

Optimal Timing of Sequential Distribution: The Impact of Congestion Externalities and Day-and-Date Strategies

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Abstract

The window between a film's theatrical and video releases has been steadily declining with some studios now testing day-and-date strategies (i.e., when a film is released across multiple channels at once). We present a model of consumer choice that examines trade-offs between substitutable products (theatrical and video forms), the possibility of purchasing both alternatives, a congestion externality affecting consumption at theaters with heterogeneous consumer groups, and a decay in the quality of the content over time. Our model permits a normative study of the impact of shorter release windows (0-3 months) for which there is a scarcity of relevant data. We characterize the market conditions under which a studio makes video release time and price selections indicative of direct-to-video, day-and-date, and delayed video release tactics. During seasons of peak congestion, we establish that day-and-date strategies are optimal for high quality films with high content durability (i.e., films whose content tends to lead consumers to purchase both alternatives) whereas prices are set to perfectly segment the consumer market for films with low content durability. We find that lower congestion effects provide studios with incentives to delay release and price the video to induce multiple purchasing behavior for films with higher content durability. However, an increase in congestion effects can, in certain cases, actually lead to higher studio profitability. We also show that, at the lower range of quality, an increase in movie quality should often be accompanied by a later video release time. Surprisingly, however, we observe the opposite result at the upper range of movie quality: an increase in quality can justify an earlier release of the video.

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1 Introduction

Over the past decade, a dramatic change has taken place in how movie studios manage film distribution. In those years, the average time between the initial theatrical release of a film and its debut on digital video has dropped forty percent – from 179 days in 1999 to only 115 days in 2013 (see Table 1). This forward shift of 2 months has partly been driven by advances in technology, especially video quality and home theater capabilities, but also by an industry that is now challenging antiquated norms (Grover 2005, Cole 2007). Is it profit-maximizing for a release window between sequential distribution channels *always* to exist? In recent years, more films have been skipping theatrical release entirely and going directly to home video as part of a direct-to-video strategy (Barnes 2008); in some cases, studios are even entertaining “day-and-date” strategies which strike at the heart of the matter (Miller 2012, Tartaglione 2013). A day-and-date strategy typically means that a product is released across two or more distinct channels on the same day. For example, in 2006, the film *Bubble* was simultaneously released across all channels by 2929 Entertainment, a company founded by Mark Cuban and Todd Wagner that has vertically integrated across production, distribution, and exhibition – an opportune proving ground for testing such strategies (Kirsner 2007). In fact, many argue that release windows are inherently inefficient since the positive impacts of early promotional spending are not fully captured (Gross 2006). Disney CEO Robert Iger even comments that a film should be released faster on digital video since it has “... more perceived value to the consumer because it’s more fresh” (Marr 2005); perhaps not surprisingly, Disney announced an early video release of *Alice in Wonderland* only 12.5 weeks after its theatrical release instead of the typical 16.5 weeks at the time (Smith and Schuker 2010). Vogel (2007) predicts that further changes in studios’ sequential distribution strategies will continue to occur; as a result, there is a strong need for new research aiming to provide a better understanding of these strategies.

Although the release window is gradually narrowing and a few films have been released using day-and-date strategies, it remains difficult to ascertain the impact of a substantial reduction in the release window on profitability. From an empirical standpoint, the average release window is still approximately three to four months, and there is very little data on any films with windows ranging from zero to three months. In prior studies, researchers have also commented on the low variance observed in the release window measure (see, e.g., Lehmann and Weinberg 2000). Thus, accurately predicting the effect of releasing a film simultaneously on video or even one month after theatrical release continues to be quite difficult, although some studies have sought to close this gap using surveys (Grover 2006, Hennig-Thurau et al. 2007).

However, one can gain significant insights into how various release strategies would tend to affect consumption and hence profitability if we enhance our understanding of the economic trade-offs consumers face when choosing between theatrical and video alternatives. In this paper, we take a normative approach to studying the theater-video windowing problem. We model the primary economic incentives of consumers making film consumption decisions and subsequently analyze

Year	Release Window (Days)	Year	Release Window (Days)
1998	200.4	2006	129.2
1999	179.1	2007	126
2000	175.7	2008	127.8
2001	165.4	2009	123.2
2002	171.4	2010	121.4
2003	153	2011	120.2
2004	145.8	2012	114.7
2005	141.8	2013	115.1

Table 1: Shrinking of industry average video-release window from 1998 to 2013 (Tribbey 2013).

how consumers behave, in equilibrium, as the video release time is varied over its full span. By taking into account strategic consumer behavior, we can explore how studios should set the video release time and price based upon market conditions, movie characteristics, and operational factors. Although consumption by moviegoers can be affected by a wide range of other considerations, in our model we restrict our attention to four primary considerations: *(i)* quality decay of the film content over time, *(ii)* the quality/price gap between theatrical and video alternatives, *(iii)* residual value of the video alternative in addition to movie consumption (multiple purchases), and *(iv)* theater congestion.

For example, all else being equal, consumers prefer to see a film earlier rather than later (Marr 2005). Whether the content is viewed in a theater or at home on video, due to “buzz” generated by marketing, film critics, and social circles, consumers derive highest perceived value at launch, which then decreases over time (Thompson 2006, Cole 2007, Smith and Schuker 2010). Since consumers who watch a film in theaters might also purchase videos of the film, the studios can affect such multiple purchasing behavior by moving up the video release date and pricing it accordingly (Moul and Shugan 2005).

On the other hand, there are two effects that can incentivize viewers to postpone consumption. First, if a film is sequentially distributed through separate channels, some consumers may prefer to wait for lower prices in the secondary channel even though the value derived from the film decays during that time. Thus, a *substitution effect* tends to shift consumption later due to lower prices in the subsequent channel. Second, a *congestion effect* can also shift consumption later in time. Many popular films sell out screenings right after theatrical release. In 2008, *The Dark Knight* was selling out so many screens at midnight on opening day that exhibitors hurried to boost capacity by adding screenings at 3 a.m. and 6 a.m. (Cieply 2008). Furthermore, when Michael Jackson’s comeback concert footage was assembled into a film titled, *This Is It*, it sold out over one thousand screens (Jurgensen 2009). Anticipating sold-out screenings, some consumers may prefer to delay their viewing. However, a film need not sell out to induce consumers to delay. Even higher utilization of theaters leads to longer waits at ticketing and concessions, poorer seat

choices in auditoriums, and other undesirable crowd externalities (e.g., crying babies, noise from conversations, and temperature problems). Hence, some consumers may avoid viewing a movie at the box office due to congestion-generated problems. Because congestion also strongly interacts with the studio's choice of video release time and its price due to substitution effects, optimal management of the entire system requires careful coordination.

In this paper, our research objective is to develop an understanding of how a studio should coordinate the video release time and price for its film as part of a comprehensive strategy to manage the film's total profitability across the theater and home video channels. Because earlier release times greatly impact cannibalization, it is critical that we capture the preferences of consumers who make multiple purchases, i.e., pay to see the film in a theater as well as purchase the video, as part of the central trade-off. Such preferences tend to mitigate cannibalization losses, but we also parameterize the delay after which multiple purchasing consumers actually obtain residual value to study regions of limited mitigation. Because delaying consumption due to congestion becomes more appealing with earlier video release times, we capture different classes of congestion sensitivity in the consumer population. Using our model, we fully characterize market conditions under which the optimal video release time and price give rise to direct-to-video, day-and-date, perfect segmentation, and delayed release tactics that maximize profitability for the studio. In each case, we provide an analytical characterization of the optimal video release time and price, and offer studios practical insights on how to utilize each lever to shape consumer demand toward preferable outcomes.

2 Literature Review

Eliashberg et al. (2006) provide an extensive review of research related to the motion picture industry. In their discussion of the distribution stage, they pose several questions in regard to the substitutability of DVDs for theatrical consumption and how consumers make trade-offs between the two product forms. We investigate these topics at the consumer level in order to study various video release strategies while generating broader implications on how to optimally manage sequential distribution; thus, our work is closest in nature to research that examines the time window between theatrical and video release.

Frank (1994) studies the timing of sequential distribution by constructing a theoretical model in which potential revenue functions for both product forms linearly decrease in time. Minimizing the sum of the opportunity cost of a video release in the theatrical market and the opportunity cost of a delay of the video release, he characterizes the optimal, positive video release time. Lehmann and Weinberg (2000) construct a reduced-form model in the context of a video rental firm managing inventory ordering decisions and also analytically characterize the optimal release. In addition to inventory ordering decision, Gerchak et al. (2006) also consider shelf-retention time, i.e., when to remove a video from front shelves, for a video rental chain. They show that a revenue sharing

contract with wholesale price augmented by a licensing fee is optimal for a studio and coordinates the channel. Studying potential consumption of both versions in a model of intertemporal movie distribution, Calzada and Valletti (2012) show that versioning can be optimal for information goods with zero marginal costs. They further establish that a monopolist, or a centralized channel, will often price and simultaneously release both versions whereas a sequential release strategy may be driven by a vertical, decentralized channel structure found in the motion picture industry. Their work is closest in spirit to the current work, and we discuss our relative contribution in greater detail below.

The empirical analysis in Luan and Sudhir (2006) is also related to our work since they account for buzz decay and multiple purchases in their utility specification. In their study, they find that both highly rated films and animated films tend to be less substitutable and that, on average, the optimal release window should be 2.5 months. In another empirical study, Hennig-Thurau et al. (2007) examine multiple channels and release orderings. Although the introduction of DVD rentals is country-dependent, they similarly find that DVD sales should optimally be delayed by three months. Nelson et al. (2007) study the time gap between the end of a film's theatrical run and its release on DVD, finding that about 30 percent of films have DVD versions released while the film is still in theaters and that the time gap is generally declining.

Our work is also related to papers that study the impact of a social presence on consumers. Harrell et al. (1980) find that perceived crowding negatively affects shopping behavior as consumers employ adaptation strategies. Hui et al. (2009) study shoppers' path behavior and zone density, finding that consumers might be attracted to higher density zones but shop less in them. Argo et al. (2005) demonstrate that increasing social presence tends to positively affect emotions initially and then to have a more negative effect as the presence gets larger. In line with their finding, we focus on the impact of congestion on the theater-viewing experience since theater owners maximize their profits by allocating screens based on movie demand. Specifically, theater owners have economic incentives to keep their capacity highly utilized, and congestion-sensitive consumers are likely to be negatively affected at these higher levels of congestion.

In a broader context, sequential product introduction is related to intertemporal price discrimination (see, e.g., Coase 1972, Bulow 1982, Gul et al. 1986, Besanko and Winston 1990, Desai and Purohit 1998, and Desai et al. 2004). The papers in this literature demonstrate that a firm competes against itself by selling across different time periods without commitment. Exploring intertemporal pricing under capacity constraint, Su (2007) shows that markdown pricing can be optimal when high valuation customers are less patient and that, otherwise, optimal prices are increasing in time. Moorthy and Png (1992) consider a seller with two substitutable and differentiated products and show that sequential introduction can both effectively reduce cannibalization and be more profitable than simultaneous introduction. Studying films with short runs, Waterman and Weiss (2010) find that the release window for videos is still long and invariant to theatrical run length. Their results suggest that studios can credibly commit to release windows. We examine

sequential introduction where, upon theatrical movie release, the studio commits to a video release time and price.

In this paper, we build on the ideas summarized above with the goal of developing a theoretical framework in which we start from the consumer’s choice problem in order to clarify the trade-offs consumers make and how demand for each product form arises as the result of equilibrium strategic behavior. Augmenting extant literature, we explicitly consider additional relevant factors on the studio’s optimal video release time and pricing strategy: (i) an endogenously arising congestion externality at theaters with heterogeneity in consumer sensitivity to congestion; (ii) a time delay after which consumers who buy both versions obtain residual value associated with the second purchase; and (iii) a decaying quality over time. By studying the decision problems faced by consumers and the studio in the presence of these factors, we provide several new insights into how optimal film release strategies should be adapted to the factors across diverse conditions.

Although the model employed in Calzada and Valletti (2012) shares some of our model’s features, our focus on the above factors leads to an enriched understanding of decision making in this context. Congestion is as important of a factor on the consumer’s decision as video release time, and it is imperative to assess how congestion effects influence the studio’s optimal release strategy. In a survey of a sample of the moviegoer population, we found that 45% out of 209 respondents of the moviegoer population chose either “Very likely to delay” or “Definitely will delay” in response to a question of whether they would consider delaying film consumption because of crowds at theaters (73% when including “Somewhat likely to delay”). Moreover, 103 respondents ranked congestion as providing greater delay incentives than the video release time itself. Our study is the first to examine this important factor, and we formally demonstrate that congestion has a subtle, moderating effect between film content durability and optimal release time. We show that for high quality films, under low congestion effects, an increase in film durability should be coupled with a *delayed* video release. However, when congestion effects are substantial, it is a more profitable strategy to release the video immediately under both high and low film durability. Even though congestion provides consumers with additional incentives to substitute toward what is typically characterized as the less profitable video channel, a studio may still find it optimal to release the video immediately under high congestion; this can be true despite substantial movie revenues being cannibalized and even in conditions where no multiple purchases are induced in equilibrium.

Calzada and Valletti (2012) establish that for information goods, versioning is optimal as long as the degree of substitutability between alternatives is not too large. We model congestion as an endogenously determined externality in equilibrium that stems from the size of moviegoer demand. When consumers can strategically respond to congestion effects and effectively separate, the studio has greater incentives to version. In fact, we establish that a versioning strategy continues to be optimal for studios on a much broader region, including the region where the degree of substitutability between alternatives is quite high.

Because the consumer value derived from watching a film drops quickly in the typical 3-4 month

release window whereas financial discounting over the same time period is relatively negligible in comparison, it is important to separate out the film’s decaying quality over time in the model. By doing so, in contrast to prior work, we find that sequencing (i.e., delaying the video release) can be optimal even if the studio and consumers are homogeneous with regard to financial discounting. Further, by capturing a time delay after which consumers subsequently derive value in the case of multiple purchases, we characterize how the sequencing decision interacts with time delays in consumer preference. Finally, due to the additional context features captured by our model, we formally establish how the optimal video release time surprisingly responds in a non-monotonic manner to changes in primitives such as the degree of substitutability and the congestion cost factor; this has not been established in prior work and highlights the importance of our focal factors. As a consequence, we add to this body of literature by providing a nuanced view of the interactive effects that film quality, congestion, and content durability have on the optimal strategy that should be pursued by studios.

3 Model and Consumer Market Equilibrium

There is a continuum of consumers who are heterogeneous in their sensitivities to the quality of a cinematic production. Each consumer’s sensitivity (i.e., her type) is uniformly distributed on $\mathcal{V} \triangleq [0, 1]$. We assume that the product can be consumed in theaters (the *movie*) for a given price $p_m > 0$ at time zero, and consumed in digital form at home (the *video*) for $p_d > 0$. In the movie industry, there are many other channels for obtaining film content including pay-per-view on-demand services (e.g., Vudu), video rental services (e.g., Netflix, Blockbuster, and Redbox), and cable services (e.g., Time Warner and Comcast). Our model simplifies this setting and focuses on clarifying the main trade-offs between a primary and secondary channel. However, the insights derived from our analysis can be readily applied to the more complex setting.

Two of the primary factors identified in Section 1 that affect consumption are the quality/price gap between alternatives and quality decay. First, to capture the former factor, we adapt a standard model of vertical product differentiation to our specific setting (Shaked and Sutton 1983, 1987). The product consumed in theaters has an inherent level of quality given by $\gamma_m > 0$. For example, the box office hit *Avatar* would be associated with a higher value for γ_m due to its special effects and 3-D features. If a consumer with quality sensitivity $v \in \mathcal{V}$ views the movie in a theater, her maximum willingness to pay is given by $\gamma_m v$. Similarly, the inherent quality of the corresponding video is given by $\gamma_d > 0$. Second, as discussed earlier, there is substantial decay in the quality of the film itself over time not because the content has changed but because it is steadily losing its relevance (akin to *vintage-use* depreciation in Desai and Purohit 1998). In that sense, we can consider the video to be essentially a different product at each moment in time. If the video is released at time $T \in [0, 1]$ and consumer v purchases only the video, her maximum willingness to pay is $\gamma_d(1 - T)v$.

The third critical factor we identified relates to the residual value of the video for consumers who make multiple purchases. Because consumers often purchase videos of films they have already seen in theaters, it is important to permit consumption of both movie and video alternatives in the model. Should a consumer opt for multiple purchases by consuming both the movie and the video, her willingness to pay for the video is modified to $\delta\gamma_d(1 - \max(T, \xi))v$, where $\delta \in [0, 1]$ represents the *durability* of the film in terms of its content, and $\xi \in [0, 1]$ denotes the minimum time beyond which the consumer is again willing to pay for the durable content. For example, a larger value for δ indicates that consumers still derive considerable value from watching the film on video again after viewing it in a theater. As δ becomes smaller, the video becomes more of a substitute for the movie since the residual value associated with multiple purchasing diminishes. Because a consumer has less incentive to consume both alternatives, she tends toward the one providing higher net utility.

In our model, the content durability is a film characteristic, but it directly interacts with consumers' heterogeneous types. Specifically, consumers with the highest types are the ones with the greatest incentive to engage in multiple purchases. However, the durability of the content itself is dependent on the type of film produced. For example, children's movies such as *Pocahontas*, *Aladdin*, and *Cars*, would likely be associated with a higher δ because their content maintains large residual value for repeated viewings. Similarly, films that have established subcultures (e.g., *Star Wars* and *Star Trek*) would also have higher durability. On the other hand, documentary films and historical dramas like *Hotel Rwanda*, in which the focus lies on being informative, may have relatively lower residual value after a first viewing in comparison to highly entertaining films. Luan and Sudhir (2006) also find that films with lower overall consumer ratings from reviews as well as films that are R-rated tend to have lower content durability.

We can now specify the timing of decisions and formally define the consumer strategy set. At the beginning stage, the production studio determines when to open its video distribution channel and sets the video price. The studio announces and commits to this video release time which is denoted by T as well as the video price p_d . Subsequent to the announcement, each consumer decides whether to consume only the movie ($s = M$), only the video ($s = D$), both the movie and video ($s = B$) or neither alternative ($s = N$). The strategy set is thus denoted by $S \triangleq \{M, D, B, N\}$, and each consumer chooses the action $s \in S$ that maximizes her payoff.

