Coordination of Price Promotions in Complementary Categories

Maxim Sinitzyn
Department of Economics, University of California, San Diego
September 2012

Abstract

In this paper, I investigate the outcome of a price competition between two firms, each producing two complementary products. Specifically, I study each firm’s decision to coordinate price promotions of its products. Consumers are divided into loyalists, who purchase both products from their preferred firm, and heterogeneous switchers, who choose between four possible bundles or buy a product in a single category. The switchers are willing to pay some price premium in order to purchase two complementary products that share the same brand name and are produced by the same firm, because they believe that these products are a better match than two complementary products with different brand names. I find that each firm predominantly promotes its complementary products together. This finding is correlationally supported by data in the shampoo and conditioner and in the cake mix and cake frosting categories.

Key words: price promotions; complementary products; heterogeneous consumers

1 Introduction

When a firm produces goods in several related categories, it has to take into account cross-category price effects in order to develop an optimal pricing strategy. Several recent empirical studies contributed to developing the framework for estimating these cross-category effects at the category level (Chintagunta and Haldar 1998, Manchanda et al. 1999, Russell and Petersen 2000, Duvvuri et al. 2007, Song and Chintagunta 2007, Niraj et. al. 2008, Sriram et al. 2010) and at the brand level (Wedel and Zhang 2004, Song and Chintagunta 2006, Mehta 2007, Ma et. al.
2011). Using estimated cross-category elasticities, it is possible to find the single optimal price for each product (Song and Chintagunta 2006) or to compute the total effect of a price promotion on profits (Manchanda et al. 1999, Duvvuri et al. 2007, Niraj et al. 2008). However, there exists little theoretical work that analyzes firms’ optimal strategies involving price promotions in related categories. The current paper fills this gap by introducing a theoretical model that predicts that complementary products that share the same brand name and are produced by the same firm go on sale at the same time. This prediction finds correlational empirical support in two pairs of complementary categories: shampoo/conditioner and cake mix/cake frosting.

In my model, there are two firms, each selling products in two complementary categories. Following Varian (1980) and Narasimhan (1988), I model price promotions (sales) as mixed strategies in price competition between these firms. There are two types of consumers: loyalists and switchers. The loyal customers buy both products from their preferred firm. The switchers mix and match between the brands offered by the two firms or buy a product in only one category. They choose the single product or the bundle that offers them the highest utility. Simester (1997) studied a similar model but assumed that the switchers purchase both products from the same firm. I allow the switchers to purchase two complementary products from different firms, but assume that they are willing to pay a price premium in order to purchase products that share the same brand name.\footnote{While the source of this consumer behavior is the complementarity in consumption, the firms, when setting their prices, care about the purchase complementarity. In this paper, I assume that the consumption complementarity leads to the purchase complementarity.} This assumption reflects a consumer belief that the complementary products carrying the same brand name are a better match. Often, firms themselves contribute to this consumer perception by placing product advertisements on the labels of their complementary products. For example, Pantene Pro-V shampoo and conditioner bottle labels encourage "For best results, try other products in our collection." The last step in the recipe on the back of a Betty Crocker cake mix box is "Frost with Betty Crocker Creamy Deluxe or Whipped Frosting," and the writing on the lid of a can of BEHR paint states "For Lifetime Guarantee, always use BEHR PRIMERS".

I further assume that the switchers have heterogeneous tastes for the different single products and bundles, and I model their demand as logit—the popular demand specification in the empirical literature. Sinitzhyn (2008a) showed that for such demands, the support of the mixed strategy Nash
Equilibrium in prices consists of a finite number of points. Subsequent research in the area of price promotions with heterogeneous consumers (Sinitsyn 2008b and 2009) outlined the techniques for studying these equilibria in various models of price competition in single categories. Here, I extend these methods to allow for competition in multiple categories. I compute the equilibria for various values of the parameters of the demand function and find that, while the firms sometimes use the price pairs in which one product goes on sale while the complementary product does not, these occurrences are rare. Most probability is assigned to the price pairs in which both products are discounted. This is the central result of the theoretical section–price promotions of complementary products are synchronized.

Next, I examine supermarket price data on the brands in three pairs of complementary categories. In two of the three pairs, shampoo/conditioner and cake mix/cake frosting, the firms use the same brand names for their products in complementary categories, and I expect that consumers gain additional utility from purchasing the complementary products sharing the same brand name. Correlationally consistent with the theory, I find that, in these category pairs, the firms often put their complementary products on sale together, and I reject the hypothesis that their promotions are independent. In another complementary products pair, detergent/fabric softener, the top two firms, Procter & Gamble and Unilever, use different brand names for the products in different categories. Hence, it is likely that, in the absence of matching brand names, consumers do not gain extra utility from purchasing the products produced by the same firm. Correspondingly, I find that there is almost no coordination of sales of detergents and fabric softeners–there is evidence to reject the hypothesis of independent price promotions in only one out of the fourteen possible brand pairs present in the data.

2 Evidence of Brand Matching Preferences

In this section, I will provide some empirical and experimental evidence for the assumption that the switching consumers are willing to pay a price premium in order to purchase two complementary products that share the same brand name and are produced by the same firm. I model this preference by explicitly adding the premium to the consumer’s utility for a bundle with matching brand names. This approach was recently employed by Ma et al. (2012), who esti-
mate a multi-category brand-choice model for cake mixes/cake frostings in which they include coefficients that capture the brand-level complementarity between brands in different categories. They find that two out of three brands (Betty Crocker and Duncan Hines) have the strongest brand-level complementarity with their umbrella brand in the complementary category. For the third brand, Pillsbury, Ma et al. (2012) find a large positive correlation between the error terms for cake mix and frosting, which means that, if a household makes a purchase of Pillsbury cake mix for some unobserved reason, it is also more likely to buy Pillsbury frosting. Simonin and Ruth (1995) conducted an experiment in which subjects evaluated bundles consisting of a tube of toothpaste and a complementary dental hygiene product, either sharing the toothpaste’s brand or not. Their results suggest that the subjects placed a premium on within-brand bundles. Mulhern and Leone (1991) and Walters (1991) estimated cross-category effects for cake mix/cake frosting categories and found "stronger complementary relationships for items that have the same brand name" (Mulhern and Leone 1991, p. 72).

I chose to supplement the evidence provided by the above studies by running an experiment with the undergraduate students at an east coast university. Sixty-one subjects answered a questionnaire with three types of questions. The questions were designed to evaluate their preferences for matching between two brands of cake mix and cake frosting: Betty Crocker (BC) and Duncan Hines (DH).

Endowment questions presented a promotion of the type “Buy one package of cake mix, get Duncan Hines frosting free,” then offered the choice of, in this example, BC or DH cake mix at equal price. Each subject was asked four endowment questions, corresponding to each of the four products. Any consumer preferences in which brand preference in each category does not vary in response to the brand chosen in the complementary category would manifest as a 50% matching rate. Yet the participants chose matching bundles 71% of the time, with the rates of matching varying from 62% to 80% among the four products—each greater than 50%, significant at the .05 level. Additionally, classifying the respondents according to the type of preferences revealed, I find that “matchers” are the modal group, comprising 42% of respondents, and that 27% of the respondents are loyal to one of the two brands. These data provide evidence validating the assumption of co-existence of loyal consumers and switchers, which is crucial to the model
presented in this paper.

Valuation questions asked subjects to provide a valuation on a 1 to 9 scale for each of the four individual products, and each of the four possible cake mix/cake frosting bundles. Using these valuations, I conducted a conjoint analysis, estimating an individual’s valuation for each bundle as a function of component product valuations and bundle-specific fixed effects. I find that the coefficient for matching brands is positive and significant at the .001 level. Additionally, by comparing the estimated value of the “matching brand” coefficient, 0.59, to the mean bundle valuation of 5.53, I find that the additional utility consumers derive from brand-level complementarity is substantial.

Discrete choice questions presented subjects with a series of purchase scenarios, in which each of the four products was listed at either regular price or sale price, and asked them to select the cake mix and cake frosting they would purchase given those prices. I modeled their decision process as multinomial logit, consistent with the empirical literature and with the theoretical model presented in this paper. Regressing the respondents’ bundle choices on the price of each bundle and the components of the bundle, I find that the coefficient for matching brands is positive and significant at the .001 level, as in the conjoint analysis. It is likely, however, that the presence of the brand-loyal consumers who choose a same-brand bundle with little or no price elasticity upwardly biases the estimated coefficient for brand complementarity. Therefore, I conducted a series of robustness checks: excluding the loyals identified using discrete choice responses, excluding the loyals or including only the matchers identified using endowment responses, and adding the component valuations as regressors. The finding of a positive and significant coefficient for brand complementarity is robust to all of these modifications. In addition, modeling the consumers’ decision process using a bivariate probit model yields qualitatively similar results, which withstand all of the same robustness checks. The discrete choice data, therefore, provide reliable evidence of consumer preference for matching brands across complementary categories, consistent with the results obtained from the conjoint and endowment choice analyses.\(^2\)

\(^2\) Although none of the experimental or empirical analyses can characterize consumer reasoning underlying brand-level complementarity, some anecdotal evidence can be found in online forums where visitors discuss their reasons for matching their brands of shampoo and conditioner. Some examples of individuals’ reasoning are: “I usually match...but I think it’s just because I like things to be uniform,” and “I prefer mine to match, as well. I feel like
3 Simple Model

Before presenting my main model that relies on numerical solutions, in this section I will present a simple, analytically tractable model that outlines the basic mechanism of price promotions in complementary categories. Assume that there are two firms, each producing two complementary products. The set of consumers has measure one, and each consumer purchases both complementary products. The firms can either charge a regular price normalized to 1 or a sale price \( p < 1 \) for each product. Thus, there are four possible price pairs available to the firms: \((p; p)\), \((p; 1)\), \((1; p)\), and \((1; 1)\). The firms are symmetric, so if the prices are identical in both markets, each firm gets 0.5 of the consumers in each market. The following variables describe the additional portion of the market a firm can gain if it runs a price promotion.

