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Step-Level Reasoning and Bidding in Auctions

Uri Gneezy
Graduate School of Business, The University of Chicago, 5807 S. Woodlawn, Suite 519,
Chicago, Illinois 60637, ugneezy@gsb.uchicago.edu

Step-level models of reasoning (SLR) proved to be very successful in predicting behavior in the beauty contest game. Recently, a quantified version of the model was suggested as a more general model of thinking. In particular, it was found that the distribution of choices could be represented by a Poisson distribution. I test the model in stylized first- and second-price common-value sealed-bid auctions. Equilibrium, for both auction types, prescribes that players undercut each other and profits are small. The SLR prediction, on the other hand, is different for the two auctions. Nash equilibrium predicts the outcomes poorly; the SLR model predicts the outcomes well in the second-price auction. However, while bids in the first-price auction could be represented by a Poisson distribution, this could not be attributed to step-level reasoning.

Key words: depth of reasoning; bidding behavior
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1. Introduction

How should—and how do—people bid in auctions? The normative answer of how people *should* bid if everyone else is rational is well developed and understood by economists. The descriptive answer of how people *do* bid is far less understood. It is not even clear if developing a general behavioral theory of auction bidding is a feasible task.[1] Recently, an energetic attempt to construct a game-theoretic model of behavior in one-shot games was made using a step-level model of reasoning (SLR). See Stahl (1993), Stahl and Wilson (1994, 1995), Nagel (1995), Duffy and Nagel (1997), Ho et al. (1998), Bosch-Domenech et al. (2003), Sonsino et al. (2000), Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2004), Haruvy et al. (2001), and Haruvy (2002). In this paper, I test the predictive power of this model in stylized auctions, with the aim of using it as a building block for developing a behavioral model of more complex auction bidding.

In the SLR model, reasoning is characterized by the number of steps of iterated thinking that players use. A zero-step player simply chooses a strategy at random; usually it is assumed that these players choose all possible strategies with equal probability. A one-step player best response to zero-step players, assuming that all other players are zero-step players. In general terms, a K-level player thinks that all other players use zero to $K-1$ steps.

The SLR model is successful in explaining observations from a "beauty contest" game (Nagel 1995). In this game, participants have to choose a number in $[0, 100]$. The player who chooses the number closest to a proportion (between 0 and 1, usually 2/3) of the average of all numbers wins a prize. By iterated elimination of weakly dominated strategies, it is easy to see that this game has a unique equilibrium in which everybody chooses 0. A zero-step player in the beauty contest game simply randomizes between numbers. A $K=1$ best response to this by choosing 50 (the average of choices by zero-level) times 2/3, i.e., 33.33, etc.

Camerer et al. (2004, 2005) (CHC hereafter) suggest that the frequency of players using a different number of steps is Poisson distributed with mean τ. Moreover, for $K>0$, players use a normalized Poisson distribution to model what other players are choosing and to compute the expected payoff. Using several laboratory experiments with normal-form games, CHC estimate τ to be between 1 and 2. Several authors (e.g., Bosch-Domenech et al. 2003, Slonim 2005) have reported results from experiments with the beauty contest game that were run with various populations (newspaper readers, students, experienced students, and game theorists). They found that behavior can be interpreted as iterative reasoning, with $K=1$, 2, or 3 players with a different τ to describe the data, depending on the subject pool.

In this paper, I use experiments done via e-mail. This method is similar to the newspaper experiments reported by Bosch-Domenech et al. (2003), in which people have more time to think about the problem than in a regular laboratory experiment. For example, Bosch-Domenech et al. (2003, p. 1694) report that

[1] That is not to say that experimental findings on auctions, and understanding of certain phenomena, do not exist. See, for example, the survey and collective work on common-value auctions in Kagel and Levin (2002).

39 participants in the newspaper experiment claimed that they had run an experiment among students, friends, and relatives, to help them decide what number to submit. As a result, a higher τ may be expected. In the demonstrations below I will use $\tau = 3$, as will be justified by the data later on.

One test of such a behavioral theory is how well it can predict behavior in new environments. Moreover, for such a theory to be interesting for economists, this new environment should have economic meaning. My goal in this paper is to test the theory in a different game with economic relevance. To achieve this goal, I test the SLR model (with the CHC formulation) using auctions. The first set of experiments involves a first-price sealed-bid auction in which two players simultaneously choose an integer (a bid) from the set $\{1, 2, \ldots, N\}$. The player who chooses the lowest bid gets a dollar amount times the number (s)he bids and the other player gets 0. In case of a tie, the earnings are split amongst both players.

One problem in empirical research on auctions is that the reservation price of bidders is unknown to the observer. The advantage of the stylized auction used in this paper is that it allows us to observe these reservation prices. This auction can simply be interpreted as selling $100 to the highest bidder (i.e., the person who is willing to accept the lowest amount). A fundamental difference between this auction game and the beauty contest game is the fact that equilibrium prediction (each player chooses 1 or each player chooses 2) differs sharply from the surplus-maximizing outcome (each player choosing N).

Another direction of investigation relates to the number of reasoning levels leading to equilibrium. In the beauty contest game studied by Nagel (1995), a relatively small number of reasoning levels leads players to equilibrium. This is similar to the first-price auction with $N = 10$. However, when increasing N to 100, the prediction of the SLR model is concentrated around the choice of 50 (the $K = 1$ choice). This is worrisome, because the generality of the model will be severely damaged if it is not able to predict behavior in games with a large number of iterations leading to equilibrium.

