International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences*

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Abstract

This paper investigates the international asset allocation effects of time-variations in higher order moments of stock returns such as skew and kurtosis. In the context of a four-moment international CAPM specification that relates stock returns in five regions to returns on a global market portfolio and allows for time-varying prices of covariance, co-skewness and co-kurtosis risk, we find evidence of distinct bull and bear regimes. Ignoring such regimes, an unhedged US investor's optimal portfolio is strongly diversified internationally. The presence of regimes in the return distribution leads to a substantial increase in the investor's optimal holdings of US stocks as does the introduction of skew and kurtosis preferences. We relate these findings to the US market portfolio's relatively attractive co-skewness and co-kurtosis properties with respect to the global market portfolio and its performance during global bear states.

Key words: International Asset Allocation, Regime Switching, Skew and Kurtosis Preferences, Home Bias.

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Abstract

This paper investigates the international asset allocation effects of time-variations in higher order moments of stock returns such as skew and kurtosis. In the context of a four-moment international CAPM specification that relates stock returns in five regions to returns on a global market portfolio and allows for time-varying prices of covariance, co-skewness and co-kurtosis risk, we find evidence of distinct bull and bear regimes. Ignoring such regimes, an unhedged US investor's optimal portfolio is strongly diversified internationally. The presence of regimes in the return distribution leads to a substantial increase in the investor's optimal holdings of US stocks as does the introduction of skew and kurtosis preferences. We relate these findings to the US market portfolio's relatively attractive co-skewness and co-kurtosis properties with respect to the global market portfolio and its performance during global bear states.

Despite the increased integration of international capital markets, investors continue to hold equity portfolios that are largely dominated by domestic assets. According to Thomas, Warnock and Wongswan (2006), by the end of 2003 US investors held only 14% of their equity portfolios in foreign stocks at a time when such stocks accounted for 54% of the world market capitalization.[1] This evidence is poorly understood: Calculations reported by Lewis (1999) suggest that a US investor with mean-variance preferences should hold upwards of 40% in foreign stocks or, equivalently, only 60% in US stocks.

Potential explanations for the home bias include barriers to international investment and transaction costs (Black (1990), Chaieb and Errunza (2007), Stulz (1981)); hedging demand for stocks that have low correlations with domestic state variables such as inflation risk or non-traded assets (Adler and Dumas (1983), Serrat (2001)); information asymmetries and higher estimation uncertainty for foreign than domestic stocks (Brennan and Cao (1997), Guidolin (2005)); and political or corporate governance risks related to investor protection (Dahlquist et al (2004)).[2]

As pointed out by Lewis (1999) and Karolyi and Stulz (2002), the first of these explanations is weakened by the fact that barriers to international investment have come down significantly over the last thirty years and by the large size of gross investment flows. Yet there is little evidence that US investors' holdings of foreign stocks have been increasing over the last decade where this share has fluctuated around 10-15%. The second explanation is weakened by the magnitude by which foreign stocks should be correlated more strongly with domestic risk factors as compared with domestic stocks. In fact, correlations with deviations from purchasing power parity can exacerbate the home bias puzzle (Cooper and Kaplanis (1994)) as can the strong positive correlation between domestic stock returns and returns on human capital (Baxter and Jermann (1997)). It is also not clear that estimation uncertainty provides a robust explanation as it affects domestic as well as foreign stocks. Finally, political risk seems to apply more to emerging and developing financial markets and is a less obvious explanation of investors' limited diversification among stable developed economies. Observations such as these lead Lewis (1999, p. 589) to conclude that "Two decades of research on equity home bias have yet to provide a definitive answer as to why domestic investors do not invest more heavily in foreign assets."

In this paper we address whether a combination of investor preferences that put weight on the skew and kurtosis of portfolio returns along with time-variations in international investment opportunities captured by regime switches can help explain the home bias and, if so, why US stocks may be more

[1]Similar home biases in aggregate equity portfolios are present in other countries, see French and Poterba (1991) and Tesar and Werner (1994).

[2]Behavioral explanations (e.g., 'patriotism' or a generic preference for 'familiarity') have been proposed by, e.g., Coval and Moskowitz (1999) and Morse and Shive (2003). Uppal and Wang (2003) provide theoretical foundations based on ambiguity aversion.

attractive to domestic investors than previously thought. Our analysis generalizes the standard international CAPM (ICAPM) specification that assumes mean-variance preferences over a time-invariant distribution of stock returns in two significant ways. First, we allow investor preferences to depend not only on the first two moments of returns but also on higher moments such as skew and kurtosis. The motivation for the generalization to higher moments arises from studies such as Harvey and Siddique (2000), Dittmar (2002) and, subsequently, Smith (2007), which, in the context of three and four-moment CAPM specifications for the cross-section of US stock returns, have found that higher order moments add considerable explanatory power and have first order effects on equilibrium expected returns. In addition, Harvey, Liechty, Liechty and Muller (2004) have found that international asset holdings can be quite different under third-moment preferences compared to the standard mean-variance case.[3]

Second, we model returns by means of a four-moment ICAPM with regimes that capture time-variations in the risk premia, volatility, correlations, skew and kurtosis (as well as co-skewness and co-kurtosis) of local equity return indices and the world market portfolio. Studies such as Ang and Bekaert (2002) have shown that regime switching models can successfully capture the asymmetric correlations found in international equity returns during volatile and stable markets, while Das and Uppal (2004) report that simultaneously occurring jumps that capture large declines in most international markets can affect international diversification. We go further than these studies and allow both the exposure of local markets to global risk factors and the world price of covariance, co-skewness and co-kurtosis risk to vary across regimes.

Both higher order preferences and regimes turn out to play important roles in US investors' international asset allocation and thus help explaining the home bias. Regimes in the distribution of international equity returns generate skew and kurtosis and therefore affect the asset allocation of a mean-variance investor differently from that of an investor whose objectives depend on higher moments of returns. This is significant since the single state model is severely misspecified and fails to capture basic features of international stock market returns. Our estimates suggest that a US mean-variance investor with access to the US, UK, European, Japanese and Pacific stock markets should hold only 30 percent in domestic stocks. The presence of bull and bear states raises this investor's weight on US stocks to 50 percent. Introducing both skew and kurtosis preferences and bull and bear states further increases the weight on US stocks to 70 percent of the equity portfolio, much closer to what is observed empirically.

[3]Harvey, Liechty, Liechty and Muller (2004) propose a Bayesian framework for portfolio choice based on Taylor expansions of an underlying expected utility function. They assume that the distribution of asset returns is a multivariate skewed normal. In their application to an international diversification problem, they find that under third-moment preferences, roughly 50 percent of the equity portfolio should be invested in US stocks.

To gain intuition for these findings, note that markets that have positive co-skewness with the global market portfolio are desirable to risk averse investors since they tend to have higher expected returns during volatile periods. Similarly, low co-kurtosis with global market returns means that local returns tend to be higher when world market returns are skewed to the left (i.e. during global bear markets), and is thus attractive since it decreases the overall portfolio risk. This turns out to be important because US stocks have attractive co-skewness and co-kurtosis properties. The US portfolio has a co-skewness of -0.05 and a co-kurtosis of 3.40 with the global market portfolio. In comparison, a fully-diversified ICAPM portfolio has lower skewness (-0.50) and higher kurtosis (4.51). Moreover, the US portfolio also has better co-skew and co-kurtosis properties than most of the other equity markets included in our analysis.

Previous studies have found that foreign stocks form an important part of US investors' optimal portfolio holdings under mean-variance preferences. However, the intuition above suggests that a US investor who dislikes negative co-skewness and high co-kurtosis with returns on the world market portfolio will put more weight on domestic stocks. The question then becomes how investors trade off between the mean, variance, skew and kurtosis properties of local market returns in an international portfolio context. Our paper addresses this issue when higher-order moments are modeled endogenously as part of an asset pricing model with regime shifts.

The contributions of our paper are as follows. First, we develop a flexible regime switching model that captures time-variations in the covariance, co-skewness and co-kurtosis risk of international stock markets with regard to the world equity portfolio. We find evidence of two regimes in the distribution of international stock returns. The first regime is a bear state with low ex-post mean returns and high volatility related to uncertainty spurred by market crashes, uncertain economic prospects during recessions or uncertainty about monetary policy. The second regime is a bull state which is associated with less volatile returns and more attractive investment opportunities. Variations in the skew and kurtosis of the world market portfolio are linked to uncertainty induced by shifts between such states. For example, the uncertainty surrounding a switch from a bull to a bear state takes the form of an increased probability of large negative returns (high kurtosis and large negative skew).

Second, we build on and generalize Harvey (1991)'s findings of time-variations in the world price of covariance risk to cover variations in the world price of co-skewness and co-kurtosis risk. We find that co-skewness and co-kurtosis risk are economically important components of the overall risk premium with magnitudes comparable to the covariance risk premium. This finding has substantial asset allocation implications and is an important difference between our study and that of Ang and Bekaert (2002) who cannot reject that expected returns are identical across different states, in part due to large estimation errors. By estimating a constrained asset pricing model which nests the two-and three-moment conditional ICAPM as special cases, we manage to identify significant variations in expected returns across the two states.

Third, we analyze the international portfolio implications of time-varying higher order moments. As in Harvey and Siddique (2000) and Dittmar (2002), our approach approximates the unknown marginal utility function by means of a Taylor series expansion of the utility function. Moreover, our analysis decomposes the effect of regimes and higher order moments on portfolio weights. We find that US stocks are more attractive than is reflected in the standard mean-variance case due to their relatively high co-skewness and low co-kurtosis with the global market portfolio. Compared with other stock markets, US stocks tend to perform better when global markets are volatile or skewed to the left, i.e. during global bear markets. This gives rise to another important difference to the analysis in Ang and Bekaert (2002) who conclude that the presence of a bear state with highly volatile and strongly correlated returns does not negate the economic gains from international diversification. We show that the relatively good performance of US stocks in the bear state can in fact help explain the higher allocation to the US market than in the benchmark single-state model. Intuition for this result comes from the higher marginal utility of additional payoffs during global bear markets which means that stock markets with good performance in these states tend to be attractive to risk averse investors.

The fourth and final contribution of our paper is to develop a new tractable approach to optimal asset allocation that is both convenient to use and offers new insights. When coupled with a utility specification that incorporates skew and kurtosis preferences, the otherwise complicated numerical problem of optimal asset allocation in the presence of regime switching is reduced to that of solving for the roots of a low-order polynomial. While papers such as Ang and Bekaert (2002) use numerical methods to solve bi- or tri-variate portfolio problems, our paper employs a moment-based utility specification that offers advantages both computationally and in terms of the economic intuition for how results change relative to the case with mean-variance preferences. The ability of our approach to solve the portfolio selection problem in the presence of multiple risky assets is important since gains from international asset allocation can be quite sensitive to the number of included assets.

The plan of the paper is as follows. Section 1 describes the return process in the context of an ICAPM extended to account for higher order moments, time-varying returns and regime switching and reports empirical results for this model. Section 2 sets up the optimal asset allocation problem for an investor with a polynomial utility function over terminal wealth when asset returns follow a regime switching process. Section 3 describes the solution to the optimal asset allocation problem, while Section 4 reports a range of robustness checks. Section 5 concludes. Appendices provide technical details.

1. **A Four-Moment ICAPM with Regime Switching in Asset Returns**

Our assumptions about the return process build on extensive work in asset pricing based on the no-arbitrage stochastic discount factor model for (gross) asset returns, R^i_{t+1}:

$$E[R^i_{t+1} m_{t+1} | \mathcal{F}_t] = 1 \quad i = 1, ..., I. \tag{1}$$

Here $E[\cdot|\mathcal{F}_t]$ is the conditional expectation given information available at time $t, \mathcal{F}_t$, and m_{t+1} is the investor's intertemporal marginal rate of substitution between current and future consumption or – under restrictions established by Brown and Gibbons (1985) – current and future wealth.

The two-moment CAPM follows from this setup when the pricing kernel, m_{t+1}, is linear in the returns on an aggregate wealth portfolio. Harvey (1991) shows that, in a globally integrated market, differences across country portfolios' expected returns should be driven by their conditional covariances with returns on a world market portfolio, R^W_{t+1}:

$$E[R^i_{t+1}|\mathcal{F}_t] - R^f_t = \frac{E[R^W_{t+1}|\mathcal{F}_t] - R^f_t}{Var[R^W_{t+1}|\mathcal{F}_t]} Cov[R^i_{t+1}, R^W_{t+1}|\mathcal{F}_t]. \tag{2}$$

Here both equity returns, R^i_{t+1}, and the conditionally risk free return, R^f_t, are expressed in the same currency (e.g. US dollars).

The two-moment ICAPM in equation (2) can be extended to account for higher order terms such as $Cov[R^i_{t+1}, (R^W_{t+1})^2|\mathcal{F}_t]$ and $Cov[R^i_{t+1}, (R^W_{t+1})^3|\mathcal{F}_t]$ that track the conditional co-skewness or co-kurtosis between the aggregate (world) portfolio and local portfolio returns. Such terms arise in a nonlinear model for the pricing kernel that depends on higher order powers of returns on the world market portfolio. Consistent with this, and building on Harvey and Siddique (2000) and Dittmar (2002), suppose that the pricing kernel can be approximated through a third-order Taylor series expansion of the marginal utility of returns on aggregate wealth:

$$m_{t+1} = g_{0t} + g_{1t} R^W_{t+1} + g_{2t} \left(R^W_{t+1}\right)^2 + g_{3t} \left(R^W_{t+1}\right)^3, \tag{3}$$

where $g_{jt} = U^{j+1}/U'$ is the ratio of derivatives of the utility function ($U^{(1)} \equiv U'$ is the first derivative, etc.) evaluated at current wealth. Assuming positive marginal utility ($U' > 0$), risk aversion ($U'' < 0$), decreasing absolute risk aversion ($U''' > 0$) and decreasing absolute prudence ($U'''' < 0$), it follows that $g_{1t} < 0$, $g_{2t} > 0$ and $g_{3t} < 0$.[4] Negative exponential utility satisfies such restrictions and the same applies to constant relative risk aversion preferences. More generally, Scott and Horvath (1980) have shown that a strictly risk-averse individual who always prefers more to less and consistently (i.e. for all wealth levels) likes skewness will necessarily dislike kurtosis.

[4] Vanden (2006) argues that investors' preference for positively skewed portfolio returns may have far-reaching implications for the stochastic discount factor, to the point of making options nonredundant so that (powers of) their returns enter the expression for the equilibrium pricing kernel.

Combining (1) with the cubic pricing kernel (3) and assuming that a conditionally risk-free asset exists, we get a four-moment asset pricing model:

$$E[R^i_{t+1}|\mathcal{F}_t]-R^f_t=\gamma_{1t}Cov(R^i_{t+1},R^W_{t+1}|\mathcal{F}_t)+\gamma_{2t}Cov(R^i_{t+1},(R^W_{t+1})^2|\mathcal{F}_t)+\gamma_{3t}Cov(R^i_{t+1},(R^W_{t+1})^3|\mathcal{F}_t), \quad (4)$$

where $\gamma_{jt} = -g_{jt}R^f_t$ $(j = 1, 2, 3)$, so $\gamma_{1t} > 0$, $\gamma_{2t} < 0$ and $\gamma_{3t} > 0$. This means that covariance and co-kurtosis risk earn positive risk premia while co-skewness risk earns a negative risk premium since an asset with a high return during times when returns on the world portfolio are highly volatile is desirable to risk averse investors. The positive premium on co-kurtosis risk suggests that the standard CAPM covariance premium carries over to 'large' returns. Co-skew earns a negative risk premium since an asset with a high return during times when the world portfolio is highly volatile is desirable to risk averse investors.

By imposing restrictions on equation (4), it is possible to obtain a variety of asset pricing models from the literature as special cases. If $\gamma_{3t} = 0$ at all times, then (4) reduces to Harvey and Siddique's (2000) three-moment framework in which only covariance and co-skewness are priced. If $\gamma_{2t} = \gamma_{3t} = 0$, then (4) becomes a time-varying, conditional ICAPM

$$E[R^i_{t+1}|\mathcal{F}_t] - R^f_t = \gamma_t Cov(R^i_{t+1}, R^W_{t+1}|\mathcal{F}_t) = \beta_t(E[R^W_{t+1}|\mathcal{F}_t] - R^f_t), \quad (5)$$

where both the risk premium and the exposure to risk (measured by the conditional beta) are time-varying.

In spite of its ability to nest a number of important asset pricing models, there are good reasons to be skeptical about the exact validity of the four-moment model in (4). On theoretical grounds, a reason for the failure of the CAPM to hold exactly in an international context is that it requires the world market portfolio to be perfectly correlated with world consumption (Stulz (1981)). Furthermore, Bekaert and Harvey (1995) show that limited international capital market integration means that terms such as $Var[R^i_{t+1}|\mathcal{F}_t]$ will affect the risk premium. On empirical grounds, conditional CAPM specifications have been tested extensively for international stock portfolios and been found to have significant limitations. Harvey (1991) reports that not all of the dynamic behavior of country returns is captured by a two-moment model and interprets this as evidence of either incomplete market integration, the existence of other priced sources of risk or model misspecification. The four-moment CAPM also ignores the presence of persistent 'regimes' documented for asset returns in papers such as Ang and Chen (2002), Engel and Hamilton (1990), Guidolin and Timmermann (2006), Gray (1996), Perez-Quiros and Timmermann (2000) and Whitelaw (2001).

1.1. *Regime Switches*

To allow for conditional time-variations in the return process and the possibility of misspecification biases, we extend the four-moment CAPM as follows. First, consistent with equations (3) and (4) we assume that returns on the world market portfolio depend not only on the conditional variance, $Var[R^W_{t+1}|\mathcal{F}_t]$, but also on the conditional skew, $Sk[R^W_{t+1}|\mathcal{F}_t]$, and kurtosis, $K[R^W_{t+1}|\mathcal{F}_t]$ of this portfolio.[5] Furthermore, to obtain a flexible representation without imposing too much structure, the price of risk associated with these moments is allowed to depend on a latent state variable, S_{t+1}, that is assumed to follow a Markov process but is otherwise not restricted. In turn this state dependence carries over to the price of the risk factors appearing in the equations for returns on the individual stock market portfolios, denoted by $\gamma_{1,S_{t+1}}$ (covariance risk), $\gamma_{2,S_{t+1}}$ (co-skewness risk) and $\gamma_{3,S_{t+1}}$ (co-kurtosis risk). Finally, consistent with empirical evidence in the literature (Harvey (1989) and Ferson and Harvey (1991)) we allow for predictability of returns on the world market portfolio through a vector of instruments, $\mathbf{z}_{t+1}$, assumed to follow some autoregressive process.

