand uncertainty in marketing durable goods. *Management Science*, 53(1):pp. 150–158, 2007.

R. J. Dolan. *Managing the new product development process*. Addison-Wesley, 1993.

T. Erdem and M. P. Keane. Decision-making under uncertainty: Capturing dynamic brand choice processes in turbulent consumer goods markets. *Marketing Science*, 15(1):pp. 1–20, 1996.

D. Forlani, J. W. Mullins, and O. C. Walker. New product decision making: How chance and size of loss influence what marketing managers see and do. *Psychology and Marketing*, 19(11):957–981, 2002.

W. B. Gartner and R. J. Thomas. Factors affecting new product forecasting accuracy in new firms. *Journal of Product Innovation Management*, 10(1):35–52, 1993.

P. E. Green and V. Srinivasan. Conjoint analysis in consumer research: Issues and outlook. *Journal of Consumer Research*, 5(2):pp. 103–123, 1978.

S. Grossman, R. Kihlstrom, and L. Mirman. A bayesian approach to the production of information and learning by doing. *The Review of Economic Studies*, 44(3):533–547, 1977.

B. Handel, K. Misra, and J. Roberts. Robust firm pricing with panel data. *Journal of Econometrics*, 174(2), 2013.

J. M. Harrison, N. B. Keskin, and A. Zeevi. Bayesian dynamic pricing policies: Learning and earning under a binary prior distribution. *Management Science*, 58(3):pp. 570–586, 2012.

T. Hayashi. Context dependence and consistency in dynamic choice under uncertainty: The case of anticipated regret. *Theory and Decision*, 70(4):399–430, 2011.

I. Hendel and A. Nevo. Measuring the implications of sales and consumer inventory behavior. *Econometrica*, 74(6):1637–1673, 2006.

G. Hitsch. Optimal dynamic product launch and exit under demand uncertainty. *Marketing Science*, 25(1):pp. 25–30, 2006.

I. L. Janis and L. Mann. *Decision making: A psychological analysis of conflict, choice, and commitment.* Free Press, 1977.

K. B. Kahn. An exploratory investigation of new product forecasting practices. *Journal of Product Innovation Management*, 19(2):133–143, 2002.

K. B. Kahn. *New-Product Forecasting.* John Wiley and Sons, Ltd, 2010. ISBN 9781444316568.

R. Kleinberg and F. T. Leighton. The value of knowing a demand curve: Bounds on regret for on-line posted-price auctions. Akamai Technologies, 2014.

F. H. Knight. *Risk, uncertainty and profit (reprint, original 1921).* Courier Dover Publications, 2012.

T. L. Lai. Adaptive treatment allocation and the multi-armed bandit problem. *The Annals of Statistics*, 15(3):pp. 1091–1114, 1987.

M. Lawrence, P. Goodwin, M. O'Connor, and D. Önkal. Judgmental forecasting: A review of progress over the last 25years. *International Journal of Forecasting*, 22(3):493–518, 2006.

Q. Liu and D. Zhang. Dynamic pricing competition with strategic customers under vertical product differentiation. *Management Science*, 59(1):pp. 84–101, 2013.

L. M. Lodish. Applied dynamic pricing and production models with specific application to broadcast spot pricing. *Journal of Marketing Research*, 17(2):pp. 203–211, 1980.

C. Manski. *Social Choice with Partial Knowledge of Treatment Response*. Princton University Press, Princton, 2005.

J. Milnor. *Games Against Nature in R.M. Thrall, C.H. Coombs, and R.L. Davis (Eds.) Decision Processes.* Wiley, New York, 1954.

L. Mirman, L. Samuelson, and A. Urbano. Monopoly experimentation. *International Economic Review*, 34(3):549–563, 1993.

H. Nair. Intertemporal price discrimination with forward-looking consumers: Application to the us market for console video-games. *Quantitative Marketing and Economics*, 5(3):pp. 239–292, 2007.

G. Perakis and G. Roels. Robust controls for network revenue management. *Manufacturing and Service Operations Management*, 12(1):56–76, 2010.

B. Robinson and C. Lakhani. Dynamic price models for new-product planning. *Management Science*, 21(10):pp. 1113–1122, 1975.

M. Rothschild. A two-armed bandit theory of market pricing. *Journal of Economic Theory*, 9(2): 185–202, 1974.

L. J. Savage. The theory of statistical decision. *Journal of the American Statistical Association*, 46 (253):pp. 55–67, 1951.

K. Schlag. Why minimax regret? http://homepage.univie.ac.at/karl.schlag/research/learning/whyMR.pdf, 2006.

D. Schwartz. *Concept testing*. AMACOM, 1987.

Z. Schwartz and E. Cohen. Hotel revenue-management forecasting: Evidence of expert-judgment bias. *Cornell Hotel and Restaurant Administration Quarterly*, 45(1):85–98, 2004.