When the product is consumed in a theater (i.e., either $s = M$ or $s = B$), congestion externalities arise due to the theater's fixed capacity and limited resources. For example, as the number of patrons seeing a movie at a theater grows large, there is an increased risk of screenings being sold out, having only poor seats remaining, and the viewing experience being degraded due to congestion externalities. Congestion is the fourth factor we capture in our model that critically affects moviegoer consumption. We use the term "congestion" in a vein similar to that of Vickrey (1955) in his study of New York City's subway system where he states, "...where congestion occurs, the fare may fail to reflect the relatively high cost either of providing additional service at such times, or of the added discomfort to existing passengers occasioned by the crowding in of additional

passengers.” We highlight this point since traditional congestion costs in the operations management literature stem from longer waiting times in service processes. In our context, however, consumers do not simply wait at theaters and incur costs until a screen becomes free; rather, they adapt by altering their consumption decision. For these reasons, we model congestion costs as proportional to the mass of consumers viewing the movie in a theater. *Ceteris paribus*, a consumer strives to consume earlier to increase her surplus due to quality decay, but congestion provides incentives to delay and substitute to alternative content forms.

Because consumers may vary in their sensitivity to congestion, we examine two classes of consumers: $\mathcal{C} = \{H, L\}$. Class H refers to consumers who are sensitive to congestion, and thus may have a positive congestion cost parameter denoted as $\alpha > 0$. We let $\sigma: \mathcal{V} \times \mathcal{C} \rightarrow \mathcal{S}$ be a strategy profile of consumer actions and denote the mass of consumers choosing in-theater consumption with $D_m(\sigma)$. A class H consumer with quality sensitivity v obtains a net payoff of $\gamma_m v - \alpha D_m(\sigma) - p_m$ if she consumes the movie only (i.e., $\sigma(v, H) = M$), for example. On the other hand, a class L consumer is less sensitive to congestion and for convenience we assume her congestion parameter is zero such that her net payoff analogously becomes $\gamma_m v - p_m$. Finally, we denote the probability any given consumer $v \in \mathcal{V}$ belongs to class L and H with $\rho \in (0, 1)$ and $1 - \rho$, respectively. Fixing all other consumers to the strategy prescribed by σ_{-v} , we can summarize the net payoff to the consumer with quality sensitivity v when undertaking action s by:

$$V(v, H, s, \sigma_{-v}) \triangleq \begin{cases} \gamma_m v - \alpha D_m(\sigma) - p_m + \delta \gamma_d (1 - \max(T, \xi)) v - p_d & \text{if } s = B; \\ \gamma_m v - \alpha D_m(\sigma) - p_m & \text{if } s = M; \\ \gamma_d (1 - T) v - p_d & \text{if } s = D; \\ 0 & \text{if } s = N, \end{cases} \quad (1)$$

for class H , and

$$V(v, L, s, \sigma_{-v}) \triangleq \begin{cases} \gamma_m v - p_m + \delta \gamma_d (1 - \max(T, \xi)) v - p_d & \text{if } s = B; \\ \gamma_m v - p_m & \text{if } s = M; \\ \gamma_d (1 - T) v - p_d & \text{if } s = D; \\ 0 & \text{if } s = N, \end{cases} \quad (2)$$

for class L . We focus our study on the parameter region where $\gamma_m > \gamma_d$ is satisfied so that the inherent quality of the theatrical experience is higher than the video (Vogel 2007). For example, going to the theater can be thought of as a complementary event that includes viewing the film and thus carries a higher quality. We also focus on the region where $\gamma_m > p_m$ is satisfied such that there exist consumers who can obtain positive surplus at the theater. Finally, we assume that the movie price p_m is high enough that the video pricing decision is not constrained from above.

3.1 Consumer Market Equilibrium and the Studio's Problem

Taking the video release time T , video price p_d and other model parameters as given, we derive the consumer market equilibrium. We can classify consumers by the product forms they consume: both the movie and the video (*both*), only the movie (*movie*), only the video (*video*), or nothing (*none*). First, we develop an understanding of what types of consumption outcomes occur under the various market conditions. For example, if a blockbuster movie is coupled with a fast video release time, to what extent will theater demand be cannibalized, particularly for high content durability films? By gaining insight into how moviegoers adjust their consumption patterns in response to durability, video release times, and congestion, we can more clearly see how release timing and pricing affect profitability, a subject we address in the next section.

Thus, taking into account a film's quality decay over time, a congestion externality, and the availability of a video alternative, each consumer chooses an action that maximizes her own surplus. An equilibrium strategy profile σ^* must satisfy the following for each $v \in \mathcal{V}$ and $c \in \mathcal{C}$:

$$V(v, c, \sigma^*(v, c), \sigma_{-v}^*) \geq V(v, c, s, \sigma_{-v}^*) \quad \text{for all } s \in S. \quad (3)$$

Because of the monotone properties of (1) and (2), it follows that the equilibrium strategy profile σ^* is characterized by thresholds. In particular, there exist threshold values $\omega_b^c, \omega_m^c, \omega_d^c > 0$ (where $c \in \mathcal{C}$ refers to the consumer class) such that the equilibrium consumer strategy profile is given by

$$\sigma^*(v, c) = \begin{cases} B & \text{if } \omega_b^c \leq v \leq 1; \\ M & \text{if } \omega_m^c \leq v < \omega_b^c; \\ D & \text{if } \omega_d^c \leq v < \omega_m^c; \\ N & \text{if } v < \omega_d^c, \end{cases} \quad (4)$$

noting that (i) consumers with the lowest sensitivity to quality (i.e., low types) remain out of the market; (ii) consumers with slightly higher quality sensitivity purchase only the video alternative; (iii) consumers with even higher sensitivity choose to view the movie in theaters; and finally (iv) consumers with the highest quality sensitivity consume both the movie and video alternatives. This threshold structure holds for both consumer classes, H and L , who vary with regard to congestion costs. Whether any particular consumer segment: *both*, *movie*, *video*, or *none* is present in equilibrium critically hinges on the underlying parameter region as well as how the studio strategically sets the video release time and price.

Taking into consideration the equilibrium consumer strategies developed above, we next layout the studio's decision problem. We compute the demand for the movie by

$$D_m \triangleq \int_{\mathcal{V}} [\rho \mathbf{1}_{\{\sigma^*(v, L) \in \{B, M\}\}} + (1 - \rho) \mathbf{1}_{\{\sigma^*(v, H) \in \{B, M\}\}}] dv, \quad (5)$$

which measures the population of consumers whose equilibrium strategy includes viewing the film

in a theater. Similarly, we denote the aggregate demand for the video by

$$D_d \triangleq \int_{\mathcal{V}} [\rho \mathbf{1}_{\{\sigma^*(v,L) \in \{B,D\}\}} + (1 - \rho) \mathbf{1}_{\{\sigma^*(v,H) \in \{B,D\}\}}] dv. \quad (6)$$

We denote the studio's share of movie and video revenues with λ_m and λ_d , respectively, where $\lambda_m, \lambda_d \in [0, 1]$. Since the marginal cost of satisfying consumers, whether in a theater or by providing a video, is fairly small, we make a simplifying assumption that it is zero. Thus, the studio's profit function can be written as

$$\Pi(T, p_d) \triangleq \lambda_m p_m D_m + \lambda_d p_d D_d. \quad (7)$$

The main objective of this paper is to develop an understanding of how consumers adapt their consumption choices to changing video release times and price, and, subsequently, to characterize how studios can manage them by optimizing video release time and pricing. Hence, we take movie prices as fixed and exogenous to the model especially because uniform pricing has been the standard in this industry since the 1970s (Orbach and Einav 2007). The studio's problem can then be written as

$$\max_{T \in [0,1], p_d > 0} \Pi(T, p_d) \quad (8)$$

$$s.t. \quad \sigma^*(\cdot | T, p_d) \text{ satisfies (3)}.$$

As can be seen in (7), prices have a direct effect on studio profits while all parameters, including release time and prices, indirectly influence profitability through their impact on the strategic consumption behavior of consumers.

4 Optimal Release Time and Pricing

In this section, we develop the solution to the studio's profit maximization problem. By the formulation in (8), the solution to the studio's problem is a couple (T^*, p_d^*) corresponding to the optimal video release time and price. The studio can vastly change the equilibrium consumer market structure induced in both consumer classes by changing its release time and video price. In the full characterization of the consumer market equilibrium, there are 15 unique structure pairs that arise in equilibrium as T and p_d are varied. As an example, we will use shorthand notation such as $[L : B-M-N]$ and $[H : D-N]$ to conveniently express that *both*, *movie* and *none* segments are represented in equilibrium in class L , whereas only *video* and *none* segments are present in class H . The optimal strategy, (T^*, p_d^*) together with the induced consumer market structure $\sigma^*(\cdot | T^*, p_d^*)$ jointly can be thought of as a *tactic* being employed by the studio in a given parameter range.

First, we briefly describe how a studio should handle a movie with a low quality parameter. Because $\gamma_m > p_m$, consumers are always guaranteed positive surplus from consuming the movie, and the studio faces a trade-off. On one hand, the inherent quality of its movie offering is higher and can earn the studio a price premium. On the other hand, in order to incentivize moviegoers

to consume the movie version, the studio necessarily must either increase the price of the video or delay its release to limit cannibalization of movie revenues. In either case, the studio's video revenues associated with the *both* and *video* consumer segments will be negatively affected. When γ_m is low, the potential market for the movie is smaller, and the trade-off shifts in favor of enhancing video revenues. It is straightforward to show that the studio's optimal strategy is given by $(T^*, p_d^*) = (0, \gamma_d/2)$, and because no one will consume the movie in equilibrium under this strategy, the studio need not release it in theaters. In this sense, the studio essentially pursues a direct-to-video tactic. In the film industry, the number of direct-to-DVD films has grown 36 percent since 2005 with 675 films being released in 2008, according to Adams Media Research, and the direct-to-DVD market generates approximately \$1 billion in annual revenues (Barnes 2008). Oftentimes, studios pursue a direct-to-video tactic for films that are of lower quality.

For the remainder of the paper, to simplify the presentation to the reader while retaining the main insights, we employ a binary discretization for several parameters at high and low values. Specifically, film content durability will take on either a high (δ_H) or low (δ_L) value; the congestion parameter will similarly take on either a high (α_H) or low (α_L) value; and the movie quality parameter will also take on either a high (γ_m^H) or intermediate (γ_m^I) value. We already argued that studios will employ a direct-to-video strategy for sufficiently low quality movies so we focus our study on movies with sufficient quality that they are released in theaters. Finally, because the effect of congestion is paramount to the current study, we will restrict our focus to a limited population of congestion-insensitive, class L customers, i.e., ρ will be kept at a lower level. To keep the mathematical analysis simple and clear, we will take $\delta_L = 0$, $\delta_H = 1$, and $\alpha_L = 0$ in the proofs in Appendix A, but all results generalize to regions of low δ_L and α_L and high δ_H (generalized proofs included in an Online Supplement to the paper). Sufficient bounds on γ_m^I and ρ , and other simplifying technical conditions are detailed in Appendix B. For the case of α_H , we employ asymptotic analysis as required.¹

To lay out how our results are organized, we initially group them into these two cases of film durability. Within each case, we cover two sub-cases of movie quality. In the formal propositions, we then study outer case/subcase combinations as we vary the operational congestion factor. We also provide additional insights by holding the congestion factor and durability constant, and discussing how the studio's optimal strategy is affected as movie quality varies. Similarly, we then hold congestion and movie quality constant and vary film durability which reveals interesting comparative statics on the studio's optimal release time behavior. Our results are summarized in Table 2 which we refer to throughout the analysis.

¹Because of the complexity in the analysis of the problem, asymptotic analysis has been commonly used in microeconomic studies, e.g., Li et al. (1987), Laffont and Tirole (1988), Muller (2000), Tunca and Zenios (2006), August and Tunca (2006), Pei et al. (2011), and August and Tunca (2011) among many others. Furthermore, comprehensive treatments of the mathematical techniques in asymptotic analysis are provided in Miller (2006).

4.1 High Film Content Durability

We begin by examining the case of high content durability (δ_H) where the film retains high residual video value for consumers who also consume the movie format.

4.1.1 High Movie Quality

We first consider the sub-case in which the quality of the theatrical offering is fairly high, i.e., γ_m^H , and the theatrical movie offering becomes a very lucrative channel for the studio.

Proposition 1 *For a film with high content durability, δ_H , and high quality, γ_m^H :*

- (i) *Under a high congestion cost factor, α_H , the studio optimally releases the video immediately at $T^* = 0$ and sets its price to $p_d^* = \frac{\delta_H \gamma_d (1-\xi)}{2(\delta_H(1-\xi)(1-\rho)+\rho)}$. The studio's optimal strategy induces a consumer market structure characterized by $[L : B-M-N]$ and $[H : D-N]$, i.e., a day-and-date tactic is employed;*
- (ii) *Under a low congestion cost factor, α_L , the studio optimally delays video release until $T^* = \xi$ and sets its price to $p_d^* = \frac{\delta_H \gamma_d (1-\xi)}{2}$. The studio's optimal strategy induces a consumer market structure characterized by $[L : B-M-N]$ and $[H : B-M-N]$, i.e., a delayed release tactic is employed.*

Proposition 1 highlights that as the congestion factor increases, a studio should optimally adjust its (T^*, p_d^*) strategy such that it makes a switch from a delayed video release to a day-and-date tactic where the video is released simultaneously in conjunction with the theatrical version. To see why congestion and the optimal release time are negatively associated in this region, we first discuss part (i) of the proposition. Congestion affects class H consumers by decreasing their utility for the movie and increasing their incentive to substitute toward the video. A delayed release can help deter substitution from the movie to the video, however it will also drive low valuation video consumers out of the market and reduce the number of multiple purchases. When congestion costs are high, the delay would need to be substantial to effectively garner movie revenues from class H , while quite detrimental to these other two segments. Hence, the trade-off favors the opposite direction toward an earlier release to preserve class H video purchases and class L multiple purchases. In particular, the studio employs the strategy $(T^*, p_d^*) = (0, \frac{\delta_H \gamma_d (1-\xi)}{2(\delta_H(1-\xi)(1-\rho)+\rho)})$ that induces no class H consumer to view the movie in theaters in equilibrium, i.e., $[H : D-N]$, but also induces a *both* segment from class L consumers, i.e., $[L : B-M-N]$, which increases profitability. This provides insight into how the studio adapts its pricing strategy as the composition of consumers changes. For example, as ρ decreases (larger class H), the studio increases p_d^* to boost video revenues from class H . On the other hand, as ρ increases (larger class L), the studio decreases p_d^* to induce a larger *both* segment from class L in equilibrium. Notably, high content durability is a critical driver of multiple purchasing behavior and integral to these arguments.

In totality, we say that the studio pursues a day-and-date tactic in this region because it releases the video immediately and sets price in order to have all consumer segments represented in equilibrium, with some consumers making multiple purchases. Part (i) of Proposition 1 implies that a day-and-date tactic can be attractive for the studio for high content durability films with high theatrical quality, when the effects of congestion are substantial. One fitting example is a blockbuster children’s movie released during periods of peak demand such as the holiday season. Commenting on the motion picture industry, Walt Disney Co. CEO Robert Iger said, “I don’t think it’s out of the question that a DVD can be released in effect in the same window as a theatrical release,” suggesting that day-and-date video release strategies are being considered. Noting that children’s movies often have high content durability, Iger also suggested selling DVDs of *Chicken Little* (2005) in theaters in which the movie was playing in an interview with The Wall Street Journal (Donaldson-Evans 2006).

One issue with employing a day-and-date tactic is potential push-back from theater owners, who are concerned that early video availability will cannibalize theatrical movie demand. A solution consistent with Iger is to sell early released DVDs *only* in theaters at patron exit areas such that these DVDs can be targeted to the *both* consumer market segment, which will not cannibalize movie demand. Another option that studios may consider to alleviate theater owners’ concerns is to implement revenue sharing for video sales with theater owners, which has also been suggested by 2929 Entertainment (Grover 2006). In addition, one ancillary benefit of using a day-and-date tactic relates to advertisement costs, which amount to half of the total production cost on average (Vogel 2007). When a studio releases a film’s video a few months after being released in theaters, it must once again incur additional advertising expenditures. Under a day-and-date tactic, a studio can consolidate a film’s marketing budget into a single, shorter period, leveraging the initial buzz effectively across both movie and video offerings.

Next, we examine the studio’s decision problem as capacity constraints play a lesser role. For instance, the shadow price of capacity is likely to be lower in winter and spring in comparison to summer and holiday seasons (Einav 2007). Other factors that may decrease the congestion parameter include how the number of screens has increased over time, as well as possible local changes in the number of seats per screen (NATO 2013). Recently, small theater chains such as Cinopolis have competed by focusing on a higher quality experience through offering luxury seating, alcoholic beverages, full-service cafes, and an extensive selection of menu options for dining (Abate 2012, Luna 2012). In such cases, although there are fewer seats per screen, every seat is a “good seat” with unobstructed views, adequate spacing, and the ability to recline, which effectively reduces the congestion cost.

When the congestion cost factor diminishes, class H consumers do not have as strong of an incentive to substitute from movie to video. Therefore, in contrast to the high congestion cost case, even a small delay in the release time can be an effective deterrent. As part (ii) of Proposition 1 conveys, in this case the studio delays release to preclude substitution by either class of consumers

toward the video and focus its strategy on expanding movie revenues. Specifically, the studio's optimal strategy is $(T^*, p_d^*) = (\xi, \frac{\delta_H \gamma_d (1-\xi)}{2})$. Notably, the studio still must limit the extent of the delay to protect the multiple purchasing behavior associated with the highly durable film content. Analytically we establish in this region that when $T \leq \xi$, p_d hinges only on ξ , hence profits are weakly increasing in T . However, when $T > \xi$, profits are decreasing in T , hence $T^* = \xi$. In Table 2, we summarize the results from Proposition 1 on the top row, left two columns.

4.1.2 Intermediate Movie Quality

Next, we study the studio's release time and pricing problem when the quality of the movie is at an intermediate level, γ_m^I . We shall see that the studio has increased incentives to delay the timing of video release in this intermediate sub-case which leads to an intriguing finding: in aggregate, the studio's optimal release time is non-monotonic as the quality of the movie increases through its feasible space, from low to intermediate to high. To see why this non-monotonicity arises, we first formalize the studio's optimal behavior for an intermediate region of movie quality.