Defining excess returns on the I individual country portfolios, $x^i_{t+1} = R^i_{t+1} - R^f_t$ $(i = 1, ..., I)$ and the world portfolio, $x^W_{t+1} = R^W_{t+1} - R^f_t$, our model is

$$\begin{aligned} x^i_{t+1} &= \alpha^i_{S_{t+1}} + \gamma_{1,S_{t+1}} Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t] + \gamma_{2,S_{t+1}} Cov[x^i_{t+1}, (x^W_{t+1})^2|\mathcal{F}_t] + \gamma_{3,S_{t+1}} Cov[x^i_{t+1}, (x^W_{t+1})^3|\mathcal{F}_t] \\ &\quad + \mathbf{b}^i_{S_{t+1}} \mathbf{z}_t + \eta^i_{t+1} \\ x^W_{t+1} &= \alpha^W_{S_{t+1}} + \gamma_{1,S_{t+1}} Var[x^W_{t+1}|\mathcal{F}_t] + \gamma_{2,S_{t+1}} Sk[x^W_{t+1}|\mathcal{F}_t] + \gamma_{3,S_{t+1}} K[x^W_{t+1}|\mathcal{F}_t] + \mathbf{b}^W_{S_{t+1}} \mathbf{z}_t + \eta^W_{t+1} \\ \mathbf{z}_{t+1} &= \boldsymbol{\mu}_{z,S_{t+1}} + \mathbf{B}_{zS_{t+1}} \mathbf{z}_t + \boldsymbol{\eta}^Z_{t+1}. \end{aligned} \tag{6}$$

Consistent with the restrictions implied by the four-moment ICAPM, the risk premia $\gamma_{j,S_{t+1}}$ $(j = 1, 2, 3)$ are common across the individual assets and the world market portfolio. However, we allow for asset-specific intercepts, $\alpha^i_{S_{t+1}}$, that capture other types of misspecification. The innovations $\boldsymbol{\eta}_{t+1} \equiv [\eta^1_{t+1} ... \eta^I_{t+1} \ \eta^W_{t+1} \ (\boldsymbol{\eta}^Z_{t+1})'] \sim N(\mathbf{0}, \boldsymbol{\Omega}_{s_{t+1}})$ can have a state-dependent covariance matrix capturing periods of high and low volatility. The predictor variables, $\mathbf{z}_{t+1}$, follow a first order autoregressive process with state-dependent parameters, $\mathbf{B}_{zS_{t+1}}$, reflecting the persistence in commonly used predictor variables.

To complete the model we assume that the state variable, S_{t+1}, follows a $K-$state Markov process with transition probability matrix, $\mathbf{P}$:

$$\mathbf{P}[i, j] = \Pr(s_{t+1} = j | s_t = i) = p_{ij}, \quad i, j = 1, .., K. \tag{7}$$

Our model can thus be viewed as a time-varying version of the multi-beta latent variable model of Ferson (1990) where both risk premia and the amount of risk depend on a latent state variable.

[5] Conditional skewness and kurtosis are defined as $Sk[R^W_{t+1}|\mathcal{F}_t] \equiv E[(R^W_{t+1} - E(R^W_{t+1}|\mathcal{F}_t))^3|\mathcal{F}_t]$ and $K[R^W_{t+1}|\mathcal{F}_t] \equiv E[(R^W_{t+1} - E(R^W_{t+1}|\mathcal{F}_t))^4|\mathcal{F}_t]$, respectively.

Country returns in the asset pricing model (6) depend on their covariances, co-skewness and co-kurtosis with returns on the world portfolio. Estimating the skew and kurtosis of asset returns is difficult (Harvey and Siddique (2000)). However, our model allows us to obtain precise conditional estimates in a flexible manner as it captures skew and kurtosis as a function of the mean, variance and persistence parameters of the underlying states. Such model-based estimates are typically determined with considerably more accuracy than estimates of the third and fourth moments obtained directly from realized returns which tend to be very sensitive to outliers. Moreover, as we show in Appendix A, when the world price of covariance, co-skewness and co-kurtosis risk is identical across all markets, the model implies a tight set of restrictions across asset returns.

To gain intuition for the asset pricing model in (6), consider the special case with a single state where the price of risk is constant and−because the innovations $\boldsymbol{\eta}_{t+1} \sim N(\mathbf{0}, \boldsymbol{\Omega})$ are drawn from a time-invariant distribution−the higher moment terms $Cov[x^i_{t+1}, (x^W_{t+1})^2|\mathcal{F}_t]$, $Cov[x^i_{t+1}, (x^W_{t+1})^3|\mathcal{F}_t]$, $Sk[x^W_{t+1}|\mathcal{F}_t]$, and $K[x^W_{t+1}|\mathcal{F}_t]$ are constant and hence do not explain variations in returns:

$$\begin{aligned} x^i_{t+1} &= \alpha^i + \gamma_1 Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t] + \mathbf{b}^i \mathbf{z}_t + \eta^i_{t+1} \\ x^W_{t+1} &= \alpha^W + \gamma_1 Var[x^W_{t+1}|\mathcal{F}_t] + \mathbf{b}^W \mathbf{z}_t + \eta^W_{t+1} \\ \mathbf{z}_{t+1} &= \boldsymbol{\mu}_z + \mathbf{B}_z \mathbf{z}_t + \boldsymbol{\eta}^Z_{t+1}. \end{aligned} \tag{8}$$

This is an extended version of the ICAPM in which instruments ($\mathbf{z}_t$) are allowed to predict returns and alphas are not restricted to be zero ex-ante. When the restrictions $\alpha^i = \alpha^W = 0$ and $b^i = b^W = 0$ are imposed on all return equations, (8) simplifies to the standard ICAPM which sets $\gamma_{1t} = E[x^W_{t+1}|\mathcal{F}_t]/Var[x^W_{t+1}|\mathcal{F}_t]$ so

$$E[x^i_{t+1}|\mathcal{F}_t] = \frac{Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t]}{Var[x^W_{t+1}|\mathcal{F}_t]} E[x^W_{t+1}|\mathcal{F}_t] \equiv \beta^i_t E[x^W_{t+1}|\mathcal{F}_t]. \tag{9}$$

There are several advantages to modelling returns according to the general specification in (6). Conditional on knowing the state next period, S_{t+1}, the return distribution is Gaussian. However, since future states are not known in advance, the return distribution is a mixture of normals with weights reflecting the current state probabilities. Such mixtures of normals provide a flexible representation that can be used to approximate many distributions. They can accommodate mild serial correlation in returns−documented for returns on the world market portfolio by Harvey (1991)−and volatility clustering since they allow the first and second moments to vary as a function of the underlying state probabilities (Timmermann (2000)). Finally, multivariate regime switching models allow return correlations across markets to vary with the underlying regime, consistent with the evidence of asymmetric correlations in Longin and Solnik (2001) and Ang and Chen (2002).

1.2. *Data*

In addition to the world market portfolio, our analysis incorporates the largest international stock markets, namely the United States, Japan, the United Kingdom, the Pacific region (ex-Japan), and continental Europe. More markets could be included but parameter estimation errors are likely to become increasingly important in such cases so we do not go beyond five equity portfolios in addition to the world market portfolio.[6]

Following common practice, we consider returns from the perspective of an unhedged US investor and examine excess returns in US dollars on Morgan Stanley Capital International (MSCI) indices.[7] The risk-free rate is measured by the 30-day US T-bill rate provided by the Center for Research in Security Prices. Our data are monthly and cover the sample period 1975:01 - 2005:12, a total of 372 observations. Returns are continuously compounded and adjusted for dividends and other non-cash payments to shareholders. A number of studies have documented the leading role of US monetary policy and the US interest rate as a predictor of returns across international equity markets so we include the short US T-bill rate as a predictor variable.[8] Again our framework allows more variables to be included at the cost of having to estimate additional parameters.

Table 1 reports summary statistics for the international stock returns, the world market portfolio and the US T-bill rate. Mean returns are positive and lie in a range between 0.37 and 0.75 percent per month. Return volatilities vary from four to seven percent per month. Comparing the performance across stock markets, US stock returns are characterized by a fairly high mean and low volatility. Returns in all but one market (Japan) are strongly non-normal, skewed and fat-tailed, suggesting that a flexible model is required to incorporate such features. While the US T-bill rate is highly persistent, there is little evidence of serial correlation in stock returns. However, many of the return series display strong evidence of time-varying volatility.

1.3. *Empirical Results*

Panel A of Table 2 reports parameter estimates for the benchmark single-state, two-moment CAPM in equation (8). Alphas are positive in five regions and economically large but imprecisely estimated and statistically insignificant. The model's failure to capture returns in Japan is consistent with the strong rejections for Japan in the two-moment CAPM tests reported in Harvey (1991) and is perhaps to be expected in view of the gradual liberalization of financial markets in Japan during the 1980s and

[6] At the end of 2005 these markets represented roughly 97% of the world equity market capitalization.

[7] This is consistent with other authors' finding that US investors predominantly hold large and liquid foreign stocks such as those that dominate the MSCI indices (Thomas, Warnock and Wongswan (2006)).

[8] See Obstfeld and Rogoff (1995) for the micro foundations of such models and Kim (2001) for empirical evidence.

the analysis in Bekaert and Harvey (1995). The negative coefficients on the lagged T-bill rate are also consistent with existing literature. At 5.3, the estimated world price of covariance risk, γ_1, is positive and significant as expected.

Next consider the asset pricing model with two states, estimates of which are shown in Panel B of Table 2. To reduce the number of parameters, we impose two sets of constraints on the general model (6). First, the regression coefficients on the lagged T-bill rate were found to be insignificant for all stock markets in the first state and hence we impose that these coefficients are zero. In the second state the coefficients on the T-bill rate are large and negative and most are significant. Second, we impose that the correlations (but not the variances) between country-specific innovations, $Corr(\eta^i_{t+1}, \eta^j_{t+1})$, are the same in the two states. This restriction is again supported by the data and does *not* imply that the correlations between country returns ($Corr(x^i_{t+1}, x^j_{t+1})$) are the same in the two states since state-dependence in both the alphas and in the $b^i_{S_{t+1}}$ and $b^W_{S_{t+1}}$ coefficients generate time-variations in return correlations.[9]

As we shall see below, the economic interpretation suggested by the estimates reported in Table 2 is that state one is a bear state where returns have low (ex-post) means, high volatility and are more strongly correlated across markets. Conversely, state two is associated with more attractive, less uncertain and less correlated return prospects.

Figure 1 shows that the two states are generally well identified with state probabilities near zero or one most of the time. The bear state occurred during the three-year period between 1979 and 1982 where the Fed changed its monetary policy and again during shorter spells in 1984, 1987, 1990/1991 and 2002. These periods coincide with global recessions (the early 1980s, 1990s and 2002 recessions) and occasions with high return volatility such as October 1987. Common to these episodes is the high degree of uncertainty about economic prospects and the associated high volatility of global equity returns. In fact, volatility is highest in the first state for all equity portfolios with the exception of the UK.[10]

The persistence of the first state (0.90) is lower than that of the second state (0.94) and so the average duration of the first state (ten months) is shorter than that of the second state (20 months). In steady state one-third and two-thirds of the time is spent in states one and two, respectively. Neither of the states identifies isolated 'outliers' or jumps – a feature distinguishing our model from that proposed by Das and Uppal (2004).

It is interesting to compare the alpha estimates for the single-state and two-state models. Alpha

[9] A likelihood ratio test of the restriction that correlations do not depend on the state, i.e. $Cov(\eta^i_{t+1}, \eta^j_{t+1}) = Cor(\eta^i_{t+1}, \eta^j_{t+1})\sigma^i_{S_{t+1}}\sigma^j_{S_{t+1}}$, produces a p-value of 0.11 and is not rejected.

[10] The finding for the UK is due to two outliers in January and February of 1975 with monthly excess returns of 44 and 23 percent. If excluded from the data, the volatilility in the first state is highest also for the UK.

estimates are negative in state 1 but positive in state 2 for all portfolios. The alphas in the two states may appear to be quite large in economic terms.[11] However, as they measure returns conditional on being in a particular state and the state is never known in advance, they are not directly comparable to the corresponding estimates from the single state model. To account for this, we simulated 50,000 returns from the two-state model over a 12-month horizon, allowing for regime shifts and uncertainty about future states. Measured this way, the 12-month alphas starting from the first and second states are 0.06 and 0.70 for the US, while those for Japan are -0.45 and 0.86. The world portfolio generates alphas of -0.13 and 0.70, starting from the first and second state, respectively. All other estimates of the alphas in the two regimes shrink towards zero. Hence, although the individual state alphas appear to be quite large conditional on knowing the true state, in many regards they imply weaker evidence of mispricing than the single-state model which assumes that non-zero alphas are constant and constitute evidence of permanent model misspecification or mispricing.

Figure 2 shows that consistent with previous studies (Ang and Bekaert (2002), Longin and Solnik (1995, 2001) and Karolyi and Stulz (1999)), return correlations are higher in the bear state than in the full sample. Pairwise correlations between US stock returns and returns in Japan, Pacific ex-Japan, UK and Europe in the bear (bull) states are 0.39 (0.27), 0.65 (0.47), 0.67 (0.48) and 0.59 (0.45) and are thus systematically higher in the bear state. This happens despite the fact that correlations between return innovations are identical in the two states. In part this is due to the higher volatility of the common world market return in the bear state. Furthermore, since mean returns are different in the two states, return correlations also depend on the extent of the covariation between these parameters.

To help interpret the two states and gain intuition for what leads to changes in skew and kurtosis, it is useful to consider the time-variation in the conditional moments of the world market portfolio. To this end, Figure 3 shows the mean, volatility, skew and kurtosis implied by our model estimates, computed using the results in Appendix A.1. Consistent with our interpretation of state 1 as a bear state, mean excess returns are lower in this state, while conversely the volatility of returns is much higher.[12] Moreover, large changes in the conditional skew and kurtosis turn out to be linked to regime switches. Preceding a shift from the bull to the bear state, the kurtosis of the world market portfolio rises while its skew becomes large and negative and volatility is reduced. Uncertainty surrounding shifts from a bull to a bear state therefore takes the form of an increased probability of large negative returns.

Once in the bear state, the kurtosis gets very low and the skew close to zero, while world market volatility is much higher than normal. Hence the return distribution within the bear state is more

[11] Furthermore, the alphas in the two states are sufficiently precisely estimated that the hypothesis that they are equal to zero is very strongly rejected by a likelihood ratio test.

[12] Notice that, consistent with basic intuition, the expected excess return on the world portfolio is never negative.

dispersed, although closer to symmetric. Finally, when exiting from the bear to the bull state, the kurtosis again rises – reflecting the increased uncertainty associated with a regime shift – while volatility and skew decline to their normal levels. These large variations in the volatility, skew and kurtosis of world market returns means that our model is able to capture the correlated extremes across local markets found to be an important feature of stock returns in Harvey et al. (2004).

1.4. *Time-Variations in Risk Premia*

Further economic intuition can be gained from studying variations in the risk premia. The premium on covariance with returns on the world market portfolio (γ_1) is positive in both states but, at 15.9, is much higher in the bull state than in the bear state for which an estimate of 9.5 is obtained. The number reported by Harvey (1991) for the subset of G7 countries is 11.5 and hence lies between these two values. Consistent with the large difference between the covariance risk premium in the bull and bear state that we find here, Harvey rejects that the world price of risk is constant.

A similar conclusion holds for the co-kurtosis premium (γ_3) which is positive and insignificant in the bear state but positive and significant in the bull state. After suitable scaling the estimates of γ_3 can be compared to the price of covariance risk, γ_2.[13] This yields a price of co-kurtosis risk of 1.7 and 12.3 in the bear and bull state, respectively, and a steady state average of 8.7. As expected, the co-skewness premium (γ_2) is negative in both states although it is only significant (and by far largest) in the bull state. When converted to the same units as the covariance risk premium, the estimates are -1.1 and -3.1 in the bear and bull state, respectively, while the steady state average is -2.4.

Both the price of risk and the quantity of risk are required to show how much co-skewness risk and co-kurtosis risk contribute to expected returns compared with covariance risk. Using the parameter estimates from Table 2, we found that covariance risk (measured relative to the world market portfolio) contributes roughly the same amount to the risk premium in all markets, namely between 2.7 and 3.3 percent per year. Co-skewness risk premia vary more cross-sectionally, namely from 0.6 percent per year in Japan to 2.6 percent for Pacific stocks. Finally, co-kurtosis risk contributes between 0.5 and 1 percent to expected returns in annualized terms. For four of the six portfolios we study here (including the US and World portfolios), the combined co-skew and co-kurtosis risk premium is within one percent of the covariance risk premium.

[13] Scaling is required for meaningful comparisons. For instance, γ_1 measures the covariance risk premium per unit of covariance risk, $Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t]$, while γ_2 measures the co-skew risk premium with reference to $Cov[x^i_{t+1}, (x^W_{t+1})^2|\mathcal{F}_t]$. Since $Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t]$ and $Cov[x^i_{t+1}, (x^W_{t+1})^2|\mathcal{F}_t]$ are measured in different units (co-skewness involves squared returns), they cannot be directly compared. Because the scale of $Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t]$ is similar to $Var[x^W_{t+1}|\mathcal{F}_t]$ and the scale of $Cov[x^i_{t+1}, (x^W_{t+1})^2|\mathcal{F}_t]$ is similar to $|Sk[x^W_{t+1}|\mathcal{F}_t]|$, the transformation $\tilde{\gamma}_2 = \gamma_2 \times Var[x^W_{t+1}|\mathcal{F}_t]/|Sk[x^W_{t+1}|\mathcal{F}_t]|$ leads to a new coefficient $\tilde{\gamma}_2$ which is comparable to γ_1. Similarly, $\tilde{\gamma}_3 = \gamma_3 \times Var[x^W_{t+1}|\mathcal{F}_t]/K[x^W_{t+1}|\mathcal{F}_t]$ can be compared to γ_1.