J. Stoye. Axioms for minimax regret choice correspondences. *Journal of Economic Theory*, 146(11): 22262251, 2011.

T. T. Tyebjee. Behavioral biases in new product forecasting. *International Journal of Forecasting*, 3(3):393–404, 1987.

A. Wald. *Statistical Decision Functions*. Wiley, New York, 1950.

B. Wernerfelt. A special case of dynamic pricing policy. *Management Science*, 32(12):pp. 1562–1566, 1986.

A Proof of Lemma 2

Proof. Sketch of proof provided here. Please see online appendix for complete proof.

First we solve for $p < v_{H-}$

As in the representative consumer model, consider the regret from over-pricing and under-pricing for each consumer. The regret from over-pricing is maximized by solving $max_v v$; subject to $v < p$, which is solved at $v = p - \epsilon$ for $\epsilon \to 0$. The regret from under-pricing is maximized by solving $max_v v - p$; subject to $v > p$, which is solved at $v = v_{iH}$. The first kind of regret is greater than the second kind of regret if $\hat{p} - \epsilon > v_{iH} - \hat{p}$ or $2\hat{p} > v_{iH} + \epsilon$ ($\epsilon \to 0$). This gives the worst case valuations for each consumer. In our model, we allow for full non parametric heterogeneity in true valuations across consumers, therefore $F_{wc}(\hat{p})$ can be found by aggregating the worst-case valuations for each consumer.

The monopolist's maximum regret for each price is:

$$MR(\hat{p}) = \hat{p}G(2\hat{p}) + \int_{2\hat{p}}^{v_{H+}} v_H \; dG(v_H) - \hat{p}[1 - G(2\hat{p})]$$
$$\Rightarrow p^* = \frac{Med(v_H)}{2}$$

In the online appendix we show that the regret from $p = \frac{Med(v_H)}{2}$ is lower than the regret from $p \geq v_{H-}$.

□

B Proof of Theorem 1

Proof. Sketch of proof provided here. Please see online appendix for complete proof.

We begin by proving in the following steps:

1. The monopolist's optimal first period price $p_1^* \geq \frac{v_H}{2}$. We show that $p_1 = \frac{v_H}{2}$ has lower regret than any $p_1 < \frac{v_H}{2}$. The intuition for this result is as follows: the static maximum regret from charging a price $\frac{v_H}{2}$, therefore maximum multi-period regret from charging $p_1 = p_2 = \frac{v_H}{2}$ is v_H. If $p_1 < \frac{v_H}{2}$, maximum regret occurs when the consumer has valuation v_H, here regret in the first period is $v_H - p_1$ and regret in the second period is $v_H - \frac{v_H}{2}$ (by lemma 1). Therefore

the multi-period regret is $\frac{3}{2}v_H - p_1 > v_H$.

2. Given that $p_1^* \geq \frac{v_H}{2}$, The maximum regret under a minimax regret first period price rule for the monopolist, conditional on a first period purchase, is equal to $2(v_H - p_1^*)$. When the consumer does not purchase, maximum regret equals $\frac{3p_1^*}{2}$

3. Since dynamic maximum regret conditional on no first period purchase is increasing in p_1 and dynamic maximum regret conditional on a first period purchase is decreasing in p_1, actual dynamic minimax regret is achieved by setting p_1 to where these two quantities are equal. Given the results of the previous claims, this occurs when:

$$\frac{3}{2}p_1 = 2(v_H - p_1)$$
$$\Rightarrow p_1^* = \frac{4v_H}{7}$$

The dynamic minimax regret value given this optimal pricing rule equals $2(v_H - p_1^*) = \frac{6v_H}{7}$, which occurs when the consumer has true valuation of either v_H or $\frac{4v_H}{7} - \epsilon$. This is $\frac{6}{7}$ of what maximum regret would be if naively applying static minimax regret twice ($MR(p^*) = v_H$).

□

C Proof of Lemma 3

Proof. Sketch of proof provided here. Please see online appendix for complete proof.