Proposition 2 *For a film with high content durability, δ_H , and intermediate quality, γ_m^I :*

(i) *Under a high congestion cost factor, α_H , the studio optimally adjusts its video release and pricing strategy depending on its relative revenue share:*

- *If $\lambda_m \geq \lambda_d \Phi$, then the studio delays release until $T^* = 1 - \delta_H(1 - \xi) - \frac{\gamma_m^I}{\gamma_d} + \frac{2p_m}{\gamma_d} \sqrt{\frac{\lambda_m \rho}{\lambda_d}} < \xi$ and sets its price to $p_d^* = \frac{\gamma_m^I + \delta_H \gamma_d (1-\xi)}{2} - p_m \sqrt{\frac{\lambda_m \rho}{\lambda_d}}$, i.e., a delayed release tactic is employed;*
- *If $\lambda_m < \lambda_d \Phi$, then the studio releases immediately with $T^* = 0$ and sets its price to $p_d^* = \frac{\gamma_d}{2}$, i.e., a day-and-date tactic is employed;*

where $\Phi = \left(\frac{\gamma_m^I - \gamma_d(1 - \delta_H(1-\xi))}{2p_m \sqrt{\rho}} \right)^2$. In both cases, the studio's optimal strategy induces a consumer market structure characterized by $[L : B-D-N]$ and $[H : D-N]$.

(ii) *Under a low congestion cost factor, α_L , the studio optimally delays video release until $T^* = \xi$ and sets its price to $p_d^* = \frac{\gamma_d(1-\xi)}{2}$. The studio's optimal strategy induces a consumer market structure characterized by $[L : B-D-N]$ and $[H : B-D-N]$.*

Part (i) of Proposition 2 stands in partial contrast to the direct-to-video tactic used by the studio under low quality γ_m^L and the day-and-date tactic employed under γ_m^H when congestion cost is high; both of these tactics involve the studio optimally releasing the video at $T^* = 0$ as part of its optimal strategy. When the movie quality is at an intermediate level, consumers have increased incentives to substitute from the movie to the video. Similar to before, the video release would need to be significantly delayed to deter this substitution which remains inefficient, and again an earlier video release can improve the video market for class H and multiple purchases in class L . The

critical difference here is that when movie quality is only at an intermediate level, the studio has to be concerned with a different kind of substitution: consumers shifting from multiple purchases to the video. Here the equilibrium consumer market structure for class L consumers satisfies

$$\sigma^*(v, L) = \begin{cases} B & \text{if } \frac{p_m}{\gamma_m^I - \gamma_d((1-T) - \delta_H(1 - \max(T, \xi)))} \leq v < 1; \\ D & \text{if } \frac{p_d}{\gamma_d(1-T)} \leq v < \frac{p_m}{\gamma_m^I - \gamma_d((1-T) - \delta_H(1 - \max(T, \xi)))}; \\ N & \text{if } v < \frac{p_d}{\gamma_d(1-T)}, \end{cases} \quad (9)$$

from which it can be seen how reducing T shifts some consumers from N (nothing) to D (video) but also others from B (both) to D (video). In this case, the studio optimally delays release in order to protect revenues from multiple purchases while sacrificing some video revenues at the low end of the consumer market, provided its share of movie revenues (λ_m) is high enough. The extent to which the video release time is optimally delayed depends critically on the composition of the consumer population. As the proportion of class H consumers increases (lower ρ), video revenues at the lower end become more impactful so the studio either delays release to a lesser extent or pursues a day-and-date tactic as before.

When the congestion cost parameter is low, even the congestion-sensitive class H consumers now have increased incentives to go to theaters. In contrast to part (ii) of Proposition 1, because the movie quality (γ_m^I) is now closer to the video quality (γ_d), it is relatively more difficult to get consumers to prefer the movie over the video. In order to do so, the video release time would need to be significantly delayed. Doing so would hamper the studio's ability to benefit from multiple purchases which can be induced in cases of high durability. Thus, in this case, the studio takes a balanced approach, using a moderate delay $T^* = \xi$. This is not sufficiently large of a delay to induce a *movie* segment, but it does help prevent substitution from *both* to *video*, thus protecting multiple purchasing behavior. In equilibrium, the studio's strategy yields $[L : B-D-N]$ and $[H : B-D-N]$, and it prices the video at the monopoly level $p_d^* = \frac{\gamma_d(1-\xi)}{2}$ associated with this optimal delay.

4.1.3 Impact of Movie Quality

As technology rapidly evolves, implementation in theater equipment often precedes consumer electronics. For example, the last few years have seen a rebirth of 3-D theatrical releases driven by improvements in 3-D technology, as seen with *Avatar*, *Alice in Wonderland*, and *Clash of the Titans*. However, 3-D televisions have just recently become available and have not yet achieved widespread adoption (Bonnington 2012). Thus, the relative quality of theatrical and video offerings can vary over time. Figure 1 illustrates how the optimal video release time changes in theatrical quality under a high congestion cost parameter, which summarizes some of the results in this section. People commonly believe that low theatrical quality films are the ones that should be released earlier to video (Epstein 2005). This intuition is often associated with the observation that lower quality films sometimes bypass theatrical release entirely and appear directly on home video. In particular,

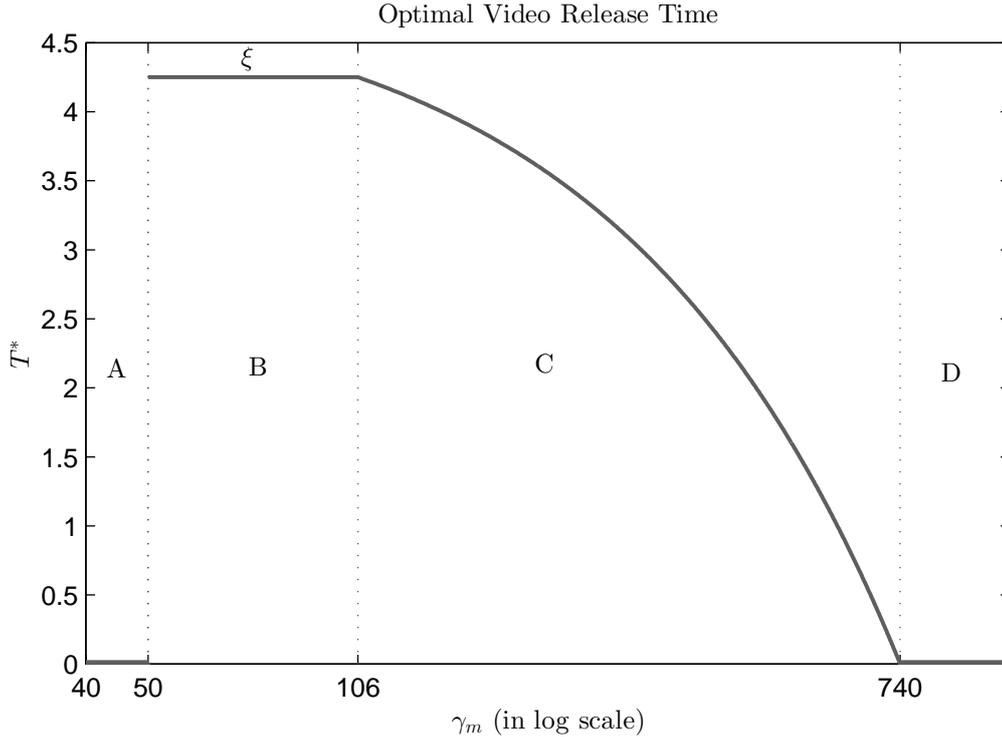


Figure 1: The impact of theatrical quality on the optimal video release time under high congestion cost.

if γ_m is sufficiently low, the studio releases the video immediately as part of a direct-to-video tactic which is illustrated in the left-hand portion of Figure 1, labeled as Region A.

Although it seems reasonable that higher quality movies are more likely to have longer release windows, this conclusion is not always justified. For instance, for a hit film with extremely high γ_m , the studio releases the video immediately, utilizing a day-and-date tactic as demonstrated in part (i) of Proposition 1; this behavior is also illustrated in Region D in Figure 1. As γ_m takes intermediate values, as in Regions B and C, the studio optimally delays the video release time. Consistent with part (i) of Proposition 2, T^* moves from ξ as seen in Region B to a release time strictly less than ξ as seen in Region C. In these regions, not only has the video release time become strictly positive, but it also continuously decreases toward a day-and-date release tactic. This result has an important empirical implication. Due to reasonably high variability on the many dimensions that characterize films, we can expect that γ_m varies significantly film by film. As the movie industry moves toward having more day-and-date releases, our model suggests that we should see video release times spanning the feasible range. As Disney CEO Robert Iger commented, “we’re not doing a one-size-fits-all approach” (Smith and Schuker 2010); we should therefore not expect videos to continue to be released according to an almost binary standard: either immediately or 3–4 months later. Instead, we anticipate seeing a more complete timespan utilized due to the diversity of film characteristics. This is a testable empirical implication of our model. Another important implication of our model is that an increase in the inherent quality of a movie, e.g., due to the 3-D

theatrical releases, should sometimes be coupled with an *earlier* video release time as illustrated in Regions B, C, and D of Figure 1. Hence, the optimal video release time is non-monotonic in the inherent quality of the theatrical version of the film.

4.2 Low Film Content Durability

A less durable film is one that does not carry as much residual value for another viewing. A film's genre, its targeted demographic, and other characteristics can all affect its level of durability. In this section, we study how a studio's optimal video release timing and pricing strategy should be adjusted when a film's content durability is at a lower level, δ_L . We compare and contrast the studio's optimal strategy under δ_L to its strategy under δ_H , and discuss the manner in which the level of content durability affects the studio's incentives.

4.2.1 High Movie Quality

We again begin by analyzing the γ_m^H sub-case.

Proposition 3 *For a film with low content durability, δ_L , and high quality, γ_m^H :*

- (i) *Under a high congestion cost factor, α_H , the studio optimally releases the video immediately at $T^* = 0$ and sets its price to $p_d^* = \frac{\gamma_d}{2}$. The studio's optimal strategy induces a consumer market structure characterized by $[L : M - N]$ and $[H : D - N]$, i.e., a perfect segmentation tactic is employed;*
- (ii) *Under a low congestion cost factor, α_L , the studio optimally adjusts its video release and pricing strategy depending on its relative revenue share:*
 - *If $\lambda_d \leq \lambda_m$, then the studio prefers not to release a video version. The studio's optimal strategy induces a consumer market structure characterized by $[L : M - N]$ and $[H : M - N]$;*
 - *If $\lambda_d > \lambda_m$, then $T^* = 0$ and*

$$p_d^* = \frac{\gamma_d(p_m(\lambda_m + \lambda_d) + \alpha_L \lambda_d(1 - \rho))}{2\lambda_d(\gamma_m^H + \alpha_L(1 - \rho))}. \quad (10)$$

The studio's optimal strategy induces a consumer market structure characterized by $[L : M - D - N]$ and $[H : M - D - N]$, i.e., a day-and-date release tactic is employed.

Under a large congestion factor, we see that similar to part (i) of Proposition 1 the studio also finds it preferable to release the video immediately even under δ_L . However, the driving force of its overall strategy is markedly different. As before, there exist incentives for releasing early to expand the both market for class L and the video market for class H . However, it is important to note that the optimal video price in the δ_H sub-case satisfies $p_d^* = \frac{\delta_H \gamma_d (1 - \xi)}{2(\delta_H (1 - \xi)(1 - \rho) + \rho)}$. Thus, in that case, coupled with an immediate release the studio also needs to adjust price in a manner

dependent on δ_H to incentivize multiple purchases. Under δ_L , the requisite price reduction would need to be significant because of lower content durability, and this would drastically hurt revenues generated from class H video consumers. In this case, the studio finds it more profitable to forgo the *both* segment and pursue a different strategy. Because of class H consumers' high congestion sensitivity, the studio is again less concerned with trying to deter cannibalization of movie demand. Instead, the studio releases the video immediately ($T^* = 0$) and sets the corresponding monopoly price for the video ($p_d^* = \gamma_d/2$). As a result, the classes separate; class L consumers compose the *movie* segment, and class H consumers compose the *video* segment. In this sense, the studio uses a perfect segmentation tactic under these conditions because of the lower content durability.

Part (ii) of Proposition 3 has both similarities and differences with part (ii) of Proposition 1 which underscore the impact of content durability on the studio's decisions. Under δ_H , we established that the studio can effectively deter substitution by delaying release because congestion costs are low, but it limits the extent of delay to maintain multiple purchases. However, as we saw above, under low content durability, it is not profit-maximizing to induce multiple purchases. But, if the revenue share from the movie market is more lucrative (i.e., $\lambda_d \leq \lambda_m$), the studio will still optimally delay video release to deter substitution. Moreover, in this case, it need not limit the amount of delay to protect multiple purchases; therefore, it instead delays video release to the point where only the movie is consumed in equilibrium. On the other hand, if the revenue share from the video market is more lucrative (i.e., $\lambda_d > \lambda_m$), the studio significantly adapts its strategy. Because it cannot induce multiple purchases with low content durability, its strategy must focus more on expanding video revenues while protecting movie revenues. Under high movie quality and low congestion costs, it is relatively difficult to induce video purchases. To achieve this expansion, the studio must both (i) release the video immediately at $T^* = 0$ and (ii) price it strategically lower at $p_d^* = \frac{\gamma_d(p_m(\lambda_m + \lambda_d) + \alpha_L \lambda_d(1 - \rho))}{2\lambda_d(\gamma_m^H + \alpha_L(1 - \rho))}$ to provide the necessary incentive to induce more video purchases. The greater the video revenue share λ_d , the more the studio is willing to cut its video price and expand the video market.

The movie industry has changed drastically over the past 15 years with many new channels (e.g., Netflix, Redbox, and online VOD such as Vudu and Hulu), new technologies (e.g., Blu-ray, HD streaming, and iTunes), and evolving consumers (e.g., higher broadband household penetration, pirated content availability, and customer impatience for content). In this dynamic environment, the studio's share of revenues in different channels also varies as players enter and exit and as negotiations take place. Part (ii) of Proposition 3 gives insight into how a studio may need to adapt its strategies when industry changes critically affect the revenue share it obtains in each channel. The results from this section are summarized in Table 2 on the top row, right two columns. The table illustrates how a decrease in congestion has different effects on the studio's strategy under high and low content durability. Under δ_H , the studio responds with a measured delay in the video release to protect the *both* market segment. However, under δ_L , due to its inability to profitably induce multiple purchases, the studio either institutes an extreme delay (completely

detering substitution to video) or switches to a video market expansion strategy depending on its video revenue share.

4.2.2 Impact of Congestion

We have shown how varying the congestion factor leads to different optimal studio strategies and their associated consumer market structures in equilibrium. Further, we also demonstrated that the effect of congestion is quite different, depending on the quality of the movie and durability of content. Next, we examine how the level of congestion impacts profitability. Because of theaters' capacity constraints, the congestion parameter can often be higher during periods when big-budget movies are released (e.g., Memorial Day, Fourth of July, Thanksgiving, and Christmas) as these films compete for a limited number of screens (Einav 2007). On the surface, one may think that an increase in the congestion cost parameter *always* leads to lower profits because it directly hurts the utility of class H consumers. However, in the following corollary, we establish that a higher congestion cost parameter can sometimes be beneficial to the studio.

Corollary 1 *For a film with low content durability, δ_L , and high quality, γ_m^H , an increase in the congestion cost parameter from α_L to α_H increases the studio's profit if either of the following conditions holds: (i) $\lambda_d > \lambda_m \max(1, \bar{\lambda}_d)$; or (ii) $\lambda_m > \lambda_d > 4\lambda_m p_m / \gamma_d$, where $\bar{\lambda}_d$ is characterized in the Appendix.*

Corollary 1 carries an important message: congestion can sometimes increase the profitability of a film. In particular, this profit improvement can occur if the studio has negotiated an increased share of the revenue in the video channel in comparison to the theatrical channel; moreover, it can also occur when the video revenue share is *less than* the movie revenue share as long as the video revenue share is not too low. We saw in part (ii) of Proposition 3 that for a high quality movie with low congestion costs, it takes an early release coupled with a price reduction in order to incentivize video purchases, which is costly to the studio. However, under high congestion costs, the congestion-sensitive consumers in class H react to the externality imposed on them by class L consumers. This provides much stronger incentives for class H to consume the video instead of the movie. With a larger congestion externality, the studio can increase its profits because it can perfectly segment the consumer market and charge class H consumers a price reflecting monopoly power over its content. Essentially, the studio leverages endogenously-determined congestion as a tool to separate the market and increase profits, but, importantly, such a strategy only makes sense for films with low content durability. Corollary 1 suggests that releasing high quality films with lower content durability during peak seasons has the potential to help increase returns to the studio.

The role of congestion here is connected to Desai and Purohit (1998) which studies the profitability of leasing versus selling strategies for a monopolist. A central element in their model is the mean depreciation factors under each strategy which determine how much residual value remains

in bought and leased products as time passes. They demonstrate that differences in depreciation rates give rise to the optimality of a combined leasing and selling strategy. An important point being made is that a high rate of depreciation of the product being sold is helpful to the firm because it makes the good effectively more of a non-durable one. In our paper, congestion negatively affects the utility associated with the movie alternative, but, similar to Desai and Purohit (1998), it relaxes incentive compatibility constraints enabling the firm to better segment the movie and video markets. An interesting point made in our paper is that such an outcome can still prevail even when consumers themselves determine the equilibrium level of congestion by their consumption behavior (e.g., even under α_H , if no consumer prefers to watch the movie then there are zero congestion costs).

4.2.3 Intermediate Movie Quality

Lastly, for the case of low content durability, we turn our attention to the final sub-case: movies with intermediate quality γ_m^I .

Proposition 4 *For a film with low content durability, δ_L , and intermediate quality, γ_m^I :*

- (i) *Under a high congestion cost factor, α_H , the studio optimally releases the video immediately at $T^* = 0$ and sets its price to*

$$p_d^* = \frac{\gamma_d(\lambda_d(1-\rho)(\gamma_m^I - \gamma_d) + \rho p_m(\lambda_m + \lambda_d))}{2\lambda_d(\gamma_m^I - (1-\rho)\gamma_d)}. \quad (11)$$

The studio's optimal strategy induces a consumer market structure characterized by $[L : M-D-N]$ and $[H : D-N]$, i.e., a day-and-date tactic is employed;

- (ii) *Under a low congestion cost factor, α_L , the studio optimally adjusts its video release and pricing strategy depending on its relative revenue share, in the same manner (i.e., under γ_m^I) given in part (ii) of Proposition 3.*

First, we discuss part (i) of Proposition 4 in relation to part (i) of Proposition 3, where we found that the studio prefers to release the video immediately as part of a perfect segmentation tactic. As the quality of the movie decreases from γ_m^H to γ_m^I , the studio still has strong incentives to release earlier; however, it becomes more difficult to induce consumption of only the *movie* option and attain perfect segmentation. In particular, if the studio releases earlier, now some class L consumers will shift consumption from *movie* to *video*. Thus, the studio faces a clear trade-off between maintaining a larger *video* market (with regard to both consumer classes) by releasing earlier and preventing cannibalization of its more valuable channel (in class L) by releasing later. In part (i) of Proposition 4, we establish that the studio's optimal strategy should still be to release immediately but then mitigate cannibalization of movie revenues by increasing the price of the video, as is characterized in (11). Releasing early maintains the quality of the video offering which

is important to class H consumers. This also enables it to price the video high and, in turn, achieves two purposes: (i) increasing revenues from the *video* segments, and (ii) reducing cannibalization from the *movie* segment to the *video* segment. Notably, this result holds true even if the revenue share for the movie is higher than for the video; that is, the studio will sacrifice movie demand and this margin to some degree by releasing early.