The second set of experiments involves second-price sealed-bid auctions. Bidding in this auction is done in a similar way to the first-price auction, and again the bidder who bids the lower number wins. However, the prize she wins is the bid made by the *other* participant. In equilibrium, participants bid 1. According to the SLR model, zero-level players will randomize over all bids, just as in the first-price auction. A level 1 (or higher) player will best respond to this by bidding 1. Using the same parameters as before, the prediction is that most players will simply bid 1, and the rest will be randomly distributed over the bid interval.

2. The First-Price Sealed-Bid Auction

2.1. Rules and Equilibrium

Each of two players simultaneously chooses an integer from the set $\{1, 2, \ldots, N\}$. The player who chooses the lowest bid gets a dollar amount times the number (s)he bids and the other player gets 0. In the case of a tie, the earnings are split between both players.

In this simple auction, the bidder who offers the lowest bid wins the auction, and is paid according to her bid. The game may also be interpreted as a duopoly market with price competition: the players are firms, the bids are prices, and payoffs are the firms' profits.[2] There are two Nash equilibria: $(1, 1)$ and $(2, 2)$, both with negligible profits. Out of these, $(1, 1)$, is strict and may be viewed as "best" in terms of various desiderata proposed in the theory of equilibrium selection (see van Damme 1987). Also, theories of sophisticated reasoning, which do not presume equilibrium behavior at all, may be applied to promote a bid of 1 as the unique procedure. The iterated dominance technique eliminates, in turn, the choice of N (as a dominated choice) and subsequently $N - 1, N - 2, \ldots, 3, 2$ by iterated eliminations. Finally, one may put forth evolutionary arguments, which do not presume any degree of sophistication whatsoever, to reach an analogous conclusion; a bid of 1 is the game's unique evolutionarily stable strategy (Maynard Smith and Price 1973). For experimental results of this game with $N = 100$ in a repeated random matching set up, see Dufwenberg and Gneezy (2000, 2002), and with group competition, see Bornstein and Gneezy (2002). Isaac and Walker (1985) study the role of feedback information in a similar game of sealed-bid private-value auctions with four bidders.

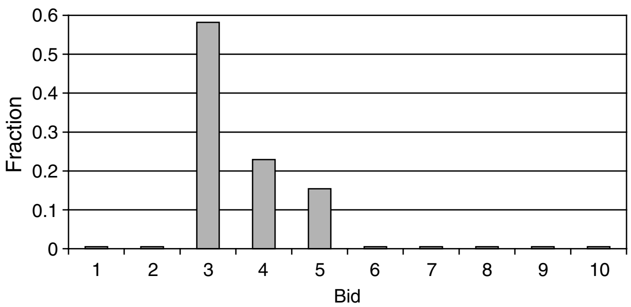
2.2. The Step-Level Reasoning Model Prediction

In this paper, I use two values of N: 10 and 100. When $N = 10$, a zero-level player will choose each of the numbers $1, 2, \ldots, 10$ with probability 0.1. The choice of a $K = 1$ player will be the best reply to this behavior, i.e., choosing that integer which maximizes the following term, where n is the integer chosen:

$$\max\left\{n \times \frac{N-n}{N} + \frac{n}{2} \times \frac{1}{N}\right\}. \tag{1}$$

[2] The assumptions underlying the game are the same as in the classical Bertrand price competition model. See the discussion regarding the relation between the game and the model in Baye and Morgan (2004) who, based on the findings of Dufwenberg and Gneezy (2000) and Abrams et al. (2000), show that only a little bounded rationality is needed to rationalize price dispersion in settings that closely approximate textbook Bertrand competition. They conclude that "bounded rationality based theories of price dispersion organize the data remarkably well" (p. 1).

Figure 1 **Prediction of the CHC Model for the $N = 10$ Game with $\tau = 3$ (First-Price Auction)**



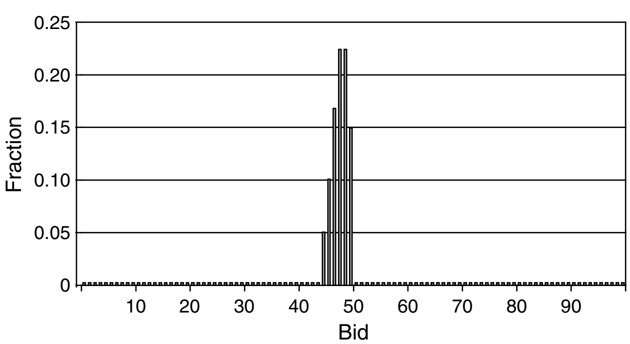
With $N = 10$, the n maximizing this argument is 5. Given the Poisson distribution suggested by CHC with $\tau = 3$, a $K = 2$ player will best respond to these strategies by choosing 4, a $K = 3$ or more player will choose 3. Note that the best response may depend on the value of τ. The prediction of the CHC model with $\tau = 3$ is presented in Figure 1.

When $N = 100$, a zero-level player will choose each of the numbers $1, 2, \ldots, 100$ with probability 0.01. The choice of a $K = 1$ player will be the best reply to this behavior, i.e., choosing 50. Given the Poisson distribution suggested by CHC, a $K = 2$ player will best respond to these strategies by choosing 49; a $K = 3$ player will choose 48, and so on (depending on τ). The prediction of the CHC model with $\tau = 3$ is presented in Figure 2.