I begin by considering the case where, in one category, a firm charges the sale price \( p \) while its rival charges 1, and the prices of the two firms are identical within the other category. The firm running a promotion in the first category gets \( s_P \) consumers in this category (the subscript \( P \) denotes the primary effect), where \( 0.5 < s_P \leq 1 \). When only one firm offers a discount in one category and the prices are identical in the other category, there is a spillover effect of the promotion on the purchase behavior in the other category. Some of the \( s_P \) consumers who purchased the product on sale in the first category would like to match it with the same brand in the second category, even though the prices in that category are identical. Therefore, a firm running a sale in the first category will also gain additional \( s_D \) consumers in the second category (the subscript \( D \) denotes the secondary effect). Note that \( s_P \) measures the total amount of consumers purchasing the product on sale \( (s_P > 0.5) \) while \( s_D \) measures the additional (over 0.5) amount of consumers gained in the second category. Restricting \( s_D \) to be less than \( s_P - 0.5 \) guarantees that the secondary effect is not larger than the primary effect, allowing for the presence of some consumers who do not care about matching the brands.

For simplicity, I also assume that, if a firm puts both products on sale, \((p; p)\), while the rival charges regular prices, \((1; 1)\), then the firm which offers promotions gets \( s_P \) in both categories.

\[ \text{it must work best as a pair, instead of using different brands and types.} \] From the forum comments, it seems that consumers prefer to match the brands of shampoo and conditioner either because of the belief that they work better together, or because of the desire to have a uniform bundle.
Also, if each firm promotes one product in different categories, there are no spillover effects—each firm gets $s_P$ consumers in the category in which it offers a promotion and $1 - s_P$ in the category in which its rival offers a promotion. The table below summarizes the outcomes and serves as a payoff matrix for the $4 \times 4$ game between the two firms.

<table>
<thead>
<tr>
<th></th>
<th>$(p; p)$</th>
<th>$(p; 1)$</th>
<th>$(1; p)$</th>
<th>$(1; 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p; p)$</td>
<td>$p$; $p$</td>
<td>$ps_p + p(0.5 + s_D)$;</td>
<td>$ps_p + p(0.5 + s_D)$;</td>
<td>$2ps_p$;</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$(p(0.5 - s_D) + (1 - s_p))$;</td>
<td>$(0.5 - s_D) + (1 - s_p)$</td>
<td>$2(1 - s_p)$</td>
</tr>
<tr>
<td>$(p; 1)$</td>
<td>$p(0.5 - s_D) + (1 - s_p)$;</td>
<td>$0.5p + 0.5$;</td>
<td>$ps_p + (1 - s_p)$;</td>
<td>$ps_p + (1.5 + s_D)$;</td>
</tr>
<tr>
<td></td>
<td>$ps_p + p(0.5 + s_D)$</td>
<td>$0.5p + 0.5$</td>
<td>$ps_p + (1 - s_p)$</td>
<td>$(1 - s_p) + (0.5 - s_D)$</td>
</tr>
<tr>
<td>$(1; p)$</td>
<td>$p(0.5 - s_D) + (1 - s_p)$;</td>
<td>$ps_p + (1 - s_p)$;</td>
<td>$0.5p + 0.5$;</td>
<td>$ps_p + (0.5 + s_D)$;</td>
</tr>
<tr>
<td></td>
<td>$ps_p + p(0.5 + s_D)$</td>
<td>$ps_p + (1 - s_p)$</td>
<td>$0.5p + 0.5$</td>
<td>$(1 - s_p) + (0.5 - s_D)$</td>
</tr>
<tr>
<td>$(1; 1)$</td>
<td>$2(1 - s_p)$;</td>
<td>$(1 - s_p) + (0.5 - s_D)$;</td>
<td>$(1 - s_p) + (0.5 - s_D)$;</td>
<td>1;</td>
</tr>
<tr>
<td></td>
<td>$2ps_p$</td>
<td>$ps_p + (0.5 + s_D)$</td>
<td>$ps_p + (0.5 + s_D)$</td>
<td>1</td>
</tr>
</tbody>
</table>

I will search for the mixed strategy equilibrium in this game that puts positive weights $\theta$, $\beta$, $\eta$, and $\lambda$ on strategies $(p; p)$, $(p; 1)$, $(1; p)$, and $(1; 1)$. I will also find the range of parameters of the model for which the promotions are correlated. To check for the correlation of promotions, I employ the following method. For each firm, the promotion of the first product occurs with probability $\theta + \beta$. The promotion of the second product occurs with probability $\eta$. If the promotion decisions were independent in these categories, the joint promotions of the two complementary products would occur with probability $(\theta + \beta)(\eta + \theta)$. The actual probability of a joint promotion is $\theta$. Thus, if $\theta > (\theta + \beta)(\theta + \eta)$, then the promotions are positively correlated. The following proposition describes the conditions under which this happens.$^3$

**Proposition 1** There exists a mixed strategy Nash Equilibrium in which the promotions are positively correlated iff $s_p < \frac{1}{1 + p^2}$ and $s_D > \frac{1 - s_p(1 + p^2)}{p}$.

If the first condition of Proposition 1 holds, that is, if $s_p < \frac{1}{1 + p^2}$, then $s_p(1 + p^2) < 1$. Thus, the second condition of Proposition 1 requires $s_D$ to be a positive number. This means that price promotions are positively correlated only if there is a cross-category spillover effect of a promotion.

$^3$All proofs are in Appendix A.
As \( s_p \) approaches \( \frac{1}{1+p^2} \) from below, \( \frac{1-s_p(1+p^2)}{p} \) (the lower bound on \( s_D \)) decreases and converges to zero. Then, for \( s_p \) sufficiently close to \( \frac{1}{1+p^2} > 0.5 \), there always exists \( s_D \) that satisfies the assumption that the secondary effect is smaller than the primary effect (\( s_D < s_p - 0.5 \)). Thus, for any discount price \( p \), there always exist parameters \( s_p \) and \( s_D \) such that the promotions are positively correlated.

The following proposition shows that the promotions become more coordinated when cross-category effect \( s_D \) increases.

**Proposition 2** The degree of correlation between promotions of complementary products \( \left( \frac{\theta}{(\theta+\beta)(\theta+\eta)} \right) \) increases with the strength of cross-category effect \( s_D \).

The intuition for this result is as follows. Assume that both firms use the same mixed strategy, and, therefore, are indifferent between using all four price pairs, including \( (p; p) \) and \( (1; 1) \). Holding everything else constant, an increase in \( s_D \) increases the profitability of a price promotion, since more consumers are captured in the complementary category. The expected profit from using \( (p; p) \) increases and the expected profit from using \( (1; 1) \) decreases. In order to restore the mixed strategy equilibrium, the payoff from \( (p; p) \) has to decrease and/or the payoff from \( (1; 1) \) has to increase. The expected payoff from using \( (p; p) \) increases with \( s_D \) because using \( (p; p) \) against the rival who uses \( (p; 1) \) or \( (1; p) \) becomes more profitable (the payoff from using \( (p; p) \) against the rival who uses \( (p; p) \) or \( (1; 1) \) does not change). Therefore, if the rival decreases the probability of using \( (p; 1) \) and \( (1; p) \), the expected profit from using \( (p; p) \) will go down. Similarly, the expected payoff from using \( (1; 1) \) decreases with \( s_D \) since using \( (1; 1) \) against the rival who uses \( (p; 1) \) or \( (1; p) \) becomes less profitable (the payoff from using \( (1; 1) \) against the rival who uses \( (p; p) \) or \( (1; 1) \) does not change). Thus, if the rival decreases the probability of using \( (p; 1) \) and \( (1; p) \), the expected profit from using \( (1; 1) \) will go up. Hence, the mixed strategy equilibrium is restored if some probability is shifted from \( (p; 1) \) and \( (1; p) \) toward \( (p; p) \) and \( (1; 1) \). Such a shift causes the promotions of complementary products to become more correlated.

The advantages of the model presented above are that it is simple, has a closed-form solution, and does not rely on functional form assumptions about consumer demand. A major disadvantage of this model, however, is that the sale price \( p \) is exogenous. In reality, the retailers would respond to changes in \( s_D \) not only by shifting the probabilities, but also by adjusting the promotion
depth. In addition, in this model the retailers are restricted to using the same sale price for
the single category discounts and for joint discounts, while, in reality, the retailers might choose
to use different discounts, depending on whether they run promotions in one category or in
both categories. In order to make the sale price endogenous, I have to specify how consumers
react to price changes, that is, I have to specify the demand function. Even very simple demand
specifications render the model analytically intractable. Since I have to proceed by using numerical
solutions, I choose to work with the logit demand specification that is often used in the empirical
literature to estimate demand for complementary products.

4 Full Model

There are two complementary product categories. Within each category, there are two brands
produced by firms A and B. The firms have constant marginal cost $c$ and compete in prices.
The highest price the firms can charge for any of their products is the reservation price $r$—the
maximum price any consumer is willing to pay. The rescaling of prices $p \to \frac{p-c}{r-c}$ normalizes the
marginal cost to zero and the reservation price to 1.

The set of consumers has measure 1. The consumers are divided into loyals and switchers.
Each firm has a share $\alpha$ of loyal consumers—these consumers buy both complementary products
from their preferred firm, provided that the price of each of the products is less than or equal to
1. The remaining $1 - 2\alpha$ consumers are the switchers. They can either buy one product or buy a
bundle. The switchers then have heterogeneous tastes for the four single products: $A0, B0, 0A,$
and $0B$ (0 denotes a no-purchase in the corresponding category) and for the four possible bundles:
$AA, AB, BA,$ and $BB$. The switching consumer $s$ has the following utility from purchasing a
bundle $ij$ (either $i$ or $j$ can be 0):

$$U_{sij} = \delta_{i,j} + \delta_{2,j} - p_{1,i} - p_{2,j} + d_c * I(i,j \neq 0) + d_b * I(i = j) + \varepsilon_{sij},$$  \hspace{1cm} (1)

where $\delta_{k,i}$ is the base utility of product $i$ in category $k$, $p_{k,f}$ ($k = 1, 2; f = A, B$) is firm $f$’s price
for its product in category $k$ ($p_{k,0} = 0$), $d_c$ is the additional utility the consumers gain from pur-
chasing products in complementary categories, $d_b$ is the additional utility the consumers gain from
purchasing the same brands in complementary categories, and $\varepsilon_{sij}$ are independently and iden-
tically Gumbel distributed with scale parameter $\mu$ ($\mu$ is the degree of consumer heterogeneity\textsuperscript{4}). Including an additional term $d_c$ in the utility function to capture the effect of complementarity at the category level is a standard specification in the empirical literature (Russell and Petersen 2000, Song and Chintagunta 2006, Gentzkow 2007, Niraj et al. 2008, Sriram et al. 2010). In addition to having category-level complementarity $d_c$, I also allow for complementarity at the brand-level by introducing $d_b$ in the utility function. The justification for this assumption is presented in Section 2 of the paper.