Second, it is worthwhile to contrast this result to that obtained under high content durability but for the same sub-case (i.e., intermediate movie quality and high congestion costs). Under high content durability, part (i) of Proposition 2 shows that the studio employs a delayed release tactic when its movie revenue share is lucrative. However, under low content durability, part (i) of Proposition 4 shows that it is never in the best interest of the studio to delay release. This difference in optimal release timing is attributable to how the studio manages cannibalization between *both* and *video* segments when multiple purchasing behavior occurs (δ_H). Because both of these market segments consume the video and incur p_d , the studio necessarily needs to use its video release timing lever to throttle substitution. On the other hand, when a *both* segment does not arise in equilibrium (δ_L), the studio can control substitution between *movie* and *video* market segments primarily using its video pricing lever.

By combining all results in Table 2, we gain a better understanding of how changes in content durability affect the video release strategy, when holding the movie quality and congestion factor classifications fixed. As an example, it is worth considering the case of a film with intermediate quality γ_m^I and under a high congestion factor α_H . Provided that the studio obtains a large share of movie revenues, we see that $T^* = 0$ for δ_L whereas $T^* = 1 - \delta_H(1 - \xi) - \frac{\gamma_m^I}{\gamma_d} + \frac{2p_m}{\gamma_d} \sqrt{\frac{\lambda_m \rho}{\gamma_d}}$ for δ_H . An immediate insight derived from this analysis is that the optimal video release time is also non-monotonic in content durability. That is, T^* increases with a shift from δ_L to δ_H , and then decreases with higher δ_H . This finding complements the work of Calzada and Valletti (2012) by demonstrating conditions under which the optimal video release time increases in content durability instead of decreases. More generally, our model suggests that if there exists sufficient variation in content durability among movies, we can expect to see video release times become more film-specific and possibly span feasible window lengths.

5 Discussion and Concluding Remarks

In this paper, we present a model of film distribution and consumption to gain insight into how studios should optimally price and time the release of video versions of their films when accounting for strategic behavior of consumers in product choice. We take a normative approach to the studio's problem and highlight the critical factors that can motivate the studio to choose video release and pricing strategies; these strategies can be characterized as direct-to-video, day-and-date, perfect segmentation, and delayed release tactics. In developing the consumer model, we incorporate several important factors: (i) quality decay of the film content over time; (ii) the quality/price gap

	$\frac{\delta_H}{\alpha_H}$		$\frac{\delta_L}{\alpha_L}$		
	α_H	α_L	α_H	α_L	
γ_m^H	$T^* = 0$ $p_d^* = \frac{\delta_H \gamma_d (1-\xi)}{2(\delta_H (1-\xi)(1-\rho) + \rho)}$ Day-and-Date [L : B-M-N] [H : D-N]	$T^* = \xi$ $p_d^* = \frac{\delta_H \gamma_d (1-\xi)}{2}$ Delayed Release [L : B-M-N] [H : B-M-N]	$T^* = 0$ $p_d^* = \frac{\gamma_d}{2}$ Perfect Segmentation [L : M-N] [H : D-N]	$\lambda_m < \lambda_d$ $T^* = 0$ $p_d^* = \gamma_d$ Day-and-Date [L : M-D-N] [H : M-D-N]	$\lambda_m \geq \lambda_d$ $T^* = 1$ $p_d^* = \gamma_d$ Movie Only [L : M-N] [H : M-N]
γ_m^I	$\lambda_m < \lambda_d \Phi$ $T^* = 0$ $p_d^* = \frac{\gamma_d}{2}$ Day-and-Date [L : B-D-N] [H : D-N]	$\lambda_m \geq \lambda_d \Phi$ $T^* = 1 - \delta_H (1 - \xi) - \frac{\gamma_m^I}{\gamma_d} + \frac{2p_m}{\gamma_d} \sqrt{\frac{\lambda_m \rho}{\lambda_d}}$ $p_d^* = \frac{\gamma_m^I + \delta_H \gamma_d (1-\xi)}{2} - p_m \sqrt{\frac{\lambda_m \rho}{\lambda_d}}$ Delayed Release	$T^* = 0$ $p_d^* = \frac{\gamma_d}{2}$ Day-and-Date [L : M-D-N] [H : D-N]	$T^* = \xi$ $p_d^* = \frac{\gamma_d (1-\xi)}{2}$ Delayed Release [L : B-D-N] [H : B-D-N]	

Table 2: Studio's Optimal Video Release Time and Pricing Strategy

between theater and video alternatives; *(iii)* consumption of both the theatrical and video version and quantification of these preferences using a notion of content durability (including delayed value realization for consumers who purchase both); and *(iv)* either negative consumption externalities associated with congestion at theaters or the absence of related costs for heterogeneous classes of consumers. These factors, which are in turn influenced by video release timing and pricing, together affect consumption behavior. Table 2 provides an overall summary of our findings. We analyze a variety of dimensions organized by case (two levels of content durability), sub-case (two levels of movie quality), and factor (two levels of congestion costs).

Using our model, we establish a wide range of relevant insights for studios. Our results also have several testable implications.

1. Focusing on intermediate-to-high movie quality: as theatrical movie quality increases, the video release time decreases.
2. Focusing on lower movie quality: as a film's theatrical movie quality decreases, it is more likely to be released using a direct-to-video tactic. Overall, the optimal video release time is non-monotonic in the quality of the movie.
3. For high quality movies with high content durability, as congestion in theater increases, e.g., due to peak season capacity constraints, more day-and-date strategies will be implemented.
4. When congestion effects are low, films with higher content durability are more likely to be released later; however, films with lower content durability should be released using a day-and-date tactic if the studio's revenue share of the video channel is more attractive than the theater channel.

These testable implications require parameters such as content durability to be measured. Such parameters can be forecasted from data on consumer viewing habits. For example, data from Nielsen can help determine how durable different genres of film are as well as how content durability might relate to a film's target audience.

A number of important questions remain for future research. In this work, we focus our attention on the optimal video release timing and pricing strategy for a single movie. This setting is important since studios have some degree of freedom in the timing of video releases, and the simpler model more readily clarifies the central trade-offs. However, competition is certainly an important topic and can even be a factor for deciding when to schedule theatrical release, particularly during peak seasons. Notably, the effect of competition on *video* release times is much weaker (Goldberg 1991). Another aspect worth investigation is the release of multiple films by the same studio over time, i.e., the repeated game aspect. Along these lines, Prasad et al. (2004) specifically consider the impact of consumer expectations over time in an aggregate model. In that context, the authors commented that an interesting avenue of future research would be to start from a consumer utility

model and capture the impact of consumer expectations which arise endogenously. Our paper may serve as a building block for further research in this direction.

Piracy is a pervasive problem in digital goods markets and is often facilitated by online systems that support illegal filesharing. Researchers who study the economics of digital piracy identify how consumers, when they pirate, often still face costs that are akin to price. For example, pirates are subject to government imposed penalties, search and learning costs, and moral costs (Connor and Rumelt 1991, August and Tunca 2008, Danaher et al. 2010, Lahiri and Dey 2013). Because would-be pirates are an important source of revenue to studios, the more general pricing problem they face must also account for the incentive compatibility constraints of consumers considering piracy. An important direction for future research is to study how movie piracy would affect the studio's setting of video release time and price controls. In particular, formal analysis of the studio's behavior as its pricing problem becomes more constrained as a consequence of combatting piracy may lead to important insights (Pogue 2012).

In this paper, we study the video release time and pricing decisions based on the studio's perspective, i.e., one decision maker. Calzada and Valletti (2012) study a decentralized channel in which an exhibitor sets the movie price and the retailer sets the video price, and show that if the studio's bargaining power relative to the exhibitor is high and content durability is relatively low, a delayed release is an optimal strategy for the studio. Furthermore, they demonstrate that if the studio's bargaining power is sufficiently higher than the exhibitor, the day-and-date strategy becomes optimal. In our model, if the channel becomes decentralized, the studio loses direct control over the video price and must then resort more toward utilizing the video release time in order to improve its revenue by inducing appropriate consumer market structures. This loss of direct video price control can lead the studio to delay the video release and sacrifice a loss in video quality so that it can ensure lucrative movie revenues. Consequently, in our setting, we expect that channel decentralization would result in greater use of a delayed released strategy for videos.

We consider a fixed simple revenue sharing contract between studios and exhibitors. In practice, conventional contracts involve time-dependent revenue sharing (i.e., a sliding, increasing percentage of revenues to exhibitors) after allowance for exhibitors' expenses. But recently, some studios and exhibitors have begun to implement "aggregate settlement," a simple revenue split without sliding scales (Vogel 2007). Our model can serve as a reasonable approximation for that scenario. We have made simplifying assumptions in order to focus primarily on the underlying trade-offs relevant to our research questions. Nevertheless, our results are robust and satisfied for wide ranges of revenue shares. One fruitful extension would be to rigorously analyze how the various contracts between studios, exhibitors, and video retailers should be designed, particularly in light of our results on the profitability of day-and-date release strategies.

We assume that movie quality is certain and common knowledge, noting that empirical evidence concerning demand uncertainty associated with quality is not as strong as popularly argued (see, e.g., Orbach 2004, Orbach and Einav 2007). Furthermore, most of the uncertainty is revealed after

the first weekend of release. Consequently, when distributors set video release dates after uncertainties are almost resolved and consumers are well-informed, most of our results are preserved. If consumers are not informed about the quality of films, then video windows determined by distributors may signal quality, a scenario which may merit study. In this direction, our paper provides an interesting observation: Contrary to conventional wisdom, longer video release windows do not necessarily signal higher movie quality.

Finally, in our study, we assume that the studio announces the video release time and price immediately and commits to both of them which seems credible given the repeated nature of the context. Commitment is a non-issue when either day-and-date or direct-to-video strategies are optimal since the video channel is opened immediately. However, a studio's inability to announce immediately can affect consumption early on during the run of a film in theaters. Extending the model to permit delayed announcements may be worth studying, especially for cases where video release times are determined to optimally be less than a month out.

This paper is a first step toward analyzing the trade-offs faced by studios as they determine an appropriate video release time and pricing strategy. Studios are demonstrably interested in the prospects of earlier release times and even day-and-date strategies, and can benefit from a better understanding of how congestion, film content durability, and movie quality interact with such strategies. Given the speed of technological advances and the enduring use of a channeling system, the film industry now has a significant opportunity to design products and make decisions on delivery systems that cater more effectively to consumer preferences. By establishing how effective day-and-date strategies are at boosting studio profits, we hope that our work helps to initiate part of this progress.

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Appendix A: Proofs of Propositions and Lemmas

For simplicity and greater clarity of argument, we focus on limiting values: $\alpha_L = 0$, $\delta_L = 0$, $\delta_H = 1$ and $\alpha_H \rightarrow \infty$ in the proofs in the main Appendix. We provide extended proofs for more general parameter values in the Online Supplement.

Proof of Proposition 1: For part (i), for class H , a high congestion cost factor α_H induces the consumer market structure $[H : D-N]$, i.e.,

$$\sigma^*(v, H) = \begin{cases} D & \text{if } \frac{p_d}{\gamma_d(1-T)} \leq v < 1; \\ N & \text{if } v < \frac{p_d}{\gamma_d(1-T)}. \end{cases} \quad (\text{A.1})$$

For $c = L$, under high content durability δ_H , high theatrical movie quality γ_m^H , and a high congestion cost factor α_H , we show that the optimal video price and the release time induce the consumer market structure $[L : B-M-N]$; specifically,

$$\sigma^*(v, L) = \begin{cases} B & \text{if } \frac{p_d}{\gamma_d(1-\xi)} \leq v < 1; \\ M & \text{if } \frac{p_m}{\gamma_m^H} \leq v < \frac{p_d}{\gamma_d(1-\xi)}; \\ N & \text{if } v < \frac{p_m}{\gamma_m^H}. \end{cases} \quad (\text{A.2})$$

To demonstrate that these equilibrium consumer market structures arise under the studio's optimal strategy requires careful examination of numerous cases and extensive algebraic comparisons. We provide these complete and rigorous arguments in an Online Supplement, while focusing on the characterization of the release time and price in the main Appendix.²

Given that the studio prefers to induce this equilibrium, its corresponding profit function is written as

$$\Pi(T, p_d) = \lambda_m p_m \rho \left(1 - \frac{p_m}{\gamma_m^H} \right) + \lambda_d p_d \left(\rho \left(1 - \frac{p_d}{\gamma_d(1-\xi)} \right) + (1-\rho) \left(1 - \frac{p_d}{\gamma_d(1-T)} \right) \right). \quad (\text{A.3})$$

Taking the derivative of $\Pi(T, p_d)$ in (A.3) with respect to p_d , we then obtain

$$\frac{\partial \Pi}{\partial p_d} = \frac{\lambda_d(1-\xi)(\gamma_d(1-T) - 2p_d) - 2\rho\lambda_d p_d(1-T - (1-\xi))}{\gamma_d(1-T)(1-\xi)}. \quad (\text{A.4})$$

Furthermore, the second-order condition becomes

$$\frac{\partial^2 \Pi}{\partial p_d^2} = -\frac{2\lambda_d}{\gamma_d} \left(\frac{1-\rho}{1-T} + \frac{\rho}{(1-\xi)} \right) < 0, \quad (\text{A.5})$$

which is satisfied, and hence the first-order condition is sufficient. Thus, $p_d^*(T)$ is an interior solution given T ; specifically, given T , $p_d^*(T) = \frac{\gamma_d(1-T)(1-\xi)}{2((1-\xi)(1-\rho) + \rho(1-T))}$. Plugging this expression into (A.3), we

²For each of the propositions, we have separated the proof in this manner between the Appendix and the Online Supplement.

obtain

$$\Pi(T, p_d^*(T)) = \frac{\rho \lambda_m p_m (\gamma_m^H - p_m)}{\gamma_m^H} + \frac{\lambda_d \gamma_d (1 - T)(1 - \xi)}{4((1 - \rho)(1 - \xi) + \rho(1 - T))}. \quad (\text{A.6})$$

Taking the derivative of this profit function with respect to T , we have

$$\frac{d\Pi(T, p_d^*(T))}{dT} = -\frac{\lambda_d \gamma_d (1 - \xi)^2 (1 - \rho)}{4((1 - \xi)(1 - \rho) + \rho(1 - T))^2} < 0. \quad (\text{A.7})$$

Hence, $T^* = 0 < \xi$, and plugging this back into $p_d^*(T^* = 0)$, we then obtain the optimal p_d^* .

Similarly, for part (ii), under a low congestion cost factor, one can prove that the optimal release time and the video price induce the consumer market structure of $[L : B-M-N]$ and $[H : B-M-N]$ (see the Online Supplement), and the corresponding profit can be written as

$$\Pi(T, p_d) = \frac{\lambda_m p_m (\gamma_m^H - p_m)}{\gamma_m^H} + \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d (1 - \xi)}\right). \quad (\text{A.8})$$

Note that this profit function does not depend on T . Since video consumption occurs at $T = \xi$ for *both* consumers, profits are weakly maximized at $T^* = \xi$. Resolving indifference at $T^* = \xi$ maintains continuity in the optimal strategy as γ_m^H decreases and is also more likely to arise due to production lead times associated with videos. Therefore, in this case, $T^* = \xi$, and by maximizing the studio's profit function over p_d , we obtain $p_d^* = \frac{\gamma_d (1 - \xi)}{2}$. Furthermore, if $T > \xi$, then the studio's profit is decreasing in T in this parameter region. Hence, under high content durability, $T^* = \xi$ and $p_d^* = \frac{\gamma_d (1 - \xi)}{2}$. ■

Proof of Proposition 2: For part (i), similar to the proof of Proposition 1, a high congestion cost factor α_H induces the consumer market structure $[H : D-N]$. Moreover, for $c = L$, under high content durability δ_H , intermediate theatrical movie quality γ_m^I , and a high congestion cost factor α_H , the optimal video price and the release time ($T \leq \xi$) induce the consumer market structure $[L : B-D-N]$. In this case, the corresponding studio's profit function can be written as

$$\begin{aligned} \Pi(T, p_d) = p_m \lambda_m \rho \left(1 - \frac{p_m}{\gamma_m^I - \gamma_d (1 - T - (1 - \xi))}\right) \\ + p_d \lambda_d \left(\rho \left(1 - \frac{p_d}{(1 - T) \gamma_d}\right) + (1 - \rho) \left(1 - \frac{p_d}{(1 - T) \gamma_d}\right)\right). \end{aligned} \quad (\text{A.9})$$

Differentiating (A.9) with respect to p_d and solving a first-order condition, it follows that

$$p_d^*(T) = \frac{(1 - T) \gamma_d}{2}. \quad (\text{A.10})$$

The second-order condition is satisfied in this case, which guarantees the optimality of (A.10). Plugging (A.10) into (A.9), we then obtain

$$\Pi(T, p_d^*(T)) = \frac{\lambda_d \gamma_d (1-T)}{4} + \rho \lambda_m p_m \left(1 - \frac{p_m}{\gamma_m^I - \gamma_d(\xi - T)} \right). \quad (\text{A.11})$$

Differentiating $\Pi(T, p_d^*(T))$ with respect to T , we obtain

$$\frac{d\Pi(T, p_d^*(T))}{dT} = \gamma_d \left(-\frac{\lambda_d}{4} + \frac{\rho \lambda_m p_m^2}{(\gamma_m^I - \gamma_d(\xi - T))^2} \right). \quad (\text{A.12})$$

It follows that $\frac{d\Pi(T, p_d^*(T))}{dT}$ is decreasing in T , hence the second-order condition is satisfied. Furthermore, if $\lambda_d \Phi \leq \lambda_m$, where $\Phi = ((\gamma_m^I - \gamma_d \xi)/(2\rho p_m))^2$, the first-order condition is satisfied at $T^* = \xi - \frac{\gamma_m^I}{\gamma_d} + \frac{2p_m}{\gamma_d} \sqrt{\frac{\lambda_m \rho}{\gamma_d}}$ and by replacing this optimal T^* , we then obtain p_d^* . Otherwise, if $\lambda_m < \lambda_d \Phi$, then (A.12) is strictly negative for all $T < \xi$ under δ_H . Therefore, the studio optimally sets $T^* = 0$ and $p_d^* = \frac{\gamma_d}{2}$, which results in day-and-date tactic.