2.3. Altruism, Risk Taking, and Equilibrium

Sometimes we can learn from listening to our subjects! As one of the participants in the treatment with $N = 10$ wrote: "If you're interested in my rationale, I think the only way I can win is by choosing a very low number (1 or 2), which wouldn't be worth playing for ($2 * \$10 * 0.10$ chance of payoff = \$2). The good feeling I get of guaranteeing a win to whoever is paired off with me is worth more. And if I'm matched with another altruist, we'll each get \$50. My decision would be different if I considered this to be a zero-sum game, but I don't consider the experimenter's payouts as a loss to himself."

Figure 2 **Prediction of the CHC Model for the $N = 100$ Game with $\tau = 3$ (First-Price Auction)**



There is a fundamental difference between the beauty contest game and the price competition game. The difference is in efficiency; whereas in the beauty contest the prize is fixed, in the auction games the prize is endogenously determined by the bids. A choice of N by both bidders maximizes the total prize.

Adjusting the model for games with tension between equilibrium and efficiency seems important. One way to do this is using altruism. I define altruistic preferences by a subject whose utility function is increasing, not only in her own payoff, but also in the other player's payoff (see Palfrey and Prisbrey 1997 for a formal definition). Charness and Rabin (2002) and Andreoni and Miller (2002) show that people are motivated to increase the total social payment in a variety of games.

McKelvey and Palfrey (1992) showed that an incomplete information game that assumes the existence of a small proportion of altruists in the population can account for many of the salient features of their data. They estimated a level of altruism of about 5%, and then modeled the reaction of other nonaltruistic players to the existence of the altruistic players. In line with this approach, one can extend the SLR model by assuming that a fraction of the players simply chooses "altruistic" bids. In the auction, this would mean people choosing a high number, even though they understand that it does not maximize their payoffs.[3]

Like in the McKelvey and Palfrey (1992) model, not all bidders must be altruistic to choose high numbers. Some players may simply try to exploit altruism by reacting to the presence of altruistic players. A related reason might be risk loving: some players may place a risky bet for potentially high return. See Baye and Morgan (2004) for a formal treatment of such behavior.

A potentially more important difference between the beauty contest game and the first price auction is that in the auction game, the equilibrium bid will be played only by level 0 players. Using the CHC parameters, a fully rational player will bid 3. This difference implies that only a level 0 bidder may choose the equilibrium. An alternative Poisson model that is tested below is that the distribution of bidders in the game "goes all the way": level 1 bidders choose 5, level 2 choose 4, level 3 choose 3, level 2 choose 2, and level 1 and higher choose equilibrium. The Poisson

[3] In the formal test below, I extend the model to games with altruistic motives by simply assuming that 0.05 of the people choose each of the numbers above 6 to 10, and that the $K > 1$ players are aware of that.

distribution with the parameters of the CHC model will be tested over this alternative prediction.

While this difference may seem minor, it has a strong implication regarding the cognitive process used. While the distribution of choices is a Poisson distribution, the underlying mechanism is very different: either people do not reason as prescribed by the SLR model or the players of level 4 and higher make a mistake in calculating the best response. I will return to this topic in the conclusion after presenting the results.

2.4. Procedure

E-mail messages were sent to students at a large university in the United States who had previously participated in laboratory and Internet experiments and who had expressed interest in further participation. All together, 400 potential participants were invited to bid in this auction, with about 200 students randomly assigned to potentially participate in each treatment. Because the vast majority of these students use e-mail, selection bias from this recruiting method should be minimal, at least with respect to standard laboratory experiments. Altogether, 74 participants replied in the $N = 10$ game, and 89 in the $N = 100$ game. It was decided to run this experiment using e-mail, instead of the traditional laboratory environment, to mimic real first-price sealed-bid auctions on the Internet.

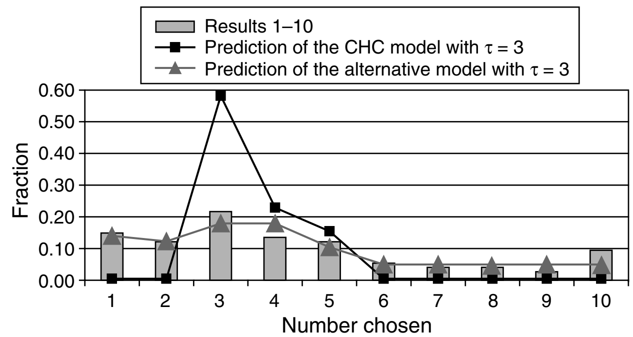
The payment to the participants was set such that in the $N = 100$ case a winning participant received a dollar amount times the number she chose. However, only 1 out of 10 participants was paid. In the $N = 10$ treatment, payoffs were normalized such that a winning participant received a dollar amount equal to 10 times her choice. This normalization was created to reduce differences between treatments. Instructions are provided in the appendix.

3. Results of the First-Price Auction

3.1. $N = 10$

As expected based on the previous studies described above, the Nash equilibrium predicted the outcomes poorly: only 15% of the participants chose 1. The distribution of actual choices and the prediction of the CHC model with $\tau = 3$ are presented in Figure 3 and Table 1.[4] Using a Kruskal-Wallis[5] test we can reject the hypothesis that the predicted outcome (of the SLR model suggested by CHC with $\tau = 3$) and the actual results are the same.