We conclude from this analysis that the coefficients on covariance, co-skewness and co-kurtosis risk have the expected signs and are economically meaningful: Investors dislike risk in the form of higher volatility or fatter tails but like positively skewed return distributions. Furthermore, the co-skew and co-kurtosis risk premia appear to be important in economic terms as they are of the same order of magnitude as the covariance risk premium.

1.5. *Are Two States Needed?*

A question that naturally arises in the empirical analysis is whether regimes are really present in the distribution of international stock market returns. To answer this we computed the specification test suggested by Davies (1977), which very strongly rejected the single-state specification.[14] Inspection of the residuals from the single-state model confirmed that this model fails to capture even the most basic properties of the international returns data while the residuals from the two-state model (standardized by subtracting the conditional mean and dividing by the conditional standard deviation) were much closer to the model assumptions.

2. **The Investor's Asset Allocation Problem**

We next turn to the investor's asset allocation problem. Consistent with the analysis in the previous section, we assume that investor preferences depend on higher order moments of returns and allow regimes to affect the return process.

2.1. *Preferences over Moments of the Wealth Distribution*

Suppose that the investor's utility function $U(W_{t+T};\boldsymbol{\theta})$ only depends on wealth at time $t+T$, W_{t+T}, and a set of shape parameters, $\boldsymbol{\theta}$, where t is the current time and T is the investment horizon. Consider an m-th order Taylor series expansion of U around some wealth level v_T:

$$U(W_{t+T};\boldsymbol{\theta}) = \sum_{n=0}^{m} \frac{1}{n!} U^{(n)}(v_T;\boldsymbol{\theta})\,(W_{t+T} - v_T)^n + \varsigma_m, \tag{10}$$

where the remainder ς_m is of order $o((W_{t+T} - v_T)^m)$ and $U^{(0)}(v_T;\boldsymbol{\theta}) = U(v_T;\boldsymbol{\theta})$. $U^{(n)}(.)$ denotes the $n-$th derivative of the utility function with respect to terminal wealth. Provided that (i) the Taylor series converges; (ii) the distribution of wealth is uniquely determined by its moments; and (iii) the order of sums and integrals can be exchanged, the expansion in (10) extends to the expected utility

[14] Regime switching models have parameters that are unidentified under the null hypothesis of a single state. Standard critical values are therefore invalid in the hypothesis test. Details of the analysis are available upon request.

functional:

$$E_t[U(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \frac{1}{n!} U^{(n)}(\upsilon_T;\boldsymbol{\theta}) E_t[(W_{t+T} - \upsilon_T)^n] + E_t[\varsigma_m], \tag{11}$$

where $E_t[.]$ is short for $E[.|\mathcal{F}_t]$. For instance, Tsiang (1972) shows that these conditions are satisfied for negative exponential utility when asset returns are drawn from a multivariate distribution for which the first m central moments exist. We thus have

$$E_t[U(W_{t+T};\boldsymbol{\theta})] \approx \hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \frac{1}{n!} U^{(n)}(\upsilon_T;\boldsymbol{\theta}) E_t[(W_{t+T} - \upsilon_T)^n]. \tag{12}$$

While the approximation improves as m gets larger – setting $m = 2$ or 3 is likely to give accurate approximations for CARA utility according to Tsiang (1972) – many classes of Von-Neumann Morgenstern expected utility functions can be well approximated using a relatively small value of m and a function of the form:[15]

$$\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \kappa_n E_t[(W_{t+T} - \upsilon_T)^n], \tag{13}$$

with $\kappa_0 > 0$, and κ_n positive (negative) if n is odd (even).

2.2. *Solution to the Asset Allocation Problem*

We next characterize the solution to the investor's asset allocation problem when preferences are defined over moments of terminal wealth while, consistent with the analysis in Section 1, returns follow a regime switching process. Following most papers on portfolio choice (e.g., Ang and Bekaert (2002) and Das and Uppal (2004)), we assume a partial equilibrium framework that treats returns as exogenous.

The investor maximizes expected utility by choosing among I risky assets whose continuously compounded excess returns are given by the vector $\mathbf{x}_t^s \equiv (x_t^1 \; x_t^2 \; ... \; x_t^I)'$. Portfolio weights are collected in the vector $\boldsymbol{\omega}_t^1 \equiv (\omega_t^1 \; \omega_t^2 \; ... \; \omega_t^I)'$ while $(1 - \boldsymbol{\omega}_t' \boldsymbol{\iota}_I)$ is invested in a short-term interest-bearing bond, where ι_I is an $I \times 1$ vector of ones. The portfolio selection problem solved by a buy-and-hold investor

[15] For power utility, Tsiang (1972) and Kraus and Litzenberger (1976) prove that the condition

$$Pr\{|h| = |W_{t+T} - \upsilon_T| \leq E_t[W_{t+T}]\} = 1$$

is required for the series

$$\frac{(E_t[W_{t+T}])^{1-\gamma}}{1-\gamma} + h(E_t[W_{t+T}])^{-\gamma} - \frac{1}{2}(E_t[W_{t+T}])^{-\gamma-1}h^2 + \frac{1}{6}(E_t[W_{t+T}])^{-\gamma-2}h^3 + ... - (-1)^m \frac{1}{m!}(E_t[W_{t+T}])^{-\gamma-m+1}h^m$$

to converge. This corresponds to imposing a bound on the amount of risk accepted by the investor. In general convergence is slower than in the exponential utility case and depends on the investment horizon, T.

with unit initial wealth then becomes

$$\max_{\boldsymbol{\omega}_t} \quad E_t\left[U(W_{t+T}(\boldsymbol{\omega}_t);\boldsymbol{\theta})\right]$$
$$s.t.\ W_{t+T}(\boldsymbol{\omega}_t) = \left\{(1-\boldsymbol{\omega}_t'\boldsymbol{\iota}_I)\exp\left(R_{t+T}^b\right) + \boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right)\right\}, \tag{14}$$

where $\mathbf{R}_{t+T}^s \equiv (\mathbf{x}_{t+1}^s + r_{t+1}^b) + (\mathbf{x}_{t+2}^s + r_{t+2}^b) + ... + (\mathbf{x}_{t+T}^s + r_{t+T}^b)$ is the vector of continuously compounded equity returns over the $T-$period investment horizon while $R_{t+T}^b \equiv r_{t+1}^b + r_{t+2}^b + ... + r_{t+T}^b$ is the continuously compounded bond return. Accordingly, $\exp(\mathbf{R}_{t+T}^s)$ is a vector of cumulated returns. Short-selling can be ruled out through the constraint $\omega_t^i \in [0,1]$ for $i = 1, 2, ..., I$.

For generality, we assume the following process for a vector of $I+1$ excess returns (the last of which can be taken to represent the risky returns on a short-term bond, $x_{t+\tau}^b = r_{t+\tau}^b$):[16]

$$\mathbf{x}_{t+1} = \tilde{\boldsymbol{\mu}}_{S_{t+1}} + \sum_{j=1}^{p} \mathbf{B}_{j,S_{t+1}}\mathbf{x}_{t-j} + \boldsymbol{\varepsilon}_{t+1}, \tag{15}$$

where $\tilde{\boldsymbol{\mu}}_{S_{t+1}} = (\mu_{s_{t+1}}^1, ..., \mu_{s_{t+1}}^{I+1})'$ is a vector of conditional means in state S_{t+1} (possibly used to "fold in" all components of the mean in state S_{t+1}), $\mathbf{B}_{j,S_{t+1}}$ is a matrix of autoregressive coefficients associated with the jth lag in state S_{t+1}, and $\boldsymbol{\varepsilon}_{t+1} = (\varepsilon_{t+1}^1, ..., \varepsilon_{t+1}^{I+1})' \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}})$ is a vector of zero-mean return innovations with state-dependent covariance matrix $\boldsymbol{\Omega}_{S_{t+1}}$.

With $I+1$ risky assets and K states, the wealth process becomes

$$W_{t+T} = \boldsymbol{\omega}_t'\exp\left[\sum_{\tau=1}^{T}(\mathbf{x}_{t+\tau} + r_{t+\tau}^b)\right] + (1-\boldsymbol{\omega}_t'\boldsymbol{\iota}_I)\exp\left[\sum_{\tau=1}^{T} r_{t+\tau}^b\right]. \tag{16}$$

We next present a simple and convenient recursive procedure for evaluating the expected utility associated with a vector of portfolio weights, $\boldsymbol{\omega}_t$, of relatively high dimension:

Proposition 1. *Under the regime-switching return process (15) and $m-$moment preferences (13), the expected utility associated with the portfolio weights $\boldsymbol{\omega}_t$ is given by*

$$\begin{aligned}\hat{E}_t[U^m(W_{t+T})] &= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}{}_nC_jE_t[W_{t+T}^j] \quad (17)\\ &= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}\binom{n}{j}\sum_{i=0}^{j}\binom{j}{i}E_t\left[\left(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right)\right)^i\right]((1\text{-}\boldsymbol{\omega}_t'\boldsymbol{\iota}_h)\exp(Tr^f))^{j-i}.\end{aligned}$$

The nth moment of the cumulated return on the risky asset portfolio is

$$E_t\left[\left(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right)\right)^n\right] = \sum_{n_1=0}^{n}\cdots\sum_{n_I=0}^{n}\lambda(n_1,n_2,...,n_I)\left(\prod_{i=1}^{I}\omega_i^{n_i}\right)M_{t+T}^{(n)}(n_1,...,n_I), \tag{18}$$

[16]This equation is more convenient to use than (6) but is fully consistent with the earlier setup if the last elements of the return vector, $\mathbf{r}_{t+1}$, are used to capture the predictor variables, $\mathbf{z}_{t+1}$, which may themselves be asset returns.

where $\sum_{i=1}^{I} n_i = n$, $0 \leq n_i \leq n$ $(i = 1, ..., I)$,

$$\lambda(n_1, n_2, ..., n_h) \equiv \frac{n!}{n_1! n_2! \ ... \ n_h!}. \tag{19}$$

and $M_{t+T}^{(n)}(n_1, ..., n_I)$ *can be evaluated recursively, using (B12) in the Appendix.*

Appendix B proves this result. The solution is in closed-form in the sense that it reduces the expected utility calculation to a finite number of steps each of which can be solved by elementary operations.

Given their recursive structure, these results are complex and difficult to analyze. Appendix B therefore uses a simple two-state model to illustrate the result with a single risky asset. Here we use the setup of the model from Table 2 to provide intuition for proposition 1 in terms of the underlying determinants of the optimal asset allocation:

1. The current state probabilities $(\pi_t, 1-\pi_t)$ are particularly important for investors with a short horizon. Starting from the bear state, investment prospects are less favorable than starting from the bull state since there is a higher chance of remaining in the initial state. Stock markets with relatively good performance in the bear state (relative to other markets) will thus be preferred when starting from this state. How far π_t is removed from zero or one reflects investors' uncertainty about the current state. The more uncertain they are, the less aggressive the resulting asset allocation.

2. State transition probabilities, p_{ij}, affect the speed of mean reversion towards the steady state investment opportunity set. The closer the "stayer" probabilities, p_{11}, p_{22} are to one, the more persistent the individual states will be and hence the more the initial state matters. Conversely, if one state has a very low "stayer" probability, then this state is more likely to capture the occasional outlier or jump in asset prices as in Das and Uppal (2004).

3. Differences between mean parameters (μ_1, μ_2) and variance parameters (σ_1, σ_2) across states are important since skewness can only arise in the regime switching model provided that expected returns differ across states, i.e. $\mu_1 \neq \mu_2$, while kurtosis is strongly affected by differences in variance parameters in the states (see Timmermann (2000)). The greater the differences between expected returns and volatilities in the bear and bull states, the larger the role played by skew and kurtosis risk. Risk averse investors prefer to invest in countries with relatively good performance in the bear state since these provide a hedge against the poor performance of the world market portfolio and since the marginal utility of payoffs are higher in this state.

4. Investor preferences, as captured in part by m, the number of higher order moments that matter to the investor, in part by the weights assigned to the various moments which we discuss further

below. Going from $m = 2$ to $m = 3$ or $m = 4$, we move from mean-variance preferences to a setup where skew and kurtosis matter as well. Moreover, as the weight in the utility function on skew and kurtosis increases, investors become more sensitive to states with high volatility and higher probability of negative returns. This means that the significance of such states in determining the optimal asset allocation grows as does the weight on countries with relatively attractive co-skewness and co-kurtosis properties.

5. The investment horizon, T, plays a role in conjunction with the average duration of the states. The shorter the investment horizon and the more persistent the states, the more sensitive the investor's asset allocation will be with respect to the current state probability. As the investment horizon grows, the return distribution will converge to its "average" value and so the asset allocation becomes less sensitive to the initial state and more sensitive to the steady state probabilities.

It is useful to compare the solution method in Proposition 1 to existing alternatives. Classic results on optimal asset allocation have been derived for special cases such as power utility with constant investment opportunities or under logarithmic utility (Merton (1969) and Samuelson (1969)). For general preferences there is no closed-form solution to (14), but given its economic importance it is not surprising that a variety of solution approaches have been suggested. Recent papers that solve the asset allocation problem under predictability of returns include Ang and Bekaert (2002), Brandt (1999), Brennan, Schwarz and Lagnado (1997), Campbell and Viceira (1999, 2001). These papers generally use approximate solutions or numerical techniques such as quadrature (Ang and Bekaert (2002)) or Monte Carlo simulations (Detemple, Garcia and Rindisbacher (2003)) to characterize optimal portfolio weights. Quadrature methods may not be very precise when the underlying asset return distributions are strongly non-normal. They also have the problem that the number of quadrature points increases exponentially with the number of assets. Monte Carlo methods can be computationally expensive to use as they rely on discretization of the state space and use grid methods.[17] Although existing methods have clearly yielded important insights into the solution of (14), they are therefore not particularly well-suited to our analysis of international asset allocation which involves a large number of portfolios.

3. International Portfolio Holdings

We next consider empirically the optimal international asset allocation under regime switching and four-moment preferences. The weights on the first four moments of the wealth distribution are determined to ensure that our results can be compared to those in the existing literature. Most studies on

[17] In continuous time, closed-form solutions can be obtained under less severe restrictions, see Kim and Omberg (1996).

optimal asset allocation use power utility so we calibrate our coefficients to the benchmark

$$U(W_{t+T};\theta) = \frac{W_{t+T}^{1-\theta}}{1-\theta},\ \theta > 0. \tag{20}$$

For a given coefficient of relative risk aversion, θ, (20) serves as a guide in setting values of $\{\kappa_n\}_{n=0}^{m}$ in (13) but should otherwise not be viewed as an attempt to approximate results under power utility. Expanding the powers of $(W_{t+T} - \upsilon_T)$ and taking expectations, we obtain the following expression for the four-moment preference function:

$$\hat{E}_t[U^4(W_{t+T};\theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W_{t+T}^2] + \kappa_{3,T}(\theta)E_t[W_{t+T}^3] + \kappa_{4,T}(\theta)E_t[W_{t+T}^4], \tag{21}$$

where[18]

$$\begin{aligned}
\kappa_{0,T}(\theta) &= \upsilon_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1) - \frac{1}{24}\theta(\theta+1)(\theta+2)\right] \\
\kappa_{1,T}(\theta) &= \frac{1}{6}\upsilon_T^{-\theta}\left[6 + 6\theta + 3\theta(\theta+1) + \theta(\theta+1)(\theta+2)\right] > 0 \\
\kappa_{2,T}(\theta) &= -\frac{1}{4}\theta\upsilon_T^{-(1+\theta)}\left[2 + 2(\theta+1) + (\theta+1)(\theta+2)\right] < 0
\end{aligned}$$

$$\begin{aligned}
\kappa_{3,T}(\theta) &= \frac{1}{6}\theta(\theta+1)(\theta+3)\upsilon_T^{-(2+\theta)} > 0 \\
\kappa_{4,T}(\theta) &= -\frac{1}{24}\theta(\theta+1)(\theta+2)\upsilon_T^{-(3+\theta)} < 0.
\end{aligned} \tag{22}$$

Expected utility from final wealth increases in $E_t[W_{t+T}]$ and $E_t[W_{t+T}^3]$, so higher expected returns and more right-skewed distributions lead to higher expected utility. Conversely, expected utility is a decreasing function of the second and fourth moments of the terminal wealth distribution. Our benchmark results assume that $\theta = 2$, a coefficient of relative risk aversion compatible with much empirical evidence.[19]

A solution to the optimal asset allocation problem can now easily be found from Proposition 1 by solving a system of cubic equations in $\hat{\boldsymbol{\omega}}_t$ derived from the first order conditions

$$\nabla_{\boldsymbol{\omega}_t}\hat{E}_t[U^4(W_{t+T};\theta)]\Big|_{\hat{\boldsymbol{\omega}}_t} = \mathbf{0}'. \tag{23}$$

At the optimum $\hat{\boldsymbol{\omega}}_t$ sets the gradient, $\nabla_{\boldsymbol{\omega}_t}\hat{E}_t[U^4(W_{t+T};\theta)]$, equal to zero and produces a negative definite Hessian matrix, $H_{\boldsymbol{\omega}_t}\hat{E}_t[U^4(W_{t+T};\theta)]$.

[18] The notation $\kappa_{n,T}$ makes it explicit that the coefficients of the fourth order Taylor expansion depend on the investment horizon through the coefficient υ_T, the point around which the approximation is calculated. We follow standard practice and set $\upsilon_T = E_t[W_{t+T-1}]$.

[19] Based on the evidence in Ang and Bekaert (2002) – who show that the optimal home bias is an increasing function of the coefficient of relative risk aversion – this is also a conservative choice.

3.1. *Empirical Results*

As a benchmark, Table 3 first reports equity allocations for the single-state model using a short 1-month and a longer 24-month horizon. Our empirical analysis considers returns on five equity portfolios and the world market. To arrive at total portfolio weights we therefore re-allocate the weight assigned to the world market using the regional market capitalizations as weights.[20] Since we are interested in the home equity bias, we report equity weights as percentages of the total equity portfolio so they sum to unity. The allocation to the risk-free asset (as a percentage of the total portfolio) is shown for interest rates that vary by up to two standard deviations from the mean. When the T-bill rate is set at its sample mean of 5.9% per annum, at the one-month horizon only 31% of the equity portfolio is invested in US stocks. Slightly less (29%) gets invested in US stocks at the 24-month horizon. Thus, in both low and high interest rate environments the fraction of the equity portfolio allocated to US stocks remains considerably short of the percentages typically reported in the empirical literature. These results support earlier findings under mean-variance preferences (e.g. Lewis (1999)) and also show that the home bias puzzle extends to a setting with return predictability from the short T-bill rate.