For part (ii), under high content durability δ_H , intermediate theatrical movie quality γ_m^I , and a low congestion cost factor α_L , the optimal video price and release time induce the consumer market structure $[L : B-D-N]$ and $[H : B-D-N]$. Consequently, the studio's resulting profit function is

$$\Pi(T, p_d) = \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d(1-T)} \right) + \frac{\lambda_m p_m (\gamma_m^I - p_m - \gamma_d(\xi - T))}{\gamma_m^I - \gamma_d(\xi - T)}, \quad (\text{A.13})$$

for $T \leq \xi$. From the first-order condition on p_d , we obtain $p_d^*(T) = \gamma_d(1-T)/2$. By plugging this optimal expression into (A.13), it follows that

$$\Pi(T, p_d^*(T)) = \frac{(\gamma_m^I - \gamma_d(\xi - T))(\lambda_d \gamma_d(1-T) + 4\lambda_m p_m) - 4\lambda_m p_m^2}{4(\gamma_m^I - \gamma_d(\xi - T))}, \quad (\text{A.14})$$

which is increasing in T for $T \leq \xi$ under the bounds on γ_m^I (see Appendix B). Similarly, for $T > \xi$, the corresponding studio's profit function for the consumer market structure $[L : B-D-N]$ and $[H : B-D-N]$ is

$$\Pi(T, p_d) = \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d(1-T)} \right) + \frac{\lambda_m p_m (\gamma_m^I - p_m)}{\gamma_m^I}. \quad (\text{A.15})$$

The optimal video price given T is the same as before, i.e., $p_d^*(T) = \gamma_d(1-T)/2$. Replacing this optimal price in (A.15), we obtain

$$\Pi(T, p_d^*(T)) = \frac{\gamma_m^I (\lambda_d \gamma_d(1-T) + 4\lambda_m p_m) - 4\lambda_m p_m^2}{4\gamma_m^I}, \quad (\text{A.16})$$

which is decreasing in T under bounds on γ_m^I . Therefore, the optimal release time is $T^* = \xi$, and the video price is $p_d^* = \gamma_d(1-\xi)/2$. ■

Proof of Proposition 3: First, for part (i), for a film with low content durability δ_L and high movie quality γ_m^H , under a high congestion cost factor α_H , the studio sets the release time and the video price in such a way that the consumer market structure at the optimum will be $[L : M-N]$ and $[H : D-N]$ (see the Online Supplement for a detailed proof that other consumer market structures are dominated by this perfect segmentation market structure). In this structure, the studio's profit can be written as

$$\Pi(T, p_d) = \rho \lambda_m p_m \left(1 - \frac{p_m}{\gamma_m^H} \right) + (1 - \rho) \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d(1-T)} \right). \quad (\text{A.17})$$

By taking the derivative of this profit function with respect to p_d , it follows that

$$\frac{\partial \Pi(T, p_d)}{\partial p_d} = \frac{(1 - \rho) \lambda_d (\gamma_d(1-T) - 2p_d)}{\gamma_d(1-T)}, \quad (\text{A.18})$$

from which we obtain $p_d^*(T) = \frac{\gamma_d(1-T)}{2}$. By plugging $p_d^*(T)$ into (A.17), we have

$$\Pi(T, p_d^*(T)) = \frac{(1 - \rho) \lambda_d \gamma_m^H \gamma_d(1-T) + 4\rho \lambda_m p_m (\gamma_m^H - p_m)}{4\gamma_m^H}, \quad (\text{A.19})$$

which is decreasing in T . Hence, $T^* = 0$ and consequently, $p_d^* = \frac{\gamma_d}{2}$.

For part (ii), under a low congestion cost factor α_L and $\lambda_d > \lambda_m$, the consumer market structure at the optimal release time and video price is $[H : M-D-N]$ and $[L : M-D-N]$. In this case, the studio's profit function is written as

$$\Pi(T, p_d) = \lambda_m p_m \frac{\gamma_m^H - p_m - (\gamma_d(1-T) - p_d)}{\gamma_m^H - \gamma_d(1-T)} + \lambda_d p_d \left(\frac{p_m - p_d}{\gamma_m^H - \gamma_d(1-T)} - \frac{p_d}{\gamma_d(1-T)} \right). \quad (\text{A.20})$$

From the first-order condition on p_d , we obtain

$$p_d^*(T) = \frac{\gamma_d p_m (1-T) (\lambda_d + \lambda_m)}{2\lambda_d \gamma_m^H}. \quad (\text{A.21})$$

Substituting the optimal video price $p_d^*(T)$ into the studio's profit function, we obtain $\Pi(T, p_d^*(T))$, which is decreasing in T ; as a result, $T^* = 0$. Therefore, $p_d^* = \frac{\gamma_d p_m (\lambda_m + \lambda_d)}{2\lambda_d \gamma_m^H}$. If $\lambda_d \leq \lambda_m$, the proof is very similar to that above. The difference is that in the corresponding parameter region, the condition $\lambda_m \geq \lambda_d$ leads to the equilibrium outcome of not releasing the video at all due to the negative impact of demand cannibalization on the studio's profits. ■

Proof of Corollary 1: We prove that under γ_m^H and δ_L ,

$$\Pi^* |_{\{\alpha=\alpha_L\}} < \Pi^* |_{\{\alpha=\alpha_H\}}, \quad (\text{A.22})$$

if either of the following conditions holds: (i) $\lambda_d > \lambda_m \max(1, \bar{\lambda}_d)$ where $\bar{\lambda}_d$ is the unique positive root of λ_d that solves

$$\gamma_d(\gamma_m^H(\gamma_m^H - \gamma_d) - p_m^2)\lambda_d^2 - 2p_m(2(\gamma_m^H)^2 - 2(\gamma_d + p_m)\gamma_m^H + p_m\gamma_d)\lambda_d - \gamma_d p_m^2 = 0, \quad (\text{A.23})$$

or (ii) $\lambda_m > \lambda_d > 4\lambda_m p_m / \gamma_d$.

First, suppose that $\lambda_d > \lambda_m$. Under α_H , δ_L , and γ_m^H , the outcome corresponds to the perfect segmentation tactic in part (i) of Proposition 3, and the corresponding studio's optimal profit is

$$\Pi^* |_{\{\alpha=\alpha_H\}} = \lambda_m p_m \rho \left(1 - \frac{p_m}{\gamma_m^H} \right) + \lambda_d (1 - \rho) \frac{\gamma_d}{4}. \quad (\text{A.24})$$

On the other hand, under α_L , the outcome corresponds to the day-and-date tactic in part (ii) of Proposition 3 under the condition of $\lambda_d > \lambda_m$. The corresponding optimal profit for the studio can be written as

$$\Pi^* |_{\{\alpha=\alpha_L\}} = p_m \left(\lambda_m - p_m \frac{4\gamma_m^H \lambda_m \lambda_d - \gamma_d (\lambda_m + \lambda_d)^2}{4\gamma_m^H \lambda_d (\gamma_m^H - \gamma_d)} \right), \quad (\text{A.25})$$

for $\alpha_L = 0$. By comparing the optimal studio's profits in (A.24) and (A.25), we obtain that if $\lambda_d > \lambda_m \bar{\lambda}_d$, the studio's profit under perfect segmentation in (A.24) is higher than the profit under the day-and-date tactic in (A.25). Moreover, there exists a unique, positive solution of λ_d in (A.23) for γ_m^H sufficiently high.

Next, suppose that $\lambda_d < \lambda_m$. In this case, under α_H , it is the same as the previous, i.e., it corresponds to perfect segmentation and the studio's profit is given in (A.24). The difference is that if $\lambda_d < \lambda_m$, then under α_L , it corresponds to a movie only release strategy in part (ii) of Proposition 3. In this region where a movie-only structure is optimal, the studio's corresponding profits become

$$\Pi^* |_{\{\alpha=\alpha_L\}} = \lambda_m p_m \left(1 - p_m / \gamma_m^H \right). \quad (\text{A.26})$$

By comparing optimal profits in those two cases, we obtain that if $\lambda_d > 4\lambda_m p_m / \gamma_d$, the studio's profit under perfect segmentation given in (A.24) is higher than the profit under a movie-only release presented in (A.26). As a result, an increase in the congestion cost parameter α can increase studio profits, which completes the proof. \square

Proof of Proposition 4: For part (i), a high congestion cost factor α_H yields $[H:D-N]$ in equilibrium. Furthermore, under δ_L and γ_m^I , the consumer market structure for class L at optimality

is $[L : M-D-N]$. Under this consumer market structure, the studio's profit can be written as

$$\begin{aligned} \Pi(T, p_d) = \lambda_d p_d \left(\rho \left(\frac{p_m - p_d}{\gamma_m^I - (1-T)\gamma_d} - \frac{p_d}{(1-T)\gamma_d} \right) + (1-\rho) \left(1 - \frac{p_d}{(1-T)\gamma_d} \right) \right) \\ + \rho \lambda_m p_m \left(1 - \frac{p_m - p_d}{\gamma_m^I - (1-T)\gamma_d} \right). \end{aligned} \quad (\text{A.27})$$

Optimizing its profit over p_d , we obtain

$$p_d^*(T) = \frac{(1-T)\gamma_d(\lambda_d(1-\rho)(\gamma_m^I - (1-T)\gamma_d) + \rho p_m(\lambda_m + \lambda_d))}{2\lambda_d(\gamma_m^I - (1-T)(1-\rho)\gamma_d)}. \quad (\text{A.28})$$

Plugging (A.28) into (A.27), and taking a derivative of $\Pi(T, p_d^*(T))$ with respect to T , we find that it is decreasing in T . Thus, it follows that $T^* = 0$. Replacing $T^* = 0$ into (A.28), we obtain (11). The proof of part (ii) directly follows from the proof of part (ii) in Proposition 3. ■

Appendix B: Characterization of Bounds

In this section, we provide detailed expressions for the bounds of γ_m^I and characterize the parameter regions that we focus on.

First, $\gamma_m^I \in [\underline{\gamma}_m, \overline{\gamma}_m]$, where $\underline{\gamma}_m = \max(\underline{\gamma}_1, \underline{\gamma}_2, \dots, \underline{\gamma}_6)$, and $\overline{\gamma}_m = \min(\overline{\gamma}_1, \overline{\gamma}_2, \dots, \overline{\gamma}_7)$, in which $\underline{\gamma}_1 = \frac{p_m}{1-\xi}$, $\underline{\gamma}_2 = \frac{1}{2} \left(p_m + \gamma_d + \sqrt{p_m^2 + \gamma_d^2} \right)$,

$$\underline{\gamma}_3 = \frac{1}{2\lambda_d(3-2\xi)} (\gamma_d \lambda_d (-2 + \xi)(-3 + 2\xi) + p_m(\lambda_d(7 - 6\xi) + \lambda_m(-3 + 2\xi)) - [(p_m^2(7\lambda_d - 3\lambda_m - 6\lambda_d\xi + 2\lambda_m\xi)^2 + 2p_m\gamma_d\lambda_d(1-\xi)(3-2\xi) \times (-\lambda_d - 3\lambda_m + 2(\lambda_d + \lambda_m)\xi) + \gamma_d^2\lambda_d^2(3-5\xi + 2\xi^2)^2)]^{1/2}),$$

$$\underline{\gamma}_4 = \frac{1}{2\lambda_d} (3p_m\lambda_d + \gamma_d\lambda_d - p_m\lambda_m - \gamma_d\lambda_d\xi - \sqrt{(p_m(-3\lambda_d + \lambda_m) + \gamma_d\lambda_d(-1 + \xi))^2 + 8p_m\gamma_d\lambda_d^2(-1 + \xi)}),$$

$$\underline{\gamma}_5 = \frac{1}{4} \left((3 + 4(1 - \xi))\gamma_d + 2p_m \left(1 - \frac{\lambda_m}{\lambda_d} \right) \right), \quad \underline{\gamma}_6 = 2p_m \frac{\lambda_m}{\lambda_m + \lambda_d},$$

$$\overline{\gamma}_1 = \gamma_d(1 - \delta(1 - \xi)) + 2p_m\delta(1 - \xi), \quad \overline{\gamma}_2 = 2p_m\sqrt{\frac{\lambda_m\rho}{\lambda_d}} - \gamma_d\delta(1 - \xi) \left(1 - \sqrt{\frac{\lambda_d}{\lambda_m\rho}} \right),$$

$$\overline{\gamma}_3 = 2p_m \left(1 + \frac{\xi\rho}{1-\xi} \right), \quad \overline{\gamma}_4 = 2\sqrt{2p_m\gamma_d(1-\xi)} - \gamma_d(1-\xi), \quad \overline{\gamma}_5 = 2p_m\lambda_m,$$

$$\overline{\gamma}_6 = \frac{1}{2\lambda_d(3-2\xi)} (\gamma_d \lambda_d (-2 + \xi)(-3 + 2\xi) + p_m(\lambda_d(7 - 6\xi) + \lambda_m(-3 + 2\xi)) + [(p_m^2(7\lambda_d - 3\lambda_m - 6\lambda_d\xi + 2\lambda_m\xi)^2 + 2p_m\gamma_d\lambda_d(1-\xi)(3-2\xi) \times (-\lambda_d - 3\lambda_m + 2(\lambda_d + \lambda_m)\xi) + \gamma_d^2\lambda_d^2(3-5\xi + 2\xi^2)^2)]^{1/2}), \quad \text{and}$$

$$\overline{\gamma}_7 = \frac{1}{2\lambda_d} (3p_m\lambda_d + \gamma_d\lambda_d - p_m\lambda_m - \gamma_d\lambda_d\xi + \sqrt{(p_m(-3\lambda_d + \lambda_m) + \gamma_d\lambda_d(-1 + \xi))^2 + 8p_m\gamma_d\lambda_d^2(-1 + \xi)}),$$

$$\overline{\gamma}_8 = p_m + \gamma_d(1 - \delta)(1 - \xi).$$

Second, we also have $\rho < \bar{\rho} = \min\left(\frac{\lambda_d(\gamma_m - \gamma_d(1-\delta)(1-\xi))^2}{4p_m^2\lambda_m}, \frac{1-\xi}{2}\right)$. And, lastly, we impose the following set of restrictions on the parameter region, which guarantees non-emptiness of the interval $[\underline{\gamma}_m, \overline{\gamma}_m]$ and helps analytical tractability: $p_m > \frac{3}{4}\gamma_d$, $\xi < 1/2$, $\lambda_m + \lambda_d > 1$, $\gamma_d\lambda_d(1 - \lambda_m)(1 - \rho) + p_m\lambda_m(\lambda_m\rho + \lambda_d(-1 - (1 - \rho)(1 - 2\lambda_m))) < 0$, $(p_m(-3\lambda_d + \lambda_m) + \gamma_d\lambda_d(-1 + \xi))^2 + 8p_m\gamma_d\lambda_d^2(-1 + \xi) > 0$, $(1 + (1 - \xi)(4 + 5(1 - \xi)))\gamma_d^2\lambda_d^2 + p_m^2(\lambda_d - \lambda_m)^2 - 2p_m(1 - \xi)\gamma_d\lambda_d(3\lambda_d + \lambda_m) < 0$, and $2p_m\gamma_d\lambda_d(-1 + \xi)(-3 + 2\xi)(-\lambda_d - 3\lambda_m + 2(\lambda_d + \lambda_m)\xi) + \gamma_d^2\lambda_d^2(3 - 5\xi + 2\xi^2)^2 + p_m^2(\lambda_m(3 - 2\xi) + \lambda_d(-7 + 6\xi))^2 > 0$. The derivation of these bounds is shown in the Online Supplement.

Online Supplement for “Optimal Timing of Sequential Distribution: The Impact of Congestion Externalities and Day-and-Date Strategies”

In this Online Supplement, we provide generalized proofs of the consumer market equilibrium characterization, optimality of direct-to-video tactics, and all of the propositions in the paper. In the paper, we provided the essence of the proofs in Appendix A for limiting parameter values. Here, we provide the complete, detailed proofs for more general parameter values.