Figure 3 **The Thicker Line Represents the Prediction of the Model with the Addition of Altruistic Players (First-Price Auction)**



The alternative model I proposed, with altruistic players and Poisson distribution over how close the bids are to equilibrium, is presented in the thick line in Figure 3. The statistical comparison of the alternative model yields no significant difference between the distributions ($p = 0.04$).

3.2. $N = 100$

The distribution of actual choices and the prediction of the CHC model with $\tau = 3$ are presented in Figure 4. Unlike the $N = 10$ case, casual inspection of the figure shows no resemblance between prediction and actual results. The statistical test indicates that the results are significantly different from the prediction of the CHC prediction with $\tau = 3$ ($p > 0.1$).

Also, unlike the $N = 10$ treatment, the alternative model does not result in a better fit.

3.3. A Comparison of the Two Treatments: Grouping the $N = 100$ Results

On a first pass the results of the $N = 10$ treatment and the $N = 100$ treatment look very different. However, the proportions of the average from the highest bid allowed are similar: the average is 4.3 in the $N = 10$ treatment, and 40 in the $N = 100$ treatment. Hence, the corresponding proportions are 0.43 and 0.4. When grouping the results of the $N = 100$ treatment in 10 groups (1–10, 11–20, ..., 91–100), the two distributions seem very similar; see Figure 5. The difference between the distributions is not statistically significant ($p = 0.05$).

The two concerns a player has when choosing a number are the probability of winning and the associated expected profits. As in the beauty contest game,

[4] I estimate τ in §3.4. For conformity, I use $\tau = 3$ for the hypothesis testing; using the estimated τ gives similar results here.

[5] The Kruskal-Wallis test is a multiple-sample generalization of the two-sample Wilcoxon rank-sum test. Samples of sizes n_j, $j = 1, \ldots, m$, are combined and ranked in ascending order of magnitude. Tied values are assigned the average ranks. Let n denote the overall sample size and let R_j denote the sum of the ranks for the jth sample. The Kruskal-Wallis one-way analysis-of-variance test H is defined as

$$H = \frac{12}{n(n+1)} \sum_{j=1}^{m} \frac{R_j^2}{n_j} - 3(n+1).$$

The sampling distribution of H is approximately chi-squared with $m - 1$ degrees of freedom.

Table 1 Results of the 1–10 Treatment and the Prediction of the CHC Model with $\tau = 3$

Bid	1	2	3	4	5	6	7	8	9	10
Results 1–10	0.15	0.12	0.22	0.14	0.12	0.05	0.04	0.04	0.03	0.09
CHC model with $\tau = 3$	0.005	0.005	0.58	0.23	0.15	0.005	0.005	0.005	0.00	0.005
Alternative model with $\tau = 3$	0.14	0.12	0.18	0.18	0.10	0.05	0.05	0.05	0.05	0.05

choosing the equilibrium number is not necessarily the best reply to the actual behavior of the other participants. Figure 6 presents the graphs of the probability of winning and the expected payoff associated with each choice for the two treatments (the $N = 100$ results are again pooled in 10 groups).

Given the fact that the distributions are not significantly different, it is not surprising that the probability of winning and the expected payoff from each choice are similar between the treatments. The maximum expected payoff is 0.18 (when the maximum pie is normalized to 1), coming from choosing either 3 or 4. The probability of winning is 54% when choosing 3, and 38% when choosing 4 (the expected payoff is calculated using Equation (1)).

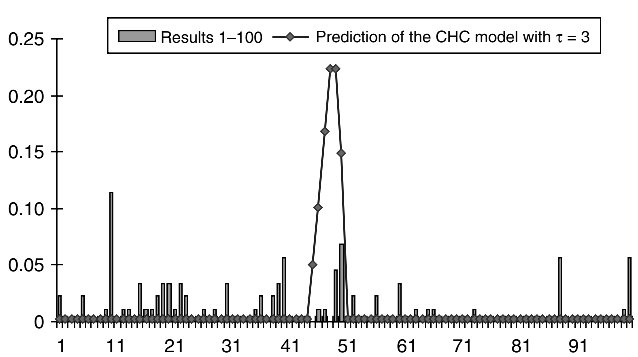
A comparison of the probability of winning with the prediction of the alternative model with $\tau = 3$ is plotted in Figure 7. The similarity between the graphs is again striking. The difference between the prediction of probability and expected payoffs and the actual ones is not statistically significant ($p < 0.01$).

Note that very much as in the beauty contest game, playing the equilibrium results in very low expected profits relative to the best response to the other actual bids.

3.4. Empirical Estimation

3.4.1. CHC Model. I estimate the parameters for the CHC model, both without altruism and with the addition of altruism. Without altruism, I need to estimate the underlying τ parameter, given the data.

Figure 4 Results of the 1–100 Treatment and the Prediction of the CHC Model with $\tau = 3$ (First-Price Auction)



If K_i represents the type of the ith player and X_i the integer chosen, then the distribution of K_i is given by

$$P(K_i = k_i) = \frac{e^{-\tau}\tau^{k_i}}{k_i!},$$

given that the choice of X_i follows the following process.