The set-up of the full model is similar to the bundling model of Matutes and Regibeau (1992). In their model, the firms are able to bundle their products, that is, to offer a discount to the consumers who purchase both products from one firm. The effect of bundling is equivalent to having $d_b$ in (1), except that the firms can choose a bundle discount, while, in my model, $d_b$ is exogenous.\textsuperscript{5} Matutes and Regibeau (1992) do not have loyal consumers; thus, the equilibrium in their model is in pure strategies. The presence of loyal consumers in my model leads to mixed strategies and allows for analysis of price promotions.

The utility from (1) gives rise to the following logit choice probability of purchasing a bundle $ij$:

$$
P_{ij} = \frac{\exp((\delta_{1,i} + \delta_{2,j} - p_{1,i} - p_{2,j} + d_c * I(i, j) \neq 0) + d_b * I(i = j) + \varepsilon_{sij})/\mu)}{\sum_{f_1, f_2 \in \{A, B, 0\}} \exp((\delta_{1,f_1} + \delta_{2,f_2} - p_{1,f_1} - p_{2,f_2} + d_c * I(f_1, f_2) \neq 0) + d_b * I(f_1 = f_2))/\mu)}
$$

(2)

Then, the profit function of firm $i$ consists of profit from the loyalists, $\alpha(p_{1,i} + p_{2,i})$, profit from the switchers who bought both products from firm $i$, $(1 - 2\alpha)(p_{1,i} + p_{2,i})P_{ii}$, and profit from the switchers who bought only one of the products from firm $i$, $(1 - 2\alpha)p_{1,i}(P_{ij} + P_{0i})$ and $(1 - 2\alpha)p_{2,i}(P_{ji} + P_{0i})$.

In order to find a pure strategy equilibrium, it is necessary to differentiate the profit functions

\textsuperscript{4}$\varepsilon_{sij}$ reflects the unobserved consumer preferences. A larger variance of $\varepsilon_{sij}$ implies more dispersed consumer tastes, and, hence, a greater consumer heterogeneity. The scale parameter of the Gumbel distribution, $\mu$, is proportional to the variance; thus, $\mu$ is also a measure of consumer heterogeneity.

\textsuperscript{5}To keep the analysis simple, I don’t let the firms offer bundle discounts, but the model can easily be extended to allow for them. Balachander et al. (2010) analyze a model in which they allow for both price promotions and bundle discounts in an unrelated pair of categories.
of both firms with respect to the two prices each firm sets and then solve the resulting system of four equations with four unknowns. However, as I will show in the following section, when consumer heterogeneity \( \mu \) is low enough, a pure strategy equilibrium does not exist, so it is necessary to search for the mixed strategy equilibria.

The demand functions and the profit functions are analytic. Sinitzyn (2008a) showed that, for price competition with analytic demands, the support of the mixed strategy equilibrium is finite.\(^6\) This means that the equilibrium strategy of firm \( i \) involves charging price vectors \( \{p_{1,i}^n, p_{2,i}^n\}_{n=1}^{N_i} \) with corresponding probabilities \( \{\gamma_i^n\}_{n=1}^{N_i} \), where \( N_i \) is the number of price vectors firm \( i \) uses. The system of equations, the solution to which is an equilibrium price distribution with finite support, is standard. It includes the first order conditions—the profit function must be maximized at each point of the support. In addition, the profits at all the points of the support of the price distribution have to be equal to each other. It is impossible to solve this system analytically, thus, I solve it numerically. Sinitzyn (2008b) and Sinitzyn (2009) provide insights on the solution procedure for the case when price has one dimension. These methods naturally extend to the current case with two-dimensional price. In the next section, I analyze the equilibrium strategies of the firms.

5 Equilibrium Strategies

The firms’ equilibrium strategies depend on the parameters of the demand function—percentage of loyal consumers \( \alpha \), consumer heterogeneity \( \mu \), category-level complementarity \( d_c \), brand-level complementarity \( d_b \), and base utility levels \( \delta_{k,i} \). To illustrate the structure of the equilibria in this model, I will fix \( \alpha = 0.25, \) \( d_c = d_b = 0.4, \) \( \delta_{k,i} = 0 \) \((k \in \{1, 2\}, i \in \{A,B,0\})\),\(^7\) and examine how

---

\(^6\)A function is analytic if it has a Taylor series about each point \( x \) that converges to this function in an open neighborhood of \( x \). The exponential function is analytic. The sums, products, compositions, and reciprocals of analytic functions are analytic; therefore, \( P_{ij} \) from (2) is analytic. Theorem 1 from Sinitzyn (2008a) states that if the strategy space is compact and the demand functions are analytic and are positive at zero for any price of the competitor, then the support of the price distribution in any mixed strategy Nash Equilibrium has a finite number of points. While the theorem is stated for the single-dimensional case, it also applies to multiple dimensions.

\(^7\)These parameters were chosen so that the percentage of joint purchase in the two categories matches the empirical estimates. For the smallest value of consumer heterogeneity at which the firms charge reservation prices
the firms’ strategies change as consumer heterogeneity declines.

When $\mu$ is high, there exists a pure strategy equilibrium. For large $\mu$, the consumer preferences are quite dispersed. Therefore, if a firm decreases its price, it gains relatively few additional consumers. Hence the demands are inelastic. With inelastic demands, price competition is relatively soft, and both firms charge the reservation price for both of their products. As $\mu$ decreases, the incentives to undercut the rival become larger, and, in a pure strategy equilibrium, both firms start charging prices that are smaller than the reservation price. The prices for two complementary products are the same, so I will label both of them $p^{D,1}$, where $D$ stands for Diagonal, as these prices lie on the diagonal of the price support square.

Figure 1 shows how prices respond to further decrease in $\mu$. The diagonal prices are represented in this figure by the solid lines. As consumers become less heterogenous, $p^{D,1}$ decreases until it becomes so low that the firms would prefer to deviate by charging the reservation prices and serving mainly their loyal consumers. Of course, charging the reservation prices with certainty cannot be an equilibrium either, as the firms will have incentives to undercut. At this point, the pure strategy equilibrium does not exist anymore, but the equilibrium is restored in mixed strategies, with both firms charging two sets of prices: $(p^{D,1}; p^{D,1})$ with probability $\gamma^{D,1}$ and $(p^{D,2}; p^{D,2}) = (1; 1)$ with remaining probability $\gamma^{D,2} = 1 - \gamma^{D,1}$.

As $\mu$ keeps decreasing, $\gamma^{D,2}$—the probability of charging the reservation prices increases, thus increasing the incentives to undercut them. When $\mu$ reaches 0.135, two new prices appear in the equilibrium. The firms find it profitable to undercut $(1; 1)$, but along only one dimension of the price support square, that is, either charging $(1; p^{O,1})$ or $(p^{O,1}; 1)$. Here, $O$ stands for Off-diagonal, as these prices lie off the diagonal of the price support square. In Figure 1, the off-diagonal prices are represented by the dotted lines. In the symmetric mixed strategy equilibrium, the firms will charge both $(1; p^{O,1})$ and $(p^{O,1}; 1)$ with equal probability. I label the total probability of charging these price pairs as $\gamma^{O,1}$ (so, $\gamma^{O,1}/2$ is the probability of charging one of these price pairs).

As $\mu$ declines from 0.135, the probability of charging the off-diagonal prices increases until it reaches 0.2635 at $\mu = 0.1277$. At this point, it becomes profitable to undercut $(1; p^{O,1})$ or $(p^{O,1}; 1)$ with a diagonal price which I label $(p^{D,3}; p^{D,3})$. The support of the firms’ optimal mixed strategies in both categories, the percentage of joint purchases that is predicted by the model is 22%. This is in line with the empirical estimates in the categories I consider in this paper, which range from 14% to 34%.
now consists of the reservation prices \((p^{D,2}; p^{D,2}) = (1; 1)\), two price pairs with identical discounts\((p^{D,1}; p^{D,1})\) and \((p^{D,3}; p^{D,3})\), and two price pairs in which only one product is discounted--\((1; p^{O,1})\) and \((p^{O,1}; 1)\).

Figure 1 shows the rest of the pricing strategies as \(\mu\) decreases until 0.0584. Figure B.1 in Appendix B gives an alternative graphic presentation of these strategies using bubble plots. Some patterns emerge that are also present for other parameters of the demand function. First, as \(\mu\) keeps decreasing, more and more diagonal prices are added to the mixed strategy equilibrium. Second, the off-diagonal prices are not always present in the mixed strategy equilibrium, and, when they are a part of the equilibrium, the probability of charging them is low. For Figure 1, the highest value for the total off-diagonal probability, \(\gamma^{O,1}\), is 0.2635, at the point right before \((p^{D,3}; p^{D,3})\) appears.

The equilibrium pricing strategies are similar for other values of \(\alpha\). Figure 2 shows the re-
regions with different mixed strategy equilibria for the different values of $\alpha$ and $\mu$. Each region is characterized by the number of diagonal ("D") and off-diagonal ("O") prices the firms use. The equilibria with up to 5 diagonal prices or 4 off-diagonal prices were computed. I conjecture that the remaining equilibria contain more than 5 diagonal prices or 4 off-diagonal prices. While a theoretical proof of this conjecture that the number of prices generally increases with a decrease in consumer heterogeneity does not exist, previous work on single category price promotions with heterogeneous consumers (Sinitsyn 2008b and Sinitsyn 2009) and multiple robustness checks in this paper indicate that, indeed, this is the case. Figure 1 suggests that no significant new insights can be gained from examining the equilibria with a larger number of prices. Also, four different levels of promotional depths are likely to be at the upper limit of the managerially relevant choice.