Proof of Consumer Market Equilibrium Characterization: We first prove the consumer equilibrium strategy σ^* characterized by thresholds, which is presented in Section 3.1. Consider class $c = L$. In this class of customers, the required conditions for $\sigma^*(v, L) = B$ are the following:

$$V(v, L, B, \sigma_{-v}^*) \geq V(v, L, M, \sigma_{-v}^*) \Leftrightarrow v \geq \frac{p_d}{\delta\gamma_d(1 - \max(T, \xi))}; \quad (\text{OS.1})$$

$$V(v, L, B, \sigma_{-v}^*) \geq V(v, L, D, \sigma_{-v}^*) \Leftrightarrow v \geq \frac{p_m}{\gamma_m - \gamma_d(1 - T - \delta(1 - \max(T, \xi)))}; \quad (\text{OS.2})$$

$$V(v, L, B, \sigma_{-v}^*) \geq V(v, L, N, \sigma_{-v}^*) \Leftrightarrow v \geq \frac{p_m + p_d}{\gamma_m + \delta\gamma_d(1 - \max(T, \xi))}. \quad (\text{OS.3})$$

As a result, it follows that $\sigma^*(v, L) = B$ if and only if $v \geq \omega_b^L$, where

$$\omega_b^L \triangleq \max \left(\frac{p_d}{\delta\gamma_d(1 - \max(T, \xi))}, \frac{p_m}{\gamma_m - \gamma_d(1 - T - \delta(1 - \max(T, \xi)))}, \frac{p_m + p_d}{\gamma_m + \delta\gamma_d(1 - \max(T, \xi))} \right). \quad (\text{OS.4})$$

Next, for $\sigma^*(v, L) = M$ consumer strategy, we need the following three conditions:

$$V(v, L, M, \sigma_{-v}^*) \geq V(v, L, B, \sigma_{-v}^*) \Leftrightarrow v \leq \frac{p_d}{\delta\gamma_d(1 - \max(T, \xi))}; \quad (\text{OS.5})$$

$$V(v, L, M, \sigma_{-v}^*) \geq V(v, L, D, \sigma_{-v}^*) \Leftrightarrow v \geq \frac{p_m - p_d}{\gamma_m - \gamma_d(1 - T)}; \quad (\text{OS.6})$$

$$V(v, L, M, \sigma_{-v}^*) \geq V(v, L, N, \sigma_{-v}^*) \Leftrightarrow v \geq \frac{p_m}{\gamma_m}. \quad (\text{OS.7})$$

As a result, $\sigma^*(v, L) = M$ if and only if

$$\max \left(\frac{p_m - p_d}{\gamma_m - \gamma_d(1 - T)}, \frac{p_m}{\gamma_m} \right) \leq v \leq \frac{p_d}{\delta\gamma_d(1 - \max(T, \xi))}. \quad (\text{OS.8})$$

Note that $\frac{p_d}{\delta\gamma_d(1-\max(T,\xi))} \leq \omega_b^L$. Third, the following three conditions are required in order to have $\sigma^*(v, L) = D$:

$$V(v, L, D, \sigma_{-v}^*) \geq V(v, L, B, \sigma_{-v}^*) \Leftrightarrow v \leq \frac{p_m}{\gamma_m - \gamma_d(1 - T - \delta(1 - \max(T, \xi)))}; \quad (\text{OS.9})$$

$$V(v, L, D, \sigma_{-v}^*) \geq V(v, L, M, \sigma_{-v}^*) \Leftrightarrow v \leq \frac{p_m - p_d}{\gamma_m - \gamma_d(1 - T)}; \quad (\text{OS.10})$$

$$V(v, L, D, \sigma_{-v}^*) \geq V(v, L, N, \sigma_{-v}^*) \Leftrightarrow v \geq \frac{p_d}{\gamma_m(1 - T)}. \quad (\text{OS.11})$$

Consequently, $\sigma^*(v, L) = D$ if and only if

$$\frac{p_d}{\gamma_m(1 - T)} \leq v \leq \min\left(\frac{p_m}{\gamma_m - \gamma_d(1 - T - \delta(1 - \max(T, \xi)))}, \frac{p_m - p_d}{\gamma_m - \gamma_d(1 - T)}\right). \quad (\text{OS.12})$$

In addition, it follows that

$$\min\left(\frac{p_m}{\gamma_m - \gamma_d(1 - T - \delta(1 - \max(T, \xi)))}, \frac{p_m - p_d}{\gamma_m - \gamma_d(1 - T)}\right) \leq \max\left(\frac{p_m - p_d}{\gamma_m - \gamma_d(1 - T)}, \frac{p_m}{\gamma_m}\right). \quad (\text{OS.13})$$

Lastly, the required conditions for $\sigma^*(v, L) = N$ are the following three:

$$V(v, L, N, \sigma_{-v}^*) \geq V(v, L, B, \sigma_{-v}^*) \Leftrightarrow v \leq \frac{p_m + p_d}{\gamma_m + \delta\gamma_d(1 - \max(T, \xi))}; \quad (\text{OS.14})$$

$$V(v, L, N, \sigma_{-v}^*) \geq V(v, L, M, \sigma_{-v}^*) \Leftrightarrow v \leq \frac{p_m}{\gamma_m}; \quad (\text{OS.15})$$

$$V(v, L, N, \sigma_{-v}^*) \geq V(v, L, D, \sigma_{-v}^*) \Leftrightarrow v \leq \frac{p_d}{\gamma_m(1 - T)}. \quad (\text{OS.16})$$

As a result, it follows that $\sigma^*(v, L) = N$ if and only if $v \leq \omega_d^L$, where

$$\omega_d^L \triangleq \min\left(\frac{p_m + p_d}{\gamma_m + \delta\gamma_d(1 - \max(T, \xi))}, \frac{p_m}{\gamma_m}, \frac{p_d}{\gamma_m(1 - T)}\right). \quad (\text{OS.17})$$

Moreover, we have $\omega_d^L \leq \frac{p_d}{\gamma_m(1 - T)}$. Using these characterizations and relationships among boundaries together with $\sigma^*(v, L) \in \{N, D, M, B\}$ for all $v \in \mathcal{V}$, (4) follows. Note that depending on the parameter values, it is possible that some strategies may not be present in equilibrium, e.g., if $\omega_m^L > \omega_b^L$, then $\sigma^*(v, L) \neq M$ for all $v \in \mathcal{V}$. Now, consider class H consumers. The only difference in this class is that for both $V(v, H, B, \sigma_{-v}^*)$ and $V(v, H, M, \sigma_{-v}^*)$, there exists an additional congestion cost, $\alpha\eta$, which is fixed given the other consumers' equilibrium strategies, i.e., σ_{-v}^* . Closely following the previous arguments for the L class consumers with this difference, we also obtain (4) for $c = H$. This completes the proof. \square

Direct-to-Video tactics: We provide the optimality of a direct-to-video tactic, i.e., $T^* = 0$ and $p_d^* = \gamma_d/2$ with a corresponding market structure of $[L : D-N]$ and $[H : D-N]$, for low quality movies.

Lemma OS.1 *There exist $\omega_1 > 1$ such that for all $\omega > \omega_1$, if $\gamma_m/p_m < \omega$, then $T^* = 0$ and $p_d^* = \gamma_d/2$. Furthermore, under $T^* = 0$ and $p_d^* = \gamma_d/2$, the resulting market structure is $[L : D-N]$ and $[H : D-N]$.*

Proof: First, suppose that $\sigma^*(1, L) \in \{B, M\}$ in equilibrium. Then, by (2), it must follow that $\gamma_m - p_m > \gamma_d(1 - \max(T, \xi)) - p_d$. Dividing by p_m and letting $\frac{\gamma_m}{p_m} = 1 + K_1\zeta$, we obtain

$$\frac{\gamma_d(1 - \max(T, \xi)) - p_d}{p_m} < \frac{\gamma_m}{p_m} - 1 = K_1\zeta. \quad (\text{OS.18})$$

By (OS.18), p_d must satisfy $p_d > \gamma_d(1 - \max(T, \xi)) - p_m K_1\zeta$. Thus, by (1), (2), (5) and (6), $D_m \leq 1 - \frac{p_m}{\gamma_m} = 1 - \frac{1}{1+K_1\zeta}$ and $D_d \leq 1 - \frac{p_d}{\gamma_d(1 - \max(T, \xi))} < \frac{p_m K_1\zeta}{\gamma_d(1 - \max(T, \xi))}$. Then, by (7),

$$\begin{aligned} \Pi(T, p_d) &= \lambda_m p_m D_m + \lambda_d p_d D_d \\ &< \lambda_m p_m \left(1 - \frac{1}{1+K_1\zeta}\right) + \lambda_d p_d \left(\frac{p_m K_1\zeta}{\gamma_d(1 - \max(T, \xi))}\right). \end{aligned} \quad (\text{OS.19})$$

Second, suppose that $\sigma^*(1, L) \in \{D\}$ in equilibrium. Then, by (4) and the previous proof, $\sigma^*(v, L) \in \{D, N\}$ and $\sigma^*(v, H) \in \{D, N\}$ for all $v \in \mathcal{V}$, and by (5), (6), and (7), we obtain

$$\Pi(T, p_d) = \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d(1 - \max(T, \xi))}\right). \quad (\text{OS.20})$$

By (8) and optimizing (OS.20), we obtain maximizers $\hat{p}_d = \frac{\gamma_d}{2}$ and $\hat{T} = 0$, hence $\Pi(\hat{T}, \hat{p}_d) = \frac{\lambda_d \gamma_d}{4}$. Comparing this expression to the upper bound on profits from (OS.19), for sufficiently small ζ , it follows that $p_d^* = \hat{p}_d$ and $T^* = \hat{T}$. \square

Proof of Proposition 1: For part (i), we first prove the following Lemma, which will be also used for the proofs of the other propositions.

Lemma OS.2 *For any $\lambda_m > 0$ and $\rho > 0$, if $\alpha > \frac{\gamma_m}{\rho}$, then $\sigma^*(v, H) \in \{D, N\}$ for all $v \in \mathcal{V}$.*

Proof: First, suppose that $\sigma^*(v, L) \in \{D, N\}$ for all $v \in \mathcal{V}$. From $V(v, H, s, \sigma_{-v}^*) \leq V(v, L, s, \sigma_{-v}^*)$ for $s \in \{M, B\}$, for all $v \in \mathcal{V}$, and $V(v, H, s, \sigma_{-v}^*) = V(v, L, s, \sigma_{-v}^*)$ for $s \in \{D, N\}$, for all $v \in \mathcal{V}$, it follows that $\sigma^*(v, H) \in \{D, N\}$ for all $v \in \mathcal{V}$. Second, suppose that $\sigma^*(v, L) \in \{B, M\}$ for some $v \in \mathcal{V}$. We then first consider the following equilibrium consumer market structures for class L : $[L : B-M-D-N]$ or $[L : M-D-N]$. In these two structures, it follows that $\eta \geq \rho(1 - \frac{p_m - p_d^*}{\gamma_m - \gamma_d(1 - T^*)})$. We

then obtain

$$\begin{aligned}
V(v, H, M, \sigma_{-v}^*) = \gamma_m v - \alpha D_m - p_m &\leq \gamma_m v - \alpha \rho \left(1 - \frac{p_m - p_d^*}{\gamma_m - \gamma_d(1 - T^*)} \right) - p_m \\
&< \gamma_m v - \gamma_m \left(1 - \frac{p_m - p_d^*}{\gamma_m - \gamma_d(1 - T^*)} \right) - p_m \\
&< \gamma_d(1 - T^*)v - p_d^* = V(v, H, D, \sigma_{-v}^*), \quad (\text{OS.21})
\end{aligned}$$

for all $v \in \mathcal{V}$, in which the second inequality follows from $\alpha > \frac{\gamma_m}{\rho}$ and the last inequality follows from $\frac{p_m - p_d^*}{\gamma_m - \gamma_d(1 - T^*)} < 1$. Hence, $V(v, H, M, \sigma_{-v}^*) < V(v, H, D, \sigma_{-v}^*)$ for all $v \in \mathcal{V}$. Next, if the market structure for class L is $[L : M-D-N]$, then from $V(v, H, B, \sigma_{-v}^*) \leq V(v, L, B, \sigma_{-v}^*)$ for all $v \in \mathcal{V}$, there cannot be *both* segment in class H . Together with (OS.21), in the case of $[L : M-D-N]$, the only possible equilibrium market structure for class H is $[H : D-N]$. Next, if the market structure for class L is $[L : B-M-D-N]$, then

$$\begin{aligned}
V(v, H, B, \sigma_{-v}^*) &= \gamma_m v - \alpha D_m - p_m + \delta \gamma_d(1 - \max(T^*, \xi))v - p_d^* \\
&< \gamma_m v - \gamma_m \left(1 - \frac{p_m - p_d^*}{\gamma_m - \gamma_d(1 - T^*)} \right) - p_m + \delta \gamma_d(1 - \max(T^*, \xi))v - p_d^* \\
&< \gamma_d(1 - T^*)v - p_d^* = V(v, H, D, \sigma_{-v}^*), \quad (\text{OS.22})
\end{aligned}$$

for all $v \in \mathcal{V}$, in which the last inequality is obtained using $V(1, L, B, \sigma_{-v}^*) > V(1, L, D, \sigma_{-v}^*)$ and the existence of strictly positive *movie* segment for class L , i.e., $\frac{p_m - p_d^*}{\gamma_m - \gamma_d(1 - T^*)} < \frac{p_d^*}{\delta \gamma_d(1 - \max(T^*, \xi))}$. As a result, in the case of $[L : B-M-D-N]$, from (OS.21) and (OS.22), it follows that $\sigma^*(v, H) \in \{D, N\}$ for all $v \in \mathcal{V}$. Similarly, for the other remaining consumer equilibrium market structures for class L , specifically, for the market structures of $[L : B-M-N]$, $[L : M-N]$, and $[L : B-D-N]$, we can similarly show $\sigma^*(v, H) \in \{D, N\}$ for all $v \in \mathcal{V}$ from $\alpha > \frac{\gamma_m}{\rho}$. \square

Since $\alpha_H > \frac{\gamma_H}{\rho}$, from Lemma OS.2, we obtain that $\sigma^*(v, H) \in \{D, N\}$ for all $v \in \mathcal{V}$. The consumer equilibrium market structure for $c = H$ becomes

$$\sigma^*(v, c = H) = \begin{cases} D & \text{if } \frac{p_d}{\gamma_d(1-T)} \leq v < 1; \\ N & \text{if } v < \frac{p_d}{\gamma_d(1-T)}. \end{cases} \quad (\text{OS.23})$$

For $c = L$, first, consider the case in which $T \leq \xi$. In this case, when $\delta = \delta_H$, consumer equilibrium market structure is as follows:

$$\left\{ \begin{array}{ll} [L : B-D-N] & \text{if } 0 \leq p_d \leq \frac{p_m \gamma_d \delta_H (1 - \xi)}{\gamma_m^H - \gamma_d(1 - T - \delta_H(1 - \xi))}; \\ [L : B-M-D-N] & \text{if } \frac{p_m \gamma_d \delta_H (1 - \xi)}{\gamma_m^H - \gamma_d(1 - T - \delta_H(1 - \xi))} < p_d \leq \frac{p_m \gamma_d (1 - T)}{\gamma_m^H}; \\ [L : B-M-N] & \text{if } \frac{p_m \gamma_d (1 - T)}{\gamma_m^H} < p_d \leq \gamma_d \delta_H (1 - \xi); \\ [L : M-N] & \text{if } \gamma_d \delta_H (1 - \xi) < p_d. \end{array} \right. \quad (\text{OS.24})$$

Consider the first region, in which $p_d \leq \frac{p_m \gamma_d \delta_H (1-\xi)}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$. If consumers prefer *movie* over *both*, it implies that $V(v, L, M, \sigma_{-v}^*) \geq V(v, L, B, \sigma_{-v}^*)$, which can be written as $v < \frac{p_d}{\delta_H \gamma_d (1-\xi)}$. Furthermore, if consumers prefer *movie* over *video*, i.e., $V(v, L, M, \sigma_{-v}^*) \geq V(v, L, D, \sigma_{-v}^*)$, we can rewrite this condition as $v > \frac{p_m - p_d}{\gamma_m^H - \gamma_d (1-T)}$. However, under the given p_d condition, $p_d \leq \frac{p_m \gamma_d \delta_H (1-\xi)}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$, it follows that $\frac{p_d}{\delta_H \gamma_d (1-\xi)} < \frac{p_m - p_d}{\gamma_m^H - \gamma_d (1-T)}$. Consequently, in this p_d region, $\sigma^*(v, L) \in \{B, D, N\}$, i.e., no consumer purchases theatrical movie offering only.

Next, in order to have $\sigma^*(v, L) = B$, we need to have the following two conditions to be satisfied: (i) $V(v, L, B, \sigma_{-v}^*) \geq V(v, L, D, \sigma_{-v}^*)$, which translates into $v \geq \frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$; (ii) $V(v, L, B, \sigma_{-v}^*) \geq V(v, L, N, \sigma_{-v}^*) = 0$, which is simplified to $v \geq \frac{p_m + p_d}{\gamma_m^H + \delta_H \gamma_d (1-\xi)}$. Using $\delta = \delta_H$, $T \leq \xi$, and the given p_d condition, we then obtain $\frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))} \geq \frac{p_m + p_d}{\gamma_m^H + \delta_H \gamma_d (1-\xi)}$. As a result, $\sigma^*(v, L) = B$ if and only if $v \geq \frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$.

Similarly, the required conditions for $\sigma^*(v, L) = D$ are: (i) $V(v, L, D, \sigma_{-v}^*) \geq V(v, L, B, \sigma_{-v}^*)$, which is equivalent to $v \leq \frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$; (ii) $V(v, L, D, \sigma_{-v}^*) \geq V(v, L, N, \sigma_{-v}^*) = 0$, which can be rewritten as $v \geq \frac{p_d}{\gamma_d (1-T)}$. Moreover, under the given conditions, it follows that $\frac{p_d}{\gamma_d (1-T)} < \frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$. Hence, we have $\sigma^*(v, L) = D$ if and only if $\frac{p_d}{\gamma_d (1-T)} \leq v \leq \frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$.

In summary, when $p_d \leq \frac{p_m \gamma_d \delta_H (1-\xi)}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}$, the consumer consumption equilibrium strategy for class L is

$$\sigma^*(v, L) = \begin{cases} B & \text{if } \frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))} \leq v \leq 1; \\ D & \text{if } \frac{p_d}{\gamma_d (1-T)} \leq v < \frac{p_m}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}; \\ N & \text{if } v < \frac{p_d}{\gamma_d (1-T)}. \end{cases} \quad (\text{OS.25})$$

In this region, the studio's profit function becomes

$$\Pi(T, p_d) = \lambda_m p_m \rho \left(1 - \frac{p_m}{\gamma_m - \gamma_d (1-T - \delta_H (1-\xi))} \right) + \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d (1-T)} \right). \quad (\text{OS.26})$$

We first consider the optimal video pricing problem given T . Taking the derivative of $\Pi(T, p_d)$ in (OS.26) with respect to p_d , we obtain

$$\frac{\partial \Pi}{\partial p_d} = \lambda_d \left(1 - \frac{2p_d}{\gamma_d (1-T)} \right), \quad (\text{OS.27})$$

which is satisfied at $p_d^*(T) = \frac{\gamma_d (1-T)}{2}$. Moreover, the second-order condition is also satisfied since $\frac{\partial^2 \Pi}{\partial p_d^2} = -\frac{2\lambda_d}{\gamma_d (1-T)} < 0$. However, in this case, since $\gamma_m = \gamma_m^H$ and $0 \leq T \leq \xi < \frac{1}{2}$, we have

$$p_d^*(T) = \frac{\gamma_d (1-T)}{2} > \frac{p_m \gamma_d \delta_H (1-\xi)}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))}. \quad (\text{OS.28})$$

Hence, $\Pi(T, p_d)$ is increasing in p_d in this region of p_d .