- If $K_i = 0$, the subject chooses each of the options $1, \ldots, 10$ with equal probability (0.1), i.e., X_i is randomized.
- If $K_i = 1$, the subject chooses 5, i.e., $X_i = 5$ deterministically.
- If $K_i = 2$, the subject chooses 4, i.e., $X_i = 4$ deterministically.

....

- If $K_i \geq 5$, the subject chooses 1, i.e., $X_i = 1$ deterministically.

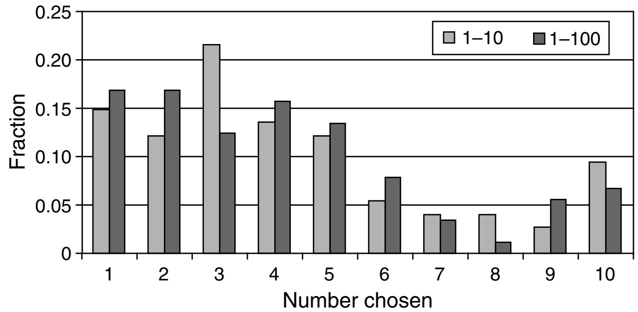
The resulting probability distribution of X_i is given by

$$\begin{aligned} P(X_i = 1) &= P(K_i \geq 5) + 0.1P(K = 0) \\ &= [1 - P(K \leq 5)] + 0.1P(K = 0) \\ P(X_i = 2) &= P(K_i = 4) + 0.1P(K_i = 0) \\ &\ldots \\ P(X_i = 5) &= P(K_i = 1) + 0.1P(K_i = 0) \\ P(X_i = y) &= 0.1P(K_i = 0) \quad \forall y \in \{6, 7, 8, 9, 10\}. \end{aligned}$$

Thus, we can estimate the τ parameter using maximum likelihood.[6] The parameter estimates of the two auctions with $N = 10$ and $N = 100$ are given in Table 2. Additionally, we also find the estimates when the $N = 100$ results are grouped. Because the parameter estimates themselves do not tell us how well the estimates fit the data, we compare the predicted distribution of choices (X_i) with the empirical distribution. The comparisons for the three models are presented in Figures 8 through 10. Table 2 also has a goodness of fit measure in terms of the mean absolute percent deviation between the predicted and empirical distributions.

[6] τ has to be reparametrized during the estimation so that it is constrained to be a positive number. However, the parameter estimates are for the true τ parameter (with the standard errors appropriately computed using the delta method).

Figure 5 A Comparison of the Results from the $N = 10$ Treatment with the Results of the $N = 100$ Treatment Grouped in 10 Different Subgroups (First-Price Auction)

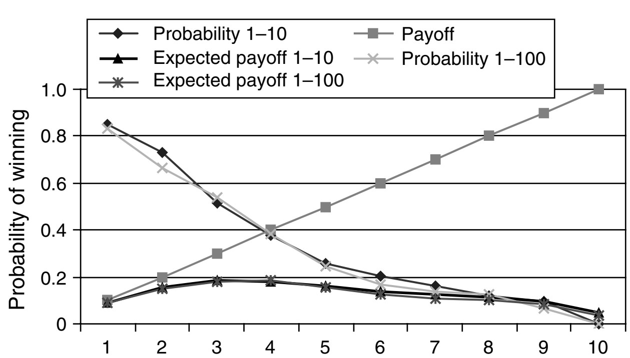


3.4.2. Alternative Model. Next, we estimate the parameters of the alternative model. To model altruism, the choice of outcomes is modeled as a mixture of the underlying process described earlier and an altruism process, by which one of the higher outcomes is chosen (with equal probability for each of these higher choices). Let α be the proportion of people who are altruistic. They are assumed to choose the higher outcomes (6 to 10 in the $N = 10$ condition) with equal probability. The $(1-\alpha)$ proportion of people are assumed to follow the earlier process.

Thus, we can derive the modified probability distribution of choices X_i as the following:

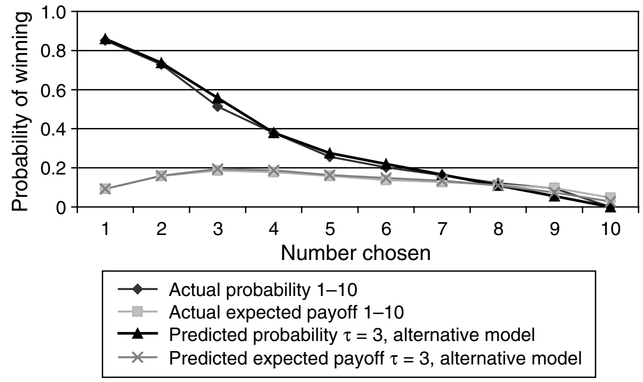
$$P(X_i = 1) = (P(K_i \geq 5) + 0.1P(K = 0)$$
$$= [1 - P(K \leq 5)] + 0.1P(K = 0)) * (1 - \alpha)$$
$$P(X_i = 2) = (P(K_i = 4) + 0.1P(K_i = 0)) * (1 - \alpha)$$
$$\ldots$$
$$P(X_i = 5) = (P(K_i = 1) + 0.1P(K_i = 0)) * (1 - \alpha)$$
$$P(X_i = y) = (0.1P(K_i = 0)) * (1 - \alpha) + 0.2 * \alpha \quad \forall y \in \{6, 7, 8, 9, 10\}.$$

Figure 6 A Comparison of the Probability of Winning with Each Number Chosen Between the Two Treatments (First-Price Auction)



Note. The $N = 100$ results are pooled in 10 groups.