Turning to the two-state model, Table 3 shows that the allocation to US stocks is much higher in the presence of regimes. This holds both when starting from the steady-state probabilities – i.e. when the investor has imprecise information about the current state – as well as in the separate bull and bear states. Under steady state probabilities and assuming an average short-term US interest rate the 1-month allocation to US stocks is 70% of the total equity portfolio. This reflects an allocation of 75% in the bear state and an allocation of 60% in the bull state.

These results show that a four-moment regime switching asset pricing model can substantially increase the optimal weights on US stocks. Moreover, this finding is robust to the level of the short US interest rate. Varying this rate predominantly affects the allocation to the risk-free asset versus the overall equity portfolio but has little affect on the regional composition of the equity portfolio.[21]

While the next section explains these results in the context of higher order moments, preliminary intuition can be gained in terms of how well the stock portfolios perform in the "bad" (bear) state. During bear markets, US stocks perform relatively better than the other markets with a higher Sharpe ratio due in part to higher mean returns and in part to lower volatility. This turns out to be especially important here since states with poor returns tend to be more heavily weighted under four-moment preferences than under mean-variance preferences and helps explain why the two-state model, which

[20] This introduces a very small approximation error as the included stock markets account for only 97% of the world market.

[21] The allocation to the short-term bond is much higher in the bear state than in the bull state. This happens because equity returns are small and volatile in the bear state and hence unattractive to risk averse investors.

distinguishes between return distributions in good and bad states, leads to higher allocations to US stocks than the single-state model which does not make this distinction. Of course, explanations based only on the first two moments merely scratch the surface of the issue here. We therefore next turn to the effect of higher order moments on international portfolio choice.

3.2. *Effects of Higher Moments*

Compared with the benchmark model, our four-moment regime switching model is able to significantly increase the allocation to US stocks. An economic understanding of the effect of skew and kurtosis on the optimal asset allocation requires studying the co-skewness and co-kurtosis properties at the portfolio level. To this end, define the conditional co-skewness of the return on market i with the world market as:

$$S_{i,W}(\mathcal{F}_t, S_t) \equiv \frac{Cov[x^i_{t+1}, (x^W_{t+1})^2 | \mathcal{F}_t, S_t]}{\{Var[x^i_{t+1}|\mathcal{F}_t, S_t](Var[x^W_{t+1}|\mathcal{F}_t, S_t])^2\}^{1/2}}. \tag{24}$$

The co-skewness is normalized by scaling by the appropriate powers of the volatility of the respective portfolios. A security that has negative co-skewness with the market portfolio pays low returns when the world market portfolio becomes highly volatile. To a risk averse investor this is an unattractive feature since global market risk rises in periods with low returns. Conversely, positive co-skewness is desirable as it means higher expected returns during volatile periods.

Similarly, define the co-kurtosis of the excess return on asset i with the world portfolio as

$$K_{i,W}(\mathcal{F}_t, S_t) \equiv \frac{Cov[x^i_{t+1}, (x^W_{t+1})^3 | \mathcal{F}_t, S_t]}{\{Var[x^i_{t+1}|\mathcal{F}_t, S_t](Var[x^W_{t+1}|\mathcal{F}_t, S_t])^3\}^{1/2}}. \tag{25}$$

Large positive values are undesirable as they mean that local returns are low (high) when world market returns are largely skewed to the left (right), thus increasing the overall portfolio risk.

Table 4 reports estimates of these moments in the bull and bear states as well as under steady state probabilities. The latter gives a measure that is more directly comparable to the full-sample estimates listed in the final column. Comparing the values implied by the two-state model to the full-sample estimates, the model generally does a good job at matching the data. Interestingly, with the exception of Japan, US stocks have the lowest co-kurtosis and highest co-skewness coefficients in both the bear state and under steady-state probabilities. Moreover, Japanese stocks are unattractive due to their low mean returns over the sample period. As we shall see, these observations help explain why domestic stocks are more attractive to US investors with skew and kurtosis preferences than in the mean-variance case.

To address the effect of higher order moments on the asset allocation, we next computed the optimal portfolio weights under mean-variance ($m = 2$) preferences:

$$\hat{E}_t[U^2(W_{t+T}; \theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta) E_t[W_{t+T}] + \kappa_{2,T}(\theta) E_t[W^2_{t+T}], \tag{26}$$

where $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta\right]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta}(1+\theta) > 0$ and $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)} < 0$. We also consider optimal allocations under three-moment preferences

$$\hat{E}_t[U^3(W_{t+T};\theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W_{t+T}^2] + \kappa_{3,T}(\theta)E_t[W_{t+T}^3] \quad (27)$$

where now $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1)\right]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta}\left[1+\theta+\frac{1}{2}\theta(\theta+1)\right] > 0$, $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)}(2+\theta) < 0$ and $\kappa_{3,T}(\theta) \equiv \frac{1}{6}\theta(\theta+1)v_T^{-(2+\theta)} > 0$.

Using steady-state probabilities, Table 5 shows that the allocation to US stocks as a portion of the overall equity portfolio is just above 50% under both mean-variance and skewness preferences. The introduction of two states on its own thus increases the allocation to US stocks from roughly 30% (as seen in Table 3) to 50%. This allocation rises further to 70% of the equity portfolio when we move to the case with skew and kurtosis preferences. Interestingly, in the bear state the large increase in the allocation to US stocks due to introducing higher moment preferences comes from the skew while the kurtosis plays a similar role in the bull state.

The correlation, co-skewness and co-kurtosis between the short interest rate and stock returns also affect asset allocations. At the 1-month horizon, the correlation between the risk-free rate and stock returns is zero since the risk-free rate is known. Future short-term spot rates are stochastic, however. This matters to buy-and-hold investors with horizons $T \geq 2$ months who effectively commit $(1-\boldsymbol{\omega}_t'\boldsymbol{\iota}_I)$ of their portfolio to roll over investments in $T-$bills $T-1$ times at unknown future spot rates. We therefore computed the co-skewness and co-kurtosis between the individual stock returns and rolling six-month bond returns assuming steady state probabilities and setting the initial interest rate at its unconditional mean. US stocks were found to generate the second-highest co-skewness coefficient (-0.06) and the second lowest co-kurtosis coefficient (4.44). Only Japanese stocks turn out to be preferable to US stocks, although their conditional mean and variance properties make them undesirable to a US investor. We conclude that the co-moment properties of US stocks against rolling returns on short US T-bills help to explain the high demand for these stocks under three- and four-moment preferences.

4. **Robustness of Results**

To summarize our results so far, we extended the standard model in two directions: First, by defining preferences over higher moments such as skew and kurtosis and, second, by allowing for the presence of bull and bear regimes tracking periods with very different mean, variances, correlations, skew and kurtosis of stock returns. In this section we consider the robustness of our results with regard to alternative specifications of investor preferences, estimation errors and dynamic portfolio choice.

4.1. *Preference Specification*

We first consider the effect of changing the coefficient of relative risk aversion from $\theta = 2$ in the baseline scenario to values of $\theta = 5$ (high) and $\theta = 10$ (very high). Ang and Bekaert (2002) and Das and Uppal (2004) found that changes in risk aversion affect their conclusions on the importance of either regime shifts or systemic (jump) risks. In unreported results that are available upon request, we found that there was no monotonic relation between θ and the weight on US stocks, although the allocation to US stocks tends to be greater for $\theta = 10$ than for $\theta = 2$. Risk aversion has a first order effect on the choice of T-bills versus stocks but has far less of an effect on the composition of the equity portfolio. Therefore, it does not seem that our conclusions depend on a particular choice of θ.

To make our results comparable to those reported in the literature which assume power utility, we also compared results under four-moment preferences to those under constant relative risk aversion. Differences between results computed under power utility and four-moment preferences were relatively minor.[22] In the bear state the allocation to US stocks was around 2-4% lower under power utility while conversely the allocation to UK stocks tended to be higher. In the more persistent bull state, allocations under the four-moment preference specification were similar to those under constant relative risk aversion.

4.2. *Precision of Portfolio Weights*

Mean-variance portfolio weights are generally highly sensitive to the underlying estimates of mean returns and covariances. Since such estimates often are imprecisely estimated, this means that the portfolio weights in turn can be poorly determined, see Britten-Jones (1999). As pointed out by Harvey, Liechty, Liechty and Muller (2004), this could potentially be even more of a concern in a model with higher moments due to the difficulty of obtaining precise estimates of moments such as skew and kurtosis.[23]

To address this concern, we computed standard error bands for the portfolio weights under the single state and two-state models using that, in large samples, the distribution of the parameter estimates from a regime switching model is

$$\sqrt{T}\left(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right) \sim N(\mathbf{0}, \mathbf{V}_{\theta}). \tag{28}$$

[22] A problem associated with low-order polynomial utility functionals is the difficulty of imposing restrictions on the derivatives (with respect to the moments of wealth) that apply globally. For example, nonsatiation cannot be imposed by restricting a quadratic polynomial to be monotonically increasing and risk aversion cannot be imposed by restricting a cubic polynomial to be globally concave (see Post and Levy (2005) and Post, van Vliet and Levy (2007)). It is therefore important to compare our results to those obtained under power utility.

[23] See also the discussion of "Omega" in Cascon, Keating and Shadwick (2003) which is used to capture sample information beyond point estimates through the cumulative density function of returns.

We set up the following simulation experiment. In the qth simulation we draw a vector of parameters, $\widehat{\boldsymbol{\theta}}^q$, from $N(\widehat{\boldsymbol{\theta}}, T^{-1}\hat{\mathbf{V}}_\theta)$ where $\hat{\mathbf{V}}_\theta$ is a consistent estimator of $\mathbf{V}_\theta$. Using this draw, $\widehat{\boldsymbol{\theta}}^q$, we solve for the associated vector of portfolio weights $\widehat{\boldsymbol{\omega}}^q$. We repeat this process Q times. Confidence intervals for the optimal asset allocation $\hat{\boldsymbol{\omega}}_t$ can then be derived from the distribution of $\widehat{\boldsymbol{\omega}}^q$, $q = 1, 2, ..., Q$. This approach is computationally intensive, as the asset allocation problem (14) must be solved repeatedly, so we set the number of simulations to $Q = 2,000$.

Results are reported in Table 6. Unsurprisingly, and consistent with the analysis in Britten- Jones (1999), the standard error bands are quite wide for the single state model. For example, at the 1-month horizon the 90% confidence band for the weight on the US market in the equity portfolio goes from 2% to 38%−a width of 36%. The width of the confidence band is roughly similar at the 24-month horizon. In comparison, the confidence band for the US weight in the two-state model under steady state probabilities only extends from 64% to 73%, a width of less than 10%. Even at longer investment horizons, the confidence bands remain quite narrow under the two-state model (e.g. from 50% to 69% under steady state probabilities when $T = 24$ months). In fact, the standard error bands for the portfolio weights are generally narrower under the two-state model than under the single-state model. This suggests that the finding that a large part of the home bias can be explained by the US stock market portfolio's co-skewness and co-kurtosis properties in bull and bear states is fairly robust.

Intuition for these findings is as follows. First, the fact that the portfolio weights do not become less precise even though we account for skew and kurtosis is related to the way we compute these moments from a constrained two-state asset pricing model. As can be seen from the time series in figures 2 and 3, these moments are well behaved without the huge spikes and sampling variations typically observed when such moments are estimated directly from returns data using rolling or expanding data windows. Second, the two-state model captures many properties of the returns data far better than the single-state model and so reduces noise due to misspecification. Third, and related to this point, one effect of conditioning on states is to capture more of the return dynamics. This means that some of the parameters in the two-state model are more precisely estimated than in the single-state model. Again this reduces the standard error bands on the portfolio weights under the two-state model.

An alternative way to measure the effect of parameter estimation error that directly addresses its economic costs is to compute the investor's average (or expected) utility when the estimated parameters as opposed to the true parameters are used to guide the portfolio selection. To this end, Panel A of Table 7 reports the outcome of a Monte Carlo simulation where returns were generated from the two-state model in Table 2. In these simulations, the parameter values were assumed to be unknown to the investor who had to estimate these using a sample of the same length as the actual data before selecting the portfolio weights assuming either a 1-month or a 24-month investment horizon. For comparison, we also report results for alternatives such as using the single state model (8) or adopting the ICAPM weights (i.e. each region is purchased in the proportion that it enters into the global market portfolio).

Even after accounting for the effect of parameter estimation errors, the two-state model produces the highest certainty equivalent return and the highest average wealth at both the 1-month and 24-month horizons. Furthermore, the improvements are meaningful in economic terms, suggesting an increase in the certainty equivalent return of about two percent per annum. Since US stock holdings are considerably higher under the two-state model, the better performance of this model again indicates that parameter estimation error does not diminish the ability of this model to explain home biases in US investors' equity holdings.

4.3. *Out-of-Sample Portfolio Selection*

Econometric models fitted to asset returns may produce good in-sample (or historical) fits and imply asset allocations that are quite different from the benchmark ICAPM portfolio. However, this is by no means a guarantee that such models will lead to improvements in 'real time' when used on future data. This problem arises, for example, when the proposed model is misspecified. It could also be the result of parameter estimation error as discussed above.

To address both concerns, we next explored how well the two-state model performs out-of-sample through the following recursive estimation and portfolio selection experiment. We first used data up to 1985:12 to estimate the parameters of the two-state model. Using these estimates, we computed the mean, variance, skew and kurtosis of returns and solved for the optimal portfolio weights at 1-month and 24-month horizons. This exercise was repeated the following month, using data up to 1986:1 to forecast returns and select the portfolio weights. Repeating this until the end of the sample (2005:12) generated a sequence of realized returns from which realized utilities and certainty equivalent returns were computed.[24]

Since this experiment does not assume that the two-state model is the 'true' model – realized returns are computed using actual data and not simulated returns – and since the sample (1986-2005) covered several bull and bear markets, this experiment provides an ideal way to test if the two-state model can add value over alternative approaches.

Results are shown in Panel B in Table 7. Again the two-state model came out ahead of the single-state model and ICAPM specifications in realized utility terms and for both investment horizons.[25]

[24] In this experiment we updated all the parameters once a year while the state probabilities were updated each month using the Hamilton-Kim filter (see Hamilton (1990) for details).

[25] An investment strategy based on the two-state model fails to produce the highest out-of-sample mean return which is now associated with the ICAPM. However, the ICAPM portfolio weights also generate return volatilities that are 2-3%

For example, at the 1-month horizon, the certainty equivalence return of the two-state model was two percent higher than under the single-state model while it exceeded that of the ICAPM by 80 basis points per annum. Results were very similar at the 24-month horizon.

4.4. *Rebalancing*

To keep the analysis simple, so far we ignored the possibility of portfolio rebalancing. However, as noted in the literature, rebalancing opportunities give investors incentives to exploit current information more aggressively. To explore the importance of this point, we therefore considered rebalancing using two investment horizons ($T = 6$ and 24 months) and various rebalancing frequencies ($\varphi = 1$, 3, 6, 12 months). To save space we simply summarize the results here, while further details are available on request. Our analysis showed that rebalancing matters most when it occurs very frequently, i.e. when φ is small. Stock allocations under rebalancing are large and always exceed 60% of current wealth. Starting from the bull state, the allocation under frequent rebalancing ($\varphi = 1$ and 3 months) differs significantly from the buy-and-hold results as the investor attempts to time the market by shifting the portfolio towards Pacific stocks and away from US and UK equities.

However, starting from the bear state or assuming that the initial state is unknown (i.e. adopting steady-state probabilities), very frequent rebalancing ($\varphi = 1$ and 3 months) increases the allocation to US stocks for long horizons ($T = 24$ months), while Japanese stocks also emerge as an attractive investment. For all possible values of φ this implies an even greater allocation to US stocks than under the buy-and-hold scenario. In fact, under frequent rebalancing a US investor with four-moment preferences and a long horizon should hold even more in US securities than under no rebalancing. For example, for $T = 24$, almost 100% of wealth goes into domestic securities, comprising between 60% and 85% in stocks (only 8-12% of total wealth goes into foreign stocks).

All told, regime shifts combined with preferences that reflect aversion against fat tails and negative skew help explain the home bias under a range of assumptions about the rebalancing frequency, especially when investors have little information about the current state (and thus adopt steady state probabilities), which seems to be a plausible assumption.

5. **Conclusion**

The composition of US investors' equity portfolio into domestic and foreign stocks depends critically on how the distribution of global equity returns is modeled and which preferences investors are assumed to have. Under mean-variance preferences and a single-state model for stock returns, we continue to

higher than the portfolio associated with the two-state model. This helps explain why the two-state portfolio attains higher realized utilities and certainty equivalent returns.

find substantial gains to US investors from international diversification and thus confirm the presence of a home bias puzzle. However, we argue that the standard two-moment ICAPM has important shortcomings since it ignores skewness and kurtosis in international stock returns and present empirical evidence that such moments are associated with significant risk premia. Once we account for US investors' dislike for negative skew and fat-tailed return distributions and incorporate the strong evidence of persistent bull and bear states in global equity returns, we find a much larger allocation to domestic stocks.

Intuition for this result comes from the attractive properties that US stocks have for an investor who – besides being risk averse – prefers positively skewed (asymmetric) payoffs and dislikes fat tails (kurtosis). For example, US stocks have relatively high co-skewness and low co-kurtosis with respect to the global market portfolio. The performance of US stocks certainly worsens during the global 'bear' state. However, compared with other international markets, the US market portfolio is relatively less affected and offers better investment opportunities when global equity markets are highly volatile or experience large negative returns.

Our empirical findings are consistent with and shed new light on recent papers that justify underdiversification from a theoretical perspective. Mitton and Vorkink (2007) propose a model in which investors' skewness preferences lead them to hold substantially under-diversified portfolios in equilibrium. Polkovnichenko (2005) shows that several forms of rank-dependent preferences generate preference for wealth skewness. For a range of plausible parameterizations, this can lead to under-diversification of the optimal portfolio.