Next, consider the second region of p_d , i.e., $\frac{p_m \gamma_d \delta_H (1-\xi)}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))} < p_d \leq \frac{p_m \gamma_d (1-T)}{\gamma_m^H}$. In this region, following the similar argument as above, we obtain the consumer purchase equilibrium strategy for class L as follows:

$$\sigma^*(v, L) = \begin{cases} B & \text{if } \frac{p_d}{\delta_H \gamma_d (1-\xi)} \leq v \leq 1; \\ M & \text{if } \frac{p_m - p_d}{\gamma_m^H - \gamma_d (1-T)} \leq v < \frac{p_d}{\delta_H \gamma_d (1-\xi)}; \\ D & \text{if } \frac{p_d}{\gamma_d (1-T)} \leq v < \frac{p_m - p_d}{\gamma_m^H - \gamma_d (1-T)}; \\ N & \text{if } v < \frac{p_d}{\gamma_d (1-T)}. \end{cases} \quad (\text{OS.29})$$

In this region, the studio's profit function then becomes

$$\begin{aligned} \Pi(T, p_d) = & \lambda_m p_m \rho \left(1 - \frac{p_m - p_d}{\gamma_m^H - \gamma_d (1-T)} \right) + \\ & \lambda_d p_d \left(\rho \left(1 - \frac{p_d}{\delta_H \gamma_d (1-\xi)} + \frac{p_m - p_d}{\gamma_m^H - \gamma_d (1-T)} - \frac{p_d}{\gamma_d (1-T)} \right) + (1-\rho) \left(1 - \frac{p_d}{\gamma_d (1-T)} \right) \right). \end{aligned} \quad (\text{OS.30})$$

Solving the optimal video pricing problem given T by taking the derivative of $\Pi(T, p_d)$ in (OS.30) with respect to p_d , we obtain

$$\frac{\partial \Pi}{\partial p_d} = \lambda_d + \frac{\rho p_m (\lambda_m + \lambda_d)}{\gamma_m^H - (1-T)\gamma_d} - \frac{2\lambda_d p_d}{\gamma_d} \left(\frac{1}{1-T} + \frac{\rho(\gamma_m^H - \gamma_d(1-T - \delta_H(1-\xi)))}{\delta_H(1-\xi)(\gamma_m^H - (1-T)\gamma_d)} \right). \quad (\text{OS.31})$$

The second-order condition is satisfied in this case:

$$\frac{\partial^2 \Pi}{\partial p_d^2} = -\frac{2\lambda_d}{\gamma_d} \left(\frac{1}{1-T} + \frac{\rho(\gamma_m^H - \gamma_d(1-T - \delta_H(1-\xi)))}{\delta_H(1-\xi)(\gamma_m^H - (1-T)\gamma_d)} \right) < 0. \quad (\text{OS.32})$$

From the first-order condition, we have

$$p_d^*(T) = \frac{\delta_H \gamma_d (1-T)(1-\xi)(\rho p_m (\lambda_m + \lambda_d) + \lambda_d (\gamma_m^H - (1-T)\gamma_d))}{2\lambda_d ((1-\xi)(\gamma_m^H - (1-T)\gamma_d)\delta_H + (1-T)(\gamma_m^H - \gamma_d(1-T - \delta_H(1-\xi)))\rho)}. \quad (\text{OS.33})$$

Using $\gamma_m = \gamma_m^H$ and $0 \leq T \leq \xi < \frac{1}{2}$, we obtain that $p_d^*(T)$ is larger than the upper bound of this corresponding region in p_d , which is $\frac{p_m \gamma_d (1-T)}{\gamma_m^H}$. As a result, the studio's profit function is also increasing in p_d in this region of p_d , i.e., when $\frac{p_m \gamma_d \delta_H (1-\xi)}{\gamma_m^H - \gamma_d (1-T - \delta_H (1-\xi))} < p_d \leq \frac{p_m \gamma_d (1-T)}{\gamma_m^H}$.

Next, if p_d increases further into the region of $\frac{p_m \gamma_d (1-T)}{\gamma_m^H} < p_d \leq \gamma_d \delta_H (1-\xi)$, again, following the similar argument above for the case of smaller p_d values, we obtain the consumer equilibrium market structure as

$$\sigma^*(v, L) = \begin{cases} B & \text{if } \frac{p_d}{\delta_H \gamma_d (1-\xi)} \leq v < 1; \\ M & \text{if } \frac{p_m}{\gamma_m^H} \leq v < \frac{p_d}{\delta_H \gamma_d (1-\xi)}; \\ N & \text{if } v < \frac{p_m}{\gamma_m^H}. \end{cases} \quad (\text{OS.34})$$

Then, the corresponding studio's profit function is written as

$$\Pi(T, p_d) = \lambda_m p_m \rho \left(1 - \frac{p_m}{\gamma_m^H} \right) + \lambda_d p_d \left(\rho \left(1 - \frac{p_d}{\delta_H \gamma_d (1 - \xi)} \right) + (1 - \rho) \left(1 - \frac{p_d}{\gamma_d (1 - T)} \right) \right). \quad (\text{OS.35})$$

Taking the derivative of this profit function $\Pi(T, p_d)$ in (OS.35) with respect to p_d , we then obtain

$$\frac{\partial \Pi}{\partial p_d} = \frac{\delta_H \lambda_d (1 - \xi) (\gamma_d (1 - T) - 2p_d) - 2\rho \lambda_d p_d (1 - T - \delta_H (1 - \xi))}{\delta_H \gamma_d (1 - T) (1 - \xi)}. \quad (\text{OS.36})$$

Furthermore, the second-order condition becomes

$$\frac{\partial^2 \Pi}{\partial p_d^2} = -\frac{2\lambda_d}{\gamma_d} \left(\frac{1 - \rho}{1 - T} + \frac{\rho}{\delta_H (1 - \xi)} \right) < 0, \quad (\text{OS.37})$$

which is satisfied, and hence the first-order condition is sufficient. Thus, $p_d^*(T)$ is an interior solution given T ; specifically, given T , $p_d^*(T) = \frac{\delta_H \gamma_d (1 - T) (1 - \xi)}{2(\delta_H (1 - \xi) (1 - \rho) + \rho (1 - T))}$. Furthermore, using $\gamma_m = \gamma_m^H$ and $0 \leq T \leq \xi < \frac{1}{2}$, it follows that $p_d^*(T) \in \left(\frac{p_m \gamma_d (1 - T)}{\gamma_m^H}, \gamma_d \delta_H (1 - \xi) \right)$. Hence, within this region, we achieve the local interior optimizer video price given T .

Lastly, if p_d increases further, i.e., into $p_d \geq \gamma_d \delta_H (1 - \xi)$, following the previous arguments and using the algebra and simplifying, we obtain the consumer equilibrium market structure as

$$\sigma^*(v, L) = \begin{cases} M & \text{if } \frac{p_m}{\gamma_m^H} \leq v < 1; \\ N & \text{if } v < \frac{p_m}{\gamma_m^H}. \end{cases} \quad (\text{OS.38})$$

In this region, the studio's profit function then becomes

$$\Pi(T, p_d) = \lambda_m p_m \rho \left(1 - \frac{p_m}{\gamma_m^H} \right) + \lambda_d p_d (1 - \rho) \left(1 - \frac{p_d}{\gamma_d (1 - T)} \right). \quad (\text{OS.39})$$

Optimizing over p_d given T , we obtain the interior optimizer as $p_d^*(T) = \frac{\gamma_d (1 - T)}{2}$, if it exists in the corresponding region. However, using $\delta = \delta_H$ and $0 \leq T \leq \xi < \frac{1}{2}$, we have $p_d^*(T) = \frac{\gamma_d (1 - T)}{2} < \gamma_d \delta_H (1 - \xi)$. Consequently, the studio's profit function in (OS.39) is decreasing in p_d given a fixed video release time T .

In summary, given T , the optimal video pricing problem has optimizer $p_d^*(T) = \frac{\delta_H \gamma_d (1 - T) (1 - \xi)}{2(\delta_H (1 - \xi) (1 - \rho) + \rho (1 - T))}$. Substituting this optimal video price $p_d^*(T)$ into the corresponding studio's profit function in (OS.35), the studio's profit function in terms of T is written as

$$\Pi(T, p_d^*(T)) = \frac{\rho \lambda_m p_m (\gamma_m^H - p_m)}{\gamma_m^H} + \frac{\delta_H \lambda_d \gamma_d (1 - T) (1 - \xi)}{4(\delta_H (1 - \rho) (1 - \xi) + \rho (1 - T))}. \quad (\text{OS.40})$$

Taking the derivative of this profit function with respect to T , we have

$$\frac{d\Pi(T, p_d^*(T))}{dT} = -\frac{\delta_H^2 \lambda_d \gamma_d (1-\xi)^2 (1-\rho)}{4(\delta_H(1-\xi)(1-\rho) + \rho(1-T))^2} < 0. \quad (\text{OS.41})$$

Hence, $T^* = 0 < \xi$, and plugging this back into $p_d^*(T^* = 0)$, we then obtain the optimal p_d^* .

So far, we have considered the case in which $T \leq \xi$. If $T > \xi$, following similar steps, we can show that under the conditions given in this proposition, in particular, under $\delta = \delta_H$ and $\gamma_m = \gamma_m^H$, the consumer market structure becomes

$$\begin{cases} [L : B-D-N] & \text{if } 0 \leq p_d \leq \frac{p_m \gamma_d \delta_H (1-T)}{\gamma_m^H - \gamma_d (1-T)(1-\delta)}; \\ [L : B-M-D-N] & \text{if } \frac{p_m \gamma_d \delta_H (1-T)}{\gamma_m^H - \gamma_d (1-T)(1-\delta_H)} < p_d \leq \frac{p_m \gamma_d (1-T)}{\gamma_m^H}; \\ [L : B-M-N] & \text{if } \frac{p_m \gamma_d (1-T)}{\gamma_m^H} < p_d \leq \gamma_d \delta_H (1-T); \\ [L : M-N] & \text{if } \gamma_d \delta_H (1-T) < p_d. \end{cases} \quad (\text{OS.42})$$

Using similar algebra to the previous case of $T \leq \xi$, we find that in this region of T , the studio's profit $\Pi(T, p_d^*(T))$ is decreasing in T . As a result, the day-and-date tactic with $T^* = 0$ is the optimal video release strategy in this case.

For part (ii), first, consider $T \leq \xi$. In this case of the parameter region, by closely following the proof of part (i), we obtain that the relevant equilibrium market structure is given in (OS.34) for class L . The equilibrium consumer market structure for class H has also the same structure as below, but with different thresholds:

$$\sigma^*(v, H) = \begin{cases} B & \text{if } \frac{p_d}{\delta_H \gamma_d (1-\xi)} \leq v < 1; \\ M & \text{if } \frac{\gamma_m^H (p_m + \alpha_L (1-\rho)) + \alpha_L \rho (\gamma_m^H - p_m)}{\gamma_m^H (\gamma_m^H + \alpha_L (1-\rho))} \leq v < \frac{p_d}{\delta_H \gamma_d (1-\xi)}; \\ N & \text{if } v < \frac{\gamma_m^H (p_m + \alpha_L (1-\rho)) + \alpha_L \rho (\gamma_m^H - p_m)}{\gamma_m^H (\gamma_m^H + \alpha_L (1-\rho))}. \end{cases} \quad (\text{OS.43})$$

In this region, the studio's profit function is written as

$$\Pi(T, p_d) = \frac{\lambda_m p_m (\gamma_m^H - p_m)}{\gamma_m^H + \alpha_L (1-\rho)} + \lambda_d p_d \left(1 - \frac{p_d}{\delta_H \gamma_d (1-\xi)} \right). \quad (\text{OS.44})$$

This profit function does not depend on T . Since video consumption occurs at $T = \xi$ for *both* consumers, profits are weakly maximized at $T^* = \xi$. Furthermore, resolving indifference at $T^* = \xi$ maintains continuity in the optimal strategy as γ_m^H decreases and is also more likely to arise due to production lead times associated with videos. So, in this case, $T^* = \xi$, and then by maximizing the studio's profit function over p_d , we obtain $p_d^* = \frac{\delta_H \gamma_d (1-\xi)}{2}$. Similarly, we can show that if $T > \xi$, the studio's profit is decreasing in T in this parameter region. Hence, under high content durability, $T^* = \xi$ and $p_d^* = \frac{\delta_H \gamma_d (1-\xi)}{2}$. In addition, the resulting market equilibrium is $[L : B-M-N]$ and $[H : B-M-N]$, which completes the proof. ■

Proof of Proposition 2: First, for part (i) of this proposition, technically, we prove that there exists $\omega_4 > 0$ such that for all $\omega > \omega_4$, if $\alpha_H > \omega$, and $\delta_H > 1 - \frac{1}{\omega}$, when $\underline{\gamma}_m < \gamma_m^I < \overline{\gamma}_m$ is satisfied, then the following holds:

- if $\lambda_m < \lambda_d \left(\frac{\gamma_m^I - \gamma_d(1 - \delta_H(1 - \xi))}{2p_m\sqrt{\rho}} \right)^2$, then $T^* = 0$ and $p_d^* = \frac{\gamma_d}{2}$;
- if $\lambda_d \left(\frac{\gamma_m^I - \gamma_d(1 - \delta_H(1 - \xi))}{2p_m\sqrt{\rho}} \right)^2 \leq \lambda_m < \lambda_d \left(\frac{\gamma_m^I - \gamma_d((1 - \delta_H)(1 - \xi))}{2p_m\sqrt{\rho}} \right)^2$, then $T^* = 1 - \delta_H(1 - \xi) - \frac{\gamma_m^I}{\gamma_d} + \frac{2p_m}{\gamma_d} \sqrt{\frac{\lambda_m \rho}{\lambda_d}} < \xi$ and $p_d^* = \frac{\gamma_m^I + \delta_H \gamma_d(1 - \xi)}{2} - p_m \sqrt{\frac{\lambda_m \rho}{\lambda_d}}$;
- if $\lambda_m \geq \lambda_d \left(\frac{\gamma_m^I - \gamma_d((1 - \delta_H)(1 - \xi))}{2p_m\sqrt{\rho}} \right)^2$, then $T^* = \xi$ and $p_d^* = \frac{\gamma_d(1 - \xi)}{2}$.

The condition $\gamma_m^I > \frac{p_m}{1 - \xi} = \underline{\gamma}_1$ guarantees that for δ_H close enough to 1, the studio induces some *both* demand, regardless of timing of the video release, as long as video price is not too high. If the video price is set high enough, then there is *movie only* demand but there is no *both* demand in equilibrium. For α sufficiently high, the congestion-sensitive class optimally forgoes *movie* and *both* options by Lemma OS.2. The possible equilibrium consumer market structures under these asymptotic δ_H and α_H regimes are $[L : M-N]$ and $[H : D-N]$, $[L : B-M-N]$ and $[H : D-N]$, $[L : B-M-D-N]$ and $[H : D-N]$, or $[L : B-D-N]$ and $[H : D-N]$. The condition $\gamma_m^I < 2\sqrt{2p_m\gamma_d(1 - \xi)} - \gamma_d(1 - \xi) = \overline{\gamma}_4$ is sufficient to rule out inducing consumer market structure $[L : B-M-D-N]$ and $[H : D-N]$ in equilibrium, since the video price the studio would optimally want to set, given this market structure, would be too low induce this market structure. Similarly, $\gamma_m^I < 2p_m \left(1 + \frac{\xi\rho}{1 - \xi} \right) = \overline{\gamma}_3$ is sufficient to rule out inducing consumer market structure $[L : B-M-N]$ and $[H : D-N]$ in equilibrium. Then the only remaining candidate equilibria involve either inducing consumer market structure $[L : M-N]$ and $[H : D-N]$ or $[L : B-D-N]$ and $[H : D-N]$.

Consider inducing market structure $[L : B-D-N]$ and $[H : D-N]$. We first examine the region in which $T < \xi$. Under the conditions given in the Proposition, the consumer equilibrium market structure is presented in (OS.24) as p_d changes given a fixed $T < \xi$. Furthermore, in this region, one can show that the profit function is decreasing in p_d if $p_d > \frac{p_m\gamma_d\delta_H(1 - \xi)}{\gamma_m^I - \gamma_d(1 - T - \delta_H(1 - \xi))}$. When $p_d \leq \frac{p_m\gamma_d\delta_H(1 - \xi)}{\gamma_m^I - \gamma_d(1 - T - \delta_H(1 - \xi))}$, the optimal video price $p_d^*(T)$ becomes an interior optimal solution; that is, given T , $p_d^*(T) = \frac{\gamma_d(1 - T)}{2} < \frac{p_m\gamma_d\delta_H(1 - \xi)}{\gamma_m^I - \gamma_d(1 - T - \delta_H(1 - \xi))}$ under the conditions given in the Proposition. Substituting $p_d^*(T)$ into the profit function and simplifying, we obtain

$$\Pi(T, p_d^*(T)) = \frac{\lambda_d\gamma_d(1 - T)}{4} + \rho\lambda_m p_m \left(1 - \frac{p_m}{\gamma_m^I - \gamma_d(1 - T - \delta_H(1 - \xi))} \right). \quad (\text{OS.45})$$

Taking the derivative of $\Pi(T, p_d^*(T))$ with respect to T , we then have

$$\frac{d\Pi(T, p_d^*(T))}{dT} = \gamma_d \left(-\frac{\lambda_d}{4} + \frac{\rho\lambda_m p_m^2}{(\gamma_m^I - \gamma_d(1 - T - \delta_H(1 - \xi)))^2} \right). \quad (\text{OS.46})$$

Note that $\frac{d\Pi(T, p_d^*(T))}{dT}$ is decreasing in T , and hence the second-order condition is satisfied. Furthermore, (OS.46) is strictly positive for all $T < \xi$ under sufficiently high δ , if $\lambda_m \geq \lambda_d \left(\frac{\gamma_m^I - \gamma_d((1-\delta_H)(1-\xi))}{2p_m\sqrt{\rho}} \right)^2$. When $T \geq \xi$, it follows that the studio's profit is decreasing in T . As a result, in this case, $T^* = \xi$. Plugging $T^* = \xi$ into $p_d^*(T)$, we obtain $p_d^*(\xi) = \frac{\gamma_d(1-\xi)}{2}$. Otherwise, if $\lambda_d \left(\frac{\gamma_m^I - \gamma_d(1-\delta_H(1-\xi))}{2p_m\sqrt{\rho}} \right)^2 \leq \lambda_m < \lambda_d \left(\frac{\gamma_m^I - \gamma_d((1-\delta_H)(1-\xi))}{2p_m\sqrt{\rho}} \right)^2$, the first-order condition is satisfied at $T^* = 1 - \delta_H(1 - \xi) - \frac{\gamma_m^I}{\gamma_d} + \frac{2p_m}{\gamma_d} \sqrt{\frac{\lambda_m \rho}{\gamma_d}}$ and by replacing this optimal T^* , we then obtain p_d^* . Lastly, if $\lambda_m < \lambda_d \left(\frac{\gamma_m^I - \gamma_d(1-\delta_H(1-\xi))}{2p_m\sqrt{\rho}} \right)^2$, then (OS.46) is strictly negative for all $T < \xi$ under sufficiently high δ . Then the studio optimally sets $T^* = 0$ and $p_d^* = \frac{\gamma_d}{2}$.

For completeness of proof, we have included all three cases here, but the focus of our paper has been ρ sufficiently low to emphasize the impact of high congestion on the studio's decision and the resulting equilibrium market structure. As such, our conditions on ρ will exclude the case when $T^* = \xi$. However, for the other two cases, it is necessary to have conditions to ensure that the optimal video pricing does indeed induce the relevant market structure. For $p_d^*(0)$ to induce $[L : B-D-N]$ and $[H : D-N]$ we need $\gamma_m^I \leq \gamma_d(1 - \delta_H(1 - \xi)) + 2p_m\delta_H(1 - \xi) = \bar{\gamma}_1$, and, similarly for the other case, we need $\gamma_m^I \leq 2p_m\sqrt{\frac{\lambda_m \rho}{\lambda_d}} - \gamma_d\delta_H(1 - \xi) \left(1 - \sqrt{\frac{\lambda_d}{\lambda_m \rho}} \right) = \bar{\gamma}_2$. Under intermediate γ_m^I , these bounds are satisfied.