Figure 7 A Comparison of the Probability of Winning with Each Number Chosen Between the $N = 10$ and the Prediction of the Alternative Model with $\tau = 3$ (First-Price Auction)



We again estimated the model using maximum likelihood.[7] These results are reported in Table 3. In this case, the model with $N = 100$ did not converge. However, it converged with the grouped $N = 100$ data. Figures 11 and 12 depict the predicted and empirical distribution of choices X_i.

Comparing the goodness-of-fit measures in Tables 2 and 3 (the mean absolute percentage deviation), we find that in both the $N = 10$ and grouped $N = 100$ cases, the model with altruism has a better fit. Further, for model selection, we compute the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Both these criteria are based on the log likelihood at the estimated parameter values. Both of them select the model with a higher likelihood. However, the BIC additionally penalizes a model with a higher number of parameters. With both these criteria, the model with a lower value is selected. The AIC and BIC for the two models (the CHC model and the CHC model with altruism) under the two conditions ($N = 10$ and grouped $N = 100$) are presented in Table 4. As we can see, the model with altruism outperforms the simple CHC model whether we use the AIC or the BIC.

4. The Second-Price Sealed-Bid Auction

4.1. Rules and Equilibrium

Each of two players simultaneously chooses an integer from the set $\{1, 2, \ldots, N\}$. The player who chooses the lowest bid gets a dollar amount times the number *the other player bid*, and the other player gets 0. In case of a tie the earnings are split between both players.

The rules of this auction are similar to those of the first-price auction, but here the winner is paid the

[7] Again, τ needs to be constrained to be positive. Additionally, in the model with altruism, α (which is a probability) needs to be constrained between 0 and 1.

Table 2 Parameter Estimates and Goodness-of-Fit Measures: CHC Model

	Parameter (standard errors are in parentheses)		
	$N = 10$	$N = 100$	$N = 100$ (grouped)
τ	2.0886** (0.0079)	0.0834** (0.0372)	2.1910** (0.2126)
Log-likelihood	−182.08	−402.68	−219.74
Mean absolute percent deviation (%)	65.60	45.58	63.25

**Significant at the 95% level.

loser's bid. This auction has a unique Nash equilibrium in which each player chooses 1, resulting in negligible profits. The second-price auction was studied by Vickrey (1961). See Lucking-Reiley (2000) for the history and use of this type of auction.

4.2. The Step-Level Reasoning Prediction

I again tested the game for $N = 10$ and $N = 100$. When $N = 10$ (100), a zero-level player will choose each of the numbers $1, 2, \ldots, 10$ (100) with probability 0.1 (0.01). The choice of a $K = 1$ player will be the best response to this behavior. Unlike the first-price auction, this choice is already the equilibrium choice! That is, the only players who will not choose the equilibrium choice of 1 are the zero-level players!

4.3. Procedure

E-mail messages were sent to about 200 students at the Technion, Israel. I received 132 students replies (59 in the $N = 10$ treatment, and 73 in the $N = 100$ treatment). Payments were set such that in the $N = 100$ treatment, a winning participant received a dollar amount times the number the losing participant chose. However, only one out of 10 participants was paid. As in the first-price auction, the $N = 10$ treatment payoffs were normalized such that a winning participant received a dollar amount equal to 10 times her choice.

Figure 8 Empirical and Predicted Probability Distributions: $N = 10$

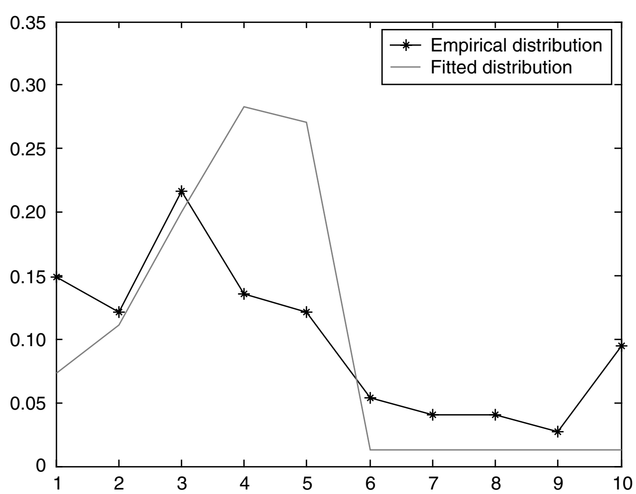
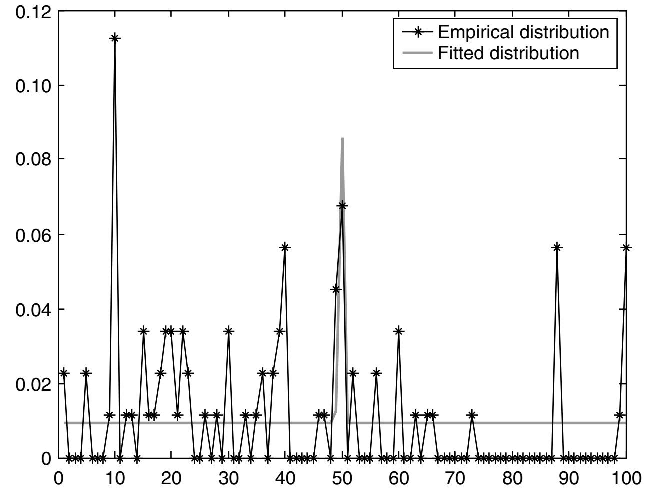


Figure 9 Empirical and Predicted Probability Distributions: $N = 100$



5. Results of the Second-Price Auction

5.1. $N = 10$

The distribution of actual choices and the prediction of the model with the parameters used in the first-price auction are presented in Figure 13; 66% of the participants chose 1, and another 14% chose 2. In the equivalent first-price auction, 74% of the participants had $k > 0$ (i.e., chose 5 or less).