For notational convenience we partition Ψ_2 into sets of feasible demand curves conditional on whether (Ψ_2^P) or not (Ψ_2^{NP}) a consumer purchased the product in the first period (see online appendix for mathematical forumation)

If $p_2 > p_1$ then maximum regret for the proportion $1-q$ of the population that did not purchase in the first period is fixed for any p_2 and equal to $(1-q)p_1$. As a result, the monopolist only considers feasible demand curves for the population that purchased in the first period when selecting a second period price. The monopolist's problem is equivalent to the static pricing problem with heterogeneity as described in section two with Ψ_2^P as the state space. The solution to lemma 2 then applies to this setting, implying $p_2^* = \frac{Med(v_H|b=1)}{2}$. Maximum regret for the second period as a function of p_2 is the fixed maximum regret for those consumers who did not purchase in the first period $(1-q)p_1$, plus the maximum regret for the static problem with heterogeneity, given Ψ_2^P, weighted by q.

If $p_2 < p_1$ then maximum regret for the proportion q of the population that did purchase will be $E[v_H|b=1]-p_2$ since the monopolist knows each person who bought in the first period will buy in the second period and maximum regret for each such buyer occurs when their valuation is as high as possible. Thus, the monopolist's maximum regret restricting $p_2 < p_1$ for this portion of the population is $[\int_{p_1}^{v_{H+}}(v_H - p_2)dG(v_H|b=1)]q$. The monopolist solves the second period minimax regret problem considering Ψ_2^{NP} along with the additional maximum regret for the high value consumers as a function of p_2.

If $q > 0.5$ then $p_2^* = p_1$: lowering price will increase maximum regret to high value consumers by $q\Delta p_2$ while the change in price can only decrease maximum regret for low value consumers by $(1-q)\Delta p_2$. If $q < 0.5$ then lowering p_2 decreases maximum regret to low value consumers by $q\Delta p_2$ until $p_2^* = \frac{p_1}{2}$ after which point lowering p_2 has no additional impact on lowering maximum regret since all low value consumers could have value $p_1 - \epsilon$. Since $(1-q)\Delta p_2 > q\Delta p_2$ for $q < 0.5$, $p_2^* = \frac{p_1}{2}$. If $q = 0.5$ then $(1-q)\Delta p_2 = q\Delta p_2$ and any price in between $\frac{p_1}{2}$ and p_1 minimizes maximum regret for $p_2 \leq p_1$. The formula for maximum regret follows immediately.

□

D Proof of Claim 1

Please see online appendix for proof.

E Proof of Lemma 4

Proof. [17] Sketch of proof provided here. Please see online appendix for complete proof.

Consider the multiperiod regret

$$R(p_1, p_2, F_{wc}(p_1, p|q)) = R(p_1, F_{wc}(p_1, p|q)) + R(p_2, F_{wc}(p_1, p|q))$$

[17] Please note in the proofs below we use $F_{wc}(p_1, p)$ and $F_{wc}(p_1)$ replaceable. This represents the worst case demand curve under a price p_1 evaluated at all prices p.

Conditional on p_1 and $q_{wc}(p_1)$ occurring, $R(p_1, F_{wc}(p_1, p|q))$ is independent of p_2. Then, picking the p_2 that minimizes maximum $R(p_2, F_{wc}(p_1, p|q))$ conditional on q, p_1, and purchase history consistent with $F_{wc}(p_1, p|q)$ is thus identical to picking p_2 to minimize maximum $R(p_1, p_2, F_{wc}(p_1, p|q))$ ex ante.

□

F Proof of Theorem 2

Proof. Outline of the main steps and results of proof provided here. Please see online appendix for complete proof.

The first five steps focus on the case where G is non-degenerate, while step 5 focuses on the degenerate case.

Step 1: Determining $F_{wc}(p_1, p|q)$ This allows us to determine the maximum regret from upward or downward pricing in the second period. We derive:

$$\begin{aligned} MR(p_1, p_2^U|q) &= 2p_1(1-q) + [2E[v_H|v_H > Q_{1-q}] - p_1 - p_2^U]q \\ MR(p_1, p_2^D|q) &= \frac{3p_1}{2}(1-q) + [2E[v_H|v_H > Q_{1-q}] - \frac{3p_1}{2}]q \end{aligned}$$

Step 2: Pricing direction over time given q **and** p_1

Define q^* as the q such that $MR(p_1, p_2^D|q) = MR(p_1, p_2^U|q)$. We find that $\forall q > q^*$, $MR(p_1, p_2^U|q) < MR(p_1, p_2^D|q)$ or the monopolist will increase price in the second time period; and $\forall q < q^*$, $MR(p_1, p_2^U|q) > MR(p_1, p_2^D|q)$ or the monopolist will increase price in second time period.