Figure 2: Regions with Mixed Strategy Nash Equilibria for $d_c = 0.4$ and $d_b = 0.4$

Figure 2 shows that for all values of $\alpha$, when consumer heterogeneity is high, there exists a pure strategy equilibrium. For smaller values of $\mu$, the equilibria are in mixed strategies and the number of diagonal prices increases as $\mu$ declines. When they are present, the number of off-diagonal prices is usually 2, with 4 appearing only for a small subset of the parameters and,
in particular, for large values of $\alpha$. The estimates in the literature (Villas-Boas 1995, Huang et al. 2006) suggest that a brand’s loyalty share is usually below 10%.\footnote{Villas-Boas (1995) shows that the percentage of loyal consumers depends on the brand, and ranges from 0.3% to 10.4% for coffee and from 0.4% to 2.2% for saltine crackers. Huang et al. (2006) estimate the total percentage of loyal consumers in the refrigerated orange juice and frozen orange juice categories to be 12.2% and 29.5%, respectively. Because there are more than three brands in each category, the average percentage of loyal consumers per brand is less than 10%.} These estimates were made for a single category, which means that the percentage of consumers loyal to the same brand in two categories is likely to be even smaller. Therefore, more relevant predictions of the model are the ones for these smaller values of $\alpha$.

The main variable of interest is not the number of off-diagonal prices, but their frequency. For each value of $\alpha$, it is possible to find the maximum value of the total probability of off-diagonal prices, $\gamma^{MAX}$, over the studied range of $\mu$. It turns out that this probability increases with $\alpha$. While for larger values of $\alpha$, $\gamma^{MAX}$ can be significant (for example, it is 0.4129 for $\alpha = 0.35$), for the empirically relevant range of $\alpha$, $\gamma^{MAX}$ is small—for example, it is 0.0267 for $\alpha = 0.05$ and 0.0893 for $\alpha = 0.1$. It is impossible to compute all equilibria for any fixed $\alpha$, as, for small levels of consumer heterogeneity, the number of prices and probabilities the firms use and, correspondingly, the number of equations in the system to solve, become too large. Therefore, the computed $\gamma^{MAX}$ covers only the shaded regions shown in Figure 2. It is unlikely, however, that there are higher values for off-diagonal probabilities in the unexplored unshaded region. For any non-large fixed $\alpha$, as consumer heterogeneity decreases and the firms start using off-diagonal prices, the maximum value of the total off-diagonal probability occurs on the lower bound of the region (2"D",2"O"), just before the third diagonal price is added to the equilibrium. As $\mu$ decreases further, although the total probability of off-diagonal prices has both increasing and decreasing regions, the general trend is downward.

The intuition for why a large probability cannot be placed on off-diagonal prices in a mixed strategy equilibrium is illustrated by the following example. Consider the extreme case, in which each firm charges two off-diagonal prices, (0.5; 1) and (1; 0.5), with equal probability. If, like in the preceding analysis, $d_b = 0.4$, then when the firms happen to charge different price pairs, the consumers will purchase a mixed bundle\footnote{For simplicity, I consider only the consumers who buy in both categories and assume that consumer hetero-}. Consumers buy the discounted products from the
different firms because the premium they are willing to pay for getting both products from the same firm, 0.4, is not large enough to overcome the price difference of 0.5 (1 for a mixed bundle versus 1.5 for a bundle from the same firm). Then the firms have a profitable deviation to a diagonal price pair (0.75; 0.75). This incentive to deviate is explained entirely by the switchers’ demands, since the price paid by the loyalists remains 1.5. When both firms use the mixed strategy involving (0.5; 1) and (1; 0.5), there is a 50% probability that they charge the same price pair, and each firm gets half the switchers, who pay 1.5. There is also a 50% probability that they charge the opposite price pairs, and each firm gets all the switchers who buy only their discounted product, paying 0.5. When one firm charges (0.75; 0.75) instead, the switchers no longer want to buy a mixed bundle: the mixed bundle will cost 1.25, the matching bundles still cost 1.5, and consumers are willing to pay 0.4 to get the matching bundle. Thus, regardless of whether the rival charges (1; 0.5) or (0.5; 1), the deviating firm will get half the switchers, who pay 1.5. Because half of the switchers paying 1.5 bring more revenue than all of the switchers paying 0.5, charging (0.75; 0.75) brings more profit than charging (1; 0.5) and (0.5; 1) with equal probabilities.\textsuperscript{10} More broadly, when the firm charges off-diagonal prices such that the switchers purchase a mixed bundle, the firm sells only the discounted product to all of them. If the firm deviates to charging a diagonal price pair while keeping the price of the bundle constant, then it will get half of the switchers, each paying more than double of the original discounted price, thereby increasing the firm’s profits.

The analysis of the simple model in Section 3 reveals that the firms coordinate their promotions more when the strength of cross-category effect increases (Proposition 2). This result is conserved in the full model. Figure 3 shows the effect of $\mu$ on total off-diagonal probability, $\gamma^O$, for different values of brand-level complementarity $d_b$ ($\alpha = 0.1$, $d_c = 0.4$, $\delta_{k,i} = 0$). Equilibria with up to 8
geneity is negligible.

\textsuperscript{10}This example can be extended to show why off-diagonal prices might be charged with small probability. If, in addition to charging (0.5; 1) and (1; 0.5), both firms charge (0.6; 0.6) with a large probability, the deviation to (0.75; 0.75) is unprofitable for some values of consumer heterogeneity. This happens because a sizable portion of the switchers will buy a mixed bundle with prices (0.5; 0.6) over the cheapest matching bundle, (0.6; 0.6), but very few will buy a mixed bundle with prices (0.75; 0.6). Therefore, while charging (0.75; 0.75) is more profitable than (0.5; 1) against the rival charging (0.5; 1) or (1; 0.5), charging (0.75; 0.75) is less profitable than (0.5; 1) against the rival charging (0.6; 0.6). If the probability of the rival charging (0.6; 0.6) is large, the deviation to (0.75; 0.75) is unprofitable, and the off-diagonal prices can exist.
diagonal prices or 6 off-diagonal prices were computed for this figure.

Figure 3: Total off-diagonal probability for different values of \( d_b \) (\( \alpha = 0.1, \ d_c = 0.4, \ \delta_{k,i} = 0 \))

Some common trends are present for all values of \( d_b \). As \( \mu \) decreases, off-diagonal prices start to appear and the total probability of charging them increases. This probability increases until reaching its maximum, \( \gamma^{O\text{MAX}} \), and then it fluctuates with small deviations around some lower level. As \( d_b \) increases, \( \gamma^{O\text{MAX}} \) decreases. For example, \( \gamma^{O\text{MAX}} = 0.56 \) for \( d_b = 0 \). Then \( \gamma^{O\text{MAX}} \) decreases to 0.2133, 0.1425, and 0.0893 for \( d_b \) equal to 0.1, 0.2 and 0.4, correspondingly.\(^{11}\) In addition to the total off-diagonal probability, the number of off-diagonal prices in relation to diagonal prices is also smaller for the larger values of \( d_b \). For example, for \( d_b = 0.4 \), the last computed equilibrium has 8 diagonal and 2 off-diagonal prices, while for \( d_b = 0 \), the last computed equilibrium has 2 diagonal and 6 off-diagonal prices. Therefore, similar to the predictions of the simple model, in the full model, an increase in the cross-category brand-level complementarity increases the coordination of price promotions.

In summary, this theoretical model predicts that while the firms sometimes place only one of the complementary products on sale, these occasions are rare. Most of the time, the complemen-

\(^{11}\)The same trend is present for the other values of \( \alpha \). For example, for \( \alpha = 0.05 \), \( \gamma^{O\text{MAX}} \) decreases from 0.1368 to 0.0915, 0.0569 and 0.0267 as \( d_b \) increases from 0 to 0.1, 0.2, and 0.4.
tary products are promoted together. The frequency of co-promotions is greater for higher values of brand-level complementarity and for smaller levels of consumer loyalty.

6 Extensions

In this section, I will show that the predictions of the base model from the previous section also hold for more realistic settings. This analysis is motivated by the industry structure in the cake mix/cake frosting categories—one of the complementary category pairs studied in Section 7. These categories exhibit an asymmetric response to a promotion in the other category (Manchanda et al. 1999); the production processes of cake mixes and cake frostings are different, likely leading to different costs; the regular prices of cake mix and cake frosting differ, and there are three major producers. I will deal with each of these issues in turn.

6.1 Asymmetric Promotional Response

In equation (1), I assume that both complementary products enter the consumers’ utility function symmetrically. Thus, the effect of a price promotion in the first category on the brands in the second category is the same as the effect of an identical price promotion in the second category on the brands in the first category. However, it has been established in the empirical literature that cross-category price effects can be asymmetric within pairs of categories (Manchanda et al. 1999, Russell and Petersen 2000, Mehta 2007, Niraj et al. 2008). This literature shows the presence of the asymmetry at the category level. Recent work by Song and Chintagunta (2006) demonstrated that this asymmetry also exists at the brand level. I also model the asymmetry in the promotional response at the brand level by modifying the utility function from equation (1) in the following way:

$$U_{sij} = \delta_{1,i} + \delta_{2,j} - tp_{1,i} - p_{2,j} + d_c * I(i, j \neq 0) + d_b * I(i = j) + \varepsilon_{sij},$$

(3)

Here, $t$ measures the relative importance of the first category product’s price in the consumers’ utility function. If $t$ is greater than 1, then the impact of a change in the price of firm $i$’s first category product on the probability of purchase of its secondary category product is greater than the impact of the change in the price of firm $i$’s second category product on the probability of
purchase of its first category product. Using the terminology from Manchanda et al. (1999), category 1 is the primary category, and category 2 is the secondary category.

In order to illustrate the equilibrium strategies for this case, I will follow the procedure used in Section 5. I fix $\alpha = 0.1$, $d_c = d_b = 0.4$, $t = 1.5$, $\delta_{k,i} = 0$ ($k \in \{1, 2\}$, $i \in \{A, B, 0\}$), and examine what happens to the firms’ strategies as consumer heterogeneity $\mu$ declines.\(^{12}\) As expected, since the products enter the firms’ profit functions asymmetrically, even if the firms discount their complementary products together, the sizes of these discounts will be different. It is, thus, unfeasible to construct an informative analog of Figure 1 for this asymmetric case. A convenient way to represent the firms’ strategies is to use the bubble graphs, which are shown in Figure B.2 in Appendix B.