5.2. $N = 100$

The distribution of actual choices and the prediction of the model with the parameters used in the

Figure 10 Empirical and Predicted Probability Distributions: $N = 100$ **and Grouped**

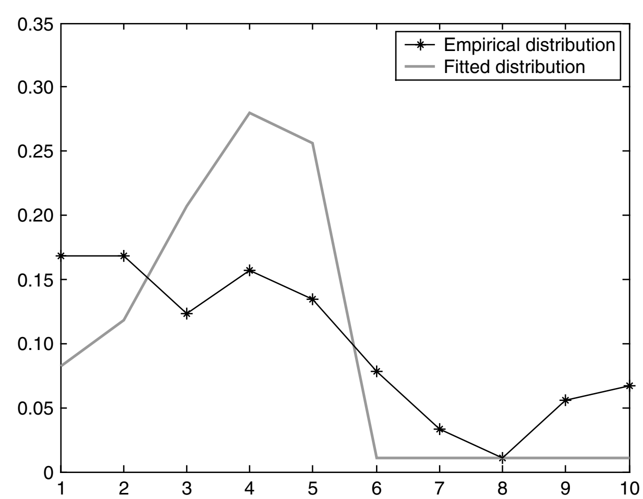


Table 3 **Parameter Estimates and Goodness-of-Fit Measures: CHC Model with Altruism**

	Parameter (standard errors in parentheses)	
	$N = 10$	$N = 100$ (grouped)
τ (Poisson mean)	3.0546** (0.2937)	3.1373** (0.7002)
α (Proportion of altruistic people)	0.2388 (0.1819)	0.2305 (0.1908)
Log-likelihood	−160.54	−193.85
Mean absolute percent deviation (%)	25.87	55.16

**Significant at the 95% level.

first-price auction are presented in Figure 14. Unlike the first-price auction, casual inspection of the figure does show resemblances between prediction and actual results; 63% of the participants chose 1, and 74% chose 5 or less. This value is not statistically different than the 75% of the participants with $k > 0$ (i.e., who chose 50 or less) in the first-price auction. Unlike the first-price auction, $k > 0$ suffices to bid the equilibrium price.

5.3. A Comparison of the Two Treatments: Grouping the $N = 100$ Results

For completeness of the comparison with the first-price auction, grouping the results of the $N = 100$ treatment in 10 groups (1–10, 11–20, ..., 91–100) and comparing them to the $N = 10$ treatment revealed no statistically significant difference ($p < 0.01$).

6. Discussion

This paper challenges the step-level model of reasoning. The data presented regarding bidding in first-price auctions suggest that while the Poisson type of response observed by CHC is replicated, it cannot directly be attributed to the quantified version of the SLR model.

This result has a strong implication regarding the cognitive process used by people in these games. While the distribution of choices is a Poisson distribution, the underlying mechanism is very different. Two types of reasons may cause this. First, the SLR type does not describe behavior well, and the Poisson distribution observed is due to some other cognitive processes. Alternatively, it might be that, while reasoning according to the SLR model, the level 4 and higher players make a mistake in calculating the best response. That is, they estimate correctly what other players will do, but miscalculate the best response to this behavior and replace it with equilibrium.

In the second-price auction, the SLR model with CHC quantification has a sharply different prediction than in the first-price auction; most people (those with $K > 0$) are predicted to be at equilibrium. In this case, the model with the CHC parameters predicted the results well. The interesting questions we are left with are—why do we observe this Poisson type of response in a variety of games, and why is it in line with the SLR model only in part of the games?

Figure 11 **Empirical and Predicted Probability Distributions: $N = 10$ Alternative Model**

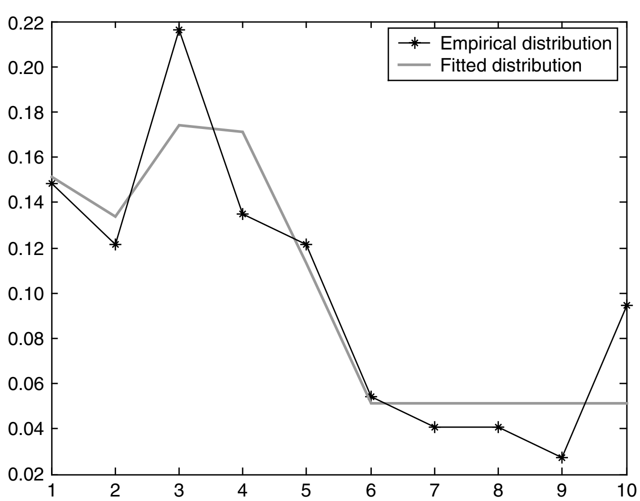


Figure 12 **Empirical and Predicted Probability Distributions: $N = 100$ and Grouped Alternative Model**