An interesting issue that goes beyond the analysis in the current paper is whether our results extend to the home bias observed in investors' equity holdings in other countries. One may conjecture that – because stock and bond markets in the same economy are more likely to be "in phase" than are markets across national borders – the finding that stock returns in one country have attractive co-moment properties with national short-term rates extends beyond our analysis for the US. This would contribute to explain the international evidence of a pervasive home bias in stock holdings.[26]

Appendix A: Conditional Moments and Estimation Procedure

This appendix describes how we derive the conditional higher order moments of stock returns and explains the econometric methodology used in estimating the asset pricing model (6).

A.1 Moments of Returns

Letting $\mathbf{y}_{t+1} = (\mathbf{x}'_{t+1}, x^W_{t+1}, \mathbf{z}'_{t+1})'$ be a vector of excess returns and predictor variables with intercepts $\boldsymbol{\mu}_{S_{t+1}} = (\alpha^1_{S_{t+1}}, .., \alpha^I_{S_{t+1}}, \alpha^W_{S_{t+1}}, \boldsymbol{\mu}'_{zS_{t+1}})'$, we can collect the conditional moments of returns and

[26] We are grateful to an anonymous referee for pointing our attention in this direction.

the world price of co-moment risk in the matrices $\mathbf{M}_{S_t}$ and $\boldsymbol{\Upsilon}_{S_{t+1}}$ as follows

$$\mathbf{M}_{S_t} \equiv \left(\left[\begin{array}{c} \begin{bmatrix} Cov[\mathbf{x}_{t+1}, x^W_{t+1}|\mathcal{F}_t] & Cov[\mathbf{x}_{t+1}, (x^W_{t+1})^2|\mathcal{F}_t] & Cov[\mathbf{x}_{t+1}, (x^W_{t+1})^3|\mathcal{F}_t] \\ Var[x^W_{t+1}|\mathcal{F}_t] & Sk[x^W_{t+1}|\mathcal{F}_t] & K[x^W_{t+1}|\mathcal{F}_t] \end{bmatrix} \\ \mathbf{O} \end{array} \right] \otimes \boldsymbol{\iota}_3' \right) \odot (\boldsymbol{\iota}_3' \otimes \mathbf{I})$$

$$\boldsymbol{\Upsilon}_{S_{t+1}} \equiv \begin{bmatrix} \gamma^1_{1,S_{t+1}} & ... & \gamma^I_{1,S_{t+1}} & \gamma^W_{1,S_{t+1}} & 0 & ... & 0 \\ \gamma^1_{2,S_{t+1}} & ... & \gamma^I_{2,S_{t+1}} & \gamma^W_{2,S_{t+1}} & 0 & ... & 0 \\ \gamma^1_{3,S_{t+1}} & ... & \gamma^I_{3,S_{t+1}} & \gamma^W_{3,S_{t+1}} & 0 & ... & 0 \end{bmatrix},$$

where $\boldsymbol{\iota}_3$ is a 3×1 vector of ones and $\mathbf{J}$ is a matrix that selects the co-moments of excess returns:

$$\mathbf{J} \equiv \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We can then write the asset pricing model (6) more compactly as

$$\mathbf{y}_{t+1} = \boldsymbol{\mu}_{S_{t+1}} + \mathbf{M}_{S_t} vec(\boldsymbol{\Upsilon}_{S_{t+1}}) + \mathbf{B}_{s_{t+1}} \mathbf{y}_t + \boldsymbol{\eta}_{t+1}. \quad \text{(A1)}$$

Here $\mathbf{B}_{S_{t+1}}$ captures autoregressive terms in state S_{t+1} and also collects the coefficients $\mathbf{b}^i_{S_{t+1}}$ and $\mathbf{b}^W_{S_{t+1}}$ that measure the impact of the lagged instruments $\mathbf{z}_t$ on the risk premia. Finally $\boldsymbol{\eta}_{t+1} \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}})$ is the vector of state-dependent innovations.

To characterize the moments of returns on the world market portfolio and its co-moments with local market returns, note that mean returns can be computed from

$$\bar{\mathbf{y}}_{t+1} \equiv E[\mathbf{y}_{t+1}|\mathcal{F}_t] = \sum_{k=1}^{K} (\boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_k) \tilde{\boldsymbol{\mu}}_k + \sum_{k=1}^{K} (\boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_k) \mathbf{A}_k \mathbf{y}_t, \quad \text{(A2)}$$

where $\boldsymbol{\pi}_t$ is the vector of state probabilities, $\mathbf{e}_k$ is a vector of zeros with a one in the k-th position so $(\boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_k)$ is the ex-ante probability of being in state k at time $t+1$ given information at time t, $\mathcal{F}_t$, and $\tilde{\boldsymbol{\mu}}_k \equiv \boldsymbol{\mu}_k + \mathbf{M}_{S_t} vec(\boldsymbol{\Upsilon}_k)$.

Because $\tilde{\boldsymbol{\mu}}_k$ involves higher order moments of the world market portfolio such as $\mathbf{M}_{S_t} vec(\boldsymbol{\Upsilon}_k)$ as well as higher order co-moments between individual portfolio returns and returns on the global market portfolio, the (conditional) mean returns $E[\mathbf{y}_{t+1}|\mathcal{F}_t]$ enter the right-hand side of (A1). For instance, computing $Cov[\mathbf{x}_{t+1}, x^W_{t+1}|\mathcal{F}_t]$ requires knowledge of the first I elements of $E[\mathbf{y}_{t+1}|\mathcal{F}_t]$. Below we explain the iterative estimation procedure used to solve the associated nonlinear optimization problem.

The conditional variance, skew and kurtosis of returns on the world market portfolio, x^W_{t+1}, can now be computed as follows:

$$\begin{aligned}
Var[x^W_{t+1}|\mathcal{F}_t] &= \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\left[\left(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_{I+1}\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t\right)^2\right] + \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)Var[\eta^W_{t+1}|S_{t+1}=k] \\
Sk[x^W_{t+1}|\mathcal{F}_t] &= \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\left[\left(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_{I+1}\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t\right)^3\right] \\
&\quad +3\sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\left[\left(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_{I+1}\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t\right)Var[\eta^W_{t+1}|S_{t+1}=k]\right] \\
K[x^W_{t+1}|\mathcal{F}_t] &= \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\left[\left(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_{I+1}\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t\right)^4\right] \qquad \text{(A3)} \\
&\quad +6\sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\left[\left(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_{I+1}\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t\right)^2 Var[\eta^W_{t+1}|S_{t+1}=k]\right].
\end{aligned}$$

Clearly the skew and kurtosis of global equity returns are functions of the mean and variance parameters $\{\tilde{\mu}_{1,k}, .., \tilde{\mu}_{I,k}, \mathbf{A}_k, \boldsymbol{\Omega}_k\}_{k=1}^K$, state probabilities, $\boldsymbol{\pi}_t$, and mean VAR coefficients, $\bar{\boldsymbol{\alpha}}_j \equiv \mathbf{e}'_j\sum_{k=1}^K(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\mathbf{A}_k$. Hence, no new parameters are introduced to capture the higher moments of the return distribution.

Similarly, the covariance between country returns, x^i_{t+1}, and the world market return, x^W_{t+1}, is

$$\begin{aligned}
Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t] &= \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)[\left(\tilde{\mu}_{i,k} - \mathbf{e}'_i\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_i\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_i)\mathbf{y}_t\right)(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+ \\
&\quad +(\mathbf{e}'_{I+1}\mathbf{A}_l - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t)] + \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)Cov[\eta^i_{t+1}, \eta^W_{t+1}|S_{t+1}=k]. \qquad \text{(A4)}
\end{aligned}$$

Given estimates of the parameters and state probabilities, $Cov[x^i_{t+1}, x^W_{t+1}|\mathcal{F}_t, S_t]$ can easily be calculated.

Finally, the co-skewness and co-kurtosis between local market returns and the world market return is

$$\begin{aligned}
Cov[x^i_{t+1}, (x^W_{t+1})^2|\mathcal{F}_t] &= \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)[\left(\tilde{\mu}_{i,k} - \mathbf{e}'_i\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_i\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_i)\mathbf{y}_t\right)(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+ \\
&\quad +(\mathbf{e}'_{I+1}\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t)^2] + \sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\left[\left(\tilde{\mu}_{i,k} - \mathbf{e}'_i\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_i\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_i)\mathbf{y}_t\right)Var[\eta^W_{t+1}|S_{t+1}=k]\right] + \\
&\quad +2\sum_{k=1}^{K}(\boldsymbol{\pi}_t'\mathbf{P}\mathbf{e}_k)\left[\left(\tilde{\mu}^W_k - \mathbf{e}'_{I+1}\bar{\mathbf{y}}_{t+1}+(\mathbf{e}'_{I+1}\mathbf{A}_k - \bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_t\right)Cov[\eta^i_{t+1}, \eta^W_{t+1}|S_{t+1}=k]\right], \qquad \text{(A5)}
\end{aligned}$$

and

$$Cov[x_{t+1}^{i},(x_{t+1}^{W})^{3}|\mathcal{F}_{t}]=\sum_{k=1}^{K}(\boldsymbol{\pi}_{t}'\mathbf{P}\mathbf{e}_{k})\left[\left(\tilde{\mu}_{i,k}-\mathbf{e}_{i}'\bar{\mathbf{y}}_{t+1}+(\mathbf{e}_{i}'\mathbf{A}_{k}-\bar{\boldsymbol{\alpha}}_{i})\mathbf{y}_{t}\right)\left(\tilde{\mu}_{k}^{W}-\mathbf{e}_{I+1}'\bar{\mathbf{y}}_{t+1}+(\mathbf{e}_{I+1}'\mathbf{A}_{k}\text{-}\bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_{t}\right)^{3}\right]$$

$$+3\sum_{k=1}^{K}(\boldsymbol{\pi}_{t}'\mathbf{P}\mathbf{e}_{k})\left[\left(\tilde{\mu}_{i,k}-\mathbf{e}_{i}'\bar{\mathbf{y}}_{t+1}+(\mathbf{e}_{i}'\mathbf{A}_{k}-\bar{\boldsymbol{\alpha}}_{i})\mathbf{y}_{t}\right)\left(\tilde{\mu}_{k}^{W}-\mathbf{e}_{I+1}'\bar{\mathbf{y}}_{t+1}+(\mathbf{e}_{I+1}'\mathbf{A}_{k}-\bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_{t}\right)Var[\eta_{t+1}^{W}|S_{t+1}{=}k]\right]+$$

$$+3\sum_{k=1}^{K}(\boldsymbol{\pi}_{t}'\mathbf{P}\mathbf{e}_{k})\left[\left(\tilde{\mu}_{k}^{W}-\mathbf{e}_{I+1}'\bar{\mathbf{y}}_{t+1}+(\mathbf{e}_{I+1}'\mathbf{A}_{k}-\bar{\boldsymbol{\alpha}}_{I+1})\mathbf{y}_{t}\right)^{2}Cov[\eta_{t+1}^{i},\eta_{t+1}^{W}|S_{t+1}=k]\right]. \quad \text{(A6)}$$

Terms such as $\left(\tilde{\mu}_{i,k}-\mathbf{e}_{i}'\bar{\mathbf{y}}_{t+1}\right)\left(\tilde{\mu}_{k}^{W}-\mathbf{e}_{I+1}'\bar{\mathbf{y}}_{t+1}\right)$ show the deviations of the state-specific mean from the overall mean and do not arise in single-state models.

A.2 Estimation

Defining $\boldsymbol{\eta}_{S_{t+1}}$ as a vector of residuals in state S_{t+1}, the contribution to the log-likelihood function conditional on being in state S_{t+1} at time $t+1$ is given (up to a constant) by:

$$\ln p(\mathbf{y}_{t+1}|\mathcal{F}_{t},S_{t+1};\boldsymbol{\lambda})\propto-\frac{1}{2}\ln|\boldsymbol{\Omega}_{S_{t+1}}|-\frac{1}{2}\boldsymbol{\eta}_{S_{t+1}}'\boldsymbol{\Omega}_{S_{t+1}}^{-1}\boldsymbol{\eta}_{S_{t+1}},$$

where $\boldsymbol{\lambda}=\{\boldsymbol{\phi}_{s},\boldsymbol{\Omega}_{s},\mathbf{P}\}_{s=1}^{K}$ collects the mean ($\boldsymbol{\phi}$), variance ($\boldsymbol{\Omega}$) and transition probability ($\mathbf{P}$) parameters of the model (A1). The expected value of the log-likelihood employed by the EM algorithm is maximized by choosing the parameters $\boldsymbol{\lambda}^{l+1}$ in the $l+1$ iteration to satisfy (see Hamilton (1990, p.51)):

$$\sum_{t=1}^{T}\sum_{S_{t+1}=1}^{K}\left.\frac{\partial\ln p(\mathbf{y}_{t+1}|\mathcal{F}_{t},S_{t+1};\boldsymbol{\lambda})}{\partial\boldsymbol{\lambda}}\right|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}^{l+1}}p(S_{t+1}|\mathbf{y}_{2},\mathbf{y}_{3},...,\mathbf{y}_{T};\boldsymbol{\lambda}^{l})=\mathbf{0}, \quad \text{(A7)}$$

where $\{p(S_{t+1}|\mathbf{y}_{2},\mathbf{y}_{3},...,\mathbf{y}_{T};\boldsymbol{\lambda}^{l})\}_{S_{t+1}=1}^{K}$ are the smoothed state probabilities for each of the K states. Letting $\mathbf{y}\equiv[\mathbf{y}_{2}'\ \mathbf{y}_{3}'\ ...\ \mathbf{y}_{T}']'$ and $\boldsymbol{\eta}\equiv[\boldsymbol{\eta}_{1}'\ \boldsymbol{\eta}_{2}'\ ...\ \boldsymbol{\eta}_{K}']'$, it is useful to re-write the log-likelihood as:

$$\begin{aligned}\ell(\mathbf{y}_{1},...,\mathbf{y}_{T}|\boldsymbol{\delta}) &\propto -\frac{1}{2}\sum_{s=1}^{K}\ln|\boldsymbol{\Omega}_{s}|\sum_{t=2}^{T}p(S_{t+1};\boldsymbol{\lambda}^{l})-\frac{1}{2}\sum_{s=1}^{K}\boldsymbol{\eta}_{s}'(\tilde{\boldsymbol{\Sigma}}_{s}\otimes\boldsymbol{\Omega}_{s}^{-1})\boldsymbol{\eta}_{s}\\ &= -\frac{1}{2}\sum_{s=1}^{K}\ln|\boldsymbol{\Omega}_{s}|\sum_{t=2}^{T}p(S_{t+1};\boldsymbol{\lambda}^{l})-\frac{1}{2}\boldsymbol{\eta}'\mathbf{W}^{-1}\boldsymbol{\eta}\end{aligned}$$

where

$$\mathbf{Z}\equiv\begin{bmatrix}\mathbf{Z}_{1}\\ \mathbf{Z}_{2}\\ \vdots\\ \mathbf{Z}_{K}\end{bmatrix};\quad \mathbf{Z}_{i}\equiv\begin{bmatrix}[\mathbf{e}_{i}'\ \mathbf{e}_{i}'\otimes\mathbf{y}_{1}']\otimes\mathbf{I}_{N}\\ [\mathbf{e}_{i}'\ \mathbf{e}_{i}'\otimes\mathbf{y}_{2}']\otimes\mathbf{I}_{N}\\ \vdots\\ [\mathbf{e}_{i}'\ \mathbf{e}_{i}'\otimes\mathbf{y}_{T-1}']\otimes\mathbf{I}_{N}\end{bmatrix};\quad \mathbf{W}^{-1}\equiv\begin{bmatrix}\tilde{\boldsymbol{\Sigma}}_{1}\otimes\boldsymbol{\Omega}_{1}^{-1} & \cdots & \mathbf{O}\\ \vdots & \ddots & \vdots\\ \mathbf{O} & \cdots & \tilde{\boldsymbol{\Sigma}}_{K}\otimes\boldsymbol{\Omega}_{K}^{-1}\end{bmatrix}$$

$$\tilde{\boldsymbol{\Sigma}}_{i}\equiv diag\{p(s_{2}=i;\boldsymbol{\lambda}^{l}),p(s_{3}=i;\boldsymbol{\lambda}^{l}),...,p(s_{T}=i;\boldsymbol{\lambda}^{l})\}.$$

The EM updating equation for the transition probabilities is based on the smoothed state probabilities and can be found in equation (4.1) of Hamilton (1990, p. 51). Filtered state probabilities are calculated as a by-product. The first order conditions for the mean and variance parameters, $\boldsymbol{\phi}$ and $\boldsymbol{\Omega}$, are

$$\frac{\partial \ln \ell(\mathbf{y}_t|\boldsymbol{\delta})}{\partial \boldsymbol{\phi}} = -\frac{1}{2}\hat{\boldsymbol{\eta}}'\hat{\mathbf{W}}^{-1}\mathbf{Z} = \mathbf{0} \tag{A8}$$

$$\frac{\partial \ln \ell(\mathbf{y}_t|\boldsymbol{\delta})}{\partial \boldsymbol{\Omega}_s} = -\frac{1}{2}\sum_{t=1}^{T} p(S_{t+1}=s;\boldsymbol{\lambda}^l)\hat{\boldsymbol{\Omega}}_s^{-1} + \frac{1}{2}\hat{\boldsymbol{\Omega}}_s^{-1}\hat{\boldsymbol{\varepsilon}}_s'\tilde{\boldsymbol{\Sigma}}_s\hat{\boldsymbol{\varepsilon}}_s\hat{\boldsymbol{\Omega}}_s^{-1} = \mathbf{O} \quad s=1,2,...,K, \tag{A9}$$

where $\hat{\boldsymbol{\varepsilon}}_s \equiv [(\mathbf{y}_2 - \mathbf{Z}_{s_2=i}\hat{\boldsymbol{\phi}})' \ (\mathbf{y}_3 - \mathbf{Z}_{s_3=i}\hat{\boldsymbol{\phi}})' \ ... \ (\mathbf{y}_T - \mathbf{Z}_{s_T=i}\hat{\boldsymbol{\phi}})']'$ are the residuals in state s and $\hat{\mathbf{W}}^{-1}$ is a function of $\{\hat{\boldsymbol{\Omega}}_s\}_{s=1}^K$. Equation (A8) implies that $\hat{\boldsymbol{\phi}}^{l+1}$ is a GLS estimator once observations are replaced by their smoothed probability-weighted counterparts:

$$\hat{\boldsymbol{\phi}}^{l+1} = (\mathbf{Z}'\hat{\mathbf{W}}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\hat{\mathbf{W}}^{-1}(\boldsymbol{\iota}_{\mathbf{k}} \otimes \mathbf{y}). \tag{A10}$$

Similarly, (A9) implies a covariance estimator

$$\hat{\boldsymbol{\Omega}}_s = \frac{\hat{\boldsymbol{\varepsilon}}_s'\tilde{\boldsymbol{\Sigma}}_s\hat{\boldsymbol{\varepsilon}}_s}{\sum_{t=1}^T p(S_{t+1};\boldsymbol{\lambda}^l)}. \tag{A11}$$

$\hat{\boldsymbol{\phi}}^{l+1}$ and $\{\hat{\boldsymbol{\Omega}}_s^{l+1}\}_{s=1}^K$ must be solved for jointly since $\hat{\boldsymbol{\varepsilon}}_s$ enters the expression for the covariance matrix and also depends on $\hat{\boldsymbol{\phi}}^{l+1}$, while the regime-dependent covariance matrices $\{\hat{\boldsymbol{\Omega}}_s^{l+1}\}_{s=1}^K$ enter (A10) via $\hat{\mathbf{W}}^{-1}$. Hence, within each step of the EM algorithm, (A10)-(A11) is iterated upon until convergence of the estimates $\hat{\boldsymbol{\phi}}^{l+1}$ and $\{\hat{\boldsymbol{\Omega}}_s^{l+1}\}_{s=1}^K$.