For high levels of consumer heterogeneity, both firms charge the reservation price for both complementary products. As $\mu$ decreases and demand becomes more elastic, both firms start discounting their primary category brands, each charging prices ($p^{O.1}; 1$). Eventually, the firms have enough incentive to undercut the price along the second dimension. In equilibrium, the firms now charge ($p^{D.1}$), where $p^{D.1}$ is a 2-dimensional vector with both prices less than 1.\(^{13}\) A decrease in $\mu$ leads to lower prices in both categories until the prices become low enough that the firms would prefer to deviate to (1; 1). At this point the pure strategy equilibrium no longer exists, and the firms use mixed strategies, charging either (1; 1) or ($p^{D.1}$)–Figure B.2a.

A further decrease in $\mu$ causes the firms to undercut (1; 1) by dropping the price of the secondary product. That is, the firms start charging (1; $p^{O.1}$) instead of (1; 1), which results in the equilibrium with price pairs ($p^{D.1}$) and (1; $p^{O.1}$)–Figure B.2b. As $\mu$ decreases further and these sale prices decrease, the firms again find a profitable deviation to (1; 1), which leads to an emergence of an equilibrium with three prices–Figure B.2c. In this equilibrium, the firms charge

\(^{12}\)It can be shown that for the utility function (3), the cross-elasticity of demand of the second category product with respect to the price of the first category product is $t$ times the cross-elasticity of demand of the first category product with respect to the price of the second category product. This ratio of these elasticities was estimated to be close to 1.5 in Manchanda et al. (1999). Therefore, I chose $t = 1.5$ for the main analysis of the firms’ strategies in this subsection.

\(^{13}\)With some abuse of notation, I will keep using the superscript $D$ to indicate a price pair in which both products are on sale, even though, since the size of the discounts is different in different categories, this price no longer lies on the diagonal of the price support square.
the reservation price for both products, put both complementary products on sale at the same time, or promote only the secondary product.

As \( \mu \) keeps decreasing, the firms add more price pairs in which both products are promoted. In Figure B.2d, the firms use 2 diagonal prices, and in Figure B.2e they use 3 diagonal prices. As \( \mu \) decreases further, more and more prices appear in the equilibria—Figure B.2f. These new price pairs constitute either promotions in both categories, \( (\mathbf{p}_D^{1,4}, (\mathbf{p}_D^{1,5}, (\mathbf{p}_D^{1,6}) \); or promotions in only the secondary category, \( 1; p^{O,2} \).

Several noteworthy patterns emerge from Figure B.2. First, for a large range of consumer heterogeneity \( \mu \), the lowest sale price of the primary product is below 0. Since marginal cost is normalized to zero, this means that the product is sold at a price below cost. This is consistent with a loss leader pricing strategy. In general, the promotion depth is larger for the primary product. Second, for a large subset of price pairs (for example, for \( (\mathbf{p}_D^{1,1}, (\mathbf{p}_D^{1,2}, (\mathbf{p}_D^{1,3}, (\mathbf{p}_D^{1,4}, (\mathbf{p}_D^{1,5}) \) in Figure B.2f), the discount depth of the secondary product is almost the same for each price pair, while the discount depth of the primary product varies. Taken together, these two observations lead to the conclusion that the firms use deeper promotions and more varied discount levels for their primary product. Also, while there are price pairs in which only the secondary product is promoted, there are no price pairs in which only the primary product is promoted.

Finally, consistent with the findings in the symmetric case, most of the probability is put on the price pairs in which both products are sold at the reservation price or promoted together. Figure 4 shows the effect of \( \mu \) on total off-diagonal probability, \( \gamma^O \), for different values of cross-category asymmetry in promotional response \( t \) (\( \alpha = 0.1, d_b = d_c = 0.4, \delta_{k,i} = 0) \). Equilibria with up to 8 diagonal prices were computed for this figure.

Figure 4 shows that the total off-diagonal probability is greater for the larger values of \( t \). But this probability is still relatively small, reaching a maximum value of 0.2294 for \( t = 2 \). Thus, for the case of asymmetric promotional response, the model also predicts that the complementary products should be promoted together.
6.2 Different Regular Prices and Costs

One of the assumptions made in Section 4 is that the two complementary products have identical reservation prices. In the subsequent analysis, I will relax this restriction and consider the case of different reservation prices. The consumers’ reservation prices for the complementary products are now $r_1 = 1$ and $r_2 \leq 1$, where $r_1$ is still normalized to 1. Figure B.3 in Appendix B illustrates the equilibrium strategies for different values of $\mu$, first when the difference in reservation prices is large ($r_2 = 0.6$; left column), and second when the difference in reservation prices is small ($r_2 = 0.8$; right column).

Both cases are characterized by an increase in the number of diagonal prices as $\mu$ decreases. In each case, single-category promotions exist for only one category, but which category is promoted on its own depends on $r_2$. The off-diagonal price ($1; p^{O,1}$) with only the second category product on sale is present only for the high values of $r_2$. This happens because the firms typically use reservation prices ($1; r_2$) in their mixed strategies, and it is not profitable to undercut this price pair in the second category if the reservation price is already small. The off-diagonal price ($p^{O,1}; r_2$) with only the first category product on sale is present only for the small values of $r_2$. This happens because, as $\mu$ decreases and new diagonal prices ($p^{Di}$) appear in the equilibrium, if $p^{Di}$ is smaller
than \( r_2 \), then the limit on the reservation price is not binding. However, if \( p^{Di} \) is greater than \( r_2 \), then the firm cannot charge \( p^{Di} \) in the second category. Therefore, the new price pair that is added to the equilibrium is \( (p^{Di}, r_2) \), in which the first category product is on promotion and the second category product is sold at the reservation price.

When the off-diagonal prices are present, the probability of charging them is once again small. Figure 5a shows the effect of \( \mu \) on total off-diagonal probability, \( \gamma^O \), for different values of reservation price \( r_2 \) (\( \alpha = 0.1 \), \( d_b = d_c = 0.4 \), \( \delta_{k,i} = 0 \)). The dependence of total off-diagonal probability on \( r_2 \) is U-shaped. Its maximum value is 0.0812 for \( r_2 = 0.8 \); then it decreases to 0.0198 for \( r_2 = 0.7 \), before increasing to 0.0450 for \( r_2 = 0.6 \) and 0.0837 for \( r_2 = 0.4 \). With all of these probabilities less than 10\%, the model predicts that the complementary products should be promoted together for the case of asymmetric reservation prices.

Figure 5: Total off-diagonal probability for different values of \( r_2 \) and \( c_2 \) (\( \alpha = 0.1 \), \( d_b = d_c = 0.4 \), \( \delta_{k,i} = 0 \))

Another assumption made in Section 4 was that the two complementary products have identical marginal costs. In order to examine the effect of different marginal costs on the firms’ strategies, I normalize the reservation price of both products to 1, normalize the marginal cost of the first category product to zero, and let the marginal cost of the second category product, \( c_2 \), be greater than zero. The equilibrium strategies (not presented in the paper, but available from the author) respond to the changes in \( \mu \) as follows: as \( \mu \) decreases, more price pairs with both products on sale are added to the equilibrium. In all of these price pairs, the first product (with the
smaller cost) is promoted more deeply than the second product. There is also an increase in the number of price pairs in which only the first product is on promotion. Figure 5b shows that the probability of charging these off-diagonal prices is higher for the larger values of $c_2$. However, even for $c_2 = 0.6$, the maximum off-diagonal probability is only 0.1675. The number of off-diagonal prices is also higher for the larger values of $c_2$—for the equilibrium with 8 diagonal prices, there are 5 off-diagonal prices for $c_2 = 0.6$, 2 off-diagonal prices for $c_2 = 0.4$, and 1 off-diagonal price for $c_2 = 0.2$.

6.3 Three Firms and Three Complementary Categories

For some of the product categories that I consider later in the paper (shampoos, conditioners, cake mixes, cake frostings), there are three major producers. This subsection examines whether the predictions of the two-firm model hold for the three-firm model. Equation (1) is easily extended to the case of 3 firms. I consider only symmetric equilibria.\footnote{Baye et al. (1992) show that the Varian model of sales with more than two firms has a continuum of asymmetric equilibria. However, they also show that the unique subgame perfect equilibrium of a metagame in which both firms and consumers are players is symmetric. For simplicity, I consider only symmetric equilibria.} The structure of these equilibria and the responses of prices and probabilities to a decrease in $\mu$ are very similar to the ones described in Section 5 for the case of two firms. In the case of three firms, for the identical parameter values ($\alpha = 0.1$, $d_c = d_b = 0.4$, $\delta_{k,i} = 0$), off-diagonal prices first appear for smaller values of $\mu$ ($\mu = 0.107$ for three firms vs. $\mu = 0.142$ for two firms). In addition, the maximum total off-diagonal probability is lower for the case of three firms—it is equal to 0.0233, while for the case of two firms it was 0.0893.

Sometimes consumers consider purchases in three complementary categories. For example, they can purchase cake mix, cake frosting, and toppings; or shampoo, conditioner, and body wash. The model is easily extended to allow for purchases in three complementary categories by specifying that consumers obtain an additional premium if they buy products from the same firm in all three categories. Analysis of the firms’ optimal strategies for this case reveals that the firms will use all possible variations of price triplets: only one product on sale, two products on sale, all three products on sale, and all three products sold at the reservation price. The probability of using the first two types of these triplets is small, however—most of the time, the
firms either charge the reservation price for the three products or promote all of them together. For example, for the same basic parameters of the demand function that were used in previous sections \((\alpha = 0.1, d_c = d_b = 0.4, \delta_{k,i} = 0)\), if the consumers get an additional utility of 0.2 for buying in all three categories and another 0.4 for matching the brands of all three complementary products, the maximum total off-diagonal probability is 0.1013.

6.4 Summary

The main prediction of the base model is that price promotions in only one category, while possible, are rare. This prediction is conserved in the various modifications of the base model, which are presented in this section. These extensions generate a number of other testable predictions of the theory, which are summarized in Table 1.