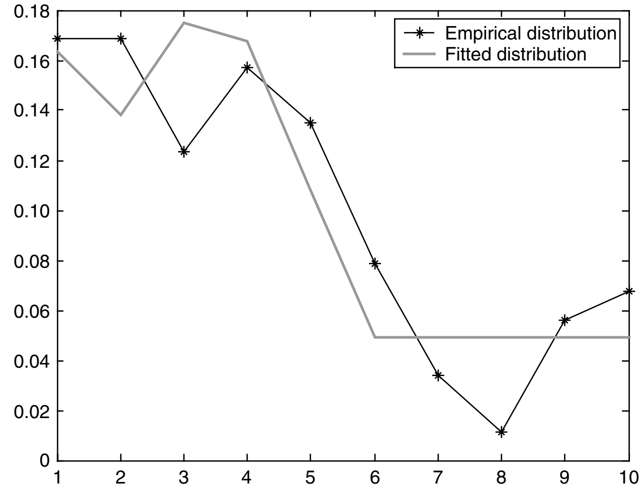
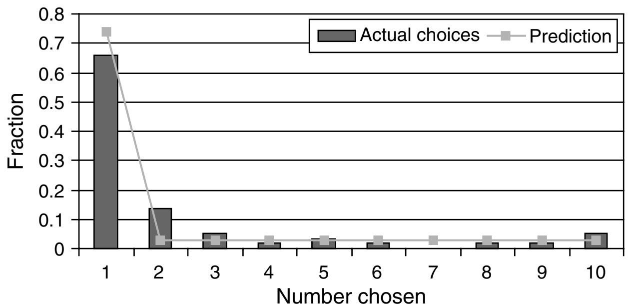


Table 4 **Model Comparisons Using Akaike Information Criterion (AIC) and Bayesian (Schwarz) Information Criterion (BIC)**

		Criterion value	
Criterion	Model	$N = 10$	$N = 100$ (grouped)
AIC	CHC model	366.16	441.48
	Alternative model	325.08	391.70
BIC	CHC model	368.65	443.97
	Alternative model	329.69	396.68

Figure 13 Prediction and Results of the 1–10 Treatment (Second-Price Auction)



The games tested in this paper have economic importance. Internet auctions capture a large and growing part of today's economy (Vulkan 2003). The advantage of the stylized auction is that it allows us to observe the reservation price of the bidders; it is symmetric, with common value and complete information. However, this is also a disadvantage. For example, in real auctions bidders seldom possess information about the reservation price of the other side (in many cases they do not even know their own reservation price). So, while the results of this paper might serve to increase optimism regarding the chances of understanding and modeling behavior in auctions, the goal is still distant. A behavioral economic model that can be used by bidders to predict outcomes of auctions, and to calculate probabilities of success and expected payoff, may be of great importance and usefulness. The goal of constructing such a model should proceed along the lines of the research presented in Kagel and Levin (2002), by trying to adapt the model to empirical evidence from more and more complex environments.

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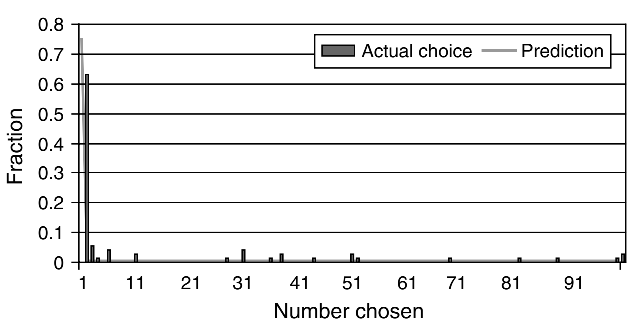
Figure 14 Prediction and Results of the 1–100 Treatment (Second-Price Auction)



Appendix. Instructions (First-Price Auction)

Dear Participant,

We would like to invite you to participate in a short experiment conducted by researchers from the University of Chicago Graduate School of Business.

All you have to do in order to participate is to read the instructions below and reply with your choice.

This task should not take more than 5 minutes, and we are going to randomly choose two people out of every 20 participants and pay them according to the instructions.

Please send your reply before December 14. We will notify the winners and inform participants of the results on December 15.

Thank you for your cooperation!

Instructions (for the 1 to 100 interval)

In the following game you are asked to choose an integer between 1 and 100. We will compare your choice to the choice of another participant chosen randomly.

If one of you chooses a lower number than the other, then he/she will win a dollar amount equal to the number he/she chose. The student who chose the higher number will not earn money.

If the two numbers chosen are equal, then each of you will get a dollar amount equal to half the number chosen.

The number I choose is: ____ .

We are going to choose one pair of participants at random, and these two participants will be paid according to the above instructions.

Instructions (for the 1 to 10 interval)

In the following game you are asked to choose an integer between 1 and 10. We will compare your choice to the choice of another participant chosen randomly.

If one of you chooses a lower number than the other, then he/she will win a dollar amount equal to 10 times the number he/she chose. The student who chose the higher number will not earn money.

If the two numbers chosen are equal, then each of you will get a dollar amount equal to 5 times the number chosen.

The number I choose is: ____ .

We are going to choose one pair of participants at random, and these two participants will be paid according to the above instructions.

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