Finally, notice that (A8) defines $\hat{\boldsymbol{\eta}}$ from

$$\boldsymbol{\eta}_{t+1} \equiv \mathbf{y}_{t+1} - \tilde{\boldsymbol{\mu}}_{S_{t+1}} - \mathbf{B}_{s_{t+1}}\mathbf{y}_t = \mathbf{y}_{t+1} - \boldsymbol{\mu}_{S_{t+1}} - \mathbf{M}_{S_t}vec(\boldsymbol{\Upsilon}_{S_{t+1}}) - \mathbf{B}_{s_{t+1}}\mathbf{y}_t,$$

so that $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$ enters $\mathbf{M}_{S_t}vec(\boldsymbol{\Upsilon}_l)$, while $\mathbf{M}_{S_t}vec(\boldsymbol{\Upsilon}_l)$ also affects $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$, creating a non-linear system of simultaneous equations. For instance, computing $Cov[\mathbf{x}_{t+1}, x_{t+1}^W|\mathcal{F}_t, S_t]$ requires knowledge of the first I elements of $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$. To make estimation possible, within the $(l+1)$th step of the EM algorithm we use an iterative scheme by which $\mathbf{M}_{S_t}vec(\boldsymbol{\Upsilon}_l)$ is first estimated using the values in $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$ from the previous optimization step, $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t; \hat{\phi}^l]$. New values of $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t; \hat{\phi}^{l+1}]$ are then computed using estimates of $\mathbf{M}_{S_t}vec(\boldsymbol{\Upsilon}_l)$ that employ $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t; \hat{\phi}^l]$. We then proceed iteratively until convergence.

Appendix B: Proof of Proposition 1

This appendix first uses a simple example to introduce our solution technique, then derives Proposition 1 and shows how to extend the results to include autoregressive terms in the return process.

B.1 Two-state example with a single risky asset

To gain intuition we first study the asset allocation problem under the simplifying assumption of a single risky asset ($I = 1$), a risk-free asset paying a constant return r_f and a regime switching process with two states:

$$\begin{aligned} x_{t+1} &= \mu_{S_{t+1}} + \sigma_{S_{t+1}}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0,1), \\ \Pr(S_{t+1} &= k|S_t = k) = p_{kk}, \ k = 1,2 \end{aligned} \tag{B1}$$

This specification is consistent with the ICAPM analysis in section 1 since the conditional moment information from (6) can be folded into $\{\mu_{S_{t+1}}, \sigma_{S_{t+1}}\}$ as described in Section 1.

With a single risky asset (stocks) and initial wealth set at unity, the wealth process is

$$W_{t+T} = \{(1-\omega_t)\exp(Tr_f) + \omega_t \exp(R_{t+T})\}, \tag{B2}$$

where R_{t+T} is the continuously compounded stock return over the T periods and ω_t is the stock holding. For a given value of ω_t, the only unknown component in (B2) is the cumulated return, $\exp(R_{t+T})$. Under the assumption of two states, $K = 2$, the nth non-central moment of the cumulated returns is given by

$$\begin{aligned} M^{(n)}_{t+T} &= E\left[(\exp(r_{t+1} + ... + r_{t+T}))^n |\mathcal{F}_t\right] \\ &= \sum_{S_{t+T}=1}^{2} E\left[(\exp(r_{t+1} + ... + r_{t+T}))^n |S_{t+T}, \mathcal{F}_t\right] \Pr(S_{t+T}|\mathcal{F}_t) \\ &\equiv M^{(n)}_{1t+T} + M^{(n)}_{2t+T}, \end{aligned} \tag{B3}$$

where $r_t \equiv x_t + r_f$. Using properties of the moment generating function of a log-normal random variable, each of these conditional moments $M^{(n)}_{kt+1}$ $(k = 1,2)$ satisfies recursions

$$\begin{aligned} M^{(n)}_{kt+T} &= E\left[\exp(n(r_{t+1} + ... + r_{t+T-1}))|S_{t+T}\right] E\left[\exp(nr_{t+T})|S_{t+T}, \mathcal{F}_t\right] \Pr(S_{t+T}|\mathcal{F}_t) \\ &= \left(M^{(n)}_{kt+T-1}p_{kk} + M^{(n)}_{-k,t+T-1}(1-p_{-k-k})\right)\exp\left(n\mu_k + \frac{n^2}{2}\sigma_k^2\right), \quad (k = 1,2) \end{aligned}$$

where we used the notation $-k$ for the converse of state k, i.e. $-k = 2$ when $k = 1$ and vice versa. In more compact notation we have

$$\begin{aligned} M^{(n)}_{1t+1} &= \xi^{(n)}_1 M^{(n)}_{1t} + \beta^{(n)}_1 M^{(n)}_{2t} \\ M^{(n)}_{2t+1} &= \xi^{(n)}_2 M^{(n)}_{1t} + \beta^{(n)}_2 M^{(n)}_{2t}, \end{aligned} \tag{B4}$$

where

$$\xi^{(n)}_1 = p_{11}\exp\left(n\mu_1 + \tfrac{n^2}{2}\sigma_1^2\right), \qquad \beta^{(n)}_1 = (1-p_{22})\exp\left(n\mu_1 + \tfrac{n^2}{2}\sigma_1^2\right),$$
$$\xi^{(n)}_2 = (1-p_{11})\exp\left(n\mu_2 + \tfrac{n^2}{2}\sigma_2^2\right), \qquad \beta^{(n)}_2 = p_{22}\exp\left(n\mu_2 + \tfrac{n^2}{2}\sigma_2^2\right).$$

Equation (B4) can be reduced to a set of second order difference equations:

$$M^{(n)}_{it+2} = (\xi^{(n)}_1 + \beta^{(n)}_2)M^{(n)}_{it+1} + (\xi^{(n)}_2\beta^{(n)}_1 - \beta^{(n)}_2\alpha^{(n)}_1)M^{(n)}_{it}, \quad (i = 1,2). \tag{B5}$$

Collecting the two regime-dependent moments in the vector $\boldsymbol{\vartheta}_{it+T}^{(n)} \equiv (M_{it+T}^{(n)}\ M_{it+T-1}^{(n)})'$, equation (B5) can be written in companion form:

$$\boldsymbol{\vartheta}_{it+T}^{(n)} = \begin{bmatrix} \xi_1^{(n)} + \beta_2^{(n)} & \xi_2^{(n)}\beta_1^{(n)} - \beta_{21}^{(n)}\xi^{(n)} \\ 1 & 0 \end{bmatrix} \boldsymbol{\vartheta}_{it+T-1}^{(n)} \equiv \mathbf{A}^{(n)}\boldsymbol{\vartheta}_{it+T-1}^{(n)}.$$

The elements of $\mathbf{A}^{(n)}$ only depend on the mean and variance parameters of the two states (μ_1, σ_1^2, μ_2, σ_2^2) and the state transitions, (p_{11}, p_{22}). Substituting backwards, we get the ith conditional moment:

$$\boldsymbol{\vartheta}_{it+T}^{(n)} = \left(\mathbf{A}^{(n)}\right)^T \boldsymbol{\vartheta}_{it}^{(n)}.$$

Applying similar principles at $T = 1, 2$ and letting $\pi_{1t} = \Pr(S_t = 1|\mathcal{F}_t)$, the initial conditions used in determining the nth moment of cumulated returns are as follows:

$$\begin{aligned}
M_{1t+1}^{(n)} &= (\pi_{1t}p_{11} + (1-\pi_{1t})(1-p_{22}))\exp\left(n\mu_1 + \frac{n^2}{2}\sigma_1^2\right), \\
M_{1t+2}^{(n)} &= p_{11}\left(\pi_{1t}p_{11} + (1-\pi_{1t})(1-p_{22})\right)\exp\left(2n\mu_1 + n^2\sigma_1^2\right) + \\
&\quad +(1-p_{22})\left(\pi_{1t}(1-p_{11}) + (1-\pi_1)p_{22}\right)\exp\left(n(\mu_1+\mu_2) + \frac{n^2}{2}(\sigma_1^2+\sigma_2^2)\right), \\
M_{2t+1}^{(n)} &= (\pi_{1t}(1-p_{11}) + (1-\pi_1)p_{22})\exp\left(n\mu_2 + \frac{n^2}{2}\sigma_2^2\right), \\
M_{2t+2}^{(n)} &= p_{22}\left(\pi_{1t}(1-p_{11}) + (1-\pi_1)p_{22}\right)\exp\left(2n\mu_2 + n^2\sigma_2^2\right) + \\
&\quad +(1-p_{11})\left(\pi_{1t}p_{11} + (1-\pi_{1t})(1-p_{22})\right)\exp\left(n(\mu_1+\mu_2) + \frac{n^2}{2}(\sigma_1^2+\sigma_2^2)\right). \qquad \text{(B6)}
\end{aligned}$$

Finally, using (B3) we get an equation for the nth moment of the cumulated return:

$$M_{t+T}^{(n)} = M_{1t+T}^{(n)} + M_{2t+T}^{(n)} = \mathbf{e}_1'\boldsymbol{\vartheta}_{1t+T}^{(n)} + \mathbf{e}_2'\boldsymbol{\vartheta}_{2t+T}^{(n)} = \mathbf{e}_1'\left(\mathbf{A}^{(n)}\right)^T\boldsymbol{\vartheta}_{1t}^{(n)} + \mathbf{e}_2'\left(\mathbf{A}^{(n)}\right)^T\boldsymbol{\vartheta}_{2t}^{(n)}, \qquad \text{(B7)}$$

where $\mathbf{e}_i$ is a 2×1 vector of zeros except for unity in the ith place.

Having obtained the moments of the cumulated return process, it is simple to compute the expected utility for any mth order polynomial representation by using (13) in the main text and (B2):

$$\begin{aligned}
\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] &= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}\binom{n}{j}E_t[W_{t+T}^j] \\
&= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}\binom{n}{j}\sum_{i=0}^{j}\binom{j}{i}\omega_t^i M_{t+T}^i\left((1-\omega_t)\exp\left(Tr_f\right)\right)^{j-i}. \qquad \text{(B8)}
\end{aligned}$$

The first order condition is obtained by differentiating this equation with respect to ω_t :

$$\sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}\binom{n}{j}\sum_{i=1}^{j}\binom{j}{i}\omega_t^{i-1}(1-\omega_t)^{j-i-1}M_{t+T}^i\exp\left((j-i)Tr_f\right)(i-j\omega_t) = 0.$$

The solution takes the form of the roots of an $m-1$ order polynomial in ω_t, which are easily obtained. The optimal solution for ω_t corresponds to the root for which (B8) has the highest value.

B.2 General Results

To derive the n-th moment of the cumulated return on the risky asset holdings in the general case with multiple risky assets (I) and states (K), notice that

$$E_t\left[\left(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}^s_{t+T}\right)\right)^n\right] = E_t\Bigg[\sum_{n_1=1}^{n}...\sum_{n_I=1}^{n}\lambda(n_1,n_2,...,n_I)(\omega_1^{n_1}\times..\times\omega_I^{n_I})\times \exp\left(\sum_{\tau=1}^{T}r^1_{t+\tau}\right)^{n_1}....\times\exp\left(\sum_{\tau=1}^{T}r^I_{t+\tau}\right)^{n_I}\Bigg]. \quad \text{(B9)}$$

where the powers $0\leq n_i\leq n$ $(i=1,...,I)$ satisfy the summing-up constraint $\sum_{i=1}^{I}n_i=n$, and the coefficients λ are given by

$$\lambda(n_1,n_2,...,n_I)\equiv\frac{n!}{n_1!n_2!\;...\;n_I!}.$$

$r^i_{t+\tau}$ $(i=1,...,I)$ represent the return on asset i in period $t+\tau$, $r^i_{t+\tau}\equiv x^i_{t+\tau}+r^b_{t+\tau}$. The sum in (B9) involves $\binom{I+n-1}{n}$ terms and requires solving for moments of the form

$$\begin{aligned} M^{(n)}_{t+T}(n_1,n_2,...,n_I) &= E_t\left[\exp\left(\sum_{\tau=1}^{T}r^1_{t+\tau}\right)^{n_1}\times....\times\exp\left(\sum_{\tau=1}^{T}r^I_{t+\tau}\right)^{n_I}\right] \\ &= E_t\left[\exp\left(\sum_{i=1}^{I}n_i\sum_{\tau=1}^{T}r^i_{t+\tau}\right)\right]. \end{aligned} \quad \text{(B10)}$$

(B10) can be decomposed as follows

$$M^{(n)}_{t+T}(n_1,n_2,...,n_I)=\sum_{k=1}^{K}M^{(n)}_{k,t+T}(n_1,n_2,...,n_I), \quad \text{(B11)}$$

where for $k=1,...,K$,

$$M^{(n)}_{k,t+T}(n_1,n_2,...,n_I)=E_t\left[\exp\left(\sum_{i=1}^{I}n_i\sum_{\tau=1}^{T}r^i_{t+\tau}\right)|S_{t+T}=k\right]\Pr(S_{t+T}=k).$$

Each of these terms satisfies the recursions

$$\begin{aligned} M^{(n)}_{k,t+T}(n_1,n_2,...,n_I) &= \sum_{j=1}^{K}M^{(n)}_{j,t+T-1}(n_1,n_2,...,n_I)E_t\left[\exp\left(\sum_{i=1}^{I}n_ir^i_{t+T}\right)|S_{t+T}=k,\mathcal{F}_t\right]p_{jk} \\ &= \sum_{j=1}^{K}p_{jk}M^{(n)}_{j,t+T-1}(n_1,n_2,...,n_I)\exp\left(\sum_{i=1}^{I}n_i\tilde{\mu}_{ki}+\sum_{i=1}^{I}\sum_{u=1}^{I}n_in_u\frac{\sigma_{k,iu}}{2}\right), \end{aligned} \quad \text{(B12)}$$

where $\tilde{\mu}_{ki}$ is the mean return of asset i in state k (inclusive of risk premia related to covariance, co-skewness and co-kurtosis) and $\sigma_{k,iu}=e_i'\Omega_ke_u$ is the covariance between r_{it+T} and r_{ut+T} in state $k=1,\,2,...,\,K$. This is a generalization of the result in (B4).

Finally, using (B9) and (B10), we get an expression for the $n-$th moment of the cumulated return:

$$E_t\left[\left(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}^s_{t+T}\right)\right)^n\right]=\sum_{n_1=0}^{n}\cdots\sum_{n_I=0}^{n}\lambda(n_1,n_2,...,n_I)(\omega_1^{n_1}\times...\times\omega_n^{n_I})M^{(n)}_{t+T}(n_1,...,n_I). \quad \text{(B13)}$$

Expected utility can now be evaluated in a straightforward generalization of (B8):

$$\begin{aligned}\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] &= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}{}_nC_jE_t[W_{t+T}^j]\\ &= \sum_{n=0}^{m}\kappa_n\sum_{j=0}^{n}(-1)^{n-j}v_T^{n-j}\binom{n}{j}\sum_{i=0}^{j}\binom{j}{i}E_t[\left(\boldsymbol{\omega}_t'\exp\left(\mathbf{R}_{t+T}^s\right)\right)^i]\left((1\text{-}\boldsymbol{\omega}_t'\boldsymbol{\iota}_h)\exp\left(Tr^f\right)\right)^{j-i}.\end{aligned}$$

Inserting (B13) into this expression gives a first order condition that takes the form of an $m-1$th order polynomial in the portfolio weights.