<table>
<thead>
<tr>
<th>An increase in</th>
<th>( \gamma^O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumer loyalty</td>
<td>increases the total off-diagonal probability</td>
</tr>
<tr>
<td>brand-level complementarity</td>
<td>decreases the total off-diagonal probability</td>
</tr>
<tr>
<td>asymmetry in promotional response</td>
<td>increases the total off-diagonal probability</td>
</tr>
<tr>
<td>difference in reservation prices</td>
<td>first decreases and then increases the total off-diagonal probability</td>
</tr>
<tr>
<td>difference in marginal costs</td>
<td>increases the total off-diagonal probability</td>
</tr>
</tbody>
</table>

The probability of putting only one product on sale is smaller when the consumer loyalty is smaller; when the brand-level complementarity is larger; when the asymmetry in promotional response or the difference in marginal costs is smaller; when the difference in reservation prices for the two products is intermediate or when there are three firms competing instead of two.

In addition, for the case of asymmetric promotional response, the firms use deeper promotions and more varied discount depths for their primary product. For some parameters of the demand function, this might include selling the primary product at a price below the marginal cost, making the primary product a loss leader. For the case of different reservation prices, when both products are on sale, their nominal sale prices are very similar, which means that the product with the
larger reservation price is discounted more in percentage terms. For the case of different marginal costs, when both products are on sale, the product with the smaller cost is promoted more deeply.

7 Empirical Evidence

In this section, I will show that the pattern of real world promotions of complementary products is correlationally consistent with the theoretical predictions of my model. The data that I use was collected by ACNielsen in a large metropolitan area in the United States between January 1993 and March 1995, for a total of 124 weeks. I examine the prices from the two largest supermarket chains, which together total 39 stores.

The three pairs of complementary categories chosen for this study are shampoo/conditioner, cake mix/cake frosting, and detergent/fabric softener. On an intuitive level, these three pairs exhibit different strengths of brand-level complementarity. In the detergent and fabric softener categories, the two leading manufacturers (Procter & Gamble and Unilever) use different names in the complementary categories (for example, P&G produces detergents Tide and Cheer and fabric softeners Bounce and Downy, while Unilever produces detergents Surf and Wisk and fabric softener Snuggle). It is likely that, in the absence of matching brand names, consumers do not gain extra utility from purchasing the complementary products that are produced by the same firm \((d_b = 0)\). This intuition is consistent with the estimates of the cross-price elasticities between detergents and fabric softeners computed by Song and Chintagunta (2006). All cross-price elasticities between a firm’s product in one category and its own and its rival’s brands in the complementary category are almost identical. For example, the effect of powder Tide’s price on the demand for liquid Downy, both of which are produced by P&G, is exactly the same as its effect on the demand for liquid Snuggle, which is produced by Unilever: \(-0.112\) (Table 5 in Song and Chintagunta 2006). Therefore, in the absence of brand-level complementarity, the theoretical model predicts that the coordination of price promotions of complementary products within the detergent and fabric softener categories should be very small or nonexistent.

On the contrary, in the shampoo/conditioner and cake mix/cake frosting categories, the firms use the same brand names for their complementary products. It is likely then that, in these categories, consumers are willing to pay a substantial premium in order to match the brand
names of complementary products. Therefore, the theoretical model predicts that the promotions of complementary products within these categories should be positively correlated.

Due to space limitations, I will present a full analysis of the cake mix/cake frosting categories and will briefly summarize the findings for other category pairs. The cake mix and cake frosting categories are dominated by three brands—Betty Crocker (BC), Duncan Hines (DH), and Pillsbury (PB), which jointly account for over 90% of the market. BC has around 50% market share, while the remaining 40% is split almost evenly between DH and PB. At the time when this data was collected, each of the brands was owned by a separate company: General Mills, P&G, and Grand Metropolitan, respectively. Each of the brands has multiple flavors, but the prices and sizes within each brand are the same. For these brands, I have data only for the first 82 weeks of the sample. Table 2 provides summary statistics.

<table>
<thead>
<tr>
<th></th>
<th>1st chain</th>
<th></th>
<th></th>
<th>2nd chain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>avg. regular</td>
<td>avg. sale price</td>
<td>avg. regular</td>
<td>avg. sale price</td>
</tr>
<tr>
<td></td>
<td>price</td>
<td>in $</td>
<td>in %</td>
<td>price</td>
</tr>
<tr>
<td>BC Cake</td>
<td>$1.47</td>
<td>$1.23 (0.18)</td>
<td>84% (12)</td>
<td>$1.47</td>
</tr>
<tr>
<td>DH Cake</td>
<td>$1.2 (0.05)</td>
<td>$0.95 (0.11)</td>
<td>79% (9)</td>
<td>$1.25 (0.09)</td>
</tr>
<tr>
<td>PB Cake</td>
<td>$1.34 (0.04)</td>
<td>$1.02 (0.25)</td>
<td>76% (19)</td>
<td>$1.35 (0.04)</td>
</tr>
<tr>
<td>BC Frosting</td>
<td>$1.83</td>
<td>$1.70 (0.16)</td>
<td>93% (9)</td>
<td>$1.83</td>
</tr>
<tr>
<td>DH Frosting</td>
<td>$1.57 (0.08)</td>
<td>$1.38 (0.13)</td>
<td>88% (8)</td>
<td>$1.58 (0.06)</td>
</tr>
<tr>
<td>PB Frosting</td>
<td>$1.76 (0.11)</td>
<td>$1.64 (0.09)</td>
<td>93% (5)</td>
<td>$1.79 (0.13)</td>
</tr>
</tbody>
</table>

Standard deviations are in parentheses. Due to the changes in pricing regimes, there is more than one regular price for some of the products.

The number of weeks during which only one of the complementary products was on sale is relatively high—27% for BC, 13% for DH, and 37% for PB. These values are so high partly because of the different frequency of sales of cake mixes and cake frostings. For example, in the second chain, DH cake mix was on sale for 26 weeks while DH frosting was on sale for 17 weeks. However, these two products were on sale together for 16 weeks. Thus, while it is evident that the price
promotions of these two brands were coordinated (if DH frosting was promoted, that promotion was almost always accompanied by a promotion on DH cake mix), there were still 11 weeks when only one of these products was on sale. Now, I will formally test the hypothesis that the price promotions of cake mixes and cake frostings are independent.

Assume that out of $W$ weeks there were $S_1$ weeks when a firm puts its product in the first category on sale and $S_2$ weeks when it puts its complementary product in the second category on sale. Then, the probability of a sale of the first product is $S_1/W$ and the probability of a sale of the second product is $S_2/W$. If the sales in these two categories were independent, the probability of a joint sale would be $(S_1/W)(S_2/W)$. So, under the null hypothesis of independent sales, the number of weeks with joint sales is distributed binomially with parameters $W$ and $(S_1/W)(S_2/W)$. In Table 3, I present the values of the CDF of this binomial distribution at the actual number of weeks both products were on sale together. High values of this CDF indicate that the hypothesis of independent sales can be rejected and that the firm puts its two complementary products on sale together.

The hypothesis of independent sales can be rejected at the 1% level for four out of six brand pairs and at the 5% level for five out of six brands pairs. This suggests that brand managers of cake mixes and cake frostings do not view these product lines separately, but rather coordinate pricing decisions. It is interesting to note that in the 2nd chain, Duncan Hines and Pillsbury—the two smaller firms—have a high concurrence of their cross-brand cross-category promotions (DH Cake/PB Frosting and PB Cake/DH Frosting) and a high concurrence of their cross-brand within-category promotions (DH Cake/PB Cake and DH Frosting/PB Frosting).

Prior research has identified cake mix as a primary category and cake frosting as a secondary category (Manchanda et al. 1999). This means that discounts on cake mixes have a larger impact on the sales of cake frosting than discounts on cake frostings have on the sales of cake mixes. The theoretical model of asymmetric promotional response (Section 6.1) predicts that the product in the primary category (cake mix) should have more varied discounts depths as well as deeper discounts. This prediction is largely supported by the data—the number of different discount prices Pillsbury used for cake mixes is 10 (in chain 1) and 8 (in chain 2), while for cake frostings it used 5 (in chain 1) and 6 (in chain 2). The corresponding numbers for Betty Crocker are 9 and 6 vs. 3.
Table 3: Joint sales of cake mixes and cake frostings

<table>
<thead>
<tr>
<th>$1^{st}$ chain</th>
<th>Weeks on sale</th>
<th>BC Cake</th>
<th>DH Cake</th>
<th>PB Cake</th>
<th>BC Frosting</th>
<th>DH Frosting</th>
<th>PB Frosting</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC Cake</td>
<td>39</td>
<td>—</td>
<td>0.7013 (15)</td>
<td>0.5725 (23)</td>
<td><strong>0.9995 (30)</strong></td>
<td>0.4079 (11)</td>
<td>0.5801 (8)</td>
</tr>
<tr>
<td>DH Cake</td>
<td>29</td>
<td>—</td>
<td>—</td>
<td>0.5671 (17)</td>
<td>0.3261 (11)</td>
<td><strong>0.9999 (22)</strong></td>
<td>0.7477 (7)</td>
</tr>
<tr>
<td>PB Cake</td>
<td>48</td>
<td>—</td>
<td>—</td>
<td>0.1482 (17)</td>
<td>0.6516 (16)</td>
<td><strong>0.8823 (13)</strong></td>
<td>—</td>
</tr>
<tr>
<td>BC Frosting</td>
<td>37</td>
<td>—</td>
<td>—</td>
<td>0.2469 (9)</td>
<td>—</td>
<td>—</td>
<td>0.8295 (7)</td>
</tr>
<tr>
<td>DH Frosting</td>
<td>26</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>PB Frosting</td>
<td>17</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$2^{nd}$ chain</th>
<th>Weeks on sale</th>
<th>BC Cake</th>
<th>DH Cake</th>
<th>PB Cake</th>
<th>BC Frosting</th>
<th>DH Frosting</th>
<th>PB Frosting</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC Cake</td>
<td>48</td>
<td>—</td>
<td>0.4305 (14)</td>
<td>0.6287 (20)</td>
<td><strong>0.9712 (25)</strong></td>
<td>0.2073 (7)</td>
<td>0.5907 (22)</td>
</tr>
<tr>
<td>DH Cake</td>
<td>26</td>
<td>—</td>
<td>0.9858 (17)</td>
<td>0.8907 (13)</td>
<td><strong>0.9999 (16)</strong></td>
<td>0.9600 (17)</td>
<td>—</td>
</tr>
<tr>
<td>PB Cake</td>
<td>33</td>
<td>—</td>
<td>0.7403 (14)</td>
<td>0.9819 (12)</td>
<td><strong>0.9953 (24)</strong></td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>BC Frosting</td>
<td>31</td>
<td>—</td>
<td>—</td>
<td>0.8079 (8)</td>
<td>0.5722 (14)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>DH Frosting</td>
<td>17</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.9207 (11)</td>
<td>—</td>
</tr>
<tr>
<td>PB Frosting</td>
<td>17</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

The number in each cell shows the CDF of a binomial distribution of independent joint sales at the number of weeks with joint sales. The number of weeks with joint sales is in parentheses. The joint sales of the products produced by the same manufacturer are in bold.

and 6, and for Duncan Hines, 8 and 6 vs. 7 and 4. With respect to the depth of the discounts, the largest discounts offered on cake mixes for Pillsbury were 50% and 62%, while its largest discounts on cake frostings were 16% and 8%. The corresponding numbers for Betty Crocker are 50% and 66% vs. 50% and 32%, and for Duncan Hines, 34% and 36% vs. 28% and 13%.