To show there is no profitable deviation, we compare revenue from inducing $[L : M-N]$ and $[H : D-N]$ versus inducing $[L : B-D-N]$ and $[H : D-N]$, for each of the two cases above. One can show that inducing *both* generates revenue that dominates inducing a market structure with no *both* demand, given that durability is sufficiently high. This concludes the proof of part (i).

For part (ii), technically, we prove that there exist $\omega_5 > 0$ such that for all $\omega > \omega_5$, if $\alpha_L < \frac{1}{\omega}$, and $\delta_H > 1 - \frac{1}{\omega}$, when $\underline{\gamma}_m < \gamma_m^I < \bar{\gamma}_m$ is satisfied, then $T^* = \xi$ and the studio sets its price to $p_d^* = \frac{\gamma_d(1-\xi)}{2}$. The studio's optimal strategy induces a consumer market structure characterized by $[L : B-D-N]$ and $[H : B-D-N]$.

Again, the condition $\gamma_m^I > \frac{p_m}{1-\xi}$ guarantees that for δ_H close enough to 1, the studio induces some *both* demand, regardless of timing of the video release, as long as video price is not too high. If the video price is set high enough, then there is *movie only* demand but there is no *both* demand in equilibrium. For small α , congestion has little effect on congestion-sensitive consumers (and still has no effect on congestion-insensitive consumers). Therefore, for sufficiently small α , Class L and Class H consumers have the same market structure. The possible equilibrium consumer market structures under these asymptotic δ_H and α_L regimes are $[L : M-N]$ and $[H : M-N]$, $[L : B-M-N]$ and $[H : B-M-N]$, $[L : B-M-D-N]$ and $[H : B-M-D-N]$, or $[L : B-D-N]$ and $[H : B-D-N]$.

The condition $\gamma_m^I < 2p_m\lambda_m$ is sufficient to rule out inducing consumer market structure $[L : B-M-N]$ and $[H : B-M-N]$ in equilibrium, since the video price the studio would optimally want to set, given this market structure, would be too low induce this market structure.

Ruling out the market structure $[L : B-M-D-N]$ and $[H : B-M-D-N]$ requires care. Similar to the argument above, we will show that the video price the studio would optimally want to set would be

too low induce this market structure. Technically, we need to have $p_d^*(T^*) < \frac{p_m \gamma_d (1-\xi)}{\gamma_m^I - \gamma_d (\xi - T^*)}$. The optimal video price in this market structure is given by $p_d^*(T) = \frac{(1-T)\gamma_d \delta_H ((p_m + \gamma_m^I - \gamma_d(1-T))\lambda_d + p_m \lambda_m)(1-\xi)}{2\lambda_d(\gamma_m^I(1-T + \delta_H(1-\xi)) - \gamma_d(1-T)^2)}$. Since the expression for T^* (found in the same way as in previous propositions) is unwieldy, we will instead require that $p_d^*(T) < \frac{p_m \gamma_d (1-\xi)}{\gamma_m^I - \gamma_d (\xi - T)}$ for all $0 \leq T \leq \xi$. Note that the relevant market structure cannot be induced in equilibrium if T is set greater than ξ when γ_m is intermediate, so it suffices to focus on $0 \leq T \leq \xi$.

Showing this reduces to showing that a cubic in T is negative over $0 \leq T \leq \xi$. The proof strategy is to first find conditions so that this cubic is negative at $T = 0$ and $T = \frac{1}{2}$ (the upper bound of the ξ range) and then to make sure that the first positive stationary point of this cubic is greater than $T = \frac{1}{2}$. This guarantees that the cubic is negative for all $T \in [0, \xi]$.

The algebra is omitted for brevity, but the following set of conditions guarantees that $p_d^*(T) < \frac{p_m \gamma_d (1-\xi)}{\gamma_m^I - \gamma_d (\xi - T)}$ for all $0 \leq T \leq \xi$:

$$\max(\underline{\gamma}_3, \underline{\gamma}_4, \underline{\gamma}_5) < \gamma_m^I < \min(\bar{\gamma}_6, \bar{\gamma}_7), \quad (\text{OS.47})$$

$$(p_m(-3\lambda_d + \lambda_m) - \gamma_d \lambda_d(1-\xi))^2 - 8p_m \gamma_d \lambda_d^2(1-\xi) > 0, \quad (\text{OS.48})$$

$$(1 + (1-\xi)(4 + 5(1-\xi)))\gamma_d^2 \lambda_d^2 + p_m^2(\lambda_d - \lambda_m)^2 - 2p_m(1-\xi)\gamma_d \lambda_d(3\lambda_d + \lambda_m) < 0, \quad (\text{OS.49})$$

$$2p_m \gamma_d \lambda_d(1-\xi)(3-2\xi)(2(\lambda_m + \lambda_d)\xi - \lambda_d - 3\lambda_m) + \gamma_d^2 \lambda_d^2(3-5\xi+2\xi^2)^2 + p_m^2(\lambda_m(3-2\xi) + \lambda_d(-7+6\xi))^2 > 0. \quad (\text{OS.50})$$

Having ruled out $[L : B-M-N]$ and $[H : B-M-N]$ as well as $[L : B-M-D-N]$ and $[H : B-M-D-N]$ being induced in equilibrium, we now proceed similarly as in part (i). Comparing the profits of $[L : B-D-N]$ and $[H : B-D-N]$ versus $[L : M-N]$ and $[H : M-N]$ completes the proof. ■

Proof of Proposition 3: For part (i) of this proposition, technically, we prove that there exists $\omega_6 > 0$ such that for all $\omega > \omega_6$, if $\gamma_m^H > p_m \omega$, $\alpha^H > \omega$, $\delta_L < \frac{1}{\omega}$, and $\rho < \frac{1}{\omega}$, then $T^* = 0$ and $p_d^* = \frac{\gamma_d}{2}$. In addition, under this optimal strategy, the studio achieves perfect segmentation of consumers, i.e., $[H : D-N]$ and $[L : M-N]$.

First, we consider the case in which $T < \xi$. Under the given parameter region, in particular, for $\gamma_m^H > p_m \omega$, $\alpha^H > \omega$, and $\delta_L < \frac{1}{\omega}$, for all $\omega > \omega_6$, the relevant consumer market structure becomes (OS.38) for class L consumers, and (OS.23) for class H consumers. Depending on the video price p_d and the video release time T , some other consumer market structures can also arise in equilibrium. However, as γ_m and α become high and δ becomes small, the potential region of p_d that induces those consumer equilibrium market structures becomes arbitrarily small, and hence, those consumer market structures will be dominated by the given relevant consumer market structure. In this consumer market structure, the movie demand is $D_m = \rho(1 - \frac{p_m}{\gamma_m^H})$, and the

studio's profit function can be written as

$$\Pi(T, p_d) = \rho \lambda_m p_m \left(1 - \frac{p_m}{\gamma_m^H}\right) + (1 - \rho) \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d(1-T)}\right). \quad (\text{OS.51})$$

By taking the derivative of this profit function with respect to p_d , we obtain

$$\frac{\partial \Pi(T, p_d)}{\partial p_d} = \frac{(1 - \rho) \lambda_d (\gamma_d(1-T) - 2p_d)}{\gamma_d(1-T)}. \quad (\text{OS.52})$$

The second-order condition becomes $\frac{\partial^2 \Pi(T, p_d)}{\partial p_d^2} = -\frac{2(1-\rho)\lambda_d}{\gamma_d(1-T)} < 0$, which is satisfied. Solving the first-order condition, we have $p_d^*(T) = \frac{\gamma_d(1-T)}{2}$, which is in the interior of the relevant region under the conditions given in the Proposition including low ρ condition. Next, by substituting this optimal video price $p_d^*(T)$ into the studio's profit function in (OS.51), we obtain

$$\Pi(T, p_d^*(T)) = \frac{(1 - \rho) \lambda_d \gamma_m^H \gamma_d(1-T) + 4\rho \lambda_m p_m (\gamma_m^H - p_m)}{4\gamma_m^H}, \quad (\text{OS.53})$$

which is decreasing in T . Hence, $T^* = 0$ and consequently, $p_d^* = \frac{\gamma_d}{2}$.

Second, consider the case in which $T \geq \xi$. In this case, similar to the previous case of $T < \xi$, under the conditions given in the Proposition, the only relevant region is (OS.38) for class L consumers, and (OS.23) for class H consumers. In this consumer market equilibrium, from class L consumers, there is only *movie* demand, and the studio's profit from this class is $\rho \lambda_m p_m \left(1 - \frac{p_m}{\gamma_m^H}\right)$ as given in (OS.51). However, this profit from class L is independent of p_d and T . In contrast, the studio's profit from class H is $(1 - \rho) \lambda_d p_d \left(1 - \frac{p_d}{\gamma_d(1-T)}\right)$, all from *video*, and it is strictly decreasing in T for all p_d . Hence, the studio's profit is strictly decreasing in T in this region.

As a result, $T^* = 0 < \xi$ and $p_d^* = \frac{\gamma_d}{2}$. In addition, the strictly positive *movie* demand is from class L and strictly positive *video* demand from the class H without *both* segment from either class.

For part (ii), we first prove the second item; technically, we prove that there exists $\omega_7 > 0$ such that for all $\omega > \omega_7$, if $\gamma_m^H > p_m \omega$, $\alpha_L < \frac{1}{\omega}$, $\delta_L < \frac{1}{\omega}$, and $\lambda_d > \lambda_m$, then $T^* = 0$ and $p_d^* = \frac{\gamma_d(p_m(\lambda_m + \lambda_d) + \alpha_L \lambda_d(1-\rho))}{2\lambda_d(\gamma_m^H + \alpha_L(1-\rho))}$. In addition, under this optimal strategy, the studio employs a day-and-day tactic and the resulting market equilibrium becomes $[H : M-D-N]$ and $[L : M-D-N]$.

First, following similar steps of the previous proofs, we establish that the relevant equilibrium consumer market structures are $[H : M-D-N]$ and $[L : M-D-N]$. Then, the corresponding theatrical movie demand is $D_m = \frac{\gamma_m^H - p_m - (\gamma_d(1-T) - p_d)}{\gamma_m^H - \gamma_d(1-T) + \alpha_L(1-\rho)}$. In addition, the video demand is

$$D_d = \rho \left(\frac{p_m - p_d}{\gamma_m^H - \gamma_d(1-T)} - \frac{p_d}{\gamma_d(1-T)} \right) + (1 - \rho) \left(\frac{1}{\gamma_m^H - \gamma_d(1-T)} \left(p_m - p_d + \frac{\alpha_L(\gamma_m^H - p_m - (\gamma_d(1-T) - p_d))}{\gamma_m^H - (1-T)\gamma_d + \alpha_L(1-\rho)} \right) - \frac{p_d}{(1-T)\gamma_d} \right). \quad (\text{OS.54})$$

Then, the studio's profit function is written as $\Pi(T, p_d) = \lambda_m p_m D_m + \lambda_d p_d D_d$, which is a concave function in p_d for a given T . Optimizing $\Pi(T, p_d)$ over p_d given T , we obtain

$$p_d^*(T) = \frac{\gamma_d(1-T)(p_m(\lambda_d + \lambda_m) + \alpha_L \lambda_d(1-\rho))}{2\lambda_d(\gamma_m^H + \alpha_L(1-\rho))}. \quad (\text{OS.55})$$

Substituting the optimal video price $p_d^*(T)$ into the studio's profit function, we obtain $\Pi(T, p_d^*(T))$. Taking the derivative of this profit function with respect to T , we have

$$\frac{d\Pi(T, p_d^*(T))}{dT} = -\frac{\gamma_d(p_m(\lambda_d - \lambda_m) + \alpha_L \lambda_d(1-\rho))^2}{4\lambda_d(\gamma_m^H - (1-T)\gamma_d + \alpha_L(1-\rho))^2} < 0. \quad (\text{OS.56})$$

As a result, the optimal video release time in this region is $T^* = 0$. Replacing $T^* = 0$ and $p_d^*(0)$ into the studio's profit function, we obtain the optimal profit function in this consumer market structure. Lastly, if $\lambda_d > \lambda_m$, the equilibrium market structure that we considered here dominates the optimal studio's profit under other equilibrium consumer market structures. Therefore, in this parameter region, $T^* = 0$ and $p_d^* = \frac{\gamma_d(p_m(\lambda_m + \lambda_d) + \alpha_L \lambda_d(1-\rho))}{2\lambda_d(\gamma_m^H + \alpha_L(1-\rho))}$.

For the first item of part (ii), the proof is very similar to the proof of the second item provided above. The difference is that in the corresponding parameter region, the condition $\lambda_m \geq \lambda_d$ leads to the equilibrium outcome of not releasing the video at all due to the negative impact of demand cannibalization on the studio's profits. ■

Proof of Proposition 4: For part (i), under sufficiently low δ (δ_L) and sufficiently high α (α_H), if $\gamma_m^I < p_m + \gamma_d(1 - \delta_L)(1 - \xi) = \bar{\gamma}_8$, then the only market structures that can be induced have $[L : M-N]$ and $[H : D-N]$, $[L : M-D-N]$ and $[H : D-N]$, or $[L : D-N]$ or $[H : D-N]$. The condition $\gamma_m^I < \bar{\gamma}_8$ with sufficiently low δ guarantees that no *both* can be induced in equilibrium. As before, sufficiently high α guarantees that the congestion sensitive class has market structure $[H : D-N]$.

First, consider the consumer market structure $[L : M-D-N]$ and $[H : D-N]$. Under this consumer market structure, the studio's profit can be written as

$$\begin{aligned} \Pi(T, p_d) = \lambda_d p_d \left(\rho \left(\frac{p_m - p_d}{\gamma_m^I - (1-T)\gamma_d} - \frac{p_d}{(1-T)\gamma_d} \right) + (1-\rho) \left(1 - \frac{p_d}{(1-T)\gamma_d} \right) \right) \\ + \rho \lambda_m p_m \left(1 - \frac{p_m - p_d}{\gamma_m^I - (1-T)\gamma_d} \right). \quad (\text{OS.57}) \end{aligned}$$

Optimizing this studio's profit over p_d , we obtain

$$p_d^*(T) = \frac{(1-T)\gamma_d(\lambda_d(1-\rho)(\gamma_m^I - (1-T)\gamma_d) + \rho p_m(\lambda_m + \lambda_d))}{2\lambda_d(\gamma_m^I - (1-T)(1-\rho)\gamma_d)}. \quad (\text{OS.58})$$

We plug (OS.58) into (OS.57) and take a derivative of $\Pi(T, p_d^*(T))$ with respect to T . Showing that the profit derivative in T is decreasing in T is equivalent to showing a cubic function in ρ is negative

for all T . Using a change of variables $\rho = \frac{x}{1+x}$, this is equivalent to showing that the transformed cubic function, defined here as $g(x)$, is negative for all $x \geq 0$. The proof strategy is to show that since this cubic has negative third-order term for all T and since its intercept is negative for all T , it suffices to show that the stationary points of $g(x)$ (if they exist) are both negative. Omitting the details, the sufficient conditions for this to happen are

$$\begin{aligned} p_m(\gamma_d(\lambda_d + \lambda_m)^2 + \gamma_m^I(\lambda_d^2 - 6\lambda_d\lambda_m + \lambda_m^2)) &> 2(\gamma_d - \gamma_m^I)\gamma_m^I\lambda_d(\lambda_d + \lambda_m), \text{ and} \\ \gamma_m^I(\gamma_m^I - 2\gamma_d)\lambda_d + 2p_m\gamma_m^I(\lambda_d + \lambda_m) &> 4p_m^2\lambda_m, \\ 2p_m\lambda_m &< \gamma_m^I(\lambda_d + \lambda_m). \end{aligned}$$

These conditions hold under the assumptions on γ_m^I and the technical restrictions in Appendix B in the paper, so we find $T^* = 0$ and $p_d^* = \frac{\gamma_d(\lambda_d(1-\rho)(\gamma_m^I - \gamma_d) + \rho p_m(\lambda_m + \lambda_d))}{2\lambda_d(\gamma_m^I - (1-\rho)\gamma_d)}$ are optimal within the market structure region of $[L:M-D-N]$ and $[H:D-N]$. Moreover, under this optimal price and release time, the resulting equilibrium market structure is indeed $[L:M-D-N]$ and $[H:D-N]$. Lastly, the parameter region guarantees that the optimal studio profits at $T^* = 0$ and $p_d^* = \frac{\gamma_d(\lambda_d(1-\rho)(\gamma_m^I - \gamma_d) + \rho p_m(\lambda_m + \lambda_d))}{2\lambda_d(\gamma_m^I - (1-\rho)\gamma_d)}$ under the corresponding equilibrium market structure of $[L:M-D-N]$ and $[H:D-N]$ dominates the interior optimal profits under the other potential consumer market structures, i.e., (i) $[L:M-N]$ and $[H:D-N]$, and (ii) $[L:D-N]$ and $[H:D-N]$, which completes the proof.

For part (ii), under sufficiently low δ (δ_L) and α (α_L), if $\gamma_m^I < \bar{\gamma}_8$, then the only market structures that can be induced are $[L:M-N]$ and $[H:M-N]$, $[L:M-D-N]$ and $[H:M-D-N]$, or $[L:D-N]$ or $[H:D-N]$. The proof follows similarly to the proofs of previous propositions, so we omit the details. If $\lambda_d > \lambda_m$ and $\gamma_m^I > \frac{1}{2}(p_m + \gamma_d + \sqrt{p_m^2 + \gamma_d^2}) = \underline{\gamma}_2$, the studio optimally induces equilibrium consumer market structure $[L:M-D-N]$ and $[H:M-D-N]$ with $T^* = 0$ and $p_d^* = \frac{\gamma_d(p_m(\lambda_m + \lambda_d) + \alpha_L\lambda_d(1-\rho))}{2\lambda_d(\gamma_m^I + \alpha_L(1-\rho))}$. On the other hand, if $\lambda_d \leq \lambda_m$ and $\gamma_m^I > \underline{\gamma}_2$, then the studio optimally sets $T^* = 1$ and $p_d^* = \gamma_d$, disincentivizing *video* purchase and inducing equilibrium market structure $[L:M-N]$ and $[H:M-N]$. By definition of intermediate γ_m^I , we always have $\gamma_m^I > \underline{\gamma}_2$. Hence, whether the studio optimally induces *video* demand when α and δ are low only depends on the studio's share of movie and video revenues, completing the proof. ■