Generalizing the results to include autoregressive terms is straightforward. To keep the notation simple, suppose $k=2$. Using (15) in the main text the n-th noncentral moment satisfies the recursions

$$\begin{aligned}M_{k,t+T}^{(n)} &= M_{k,t+T-1}^{(n)}(n)p_{kk}\exp\left(n\tilde{\mu}_k+n\sum_{j=1}^{p}b_{j,k}E_t[r_{t+T-j}]+\frac{n^2}{2}\sigma_k^2\right)+\\ &\quad +M_{-k,t+T-1}^{(n)}(n)(1-p_{-k-k})\exp\left(n\tilde{\mu}_k+n\sum_{j=1}^{p}b_{j,k}E_t[r_{t+T-j}]+\frac{n^2}{2}\sigma_k^2\right)\end{aligned}$$

or

$$\begin{aligned}M_{1,t+1}^{(n)} &= \tilde{\xi}_1^{(n)}M_{1,t}^{(n)}+\tilde{\beta}_1^{(n)}M_{2,t}^{(n)}\\ M_{2,t+1}^{(n)} &= \tilde{\xi}_2^{(n)}M_{1,t}+\tilde{\beta}_2^{(n)}M_{2,t}^{(n)},\end{aligned}$$

where now

$$\begin{aligned}\tilde{\xi}_1^{(n)} &= p_{11}\exp\left(n\mu_1+n\sum_{j=1}^{p}b_{j,1}E_t[r_{t+T-j}]+\frac{n^2}{2}\sigma_1^2\right)\\ \tilde{\beta}_1^{(n)} &= (1-p_{22})\exp\left(n\mu_1+n\sum_{j=1}^{p}b_{j,1}E_t[r_{t+T-j}]+\frac{n^2}{2}\sigma_1^2\right)\\ \\ \tilde{\xi}_2^{(n)} &= (1-p_{11})\exp\left(n\mu_2+n\sum_{j=1}^{p}b_{j,2}E_t[r_{t+T-j}]+\frac{n^2}{2}\sigma_2^2\right)\\ \tilde{\beta}_2^{(n)} &= p_{22}\exp\left(n\mu_2+n\sum_{j=1}^{p}b_{j,2}E_t[r_{t+T-j}]+\frac{n^2}{2}\sigma_2^2\right).\end{aligned}$$

Subject to these changes, the earlier methods can be used with the only difference that terms such as $\exp\left(n\tilde{\mu}_k+\frac{n^2}{2}\sigma_k^2\right)$ have to be replaced by

$$\exp\left(n\tilde{\mu}_k+n\sum_{j=1}^{p}b_{j,k}E_t[r_{t+T-j}]+\frac{n^2}{2}\sigma_k^2\right).$$

The term $\sum_{j=1}^{p} b_{j,k} E_t[r_{t+T-j}]$ may be decomposed in the following way:

$$\sum_{j=1}^{p} b_{j,k} E_t[r_{t+T-j}] = \mathcal{I}_{\{j>T\}} \sum_{j=1}^{p} \left(\mathcal{I}_{\{j\geq T\}} b_{j,k} r_{t+T-j} + I_{\{j<T\}} b_{j,k} E_t[r_{t+T-j}] \right),$$

where $\mathcal{I}$ is an indicator function and $E_t[r_{t+1}]$, ..., $E_t[r_{t+T-1}]$ can be evaluated recursively, see Doan et al. (1984):

$$\begin{aligned}
E_t[r_{t+1}] &= \pi_{1t} \left(\tilde{\mu}_1 + \sum_{j=1}^{p} b_{j,1} r_{t-j} \right) + (1 - \pi_{1t}) \left(\tilde{\mu}_2 + \sum_{j=1}^{p} b_{j,2} r_{t-j} \right) \\
E_t[r_{t+2}] &= \boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_1 \left(\tilde{\mu}_1 + \sum_{j=1}^{p} b_{j,1} E_t[r_{t+1}] \right) + (1 - \boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_1) \left(\tilde{\mu}_2 + \sum_{j=1}^{p} b_{j,2} E_t[r_{t+1}] \right) \\
&\vdots \\
E_t[r_{t+T-1}] &= \boldsymbol{\pi}_t' \mathbf{P}^{T-1} \mathbf{e}_1 \left(\tilde{\mu}_1 + \sum_{j=1}^{p} b_{j,1} E_t[r_{t+T-2}] \right) + (1 - \boldsymbol{\pi}_t' \mathbf{P}^{T-1} \mathbf{e}_1) \left(\tilde{\mu}_2 + \sum_{j=1}^{p} b_{j,2} E_t[r_{t+T-2}] \right).
\end{aligned}$$

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Table 1

Summary Statistics for International Stock Returns

This table reports sample statistics for six international MSCI portfolios. Returns are monthly, denominated in US dollars, include dividends and are in excess of the 1-month US T-bill rate. The sample period is 1975:01 – 2005:12. Jarque-Bera is a test for normality based on the skew and kurtosis. Ljung-Box and Ljung-Box squares denote tests for fourth order serial correlation in returns and squared returns, respectively.

Portfolio	Mean	St. Dev.	Skewness	Kurtosis	Jarque-Bera	Ljung-Box	Ljung-Box squares
MSCI United States	0.5415	4.4825	-0.7084	5.9138	162.71**	1.8775	2.4714
MSCI Japan	0.3733	6.4830	0.0700	3.5044	4.2475	6.5087	11.888*
MSCI Pacific ex-Japan	0.3892	7.0538	-2.2723	22.297	5655.6**	2.7472	0.4998
MSCI Europe ex-UK	0.4158	5.0578	-0.5672	4.6124	60.246**	5.9087	12.560*
MSCI United Kingdom	0.7503	6.1898	0.7587	10.316	865.27**	4.1915	19.845**
MSCI World	0.4560	5.1740	-0.8711	6.9133	282.88**	2.3197	1.9827
US 1-month T-bills	0.4906	0.2517	0.8319	3.9949	58.250**	1248.2**	1084.5**

* denotes significance at the 5% level; ** denotes significance at the 1% level.

Table 2

Parameter Estimates for Single State and Two-State Models

Panel A reports parameter estimates for the extended single state ICAPM

$$x^{i}_{t+1} = \alpha^{i} + \gamma^{1} Cov_t[x^{i}_{t+1}, x^{W}_{t+1}] + b^{i} r^{US}_{t} + \eta^{i}_{t+1}$$
$$x^{W}_{t+1} = \alpha^{W} + \gamma^{1} Var_t[x^{W}_{t+1}] + b^{W} r^{US}_{t} + \eta^{W}_{t+1}$$
$$r^{US}_{t+1} = \alpha^{Z} + b^{Z} r^{US}_{t} + \eta^{Z}_{t+1}$$

where x^i_t and x^W_t consist of monthly excess returns on the MSCI stock index portfolios (in US dollars), *i* = US, Japan, Asia-Pacific (ex-Japan), United Kingdom, and Europe (ex-UK), '*W*' stands for the world market portfolio, and r^{US}_t is the 1-month US T-bill rate. Panel B of the table reports maximum likelihood estimates for the two-state regime switching model:

$$x^{i}_{t+1} = \alpha^{i}_{S_{t+1}} + \gamma^{1}_{S_{t+1}} Cov_t[x^{i}_{t+1}, x^{W}_{t+1}] + \gamma^{2}_{S_{t+1}} Cov_t[x^{i}_{t+1}, (x^{W}_{t+1})^2] + \gamma^{3}_{S_{t+1}} Cov_t[x^{i}_{t+1}, (x^{W}_{t+1})^3] + b^{1}_{S_{t+1}} r^{US}_{t} + \eta^{i}_{t+1}$$
$$x^{W}_{t+1} = \alpha^{W}_{S_{t+1}} + \gamma^{1}_{S_{t+1}} Var_t[x^{W}_{t+1}] + \gamma^{2}_{S_{t+1}} Skew_t[x^{W}_{t+1}] + \gamma^{3}_{S_{t+1}} K_t[x^{W}_{t+1}] + b^{W}_{S_{t+1}} r^{US}_{t} + \eta^{W}_{t+1}$$
$$r^{US}_{t+1} = \alpha^{Z}_{S_{t+1}} + b^{Z}_{S_{t+1}} r^{US}_{t} + \eta^{Z}_{t+1}$$

where $\boldsymbol{\eta}_{t+1} \equiv [\eta^{US}_{t+1} \;\; \eta^{Jap}_{t+1} \;\; \eta^{Pac}_{t+1} \;\; \eta^{UK}_{t+1} \;\; \eta^{EU}_{t+1}] \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}})$ is the vector of unpredictable return innovations with regime-specific (heteroskedastic) variances but constant correlations across states. The coefficients b^i_S and b^W_S are set to zero in the first regime. $\text{Var}_t[x^W_{t+1}]$, $\text{Skew}_t[x^W_{t+1}]$ and $K_t[x^W_{t+1}]$ are the conditional variance, skew and kurtosis of excess returns on the world portfolio. Risk premium estimates are reported per unit of covariance, skewness, and kurtosis scaled by the appropriate powers (1, 1.5, and 2) of the variance of excess returns on the world market portfolio. For the covariance matrix we report monthly volatilities on the main diagonal and correlations off the diagonal. Standard errors are reported in parentheses below parameter estimates. The sample period is 1975:01 – 2005:12.

	U.S.	Japan	Pacific ex-Japan	Europe ex-UK	United Kingdom	World	U.S. T-bill
Panel A – Single State Gaussian Model							
Cross-sectional risk premia							
Covariance (γ_1)	5.303 (2.574)						
1. Intercepts (α's)	0.463 (0.632)	0.369 (0.691)	1.076 (0.673)	0.726 (0.651)	0.659 (0.652)	0.302 (1.027)	0.022 (0.580)
2. VAR coeffs.							
U.S. T-bill	-0.655 (0.324)	-0.866 (0.523)	-2.500 (1.047)	-1.472 (1.045)	-1.055 (1.059)	-0.843 (0.577)	0.955 (0.105)
3. Volatilities	14.916***	22.112***	23.106***	17.237***	19.903***	14.131***	0.249***
4. Correlations							
U.S.	1						
Japan	0.308**	1					
Pacific ex-Japan	0.540***	0.368**	1				
Europe ex-UK	0.587***	0.476**	0.538***	1			
United Kingdom	0.534***	0.397**	0.555***	0.632***	1		
World	0.845***	0.684***	0.628***	0.790***	0.699***	1	
U.S. T-bill	-0.103	-0.014	-0.131*	-0.046	-0.093	-0.102	1

Table 2 (continued)

Estimates of a Two-State Regime Switching Model

	Panel B – Two State Model						
	Cross-sectional risk premia						
	Bear State				**Bull State**		
Covariance ($\gamma_{1,S_{t+1}}$)	9.460 (5.114)				15.874 (5.088)		
Co-skewness ($\gamma_{2,S_{t+1}}$)	-1.077 (1.050)				-3.111 (1.266)		
Co-kurtosis ($\gamma_{3,S_{t+1}}$)	1.669 (2.898)				12.302 (5.048)		
	U.S.	Japan	Pacific ex-Japan	Europe ex-UK	United Kingdom	World	U.S. T-bill
1. Intercepts (α's)							
Bear State	-0.591 (0.323)	-1.756 (0.402)	-0.723 (0.393)	-0.720 (0.339)	-0.776 (0.360)	-0.968 (0.313)	0.0002 (0.0002)
Bull State	1.079 (0.191)	1.621 (0.263)	0.813 (0.274)	0.867 (0.220)	1.218 (0.297)	1.186 (0.236)	0.077 (0.032)
2. VAR coeffs.							
Bear State:							
U.S. T-bill	—	—	—	—	—	—	0.999 (0.064)
Bull State:							
U.S. T-bill	-0.588 (0.039)	-1.118 (0.565)	-1.143 (0.588)	-0.814 (0.475)	-0.527 (0.321)	-0.902 (0.373)	0.864 (0.180)
3. Volatilities							
Bear State	10.883***	15.729***	17.864***	12.242***	11.231***	10.263***	0.439***
Bull State	9.165***	13.340***	13.785***	11.155***	15.065***	8.480***	0.546***
4. Correlations							
U.S.	1						
Japan	0.276*	1					
Pacific ex-Japan	0.552***	0.361**	1				
Europe ex-UK	0.619***	0.447**	0.561***	1			
United Kingdom	0.594***	0.393**	0.645***	0.716***	1		
World	0.836***	0.670***	0.639***	0.773***	0.750***	1	
U.S. T-bill	-0.079	-0.122	-0.115	-0.114	-0.097	-0.154	1

4. Transition probabilities	Bear State	Bull State
Bear State	0.899 (0.205)	0.1011
Bull State	0.0590	0.941 (0.146)

*denotes significance at the 10% level, ** significance at the 5% level and *** at the 1% level.

Table 3

Optimal Portfolio Weights Under the Single-State and Two-State Models

Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio. Allocations are computed under interest rates that can deviate by up to two standard deviations from their mean.

Panel		Mean – 2 x SD.	Mean – 1 x SD.	Mean	Mean + 1 x SD.	Mean + 2 x SD.
Panel A - T = 1 month				**Single State Benchmark**		
	United States	0.228	0.346	0.313	0.156	0.101
	Japan	0.261	0.309	0.375	0.375	0.499
	Pacific (ex-Japan)	0.348	0.222	0.104	0.000	0.000
	United Kingdom	0.130	0.099	0.042	0.000	0.000
	Europe (ex-UK)	0.033	0.025	0.167	0.469	0.400
	US T-bills	0.082	0.191	0.515	0.681	0.800
				Bear State (π = 1)		
	United States	0.697	0.722	0.746	0.772	0.796
	Japan	0.121	0.093	0.063	0.070	0.093
	Pacific (ex-Japan)	0.061	0.056	0.048	0.035	0.037
	United Kingdom	0.030	0.056	0.048	0.070	0.074
	Europe (ex-UK)	0.091	0.074	0.095	0.053	0.000
	US T-bills	0.668	0.462	0.370	0.431	0.460
				Steady-state probs. (π = 0.33)		
	United States	0.625	0.696	0.685	0.817	0.851
	Japan	0.125	0.101	0.110	0.070	0.060
	Pacific (ex-Japan)	0.063	0.072	0.082	0.070	0.060
	United Kingdom	0.016	0.000	0.000	0.000	0.000
	Europe (ex-UK)	0.172	0.130	0.123	0.042	0.030
	US T-bills	0.357	0.312	0.269	0.289	0.333
				Bull State (π = 0)		
	United States	0.535	0.537	0.598	0.656	0.713
	Japan	0.198	0.116	0.098	0.086	0.085
	Pacific (ex-Japan)	0.116	0.074	0.054	0.043	0.032
	United Kingdom	0.023	0.021	0.022	0.011	0.011
	Europe (ex-UK)	0.128	0.253	0.228	0.204	0.160
	US T-bills	0.142	0.053	0.078	0.070	0.062
Panel B - T = 24 months				**Single State Benchmark**		
	United States	0.310	0.367	0.286	0.000	0.000
	Japan	0.310	0.304	0.381	0.306	0.367
	Pacific (ex-Japan)	0.230	0.177	0.190	0.000	0.000
	United Kingdom	0.149	0.152	0.095	0.056	0.000
	Europe (ex-UK)	0.000	0.000	0.048	0.639	0.633
	US T-bills	0.132	0.208	0.578	0.639	0.698
				Bear State (π = 1)		
	United States	0.595	0.603	0.623	0.618	0.597
	Japan	0.139	0.141	0.130	0.118	0.125
	Pacific (ex-Japan)	0.089	0.090	0.091	0.079	0.069
	United Kingdom	0.127	0.128	0.117	0.118	0.125
	Europe (ex-UK)	0.051	0.038	0.039	0.066	0.083
	US T-bills	0.208	0.22	0.226	0.240	0.278
				Steady-state probs. (π = 0.33)		
	United States	0.590	0.593	0.635	0.627	0.622
	Japan	0.108	0.105	0.094	0.108	0.110
	Pacific (ex-Japan)	0.048	0.058	0.059	0.072	0.085
	United Kingdom	0.084	0.081	0.071	0.060	0.061
	Europe (ex-UK)	0.169	0.163	0.141	0.133	0.122
	US T-bills	0.168	0.142	0.149	0.168	0.179
				Bull State (π = 0)		
	United States	0.565	0.596	0.640	0.655	0.678
	Japan	0.087	0.090	0.093	0.080	0.080
	Pacific (ex-Japan)	0.054	0.056	0.058	0.057	0.046
	United Kingdom	0.054	0.045	0.023	0.023	0.034
	Europe (ex-UK)	0.239	0.213	0.186	0.184	0.161
	US T-bills	0.083	0.109	0.140	0.132	0.131

Table 4

Estimates of Co-Skew and Co-Kurtosis Coefficients with World Market Portfolio

This table reports sample co-skew and co-kurtosis coefficients for returns on individual market portfolios (i) versus the world market portfolio (w),

$$S_{i,W} \equiv \frac{Cov[x_t^i (x_t^W)^2 \mid \Im_t]}{\{Var[x_t^i \mid \Im_t]\}^{1/2} Var[x_t^W \mid \Im_t]}$$

$$K_{i,W} \equiv \frac{Cov[x_t^i (x_t^W)^3 \mid \Im_t]}{\{Var[x_t^i \mid \Im_t]\}^{1/2} \{(Var[x_t^W \mid \Im_t])^3\}^{1/2}} \quad (i = \text{US, JP, Pac, UK, EU})$$

We also show the coefficients implied by a two-state regime switching model:

$$x_{t+1}^i = \alpha_{S_{t+1}}^i + \gamma_{S_{t+1}}^1 Cov_t[x_{t+1}^i, x_{t+1}^W] + \gamma_{S_{t+1}}^2 Cov_t[x_{t+1}^i, (x_{t+1}^W)^2] + \gamma_{S_{t+1}}^3 Cov_t[x_{t+1}^i, (x_{t+1}^W)^3] + b_{S_{t+1}}^1 r_t^{US} + \eta_{t+1}^i \quad i = 1,\dots,5$$

$$x_{t+1}^W = \alpha_{S_{t+1}}^W + \gamma_{S_{t+1}}^1 Var_t[x_{t+1}^W] + \gamma_{S_{t+1}}^2 Skew_t[x_{t+1}^W] + \gamma_{S_{t+1}}^3 K_t[x_{t+1}^W] + b_{S_{t+1}}^W r_t^{US} + \eta_{t+1}^W$$

$$r_{t+1}^{US} = \alpha_{S_{t+1}}^Z + b_{S_{t+1}}^Z r_t^{US} + \eta_{t+1}^Z$$

The coefficients are calculated both conditional on the current state and under steady state probabilities

		Bear state	**Bull state**	**Steady-state probs.**	**Data**
United States	Co-skew	0.151	-0.127	-0.128	-0.052
	Co-kurtosis	3.200	3.434	3.408	3.401
Japan	Co-skew	0.018	-0.001	0.016	0.004
	Co-kurtosis	2.207	2.294	3.303	3.428
Pacific ex-Japan	Co-skew	-0.161	-0.567	-0.677	-0.535
	Co-kurtosis	4.522	5.782	6.561	6.704
United Kingdom	Co-skew	-0.066	-0.252	-0.339	-0.321
	Co-kurtosis	5.297	5.207	5.230	4.910
Europe ex-UK	Co-skew	0.114	-0.167	-0.222	-0.227
	Co-kurtosis	4.192	4.095	4.116	4.113

Table 5

Effects of Preferences (*m)* on Portfolio Choice

This table reports the optimal allocation to international stocks as a function of the state probability for three choices of the order of the preference polynomial, *m*: *m*=2 (mean-variance preferences), *m* = 3 (three-moment or skew preferences), and *m* = 4 (four-moment or skew and kurtosis preferences). T is the investment horizon. Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio.