In the shampoo/conditioner categories, the prices that the firms charged for their complementary products were almost always identical. In fact, the value of the CDF of the binomial distribution under the hypothesis of independent sales is 0.9999 for the three leading brands that I studied: Pantene (produced by P&G), Suave (at the time produced by Helene Curtis), and White Rain (at the time produced by Gillette). The hypothesis of independent sales is rejected in favor of the conclusion that the price promotions of shampoos and conditioners happened simultaneously, consistent with the theoretical predictions. Moreover, the coordination of price promotions was stronger for the shampoo/conditioner categories than it was for the cake mix/cake frosting categories. The theoretical model predicts that this should happen if brand-level complementarity is
stronger for the brands in the shampoo/conditioner categories than it is for the brands in the cake mix/cake frosting categories. While I do not have a measure of the brand-level complementarity in these categories, it is plausible that consumers place a larger premium on matching the brands of shampoos and conditioners based on the notion of uniformity in the bundle. The shampoo and conditioner bottles sharing the same brand name have a very similar design and are typically located next to each other on the shelf. The cake mixes and cake frostings of the same brand, on the other hand, have a less uniform design (cake mixes come in boxes while cake frostings come in cans) and are not typically placed next to each other (all brands of cake mixes are located together and all brands of cake frostings are located together).

For the final category pair, detergents and fabric softeners, the complementary brands produced by the same firm have different names, and thus, we expect that there is little or no coordination of price promotions. Indeed, the hypothesis that the sales are independent can be rejected for only one out of fourteen possible brand pairs at the 5% level, and for three out of fourteen possible brand pairs at the 10% level. In summary, the empirical evidence from three pairs of complementary categories is correlationally consistent with the predictions of the theoretical model. In the category pairs for which we expect a large degree of brand-level complementarity, the firms promoted their complementary products together, while in the category pair in which brand-level complementarity is likely to be absent, there was almost no evidence of joint promotions.

8 Concluding Remarks

This paper presents a theoretical model of price promotions of complementary products. I assume that consumers are willing to pay a price premium for purchasing complementary products that share the same brand name and are produced by the same firm, and I find that each firm predominantly promotes its complementary products together. The supermarket price data in the shampoo/conditioner and cake mix/cake frosting categories is correlationally consistent with the predictions of the theoretical model—in these categories, the firms usually put their complementary products on sale at the same time.

There are, however, alternative explanations for the observed simultaneity of price promotions
of complementary products. If a firm wants to increase the perception of complementarity of its products, simultaneous promotions encourage joint purchase and, hence, joint consumption of its products, leading to the desired result. On the demand side, consumers might respond more strongly to a joint promotion than to two separate single product promotions. For example, Chintagunta and Haldar (1998) found some synergies to promoting pasta and pasta sauce jointly—a price promotion in both categories led to a larger increase in purchase probability of both products than the combined effect of two separate price promotions of the same depth. Similarly, on the cost side, there likely exist economies of scope in promotions—it is cheaper to include both products in one promotion rather than run two separate promotions.

However, the reasoning of these alternative explanations does not require that the complementary products share the same brand name, and should thus be equally applicable to the coordination of price promotions in detergent/fabric softener categories. Since the promotions of detergents and fabric softeners are not coordinated, I regard these alternatives as less likely explanations of the observed simultaneity of price promotions of complements than the proposed model.

It is possible that economies of scope in promotions are stronger for brands that share the same brand name. In addition, the incentives for the coordination of promotions in the two categories might be driven by similar inventory cycles or observations of common demand shocks that affect both categories. It might be easier to recognize these effects and implement a joint management strategy in the shampoo/conditioner and cake mix/cake frosting categories, where the complementary brands share the same name.  

While I cannot rule out these alternative explanations, I have investigated whether sharing a brand name in multiple categories drives the coordination of promotions. I examined the prices of products that have the same brand name as the products studied in Section 7, but belong to other, non-complementary categories. I found only three categories where these products are present, and I considered each firm’s leading product in these categories. Those products are Betty Crocker Dunk-a-Roos 6 oz (cookies), Pillsbury Frozen Microwaveable Buttermilk Pancakes

\[15\] It is also possible that, historically, the products in detergent and fabric softener categories were set up under differing brand names, leading to several strong brand management teams pursuing separate objectives without coordination.
15.2 oz, and Pillsbury Hungry Jack Maple Syrup 24 oz. Using the same procedure to test for the independence of the price promotions as in Section 7, I computed the values of the CDF for the joint sales of each firm’s non-complementary products. For BC Cake Mix and BC Dunk-a-Roos, they are 0.8457 (1st chain) and 0.8260 (2nd chain). For BC Frosting and BC Dunk-a-Roos, they are 0.6346 and 0.2801. For PB Cake Mix and PB Pancakes, they are 0.9289 and 0.6234. For PB Frosting and PB Pancakes, they are 0.6578 and 0.5271. For PB Cake Mix and PB Syrup, they are 0.8912 and 0.1778. For PB Frosting and PB Syrup, they are 0.8621 and 0.4433. The hypothesis that the promotions of two products with the same brand name in unrelated categories are independent can be rejected for only one brand pair out of twelve at the 10% level.\textsuperscript{16} Simply sharing a brand name thus appears to be insufficient to cause coordinated promotions.

This paper has several limitations, some of which could be addressed in future work. First, I model the competition between manufacturers and do not consider their interactions with retailers. Intuitively, the qualitative predictions should not change if I add to the model non-strategic retailers who set their prices by charging a certain mark-up over the wholesale price (the mark-ups can be different for the regular and the sale prices). Including one or more strategic retailers would be a useful extension of the current model. Second, I provided only correlational empirical evidence in support of the main prediction of the theoretical model. Further work in this direction involves empirical testing of other implications of the model that are summarized in Table 1. Third, since the full model does not have closed form solutions, the theoretical predictions are based on numerical computations for a finite set of parameters.

Another avenue for future research could involve a reexamination of the role of umbrella branding in light of the findings that complementary products sharing the same brand name are generally promoted together. Finally, a natural complement to the current paper would be the

\textsuperscript{16}While Pillsbury Pancakes and Syrup are also complements, their promotions are not coordinated – the values of the CDF for the joint sales are 0.8297 and 0.2621. This finding does not necessarily contradict my model, because the industry structure in these categories is different from the ones I examined in this paper. The markets for frozen waffles/pancakes and syrups are dominated by the firms that produce in only one category. In addition to Pillsbury, only Aunt Jemima produces in both categories, but the shares of both companies are small: Pillsbury has 1.9% in the frozen waffles/pancakes market and 3% in the syrup market, while Aunt Jemima has 12.8% and 10.4% shares in the respective markets.
analysis of coordination of price promotions by a firm producing substitute products. Combining such an analysis with the results of the current paper would further advance our understanding of price promotion strategies for firms producing multiple products in each of several related categories.

Acknowledgements

The author thanks the department editor (Pradeep Chintagunta), the associate editor, and three anonymous referees for their guidance in improving the paper. The author benefited from the helpful comments of Robert Blattberg, Jim Dana, Nancy Eichner, Anne Gron, Ryan Kasprzak, and Robert Porter. Financial support from the Fonds Québécois de la Recherche sur la Société et la Culture is gratefully acknowledged.
References


Appendix A

Proof of Proposition 1. Assume that the second firm chooses its four strategies with probabilities $\theta$, $\beta$, $\eta$, and $\lambda = 1 - \theta - \beta - \eta$. Then the profit of the first firm, if it chooses $(p; p)$, is

$$\theta(p) + \beta(ps_p + p(.5 + s_D)) + \eta(ps_p + p(.5 + s_D)) + (1 - \theta - \beta - \eta)(2ps_p).$$

The profit from choosing $(p; 1)$ is

$$\theta(p(.5 - s_D) + 1 - s_p) + \beta(.5p + .5) + \eta(ps_p + 1 - s_p) + (1 - \theta - \beta - \eta)(ps_p + (.5 + s_D)).$$

The profit from choosing $(1; p)$ is

$$\theta(p(.5 - s_D) + 1 - s_p) + \beta(ps_p + 1 - s_p) + \eta(.5p + .5) + (1 - \theta - \beta - \eta)(ps_p + (.5 + s_D)),$$

and, finally, the profit from choosing $(1; 1)$ is

$$\theta(2(1 - s_p)) + \beta(1 - s_p + (.5 - s_D)) + \eta(1 - s_p + (.5 - s_D)) + (1 - \theta - \beta - \eta)(1).$$

Equating all four profits and solving for the probabilities, we get $\theta = \frac{s_D + ps_D + p^2 s_p - 1 + s_p}{s_D(1 + p)^2}$, $\beta = \eta = \frac{1 - s_p - p^2 s_p}{s_D(1 + p)^2}$, and $\lambda = 1 - \theta - \beta - \eta = \frac{ps_D + p^2 s_D + p^2 s_p - 1 + s_p}{s_D(1 + p)^2}$.