Step 3: $MR(p_1, p_2^D|q)$ **is increasing over the range** $[0, q^*]$ **for each** p_1

We show that for any p_1 the maximum regret over the range $[0, q^*]$, where the monopolist will price downwards over time under worst case demand (see step 2), will occur at q^*

Step 4: $\exists q > q^*$ **such that** $MR(p_1, p_2^U|q) > MR(p_1, p_2^U|q^*)$**, implying that** $p_1^* < p_2^*(\Psi_2(F_{wc}(p_1^*, p), p_1^*))$ **along the dynamic minimax regret price.**

We show that $\exists q > q^*$ such that $MR(p_1, p_2^U|q) > MR(p_1, p_2^U|q^*) = MR(p_1, p_2^D|q^*)$. This will imply that under the dynamic minimax regret first period solution p_1^*, $p_2^*(\Psi_2(F_{wc}(p_1^*, p), p_1^*)) > p_1^*$ i.e. the monopolist will always price upwards when a purchase history consistent with $F_{wc}(p_1, p)$ is observed in period one.

Step 5: $p_1^* < p^*$, The monopolist price experiments downward

Given the result in step 4, we find that the ability to learn by increasing price in the second period results in a monopolist decreasing price in the first period.

Step 6: When G is degenerate, $p_1^* = p^* = p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*))$.

This shows that our result depends critically on ex-ante heterogeneity. In the absence of ex-ante heterogeneity the firm will not price experiment.

□

G Proof of Theorem 3

Proof. Sketch of proof provided here. Please see online appendix for complete proof.

Claim 1 implies that under worst case demand for dynamic minimax regret price p_1 there will be two groups of consumers, those who purchase the product in both periods and those who never purchase. The threshold to determine who buys and who does not in the first period under worst case demand for a given p_1 solves:

$$v_H^* = \frac{3p_1 + p_2^*(\Psi_2(\Psi_1, p_1)))}{2} = \frac{12p_1 + v_{H+}}{7} \tag{1}$$

This implies that the maximum regret conditional on a given first period price p_1 is:

$$MR(p_1, \cdot) = \frac{\frac{144p_1^2}{7} - \frac{46p_1 v_{H+}}{7} - 14p_1 v_{H-}}{7(v_{H+} - v_{H-})}$$

The first period dynamic minimax regret price can then be determined by solving:

$$\min p_1 MR(p_1) \Rightarrow p_1^* = \frac{23v_{H+} + 49v_{H-}}{144}$$

Since $v_{H+} > v_{H-}$, $\frac{23v_{H+}+49v_{H-}}{144} < \frac{v_{H-}+v_{H+}}{4} = p^*$.

$p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*))$ can be found by inserting the value of v_H^* determined as a function of p_1^*

into the formula for contingently optimal pricing under lemma 3:

$$\begin{aligned} v_H^* = & \quad \frac{12p_1 + v_{H+}}{7} = \frac{420v_{H+} + 588v_{H-}}{1008} \\ p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*)) & = \frac{v_{H+} + v_H^*}{4} = \frac{714v_{H+} + 294v_{H-}}{2016} > p^* \end{aligned}$$

□

H Analysis of an uninformed Bayesian decision maker with uniform ex-ante heterogeneity

In this section, we consider the model with uniform ex-ante heterogeneity, or $v_{Hi} \sim U[v_{H-}, v_{H+}]$. We derive the optimal Bayesian price assuming an uninformative prior. Explicitly here the decision maker assumes, based on a *subjective prior*, that each consumers valuations $v_i \sim U[v_L, v_{Hi}]$. The Lemma below derives the optimal Bayesian price assuming an uninformed prior.

Lemma. *If the monopolist instead assumes a flat subjective uniform prior for each consumer i between $[v_L, v_{iH}]$. The optimal price that maximizes expected profits for the heterogeneous consumers model is is given by*

$$p_B^* = \frac{1}{2}\left(v_L + \frac{v_{H+} - v_{H-}}{log(v_{H+} - v_L) - log(v_{H-} - v_L)}\right)$$

Proof. Sketch of proof provided here. Please see online appendix for complete proof.

$$\pi_B = \int_{v_{H-}}^{v_{H+}} \left(\frac{v_H - p}{v_H - v_L}\right) p dv_H$$

The first order conditions give us

$$\begin{aligned} & 2p \int_{v_{H-}}^{v_{H+}} \left(\frac{1}{v_H - v_L}\right) dv_H = \int_{v_{H-}}^{v_{H+}} \left(\frac{v_H}{v_H - v_L}\right) dv_H \\ & \Rightarrow p = \frac{1}{2}(v_L + \frac{v_{H+} - v_{H-}}{log(v_{H+} - v_L) - log(v_{H-} - v_L)}) \end{aligned}$$

□

Corollary. *In the heterogeneous consumer model, the optimal Bayesian solution is strictly decreasing in* v_L*, or* $\frac{\partial p_B^*}{\partial v_L} < 0$

Proof. Please see online appendix for proof. □