	m	U.S.	Japan	Pacific ex-Japan	UK	EU	US T-bills
Bear State (π = 1)							
T=1	*m* = 2	0.661	0.143	0.036	0.036	0.125	0.441
	m = 3	0.841	0.079	0.016	0.063	0.000	0.373
	m = 4	0.746	0.063	0.048	0.048	0.095	0.370
T=6	*m* = 2	0.778	0.032	0.016	0.000	0.175	0.369
	m = 3	0.721	0.088	0.000	0.162	0.029	0.320
	m = 4	0.653	0.153	0.056	0.056	0.083	0.282
T=24	*m* = 2	0.594	0.000	0.058	0.000	0.348	0.309
	m = 3	0.550	0.163	0.013	0.200	0.075	0.201
	m = 4	0.623	0.130	0.091	0.117	0.039	0.226
Steady-state state probs. (π = 0.33)							
T=1	*m* = 2	0.536	0.014	0.014	0.000	0.435	0.310
	m = 3	0.521	0.366	0.000	0.113	0.000	0.289
	m = 4	0.685	0.110	0.082	0.000	0.123	0.269
T=6	*m* = 2	0.500	0.000	0.056	0.000	0.444	0.282
	m = 3	0.532	0.286	0.000	0.169	0.013	0.231
	m = 4	0.646	0.127	0.076	0.051	0.101	0.209
T=24	*m* = 2	0.525	0.000	0.050	0.013	0.413	0.198
	m = 3	0.519	0.210	0.012	0.247	0.012	0.190
	m = 4	0.635	0.094	0.059	0.071	0.141	0.149
Bull State (π = 0)							
T=1	*m* = 2	0.262	0.299	0.020	0.181	0.238	0.000
	m = 3	0.131	0.446	0.002	0.408	0.013	0.000
	m = 4	0.598	0.098	0.054	0.022	0.228	0.078
T=6	*m* = 2	0.189	0.232	0.042	0.147	0.389	0.048
	m = 3	0.215	0.398	0.000	0.366	0.022	0.069
	m = 4	0.632	0.092	0.057	0.023	0.195	0.125
T=24	*m* = 2	0.427	0.012	0.049	0.049	0.463	0.180
	m = 3	0.422	0.241	0.012	0.301	0.024	0.171
	m = 4	0.640	0.093	0.058	0.023	0.186	0.140

Table 6

Confidence Bands for Portfolio Weights

The table reports simulated confidence bands for optimal portfolio weights under either a two-state regime switching model or a single-state model. The weights are calculated assuming the 1-month US T-bill rate is set at its mean. The weight on the world market portfolio is re-allocated to the five regional portfolios using their relative market capitalizations as of 2005:12. T is the investment horizon. Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio..

	T = 1 month		**T = 6 months**		**T = 24 months**	
	5% Lower Bound	**95% Upper Bound**	**5% Lower Bound**	**95% Upper Bound**	**5% Lower Bound**	**95% Upper Bound**
			Single-State Model			
United States	0.024	0.379	0.000	0.371	0.000	0.350
Japan	0.088	0.469	0.075	0.451	0.137	0.404
Pacific (ex-Japan)	0.000	0.217	0.000	0.240	0.002	0.259
United Kingdom	0.000	0.314	0.000	0.331	0.000	0.414
Europe (ex-UK)	0.000	0.320	0.000	0.307	0.000	0.272
US T-bills	0.418	0.861	0.423	0.897	0.475	0.937
			Two-State Model			
			Bear Regime (π = 1)			
United States	0.586	0.834	0.438	0.964	0.416	0.845
Japan	0.037	0.095	0.045	0.270	0.029	0.196
Pacific (ex-Japan)	0.033	0.065	0.000	0.114	0.006	0.137
United Kingdom	0.020	0.081	0.000	0.150	0.012	0.184
Europe (ex-UK)	0.013	0.220	0.000	0.382	0.000	0.201
US T-bills	0.311	0.381	0.201	0.440	0.138	0.343
			Steady-state probs. (π = 0.33)			
United States	0.636	0.727	0.606	0.665	0.504	0.691
Japan	0.090	0.127	0.107	0.134	0.060	0.108
Pacific (ex-Japan)	0.070	0.093	0.052	0.067	0.038	0.067
United Kingdom	0.004	0.015	0.029	0.059	0.037	0.083
Europe (ex-UK)	0.065	0.225	0.064	0.207	0.114	0.267
US T-bills	0.256	0.289	0.209	0.216	0.047	0.194
			Bull Regime (π = 0)			
United States	0.484	0.744	0.578	0.674	0.518	0.736
Japan	0.015	0.145	0.073	0.106	0.060	0.113
Pacific (ex-Japan)	0.082	0.118	0.079	0.098	0.069	0.109
United Kingdom	0.000	0.047	0.000	0.027	0.000	0.036
Europe (ex-UK)	0.000	0.407	0.000	0.308	0.000	0.317
US T-bills	0.090	0.226	0.124	0.142	0.050	0.230

Table 7

Out-of-Sample Portfolio Performance

The table reports summary statistics for realized utility (using four-moment preferences) and (annualized) portfolio returns based on the portfolio weights associated with the recursive estimates of a two-state regime switching model, a single-state VAR(1) model, and a static ICAPM in which all international portfolios are bought in proportion to their weight in the world market portfolio. Asset allocations across international equity markets are calculated for two investment horizons, T = 1 month and T = 24 months. The weight on the world market portfolio is re-allocated to the five regional portfolios using their relative market capitalization. SD denotes standard deviations; the CEV is the annualized percentage certainty equivalent of a given mean realized utility. 'Equal weights' is a portfolio that assigns equal weight to all international equity portfolios such that the holdings in 1-month US T-bills matches those from the two-state model. Panel A reports portfolio performance from a simulation experiment in which the data generating process is the two-state regime switching model of Table 2. Panel B uses actual MSCI returns data from the sample period 1986:01 - 2005:12.

	Panel A - Simulated Data									
	T=1 month					**T=24 months**				
	Realized Utility			**Annualized Returns**		**Realized Utility**			**Annualized Returns**	
	Mean	**SD**	**CEV**	**Mean**	**SD**	**Mean**	**SD**	**CEV**	**Mean**	**SD**
Two-state RS	-0.987	0.021	16.42	16.77	7.28	-0.722	0.108	17.69	18.79	12.59
VAR(1)	-0.992	0.017	9.89	11.35	5.89	-0.799	0.070	11.89	12.38	7.99
ICAPM	-0.989	0.011	14.22	14.03	4.16	-0.764	0.094	14.42	15.15	10.11
Equal weights	-0.991	0.015	11.95	12.68	5.54	-0.802	0.066	11.63	12.03	7.50
	Panel B - Actual Data									
	T=1 month					**T=24 months**				
	Realized Utility			**Annualized Returns**		**Realized Utility**			**Annualized Returns**	
	Mean	**SD**	**CEV**	**Mean**	**SD**	**Mean**	**SD**	**CEV**	**Mean**	**SD**
Two-state RS	-0.993	0.029	8.22	8.73	10.05	-0.849	0.158	8.54	10.09	13.08
VAR(1)	-0.995	0.022	6.18	8.72	7.62	-0.872	0.103	7.09	7.89	9.69
ICAPM	-0.994	0.039	7.44	11.35	12.82	-0.850	0.223	7.72	11.45	16.76
Equal weights	-0.994	0.031	7.63	10.03	10.74	-0.849	0.154	7.33	8.72	12.30

Figure 1

Bear state probabilities

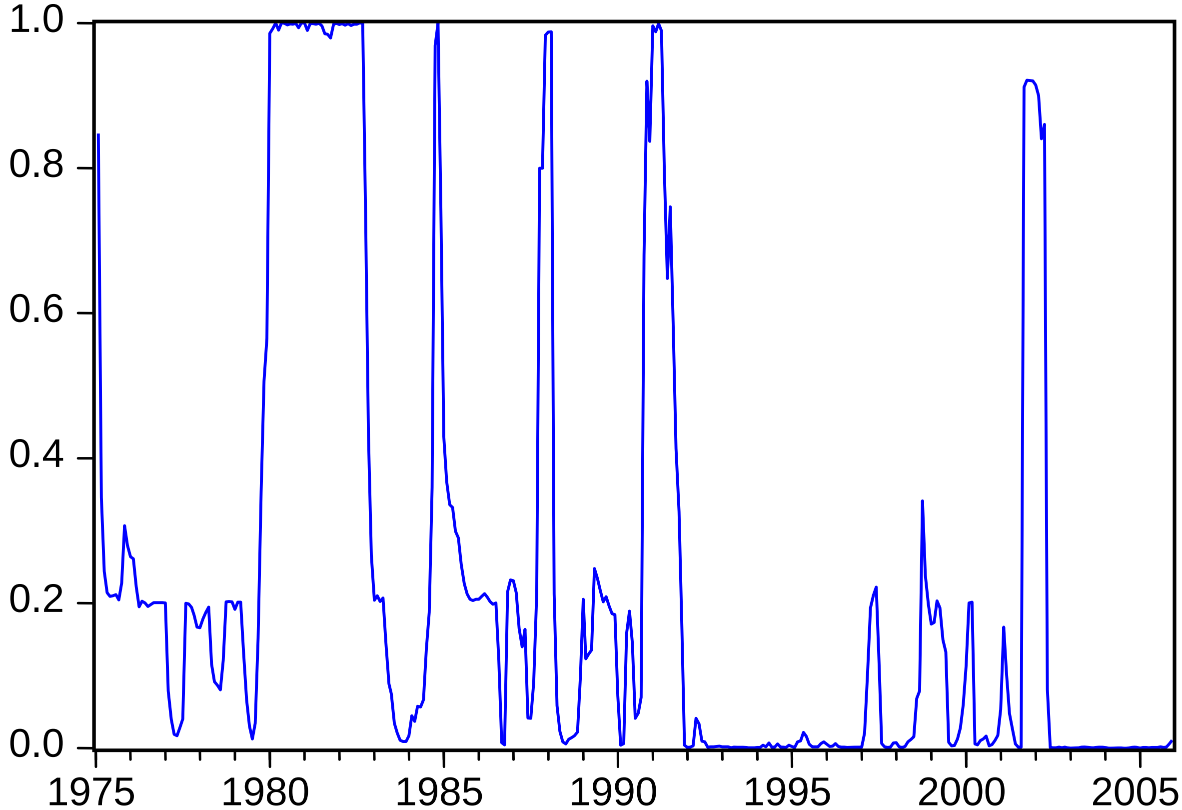
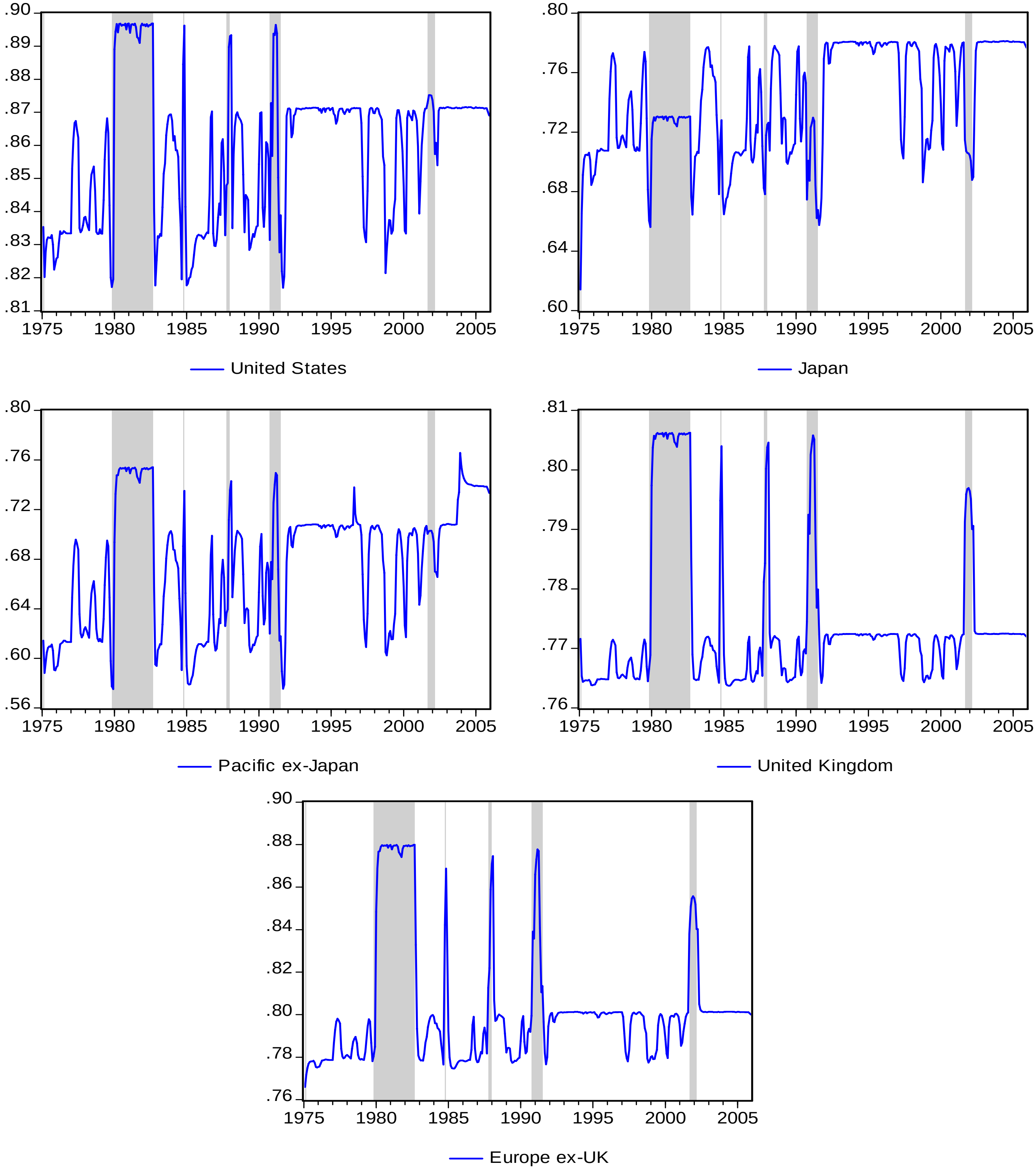


Figure 2

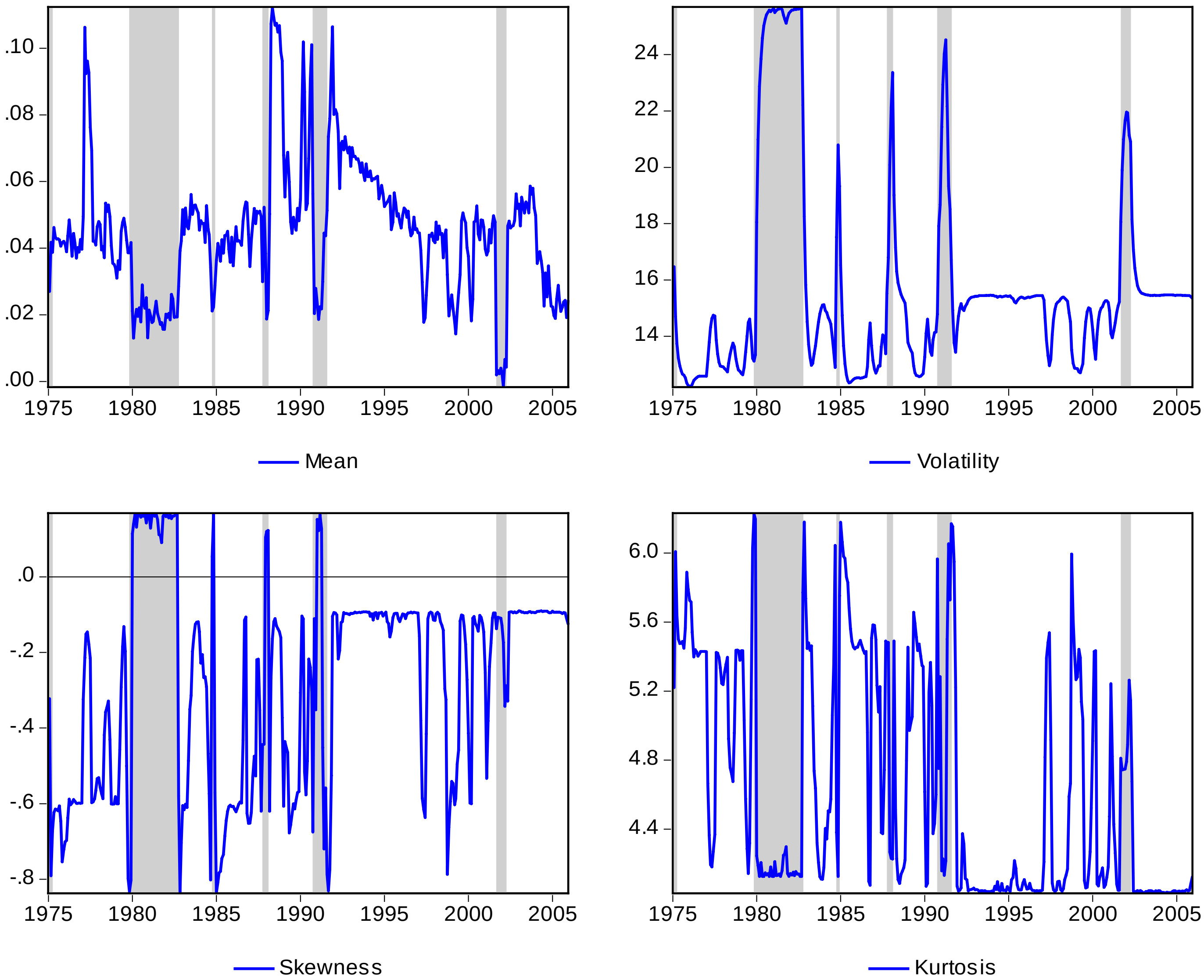
Correlations between world market and regional market returns periods



Note: Periods where the global bear state is most likely are shown in grey shades.

Figure 3

Mean excess returns, volatility, skew and kurtosis of the world market portfolio implied by the two-state model (annualized figures)



Note: Periods where the global bear state is most likely are shown in grey shades.