Now, we need to check that all probabilities are greater than zero. $\theta > \lambda$ (since $p < 1$), so it is left to check that $\beta > 0$ and $\lambda > 0$. $\beta > 0$ if $1 - s_p - p^2 s_p > 0$ or $s_p < \frac{1}{1 + p^2}$. $\lambda > 0$ iff $ps_D + p^2 s_D + p^2 s_p - 1 + s_p > 0$ or $s_p(1 + p^2) > 1 - ps_D - p^2 s_D$ or $s_D > \frac{1 - s_p(1 + p^2)}{p(1 + p)}$. In addition, the sales are positively correlated iff $\theta > (\theta + \beta)(\theta + \eta)$. Using formulas for $\theta$, $\beta$, and $\eta$ obtained above and noting that $\theta + \beta = \theta + \eta = \frac{1}{1 + p^2}$, we get $\frac{ps_D + ps_D + p^2 s_p - 1 + s_p}{s_D(1 + p)^2} > \left(\frac{1}{1 + p}\right)^2$. Solving for $s_p$, we get $s_p > \frac{1 - ps_D}{1 + p^2}$ which is equivalent to $s_D > \frac{1 - s_p(1 + p^2)}{p}$. ■

Proof of Proposition 2. From the formulas for the probabilities given in the proof of Proposition 1, $\frac{\theta}{(\theta + \beta)(\theta + \eta)} = \frac{(s_D + ps_D + p^2 s_p - 1 + s_p)(1 + p)^2}{s_D(1 + p)^2} = 1 + p - \frac{1 - s_p - p^2 s_p}{s_D} \frac{1}{p(1 + p)}$ which increases with $s_D$ since $1 - s_p - p^2 s_p > 0$ for $s_p < \frac{1}{1 + p^2}$. ■

Appendix B
Figure B.1: Equilibrium Strategies for $\alpha = 0.25$, $d_c = 0.4$, and $d_b = 0.4$ (size of bubble denotes probability of charging a price pair)
Figure B.2: Equilibrium Strategies for $\alpha = 0.1$, $d_c = 0.4$, $d_b = 0.4$, and $t = 1.5$ (size of bubble denotes probability of charging a price pair)
Figure B.3: Equilibrium Strategies for $\alpha = 0.1$, $d_c = 0.4$, $d_b = 0.4$, and two values for $r_2$, 0.6 and 0.8 (size of bubble denotes probability of charging a price pair)
Coordination of Price Promotions Within a Product Line

Maxim Sinitsyn
Department of Economic, University of California, San Diego
October 2012

Abstract

This motivating example considers two firms each producing two products. The products are horizontally differentiated with respect to one characteristic, for example, fat content. Each firm offers one low-fat version of the product and one regular version. As this characteristic becomes more important in the overall consumer demand (as measured by the number of consumers who will buy only the product with the characteristic they prefer), the firms put more emphasis on promoting both versions of their products together. Further treatment of this problem will involve analysis of firms’ strategies for more flexible demand specification, namely a nested logit.

1 Model

Lal and Villas-Boas (1998) study the model, in which two manufacturers produce two products, which they sell to the two retailers. The two retailers then sell the products to the consumers. I am interested in this second stage of this model. This stage basically models the competition between two firms each carrying two substitute products. Following the notation in Lal and Villas-Boas (1998), the firms 1 and 2 will set the prices $P_{A_1}$ and $P_{A_2}$ (firm 1), and $P_{B_1}$ and $P_{B_2}$ (firm 2). In Lal and Villas-Boas (1998) notation, $A$ and $B$ referred to the manufacturers. Here, these letters refer to some common characteristics of the products, for example, $A_1$ and $A_2$ can be regular Coke and Pepsi while $B_1$ and $B_2$ can be diet Coke and diet Pepsi. The consumers are separated into 7 segments. The price sensitive consumers will buy the product with the lowest price. The size of this segment is $S$. There are 4 segments of loyal consumers who buy only their preferred
product. Each of these segments has size $I$. Finally, there are two segments of consumers who are loyal to the product characteristic and will choose the firm that offers the product with their preferred characteristic at a lower price. The size of these segments is $M$. Lal and Villas-Boas (1998) also have two segments of size $R$ that consist of consumers loyal to the specific firm. To keep the model as simple as possible, I will not consider these segments and set $R$ equal to zero. The reservation price of consumers is $r$, which I normalize to 1.

First, I will consider the case when the costs of all four products is identical ($w_A = w_B$ in Lal and Villas-Boas (1998)). Since, $M/I$ is less than $(M + S)/I$, the conditions of Proposition 3 in Lal and Villas-Boas (1998) are satisfied, and the equilibrium strategies are outlined in the following proposition.

**Proposition 1** (Proposition 3 from Lal and Villas-Boas (1998) with $r = 1$ and $R = 0$). The equilibrium strategies of the firms are

$$F(p, 1) = F(1, p) = 1 - \frac{I(1-p)}{Mp} \quad \text{for } p \in [p^*; 1];$$

$$F(p, 1) = F(1, p) = \frac{S+M}{2S+M} - \frac{I(1-p)}{(2S+M)p} \quad \text{for } p \in [p; p^*];$$

$$F(p, p) = 1 - \frac{2I(1-p)}{Mp} \quad \text{for } p \in [p^*; 1],$$

where $p = \frac{I}{I+S+M}$ and $p^* = \frac{2I}{2I+M}$.

Proposition 3 provides the formulas for $F(p, p)$ and for the marginal distributions $F(p, 1)$ and $F(1, p)$. The CDF of the price distribution at any point $(p_1, p_2)$ cannot be identified. However, it is possible to make some conclusions about the coordination of price promotions without the knowledge of $F(p_1, p_2)$. First, note that $F(p^*, 1) = F(1, p^*) = 1 - \frac{I(1-\frac{2I}{2I+M})}{M\frac{2I}{2I+M}} = \frac{1}{2}$. Given that $F(p^*, p^*) = 0$, this means that $F(p^*, 1) - F(p^*, p^*) = F(1, p^*) - F(p^*, p^*) = \frac{1}{2}$. Therefore, the pdf of charging a price pair $(p_1, p_2)$ is positive only in the upper-left and bottom-right corners of the price support square, that is when one of the prices is higher than $p^*$ and another price is lower than $p^*$. There is zero probability assigned to the price pairs, in which both of the prices are smaller than $p^*$ or both of the price are larger than $p^*$. This characterization reveals that the firms always promote one of the products deeper than another.

Even though the promotion depths are never identical for the two products, it is possible that most of the weight can be placed on promotions of similar depths. In order to examine how the percent of the consumers who are loyal to the product characteristic, $M$, changes the
firms’ strategies of coordinating promotions, I will examine the variable $\delta$, which is equal to $F\left(1, \frac{1+p^1}{2}\right) - \frac{1}{2} - F\left(\frac{p^1+p^2}{2}, 1\right)$. In figure 1, $\delta$ represents the difference in the probability of price pairs falling into region D and the probability of price pairs falling into region B. Region D represents price pairs with relatively similar depths of price promotions. On the contrary, in region B, there is a deep discount on one of the products and a shallow discount on another.

![Figure 1: Support of $F$](image)

It is possible to write out an explicit formula for $\delta$ using the formulas from Proposition 1, and substituting $1 - 4I - 2M$ for $S$ in order to normalize the total number of consumers to 1. However, it is not possible to analytically determine whether $\partial \delta / \partial M$ is greater or smaller than zero. I performed this analysis numerically for the values of $I$ between 0.01 and 0.24 with a step of 0.01 and for the values of $M$ between 0.001 and $(1 - 4I)/2$ with a step of 0.001. It turns out that everywhere on this grid $\partial \delta / \partial M$ is greater than zero. This means that as the number of consumers who are loyal to the product characteristic increases, the firms are more likely to offer similar promotions for their substitute products.
Appendix.

**Proof of Proposition 1.** Assume that both firms are charging the prices from a binomial distribution \( F(p_A, p_B) \). If the firm charges the reservation prices for both of its products, \((1; 1)\), it will serve only its loyal consumers, and its profit is \( 2I \). When the firm charges identical prices for both products, \((p; p)\), it will win all the switchers when \( p \) is the lowest price. This happens with probability \( 1 - F(p, 1) - F(1, p) + F(p, p) \). It will also win the consumers loyal to one of the characteristics with probability \( 1 - F(p, 1) \). Then, the expected profit from charging \((p; p)\) is

\[
\pi(p, p) = p \left( 2I + M \left( 2 - F(p, 1) - F(1, p) \right) + S \left( 1 - F(p, 1) - F(1, p) + F(p, p) \right) \right).
\]  

(1)

When the firm charges prices \((p; 1)\), it wins all the switchers with probability \( 1 - F(p, 1) - F(1, p) + F(p, p) \) and the consumers loyal to the first type of product with probability \( 1 - F(p, 1) \). Given that it charges 1 for the second type of product, it never wins the consumers who are loyal to the second type of product. Then, the expected profit from charging \((p; 1)\) is

\[
\pi(p, 1) = p \left( I + M \left( 1 - F(p, 1) \right) + S \left( 1 - F(p, 1) - F(1, p) + F(p, p) \right) \right) + I.
\]  

(2)

After equating (2) and (1), and setting \( F(p, 1) = F(1, p) \), we get \( p \left( 2I + M \left( 2 - 2F(p, 1) \right) \right) = p \left( I + M \left( 1 - F(p, 1) \right) \right) + I \), from where \( F(p, 1) = 1 - \frac{I(1-p)}{Mp} \). Substituting this formula in equation \( 2I = \pi(p, p) \), we get \( 2I = p \left( 2I + \frac{2I(1-p)}{p} + S \left( \frac{2I(1-p)}{Mp} - 1 + F(p, p) \right) \right) \), from where \( F(p, p) = 1 - \frac{2I(1-p)}{Mp} \). \( F(p, p) = 0 \) at \( p = p^* = \frac{2I}{2I+M} \). \( F(p^*, 1) \) is greater than zero, therefore, another equation, \( \pi(p, 1) = 2I \) with \( F(p, p) = 0 \), describes \( F(p, 1) \) for \( p < p^* \). \( p \left( I + M \left( 1 - F(p, 1) \right) + S \left( 1 - 2F(p, 1) \right) \right) + I = 2I \), from where \( F(p, 1) = F(1, p) = \frac{S+M}{2S+M} - \frac{I(1-p)}{(2S+M)p} \). \( F(p, 1) = 0 \) at \( p = p = \frac{I}{I+S+M} \). \( \blacksquare \)